



THE DISTRIBUTION OF THE RETURNS OF JAPANESE STOCKS AND PORTFOLIOS

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ABSTRACT

The behaviour of the distribution of stock returns is of fundamental importance in financial economics, in view of its direct bearing on the descriptive validity of any theoretical model. We analysed the behaviour of Japanese stock return distributions using the Pearson system of frequency curves to determine whether a) the distributions of the returns of the shares listed in the Nikkei 225 can be described by a single type of distribution; b) the length of the time period used for the analysis affects the behaviour of the distributions, and c) the distributions of the returns of portfolios of Japanese stocks follow similar patterns of behaviour. We found that all the shares listed on the Nikkei 225 may be described by the Pearson Type IV distribution. Other behaviours are occasionally observable but only when short time periods are used in the analysis, suggesting that the length of the period is not a variable that has any significant effect on the behaviour of Japanese stock returns. When the returns of portfolios of Japanese stocks are examined, the results are more robust and exceptions to the Pearson type IV rule are less common and are confined to very short time periods of analysis. We discuss the implications of our findings for financial modelling. To the best of our knowledge, we provide the first such analysis for the Japanese market.

Keywords: Pearson system of frequency curves, Type IV distribution, Japanese equity market, Stock return distribution, Portfolio return distribution, Financial modelling.

JEL Classification: G10

INTRODUCTION

Modelling time series of stock returns is a crucial part of modern financial economics. The use of a classical Gaussian approach leads to approximations that are no longer appropriate. In all fields of financial economics, academicians and practitioners are involved in developing models that

account for the higher moments of financial returns time series. However, none of these models is generally regarded to be prominent. We believe that such a task cannot be completed successfully without obtaining satisfactory answers to two preliminary questions: (1) is there a distribution that describes stock return time series adequately, for all possible time periods considered? and (2) are there differences in the distributional behaviour if the returns of equity portfolios are considered rather than those of single stocks?

Beginning with the seminal works of [Mandelbrot \(1963\)](#) and [Fama \(1965\)](#), many researchers have addressed these issues, but in our opinion none have managed to produce conclusive results. In an attempt to make further progress, we herein approach the topic from a different perspective. Rather than trying to fit stock return time series with a known distribution, as has been attempted by a majority of other authors, we herein ask whether the time series can be described for a large set of different time periods, in terms of a system of distributions. The novelty of our approach is important from both methodological and practical standpoints, because usually the classes of a system of distributions have parameters that are sufficiently flexible to take account of and conveniently model time-varying behaviours that can be observed in the shape, scale and location of financial return time series.

[Pearson \(1895; 1901; 1916\)](#) introduced a system of continuous probability curves that can catch all possible variations in the first four moments of a distribution. In general, the Pearson curves provide a satisfactory approximation to the thickness and length of the tails of a distribution, which means that in theory, they can be used to good effect in financial analysis. Moreover, ongoing improvements to information technology have provided notable benefits in the practical treatment of the distribution types of the Pearson system, which make its potential for use greater than ever. We herein focus our analysis on the Japanese market. We use the Pearson system of continuous probability distribution to determine whether a) the distributions of the returns of the shares listed in the Nikkei 225 can be described by a single type of distribution; b) the length of the time period used for the analysis affects the behaviour of the distributions, and c) the distributions of the returns of portfolios of Japanese stocks follow similar patterns of behaviour.

The contribution of the present paper is threefold. First of all, to the best of our knowledge, it provides the first such analysis for the Japanese market. Second, it shows that Type IV of the Pearson system of frequency curves describes the distribution of the returns of all the shares listed in the Nikkei 225 index. Exceptions to this behaviour are occasional and occur only when the period of the analysis does not exceed 1000 observations. Third, it shows that when the returns of portfolios of Japanese stocks are examined, the results are more robust, and exceptions to the Pearson type IV rule are less common, than equity analysis and are confined to very short time periods of analysis (no more than 250 observations).

The remainder of the paper is organised as follows. In Section 2, we review the relevant literature. In Section 3, we present the methodology and the data. In Section 4, we present the results. Section 5 concludes.

LITERATURE REVIEW

Although widely used in scientific analysis across a range of disciplines, the Pearson system has received less than the attention it deserves in financial economics. Suggested, among other density functions, by [Lau *et al.* \(1990\)](#) as a fruitful alternative to stable distributions, to our knowledge the Pearson system was first used to model financial returns by [Hirschberg *et al.* \(1992\)](#). They analysed monthly returns for a sample of US shares and found that in a majority of cases the behaviour was of type IV. [Brannas and Nordman \(2003\)](#) conveniently described the behaviour of NYSE composite returns for the years 1981-1999 by means of a Pearson type IV density. Recently, [Pizzutilo \(2012\)](#) concluded that over finite time intervals, the behaviour of European stock returns is of type IV, and the hypothesis cannot be rejected that over infinite or very long periods of analysis the distributions may be assumed to be of type VII.

With reference to the Japanese market, [Nagahara \(1995\)](#) assumed Pearson type IV and type VII to be the distributions of daily returns of the Nikkei 225, the Topix and the S&P 500 composite. He found that these provided good fits to the data, and are better models to use than the Gaussian. [Errunza *et al.* \(1996\)](#) examined national equity market indexes, finding that developed markets including the Japanese one, show type IV behaviour whilst emerging markets exhibit significant cross-sectional differences. [Nagahara \(1999\)](#) discussed a method for overcoming some of the difficulties in the practical treatment of the Pearson Type IV curve and applied it to model one year of daily returns of the Topix and S&P500.

To our knowledge, there have to date been no other reports of investigations of the Japanese stock market that have made use of the Pearson system of frequency curves, although other distributional forms have been used. However, most of these studies have analysed the distribution of Japanese stock index returns alongside other international stock market indexes. Only rarely have authors focused on the behaviour of the distribution of the returns of shares listed only on the Japanese market. [Aggarwal *et al.* \(1989\)](#) found that the distribution of equity returns on the Tokyo Stock Exchange from 1965-1984 deviates significantly from normality, and has a significant and persistent skewness and kurtosis. By assuming a normal-variance mixture distribution, [Hurst and Platen \(1997\)](#) considered nine theoretical curves in order to identify that which provided the best fit to the empirical returns of the MSCI index for Japan and four other countries; he concluded that the Student t distribution was preferable to any of the others. [Mittnik *et al.* \(1998\)](#) investigated alternative unconditional and conditional distributional models for the returns on the Nikkei 225, finding that of the eight distributions considered, the partially asymmetric Weibull, the Student's t

and the asymmetric α -stable were the most viable candidates in terms of the overall fit, while the tails of the sample distributions were best approximated by the asymmetric α -stable distribution. Theodossiou (1998) concluded that the skewed generalised t -distribution appeared to provide a good fit to the daily log-returns of the Topix stock market index. Watanabe (2000) found that the Student's t distribution conveniently captured the excess kurtosis of the conditional distribution for daily Japanese stock returns. Kaizoji and Kaizoji (2003) deduced that the behaviour of the daily returns of the Nikkei 225 index for 1990-2003 could best be described by an exponential distribution. Gettinby *et al.* (2006) tried to characterise the distribution of extreme share returns in the S&P 500 composite, the FTSE All Share and the Nikkei 225, by using a total of seven different curves. They concluded that the Generalised Logistic distributions provided a best fit to the three indexes for all the intervals considered. Eryigit *et al.* (2009) modelled the tail distributions of the returns of a large number of world indexes (11 of them Japanese) using the Log-normal, Weibull, power law, and power law with cutoff. They concluded that the choice of the threshold of the tail had a significant influence on the results. Liu *et al.* (2012) introduced an intermediate distribution between the Gaussian and the Cauchy distributions. They found that the intermediate and q-Gaussian distributions were more suited to fitting daily returns on S&P 500, Nasdaq composite, Nikkei 225 and Dow Jones Index than the Gaussian distribution.

METHODOLOGY AND DATA

Pearson curves are probability density functions that are solutions of the following differential equation:

$$\frac{1}{y} \frac{dy}{dx} = -\frac{(a+x)}{b_0 + b_1x + b_2x^2} \quad (1)$$

where: a , b_0 , b_1 and b_2 are fixed coefficients that, assuming their existence, can be expressed in terms of the first four central moments:

$$b_0 = \frac{4\gamma_2 - 3\gamma_1}{10\gamma_2 - 12\gamma_1 - 18} \mu_2$$

$$a = b_1 = \frac{\sqrt{\gamma_1}(\gamma_2 + 3)}{10\gamma_2 - 12\gamma_1 - 18} \sqrt{\mu_2}$$

$$b_2 = \frac{2\gamma_2 - 3\gamma_1 - 6}{10\gamma_2 - 12\gamma_1 - 18}$$

γ_1 is the coefficient of skewness and γ_2 is the coefficient of kurtosis:

$$\gamma_1 = \frac{\mu_3^2}{\mu_2^3}; \quad \gamma_2 = \frac{\mu_4}{\mu_2^2}$$

where μ_i is the i th central moment.

The system classifies all continuous probability distribution into 12 types, including three main types (Type I, Type IV and Type VI) that describe the majority of empirical distributions, and nine transitional types that can be considered in general terms to be special cases of the main three. One of these special cases is the Gaussian, which is a single distribution in an infinite set of the other possible empirical curves. The system is intended to gather all possible variations in the first four moments of every continuous probability distribution.

For each share listed on the Nikkei 225 on 30th June 2011, we estimated the Pearson type: a) for the whole series of daily unconditional returns and b) for non-overlapping subperiods of 250, 500, 1000, 2000, 3000, 4000, 5000, 6000, 7000 and 8000 consecutive trading days. The significance of the estimates was assessed through a bootstrap simulation of 5000 repetitions of the same length as the original series using fixed blocks of 125 observations.

We then formed portfolios of 5, 10, 20 and 30 shares randomly chosen from those for which daily data had been available since 1974. The sole restriction imposed was that companies in the same sector must account for no more than 20% of any portfolio. To form the portfolios, each random group of shares was weighted in 12 different ways: 1) according to market capitalisation, 2) with the weight $1/n$, 3-7) with random positive weights, and 8-12) with random weights between 1 and -1. Except for case 2), weights were revised every calendar year. A total of 48 portfolios were constructed, of which four were weighted by market capitalisation, four were weighted by $1/n$, 20 were randomly weighted with short selling not allowed, and 20 were randomly weighted with short selling allowed but limited to a maximum of the value of the portfolio.

Data were collected from Bloomberg, and consisted in daily closing prices, corrected for corporate actions and dividends, of all shares listed on the Nikkei 225 index on 30th June 2011. Returns were calculated as the natural logarithm of the ratio of the closing prices of two consecutive trading days. For all stocks, we analysed the whole period of daily closing prices covered by the Bloomberg dataset, i.e., from 10th September 1974 (or the start of share listing if later) to 30th June 2011. Jarque-Bera test reject the normality hypothesis for all stocks in the sample with p-values of less than $2.2e-16$.

RESULTS

All the stocks in our sample could be described by the Pearson Type IV distribution; of these, 220 out of 225 could be fitted with a 99% significance level, 1 with a 95% significance level, and only

4 with a significance level of less than 90% (of this last group, 3 are newly listed shares with fewer than 550 days of observations).

When we restricted the analysis to shorter time periods, we observed Pearson Type I and Type VI behaviours, along with Type IV, but only rarely, and only when periods of not more than 1000 observations were examined. A total of 14,234 subperiodical analyses were undertaken. Of these, only 1.64% showed Type VI behaviour and 1.19% showed Type I behaviour. The results are summarised in Table 1.

Our analysis of the portfolios revealed a similar set of results, but with fewer exceptions to the Type IV rule. All the Type IV estimates had a 99% significance level, and Type I and Type VI behaviours were observable only when subperiods of 250 observations were used. The details are given in Table 2.

The probability density function of the Pearson Type IV is generally represented as:

$$p(x) = \frac{\left| \frac{\Gamma\left(m + \frac{v}{2}i\right)}{\Gamma(m)} \right|^2}{aB\left(m - \frac{1}{2}, \frac{1}{2}\right)} \left[1 + \left(\frac{x - \lambda}{a}\right)^2 \right]^{-m} e^{-v \arctan^{-1}\left(\frac{x - \lambda}{a}\right)} \tag{2}$$

for $a > 0$, $m > 1/2$, $v \neq 0$ and λ location parameter.

The Pearson Type IV distribution has an unlimited positive and negative range, is bell-shaped,

$\frac{va}{r}$

skewed and unimodal. Its origin is r above the mean and its inflection points are equidistant from the mode. There is no common statistical distribution that falls into Type IV. Difficulties in computing the normalising constant cause problems in the practical use of the Pearson Type IV distribution. However, [Gray and Wang \(1991\)](#) developed a method that can satisfactorily be used to approximate the tail probabilities, while [Nagahara \(2003\)](#) showed how to overcome some recursive problems. Nevertheless, the Pearson Type IV distribution gives great flexibility in modelling the parameters. This flexibility can be of the utmost importance for financial modelling, given the heterogeneity of the parameters of the actual distributions of stock returns. Table 3 shows the variability of the parameters for the estimates of the Type IV curves for the sample analysed. The table refers to the whole period analysed. When portfolios of Japanese stocks are examined, the variability of the parameters is less pronounced. Those who are interested in viewing the complete data for each stock or portfolio of the sample are invited to contact the author. Estimates of the parameters were calculated using the method of moments according to [Johnson et al. \(1994\)](#).

SUMMARY AND CONCLUSIONS

We have herein concentrated our analysis on the Japanese stock market, because to our knowledge, there have been no previous attempts to undertake an in-depth analysis of the behaviour of the distribution of Japanese shares. We employed the Pearson system of continuous probability distribution, and verified that the Type IV frequency curve describes the distribution of the unconditional returns of all the shares that comprise the Nikkei 225 index. When short time periods are used, behaviours that are not of Type IV may be observed occasionally, but only when periods containing fewer than 1000 observations are analysed. This finding suggests that the length of the period is not a variable that has any significant effect on the behaviour of Japanese stock returns. Finally, we showed that even when portfolios of Japanese equities are considered, the distributions of their returns are of Type IV, and that exceptions are less common and confined to very short time periods of analysis (i.e., no more than 250 observations).

The behaviour of the distribution of stock returns is of fundamental importance in financial economics, in view of its direct bearing on the descriptive validity of any theoretical model. Moreover, many of the statistical inferences that are typical in modern empirical studies of financial economics are also conditional on the assumptions made on the distribution of the returns.

It is nowadays generally accepted that the use of VaR models based on the normality assumption of the returns of the underlying assets leads to an under- or overestimate of the true value at risk. Neither case is welcome news for risk managers. Underestimation is what financial regulators seek to avoid because it can result in huge losses and illiquidity for financial intermediaries, as dramatically experienced during the current financial crisis. Alternative VaR models have been developed with the aim of improving the accuracy of the estimates made. Our results support the principle of capturing the thickness of the left tail and the asymmetrical characteristics of financial returns using a Pearson type IV specification, as undertaken by [Grigoletto and F. Lisi \(2011\)](#), for example.

Although models of portfolio selection are very sensitive to the assumptions made about the dynamic and distributional behaviour of returns on assets, most asset managers and practitioners use the classical mean-variance framework to allocate wealth, and the classical capital asset pricing model to estimate expected returns and measure investment performance. Both models assume, among other things, either that asset returns are normally distributed or that investors have quadratic preferences (and therefore ignore higher moments of return distributions). A huge effort has been made in the literature to develop models for pricing and allocating financial assets that overcome these shortcomings, and to provide a more realistic framework within which portfolio managers can operate. However, none of these models or frameworks has achieved particular

prominence or has, to our knowledge, used a Pearson type IV specification, as suggested herein for the Japanese market.

Understanding the behaviour of the returns of the underlying asset is also important in the pricing of options. The widely used Black & Scholes (B&S) model frequently and substantially misprices deep-in-the-money and deep-out-of-the-money options, given the skewness and kurtosis of the underlying distribution of returns. Most of the alternative models that introduced adjustments for skewness and kurtosis to the B&S formula showed that the adjustments result in significant improvements to the accuracy of the pricing. However, to our knowledge, none of these models considers Pearson Type IV behaviour for the distribution of the returns on the underlying shares.

Finally, Auto-Regressive Conditional Heteroskedastic (ARCH) models of financial return time series assume an underlying distribution for the error term. Many empirical studies have shown that conditional normal, Student's t or generalised error distributions are not adequate to represent the excess kurtosis of the implied unconditional distribution. Moreover, these conditional specifications cannot model the asymmetry of financial data. Our results support the use of a Pearson Type IV distribution for the innovation term of an ARCH-like model, which can then capture both the skewness and kurtosis of stock return time series conveniently, as was first proposed by Premaratne and Bera (2001).

Table-1. Pearson system share analysis

Observ.	Type IV		Type I		Type VI	
250	6470	94.87%	190	2.79%	160	2.35%
500	3341	98.64%	37	1.09%	9	0.27%
1000	1666	99.52%	7	0.42%	1	0.06%
2000	747	100.00%	0	0.00%	0	0.00%
3000	527	100.00%	0	0.00%	0	0.00%
4000	361	100.00%	0	0.00%	0	0.00%
5000	184	100.00%	0	0.00%	0	0.00%
6000	181	100.00%	0	0.00%	0	0.00%
7000	179	100.00%	0	0.00%	0	0.00%
8000	174	100.00%	0	0.00%	0	0.00%
Whole	225	100.00%	0	0.00%	0	0.00%

Table-2. Pearson system portfolio analysis

Observ.	Type IV		Type I		Type VI	
250	1648	95.37%	11	0.64%	69	3.99%
500	864	100.00%	0	0.00%	0	0.00%
1000	432	100.00%	0	0.00%	0	0.00%
2000	192	100.00%	0	0.00%	0	0.00%
3000	144	100.00%	0	0.00%	0	0.00%
4000	96	100.00%	0	0.00%	0	0.00%

5000	48	100.00%	0	0.00%	0	0.00%
6000	48	100.00%	0	0.00%	0	0.00%
7000	48	100.00%	0	0.00%	0	0.00%
8000	48	100.00%	0	0.00%	0	0.00%
Whole	225	100.00%	0	0.00%	0	0.00%

Table-3. Estimates of the parameters of the Type IV curves for Nikkei 225 shares.

Parameter	Mean	St. Dev	Min	Max
<u>STOCKS</u>				
M	3.063	0.207	2.591	3.950
v	-0.140	0.493	-1.228	3.461
λ	-0.002	0.005	-0.013	0.024
A	0.042	0.010	-0.051	0.081
<u>PORTFOLIOS</u>				
M	2.917	0.114	2.731	3.267
v	0.040	0.155	-0.406	0.249
λ	0.000	0.001	-0.005	0.002
A	0.031	0.011	0.021	0.053

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