



THE ROLE OF PRODUCTIVITY IN ECONOMIC GROWTH AND EQUILIBRIUM

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ABSTRACT

This study reexamines the evidence for the Balassa-Samuelson effect for the 1985-2007 period. Cointegrating relationships between the real exchange rate and productivity, real price of oil and government spending are estimated using the Johansen and Stock-Watson procedures. The findings show that for each percentage point in the US-Euro area productivity differential there is a three percentage point change in the real dollar/euro valuation. These findings are robust to the estimation methodology, the variables included in the regression, and the sample period. We suggest that economic disequilibrium can result in a decline in economic growth. This study will utilize von Neumann's "A Model of General Economic Equilibrium" as an economic equilibrium standard.

Keywords: Foreign exchange rates, Labor productivity, American dollar.

INTRODUCTION

The euro greatly depreciated against the dollar during the period 1995-2001. This decline has often been associated with relative productivity changes in the United States and the euro area over this time period. During this time period in particular, average labor productivity accelerated in the United States, while it decelerated in the euro area. Economic theory suggests that the equilibrium real exchange rate will appreciate after an actual or expected shock in average labor productivity in the traded goods sector. Such an equilibrium appreciation may be influenced in the medium term by demand side effects. Thus, productivity increases raise expected income, which leads to an increased demand for goods. However, the price of goods in the traded sector is determined more by international competition. By contrast, in the non-traded sector, where industries are not subject to the same competition, goods prices tend to vary widely and independently across countries. The work of Harrod (1939), Balassa (1964), Samuelson (1964) and Olson (2012) show that productivity

growth will lead to a real exchange rate appreciation only if it is concentrated in the traded goods sector of an economy. Productivity growth that has been equally strong in the traded and non-traded sectors will have no effect on the real exchange rate.

This paper analyses the impact of relative productivity developments in the United States and the euro area on the dollar/euro exchange rate. This paper then provides evidence on the long-run relationship between the real dollar/euro exchange rate and productivity measures with and without the oil prices and government spending variables. Importantly, to the extent that traders in foreign exchange markets respond to the available productivity data stresses the importance of reliable models. From the first to the second half of the 1990's, average productivity accelerated in the United States, while it decelerated in the euro area. This relationship has stimulated a discussion on the relationship between productivity and appreciation of the dollar during this time period. Also, of equal importance is the depreciation of the dollar during the early part of the 2000's (United States productivity increased slowly while the euro area productivity increased more rapidly). [Bailey and Wells \(2001\)](#), for instance, argue that a structured improvement in US productivity increased the rate of return on capital and triggered substantial capital flows in the United States, which might explain in part the appreciation of the US dollar during the early part of the 2000's. [Tille and Stoffels \(2001\)](#) confirm empirically that developments in relative labor productivity can account for part of the change in the external value of the US dollar over the last 3 decades. [Alquist and Chinn \(2002\)](#) argue in favor of a robust correlation between the euro area United States labor productivity differential and the dollar/euro exchange rate. This would explain the largest part of the euro's decline during the latter part of the 1990's. This paper presents the argument that the euro's persistent weakness in the 1995-2001 period and its strength during the 2001-2007 period can be partly explained by taking into consideration productivity differentials. In particular, the study analyses in detail the impact of relative productivity developments in the United States and the euro area on the dollar/euro exchange rate. The paper is organized with the first part being the introduction. The next section explains the relationship between productivity advances and the real exchange rate along with the data gathering process. Section 3 deals with the estimation, the structural VECM and impulse response analysis. Section 4 deals with tests for nonnormality and forecast error variance decomposition. Section 5 deals with exchange rate disequilibrium and the von Neumann Model and a discussion of the results.

This study is a revision of the [Olson 2012](#) study, "Productivity Growth and Its Influence on the Dollar/Euro Real Exchange Rate". This study expands the original study and includes the cointegrating relationships between the traded goods differentiated productivities of the United States and the Euro Countries and the dollar/euro real exchange rates. This study also suggests that economic disequilibrium can result in a

decline in economic growth. Von Neumann's "A Model of General Economic Equilibrium" as an economic equilibrium standard.

Background for the Balassa-Samuelson Model

The theoretical relationships that link fundamentals to the real exchange rate in the long-run center around the Balassa-Samuelson model, portfolio balance considerations as well as the uncovered (real) interest rate parity condition. This study will focus on the role of productivity differentials in the determination of the dollar/euro exchange rate. The intuition behind the Balassa-Samuelson effect is rather straight-forward. Assuming, for instance of simplicity, that productivity in the traded goods sector increases only in the home country, marginal costs will fall for domestic firms in the traded-goods sector. The results of the increased demand leads to a price shift and real appreciation. If labor is mobile between sectors in the economy, workers shift from the non-traded sector to the traded sector in response to the higher wages. This triggers a wage rise in the non-traded goods sector as well, until wages equalize again across sectors. However, since the increase in wages in the non-traded goods sector is not accompanied by productivity gains, firms need to increase their prices, which do not jeopardize the international price competitiveness of firms in the traded goods sector [Harrod \(1939\)](#), [Balassa \(1964\)](#) and [Samuelson \(1964\)](#).

[Tille and Stoffels \(2001\)](#) revealed that nearly two-thirds of the appreciation of the dollar was attributable to productivity growth differentials (using the traded and nontraded differentials). However, it is important to note that [Engel \(1999\)](#) found that the relative price of non-traded goods accounts almost entirely for the volatility of US real exchange rates. Accordingly, there should be a proportional link between relative prices and relative productivity. Labor productivity, however, is also influenced by demand-side factors, though their effect should be of a transitory rather than of a permanent nature. In particular, as the productivity increases raise future income, and if consumers value current consumption more than future consumption, they will try to smooth their consumption pattern as argued by [Bailey and Wells, 2001](#)). This leads to an immediate increased demand for both traded and non-traded goods. The increase in demand for traded goods can be satisfied by running a trade deficit. The increased demand for non-traded goods, however, cannot be satisfied and will lead to an increase in prices of non-traded goods instead. The results of the increased demand leads to a price shift and real appreciation.

The Asymptotically Stationary Process of the Model

This section presents evidence in favor of stable long-run relationships between the real dollar/euro exchange rate, the productivity measure, and the other variables. One model specification was estimated for the productivity measure. The sample covers the period from 1985 to 2007. The general model includes all variables discussed above as well as deterministic components.

The results of the autocorrelations and partial autocorrelations in figures 1-3 show that the autocorrelations typically die out over time with increasing time as in the GDP, oil prices and US productivity variables. The dashed lines are just $\pm 2/\sqrt{T}$ lines; consequently, they give a rough indication of whether the autocorrelation coefficients may be regarded as coming from a process with true autocorrelations equal to zero. Clearly, all of the series are not likely to be generated by a white noise process because the autocorrelations reach outside the area between the dashed lines for more than 50% of the time series. On the other hand, all coefficients at higher lags are clearly between the lines. Hence, the underlying autocorrelation function may be in line with a stationary data gathering process. The partial correlations convey basically the same information on the properties of the time series.

Lutkepohl (2004) states that autocorrelations and partial autocorrelations provide useful information on specific properties of a data gathering process other than stationarity. According to Lutkepohl, consistency and asymptotic normality of the maximum likelihood estimators are required for the asymptotic statistical theory behind the tests to be valid. The results of these tests are shown in the appendix (table 6). They consist of an LM test of no error autocorrelation, an LM-type test of no additive nonlinearity, and another LM-type test of parameter constancy. Bartlett (1950) and Parzen (1961) have proposed spectral windows to ensure consistent estimators. The autocorrelations of a stationary stochastic process can be defined as

$$F_y(Y) = (2P)^{-1} \sum y/e^{-iYj} = (2P)^{-1} [y_0 + 2 \sum y_j \cos(Yj)] \tag{1}$$

Where $I = \sqrt{-1}$ is the imaginary unit, $Y \in [-Y, Y]$ is the frequency, that is, the number of cycles in a unit of time measured in radians, and the y_j 's are the autocovariances of y_t as before. It can be shown that

$$Y_j = \int F_y(Y) dY \tag{2}$$

Thus, the autocovariances can be recovered from the spectral density function integral as follows:

$$Y_0 = \alpha \int y(Y) dY \tag{3}$$

Graph 1 shows the log of the smoothed spectral density estimator based on a Bartlett window with window width $M_r = 20$.

Many economic time series have characteristics incompatible with a stationary data gathering process. However, Lutkepohl (2004) recommends the use of simple transformations to move a series closer to stationarity. A logarithmic transformation may help stabilize the variance. In figure 4 the logarithms of the US productivity, M2, oil prices, US GDP, US/euro exchange rate and government spending are plotted. The logarithm is used as it ensures that larger values remain

larger than smaller ones. Even though the relative size is reduced the series reveals an upward trend and a seasonal pattern. It appears to be a stationary series.

Unit Roots

Fuller (1976) and Dickey & Fuller (1979) proposed the augmented Dickey-Fuller (ADF) test for the null hypothesis of a unit root. It is based on the t-statistic of the coefficient ϕ from an OLS estimation (see table 1). Schmidt & Phillips (1992) propose another group of tests for the null hypothesis of a unit root when a deterministic linear trend is present.

The empirical analysis employs cointegration tests as developed by Johansen (1995). In the present setting, some variables would theoretically be expected to be stationary, but appear to be near-integrated processes empirically. The presence of the cointegration relationships is tested in a multivariate setting. Table 2 and 3 show the results of the cointegration tests. Over all, the results suggest that it is reasonable to assume a single cointegration relationship between the variables and suggest being viewed as an order of $I(1)$.

Data Sources

Much of the data for this study is the same as my previous study so many of the figures, graphs and charts may be duplicated. However, the interpretations of the data is concentrated on the traded goods productivity differentials of the United States and the Euro countries and the cointegrating relationships to the dollar/euro real exchange rates.

For the period prior to 1999, the real dollar/euro exchange rate was computed as a weighted geometric average of the bilateral exchange rates of the euro currencies against the dollar. In addition, the model was estimated controlling for several other variables, which included US productivity, M2, oil prices, government spending and US GDP. As regards the real price of oil, its usefulness for explaining trends in real exchange rates is documented. For example, (Amano and Van Norden, 1998b) Amano and Van Norden (1998a) found strong evidence of a long-term relationship between the real effective exchange rate of the US dollar and the oil price. As regards government spending, the fiscal balance constitutes one of the key components of national saving. In particular, Frenkel and Mussa (1995) argued that a fiscal tightening causes a permanent increase in the net foreign asset position of a country, and consequently, an appreciation of its equilibrium exchange rate in the long term. This will occur provided that the fiscal consolidation is considered to have a long-run affect. Explaining the Euro Volatility by Productivity Developments during 1995-2001 and 2001-2007 this study shows how much of the decline of the euro against the US dollar during the 1995-2001 period can be attributed to relative changes in productivity in the

United States and the euro area. While the estimation covers the period 1985-2007, the following analysis concentrates on two distinct periods.

Period 1 (1995-2001) covers the US dollar appreciation against the euro. Moreover, it encompasses the period during which the productivity revival in the United States has taken place. Over this period, the dollar appreciated by almost 41%. During the first three years (1998-2001) of the euro, it depreciated by almost 30% against the US dollar. Figure 5 shows the impact of a change in relative productivity developments over these periods on the equilibrium real exchange rate. The contribution of the relative developments in productivity on the explanation of the depreciation of the euro against the US dollar since 1995 is significant. However, these developments are far from explaining the entire euro decline. Figures 6 and 7 show the impact of a change in relative US GDP and Euro GDP on the equilibrium dollar/euro real exchange rate. Period 2 (2001-2007) covers the US dollar depreciation against the euro. Figure 8 also shows the impact of a change in relative productivity developments over these periods on the equilibrium real exchange rate. The impact of productivity on the real exchange rate is significant.

Estimation and the Structural VECM

This study utilizes the basic vector autoregressive and error correction model suggested by [Lutkepohl \(2004\)](#). The VAR model is general enough to accommodate variables with stochastic trends. This model was used in my previous study ([Olson 2012](#)) and employs the same equations. The following VECM form is a convenient model setup for cointegrating analysis:

[Lutkepohl \(2004\)](#) suggests the following basic vector autoregressive and error correction model (neglecting deterministic terms and exogenous variables):

$$\text{For a set of } K \text{ times series variables}$$

$$y_t = A_1 Y_{t-1} + \dots + A_p Y_{t-p} + \mu_t \quad (4)$$

The VAR model is general enough to accommodate variables with stochastic trends, it is not the most suitable type of model if interest centers on the cointegration relations because they do not appear explicitly. The following VECM form is a more convenient model setup for cointegration analysis:

$$y_t = \Pi y_{t-1} + \Delta t^{-1} + \dots + I_{p-1} \Delta t^{-p+1} + \mu_t \quad (5)$$

Deterministic Terms

Several extensions of the basic model are usually necessary to represent the main characteristics of a data set. It is clear that including deterministic terms, such as an intercept, a linear trend term, or

seasonal dummy variables, may be required for a proper representation of the data gathering process. One way to include deterministic terms is simple to add them to the stochastic part,

$$y_t = \mu_t + x_t \tag{6}$$

Here μ_t is the deterministic part and x_t is a stochastic process that may have a VAR or VECM representation.

A VAR representation for y_t is as follows:

$$y_t = v_0 + v_1 t + A y_{t-1} + \dots + A_p y_{t-p} + \mu_t \tag{7}$$

A VECM (p-1) representation has the form

$$y_t = v_0 + v_1 t + \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + \mu_t \tag{8}$$

Exogenous Variables

Lutkepohl (2004) recommends further generalizations of the model to include further stochastic variables in addition to the deterministic part. A rather general VECM form that includes all these terms is

$$y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + C D_t B z_t + \mu_t \tag{9}$$

where the z_t are unmodeled stochastic variables, D_t contains all regressors associated with deterministic terms, and C and B are parameter matrices. The z_t 's are considered unmodeled because there are no explanatory equations for them in the system.

Estimation of Vecm's

Under Gaussian assumptions estimators are ML estimators conditioned on the presample values Johansen (1988b). They are consistent and jointly asymptotically normal under general assumptions,

$$V^{-1/2} \text{VEC}([\Gamma_t \dots \Gamma_{p-1}] - [\Gamma_t \dots \Gamma_{p-1}]) \rightarrow_d N(0, S_t) \tag{10}$$

Reinsel (1992) gives the following:

$$\text{VEC}(\beta_k \mu - r) @ N(\text{VEC}(\beta_k - r), \{y_{2-1} \text{ M} y_{2-1}\}^{-1} \phi \{\alpha' S \mu^{-1} \alpha\}^{-1}) \tag{11}$$

Adding a simple two-step (S2S) estimator for the cointegration matrix.

$$y_t - \Pi y_{t-1} - \Gamma x_{t-1} = \Pi^2 y_{t-2} + \mu_t \tag{12}$$

The restricted estimator $\beta_{k-r} R$ obtained from $\text{VEC}(\beta_k - r) R = \square \square + h$, a restricted estimator of the cointegration matrix is

$$BR = [Ir : BK-r] \tag{13}$$

Estimation of Models with more General Restrictions and Structural Forms

The first stage estimator β^* is treated as fixed in a second-stage estimation of the structural form because the estimators of the cointegrating parameters converge at a faster rate than the estimation of the short-term parameters (Lutkepohl, 2004). In other words, a systems estimation procedure may be applied to

$$A\Delta y_t = \alpha^*\beta^* y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + C^*D_t + B^*z + v_t \tag{14}$$

As suggested by King *et al.* (1991) the following procedure is used for the estimation of the model: Using economic theory we can infer that all three variables should be I(1) with $r = 2$ cointegration relations and only one permanent shock. The variables in this model include government spending, US productivity and oil prices. Because $k^* = 1$, the permanent shock is identified without further assumptions ($k^* - 1)/2 = 0$). For identification of the transitory shocks a further restriction is needed. If we assume that the second transitory shock does not have an instantaneous impact of the first one, we can place the permanent shock in the e_t vector. These restrictions can be represented as follows in this framework:

$$XB = \begin{bmatrix} [*00] & B [***] \\ [*00] & [**0] \\ [*00] & [***] \end{bmatrix}$$

Asterisks denote unrestricted elements. Because XB has rank 1, the new zero columns represent two independent restrictions only. A third restriction is placed on B, and thus we have a total of $K(K-1)/2$ independent restrictions as required for just-identification.

The Breusch-Godfrey test for autocorrelation (Godfrey, 1988) for the h th order residual autocorrelation assumes this model

$$V_t : B_t \mu_{t-1} + \dots + B_h \mu_{t-h} + \text{error} \tag{15}$$

For the purpose of this model the VECM form is as follows:

$$\mu_t = \alpha\beta y_{t-1} + \Gamma_1 \Delta y_{t-1} + \dots + \Gamma_{p-1} \Delta y_{t-p+1} + CD_t + B_t \mu_{t-1} + \dots + B_h \mu_{t-h} + e_t \tag{16}$$

Impulse Response Analysis-Stationary VAR Processes

Following Lutkepohl (2004), if the process y_t is I(0), the effects of shocks in the variables of a given system are most easily seen in its Wold moving average (MA) representation as follows:

$$y_t = \phi_0\mu_t + \phi_1\mu_{t-1} + \phi_2\mu_{t-2} + \dots, \tag{17}$$

where $\phi_s = S \phi_s \dots A_j$ $S=1,2,\dots$,

The coefficients of this representation may be interpreted as reflecting the responses to impulses hitting the system. The effect on an impulse is transitory as it vanishes over time. These impulse responses are sometimes called forecast error impulse responses because the μ_t S are the 1-step ahead forecast errors. Occasionally, interest centers on the accumulated effects of the impulses. They are easily obtained over all periods. The total long-run effects are given by

$$\phi_s = S \phi_s = (I_k - A_1 - \dots - A_p)^{-1} \quad (18)$$

This matrix exists if the VAR process is stable.

Lutkepohl (2004) criticizes the forecast error impulse response method in that the underlying shocks are not likely to occur in isolation if the components of μ are instantaneously correlated. Therefore, orthogonal innovations are preferred in an impulse response analysis. One way to get them is to use a Choleski decomposition of the covariance matrix S_μ . If B is a lower triangular matrix such that $S_\mu = B^{-1}\mu$, we obtain the following:

$$y_t = \phi_0 \epsilon_t + \phi_1 \epsilon_{t-1} + \dots, \quad (19)$$

Sims (1981) recommends trying various triangular orthogonalizations and checking the robustness of the results with respect to the ordering of the variables if no particular ordering is suggested by subject matter theory.

Forecasting VECM Processes

Once an adequate model for the data gathering process of a system of variables has been constructed, it may be used for forecasting as well as economic analysis. The concept of Granger-causality, which is based on forecast performance, has received considerable attention in the theoretical and empirical literature. Granger (1969) introduced a causality concept whereby he defines a variable y_{2t} to be casual for a time series variable y_{1t} if the former helps to improve the forecasts of the latter. In Table 5 the test for Granger-Causality reveals none of the p-values are smaller than 0.05. Therefore, using a 5% significance level, the null hypothesis of noncausality cannot be rejected. However, in the test for instantaneous causality there is weak evidence of a Granger-causality relation from US productivity differentials \rightarrow dollar/euro exchange rate because the p-value of the related test is at least less than 10%.

This procedure can be used if the cointegration properties of the system are unknown. If it is known that all variables are at most $I(1)$, an extra lag may simply be added and the test may be performed on the lag-augmented model. Park and Phillips (1989) and Sims et al (1990) argue that the procedure remains valid if an intercept or other deterministic terms are included in the VAR

model. Forecasting vector processes is completely analogous to forecasting univariate processes. It is assumed the parameters are known. The identification of shocks using restrictions on their long-run effects are popular. In many cases, economic theory suggests that the effects of some shocks are zero in the long-run. Therefore, the shocks have transitory effects with respect to some variables. Such assumptions give rise to nonlinear restrictions on the parameters which may in turn be used to identify the structure of the system. The impulse responses obtained from a structured VECM usually are highly nonlinear functions of the model parameters. This should be considered when drawing inferences related to the impulse responses.

Estimation of Structural Parameters

Following the procedure recommended by Lutkepohl (2004), the estimation of the SVAR model is equivalent to the problem of estimating a simultaneous equation model with covariance restrictions. First, consider a model without restrictions on the long-run effects of the shocks. It is assumed that ϵ_t is white noise with $\epsilon_t \sim N(0, I_k)$ and the basic model is a VAR; thus the structural form is

$$A y_t = A[A_1, \dots, A_p] Y_{t-1} + B \epsilon_t \tag{20}$$

The concentrated log-likelihood is as follows:

$$l_c(a, B) = \text{constant} + T/2 \log |A| - T/2 \log |B| - T/2 m (A' B'^{-1} A S_\mu) \tag{21}$$

where $S_\mu = T^{-1}(Y - A Z)(Y - A Z)'$ is just the estimated covariance matrix of the VAR residuals as argued by Breitung (2001).

Lutkepohl (2004) recommends that continuation of the algorithm stops when some prespecified criterion are met. An example would be a relative change in the log-likelihood and the relative change of the parameters. The resulting ML estimator is asymptotically efficient and normally distributed, where the asymptotic covariance matrix is estimated by the inverse of the information matrix. Moreover, the ML estimator for S_μ is

$$S_\mu = \hat{A}^{-1} \hat{B}^{-1} \hat{B} \hat{A}^{-1}$$

Where \hat{A} and \hat{B} are estimators of A and B , respectively. Note that S_μ only corresponds to the reduced-form estimate S_μ if the SVAR is exactly identified. In the presence of over-identifying restrictions, an LR test statistic for these restrictions can be constructed in the usual way as

$$LR = T(\log |S_\mu^*| - \log |S_\mu|) \tag{22}$$

For VECM'S the concentrated likelihood function

$$l_c(A, B) = \text{constant} + T/2 \log |A| - T/2 \log |B| - T/2 m (A' B'^{-1} A S_\mu) \tag{23}$$

can be used for estimating the structural parameters A and B. If no restrictions are imposed on the short-run parameters, the S_{μ} matrix represents the residual covariance matrix obtained from a reduced rank regression. If the short-run parameters are restricted or restrictions are placed on the cointegration vectors, some other estimator may be used instead of the ML estimator, and S_{μ} may be estimated from the corresponding residuals.

Generally, if long-run identifying restrictions have to be considered, maximization of the above formula is a numerically difficult task because these restrictions are typically highly nonlinear for A, B, or both. In some cases, however, it is possible to express these long-run restrictions as linear restrictions, and maximization can be done using the scoring algorithm defined above. When considering a cointegrated VECM where $A = I_k$, it follows that the restrictions on the system variables can then be written in implicit form as

$$RX\text{vec}(XB) = 0 \quad (24)$$

Where RX is an appropriate restriction matrix. Following the suggestions of Vlaar (1998) we can reformulate these restrictions as

$$RX(I_k \otimes X \text{vec}(B)) = RX\text{vec}(XB) = 0 \quad (25)$$

Replacing X by an estimator obtained from the reduced form we obtain $R_{B,1} = RX(I_k \otimes \hat{X})$, which is a stochastic restriction matrix. These implicit restrictions can be derived. Here $t^{*y/2}$ and $t^{*1-y/2}$ are the $y/2$ and $(1 - y/2)$ equations, respectively, of the empirical distribution of $(\hat{\phi}^* - \phi)$

Impulse Responses

Figures 9-10 display the impulse responses of the dollar/euro exchange rate to a one standard deviation change in the US productivity, M2, oil prices, and government spending. The responses are significant at the 95% level. Table 8 (in the appendix) displays the point estimates of the impulse responses of the real exchange rate to the one-standard deviation US productivity shocks. Also note that the results are relatively robust with the individual impulse responses falling within the 5% significant tests. Figure 9 shows that for the exchange rate these shocks have a highly significant impact over the 10-year time period and the correlation between these impulse responses is high. They show that productivity shocks have a very significant long-run impact on the dollar/euro exchange rate. The results follow those of [Clarida and Gali \(1992\)](#). The point estimates in table 8 show that for each percentage point in the US-Euro area productivity differential there is a three percentage point real change in the dollar/euro valuation. This suggests that fundamental real factors are significant in the long-run fluctuations in real exchange rates.

Refer to the appendix (figures 11-12) for the US and Euro productivity differentials. Figure 11 shows the long-run impact of productivity shocks on the dollar/euro real exchange rate. Figure 12 shows the significance of large gaps in the euro and US productivity differentials especially around the years 2000-2001 when the dollar started to depreciate against the euro.

Summarizing Impulse Responses with Forecast error variance decomposition

Forecast error variance decomposition is a special way of summarizing impulse responses. Following [Lutkepohl \(2004\)](#) the forecast error variance decomposition is based on the orthogonalized impulse responses for which the order of the variables matters. Although the instantaneous residual correlation is small in our subset VECM, it will have some impact on the outcome of a forecast error variance decomposition.

Lutkepohl (2004) suggests the forecast error variance as

$$\partial^2 k(h) = \sum (Y_{2k1,n} + \dots + Y_{2k,n}) = Y_{2kjo} + \dots Y_{2kh-1} \quad (26)$$

The term $(Y_{2k1,n} + \dots + Y_{2k,n})$ is interpreted as the contribution of variable j to the h -step forecast error variance of variables k . This interpretation makes sense if the $\epsilon_{\mu s}$ can be viewed as shocks in variable i . Dividing the preceding by $\partial^2 k(h)$ gives the percentage contribution of variable j to the h -step forecast error of variable h .

$$(t)(h) = Y_{2kjo} + \dots Y_{2kh-1} / \partial^2 k(h) \quad (27)$$

Chart 1 shows the proportion of forecast error in the dollar/euro accounted for by US productivity, government spending, M2, oil prices and US GDP. The US productivity accounts for 28% over the 20 year time interval with a sharp rise of 21% during the first 5 years. This shows that productivity shocks have a very significant short-run impact on the dollar/euro exchange rate while the long-run impact is more transitory in nature. On the basis of the appropriate p -values, the bootstrap findings of the sample-split. Chow tests (table 7) do not reject stability in the model even with the structural break in 2001.

Nonnormality Tests

The following test for residual autocorrelation is known as the Portmanteau test statistic. The null hypothesis of no residual autocorrelation is rejected for large values of Q_h (test statistic). The p -value is relatively large: consequently, the diagnostic tests indicate no problem with the model

[Lominski \(1961\)](#) and [Jarque and Bera \(1987\)](#) propose a test for nonnormality based on the skewness and kurtosis for a distribution. The Jarque & Bera tests in table 9 show some nonnormal residuals for two variables (oil prices and government spending (u_4 and u_6)). [Lutkepohl \(2004\)](#) states that if nonnormal residuals are found, this is often interpreted as a model defect. However,

much of the asymptotic theory on which inference in dynamic models is based works also for certain nonnormal residual distributions. The effects of nonnormal residuals can be resolved with the multivariate ARCH-LM tests. These tests were performed with the results shown in Table 10. The results indicate the p-value is relatively large: consequently, the diagnostic tests indicate no problem with the model

The General Economic Equilibrium Model

We assume von Neumann's "Model of General Equilibrium" as the economic equilibrium standard for the domestic country. The following section describes the model and its application to this study.

The supreme merit of John von Neumann's "Model of General Economic Equilibrium" lies in the elegance of the mathematical solution of a highly generalized problem in theoretical economics. However, the paper is of considerable interest to economists as well as to mathematicians, because it deals simultaneously with questions on several fields of economics. For example, in this paper von Neumann considers which goods will be free goods, and the determination of the prices of goods which are not free. He examines which productive processes and scales of production will be optimum and which will be unprofitable. He also examines the degree in which each optimum process will be used and the relative amounts of different goods that will be produced. At the same time he demonstrates the mechanism which determines the rate of interest and the rate of expansion of the whole economy. He gave an absolutely rigorous mathematical argument and stated his assumptions completely and without ambiguity.

John von Neumann is not concerned with short period problems in the model, but with the properties of the economic system when it has settled down to an equilibrium position which may be described as a quasi-stationary state His direct interpretation of the functions of the model appears to be similar to the thermodynamic potentials in an economic system and that the similarity can be extended to full phenomenological generality (independently of the model's assumed restrictions).

An economic system in economic equilibrium will produce the following:

1. The greatest factor of expansion of the whole economy.
2. The lowest interest factor at which a profitless system of prices is possible.

When the domestic country is in equilibrium [reference to (eq. 28) $S(t) = Y + \beta M(t)$] it is assumed this meets the conditions of economic equilibrium in von Neumann's "Model of General Economic Equilibrium". If the domestic country is not in economic equilibrium (i.e. as defined by von

Neumann's model) then the domestic country will be in economic disequilibrium by definition. In this disequilibrium position the domestic country will have the following:

1. Interest rates that are higher than optimum.
2. Lower than optimum rate of expansion of the economy.

Speculators will view this as a weakening position by the domestic government in maintaining the fixed exchange rate [In the model, Y will increase and $S(t)$ will increase. The economic disequilibrium position of the domestic country will make it more difficult for the government to maintain the peg and prevent the eventual collapse of the fixed exchange rate regime. The next section explains arbitrary speculative behavior in more detail and how it may cause an indeterminate path of the post-collapse floating exchange rate.

The Role of Arbitrary Speculative Behavior

We have based our results on an assumption that the solution for the exchange rate depends only on market fundamentals. In general, however, the exchange rate obeys the following dynamic law:

With foreign reserves of R_0 the exchange rate is now:

$$S(t) = Y + bM(t), \quad (28)$$

where Y previously set to zero, is an arbitrary constant determined at time z (t_z). The constant Y captures the arbitrary speculative behavior, which may cause an indeterminate path of the exchange rate.

The possibility that arbitrary speculative behavior can cause the collapse of a currency bears on a traditional argument favoring flexible exchange rates. The argument states that since a flexible exchange rate may be subject to arbitrary fluctuations, the exchange rate should be fixed in order to protect the real sectors of an economy. Arbitrary speculative behavior identical in nature to that which may manifest under floating rates can also render arbitrary and indeterminate the time of a fixed exchange rate. Arbitrary speculative behavior, if present, is an economic force that is masked, not purged, by the fixing of exchange rates.

(Eq. 28) reveals that the exchange rate $S(t)$ is directly proportional to $M(t)$ and the arbitrary constant Y . The arbitrary constant Y captures the speculative behavior, which may cause an undetermined path of the exchange rate. Notice that if a collapse occurs due an increase in the arbitrary constant Y , then the post-collapse exchange rate must be expected to follow CEEG in Figure 3a.

As a special case, we suppose that $m = 0$. Then $M(t)$ remains constant in which case the exchange rate $S(t)$ need never change in the absence of arbitrary speculative behavior. Therefore, a change in the exchange rate $S(t)$ may occur anytime that agent's speculative behavior sets $Y > 0$. If the speculators have the resources and choose to purchase all of the foreign reserves held by the central bank, the government will give up its defense of the currency and the exchange rate will jump up to $S1 > S^*$, as represented by point F in Figure 3a. This devaluation occurs despite the fact that the economy is fundamentally solid, with no domestic credit creation due to deficit financing.

SUMMARY OF THE RESULTS

This paper provides evidence on the long-run relationship between the real dollar/euro exchange rate and productivity measures, controlling for the real price of oil, relative government spending and M2. However, the results imply that the productivity measure can explain only about 27% of the actual amount of depreciation of the euro against the US dollar for the period 1995-2001. This outcome is confirmed by a specification in this study. Figure 18 shows that the productivity can explain only about 28% of the appreciation of the euro during the period 1995-2007 (appendix table 6 for point estimate).

Evidently, productivity is not the only variable affecting the real exchange rate in the model specified. The other variables identified also affected the dollar/euro exchange rate. In particular, the surge in oil prices since early 1999 seems to have contributed to the weakening of the euro. The magnitude of the long-run impact of changes in the real price of oil on the dollar/euro exchange rate is certainly significant. Between 1997 and 2001, the model indicates on the average that the equilibrium euro depreciation related to oil prices developments could have been around 20% (refer to table 8 for point estimate and figure 21). These results are based on long-term relationships. Overall, the model is surrounded by significant uncertainty, reflecting the inherent difficulty of modeling exchange rate behavior. While we find that in 1995-2001 the euro traded well below the central estimates derived from these specifications, this uncertainty precludes any quantification of the precise amount of over or under valuation at any point in time. This point is also made clear by [Detkin *et al.* \(2002\)](#), who employed a wide range of modeling strategies to show that the deviation from the estimated equilibrium differs widely across models and is surrounded by some uncertainty. Moreover, the results provided by [Maeso-Fernandez *et al.* \(2001\)](#) find various reasonable but non-encompassing specifications leading to different exchange rate equilibria. Again, this suggests a very cautious interpretation of the magnitude of over/under valuation. This study also suggests that arbitrary speculative behavior relating to the timing of the collapse is associated with the strong pegged to currency of the fixed exchange rate. The arbitrary use of von Neumann's "A Model of General Economic Equilibrium" as an economic equilibrium standard is

an attempt to incorporate a mathematical solution of a highly generalized problem in theoretical economics. Von Neumann's rigorous mathematical argument is proof that there is at least one possible position of quasi-stationary state equilibrium. His model is a very long-run equilibrium model and does not answer the question of whether it is applicable to systems, which are only in an approach to equilibrium. Any rigorous examination of the properties of such a system in disequilibrium would be very complicated.

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Appendices

While this study utilizes some of the data and material from the previous study, it is with this data and material that the present study can more readily examine the effect of productivity on real exchange rates and economic growth.

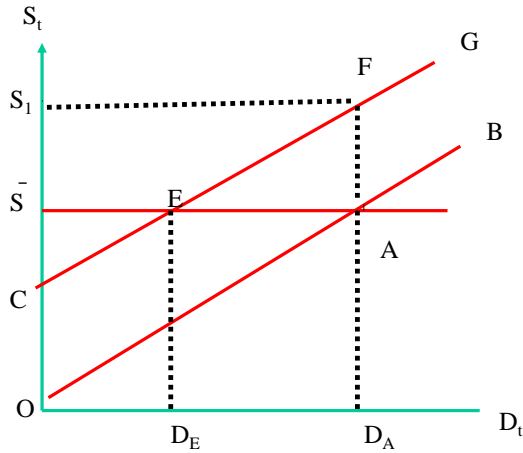


Figure 3a

Figure-1. Gov Spending

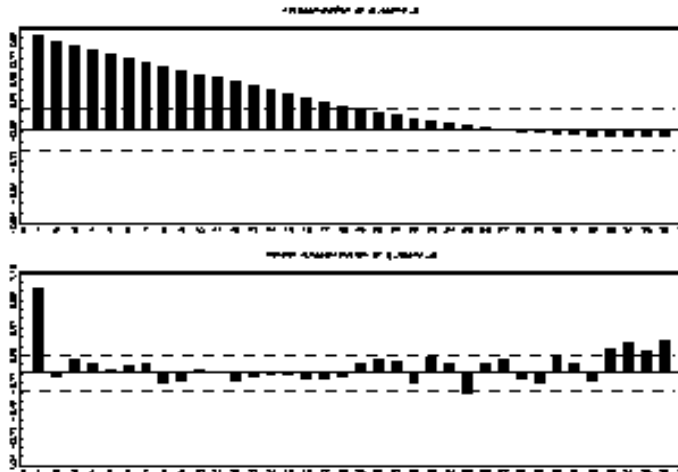


Figure-2. US Productivity

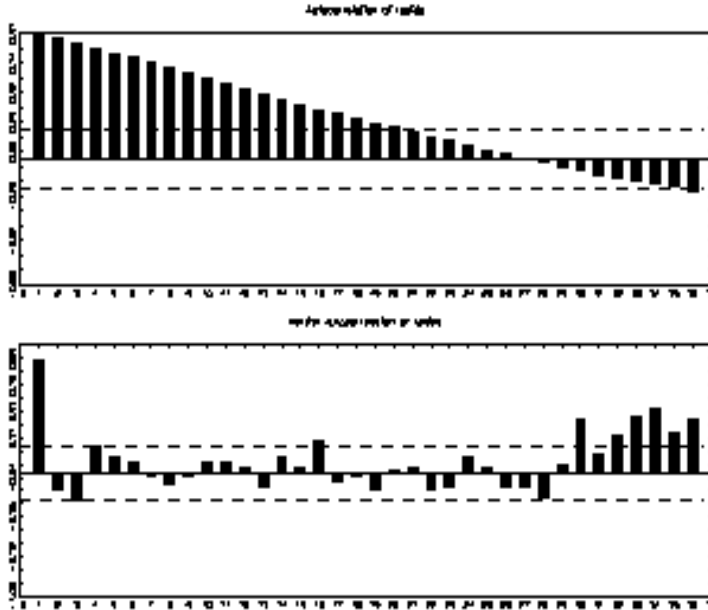


Figure-3. Oil Prices

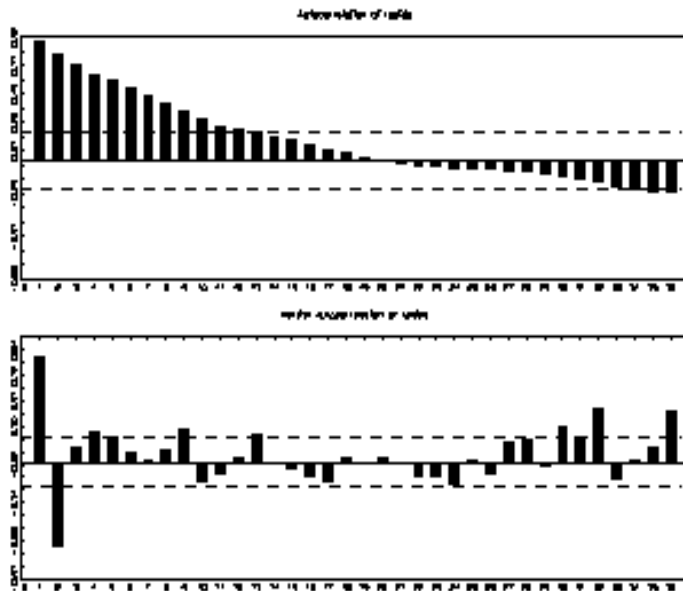
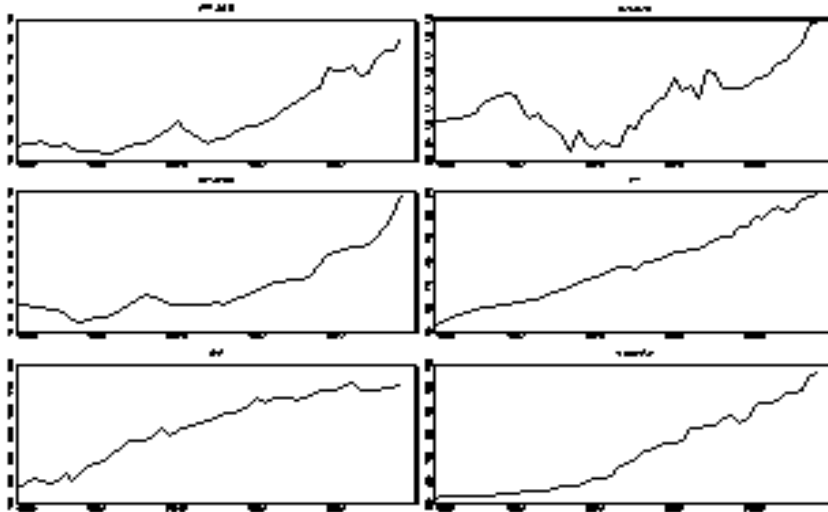


Figure -4. Time Series

US PPI_ US/Euro Exc. Rate
 Oil prices CPI
 M2 Gov_Spending

Plot of Time Series 1996.2-2008.2. T=49



Graph1

Kernel Density Estimation
 (Gaussian kernel)

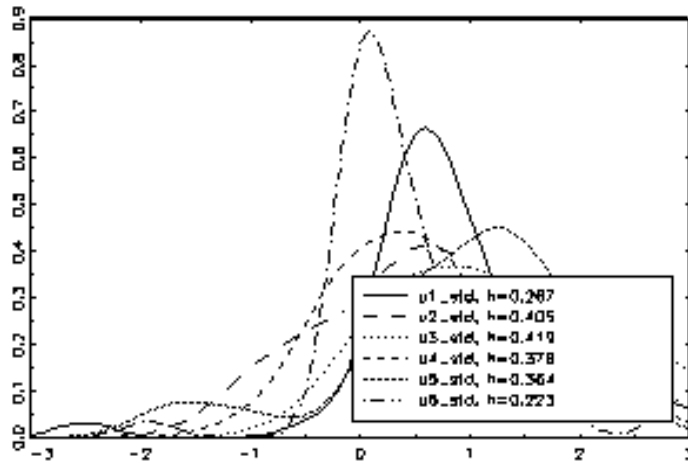


Figure-5. US Prod > USD/EURO Exchange Rate

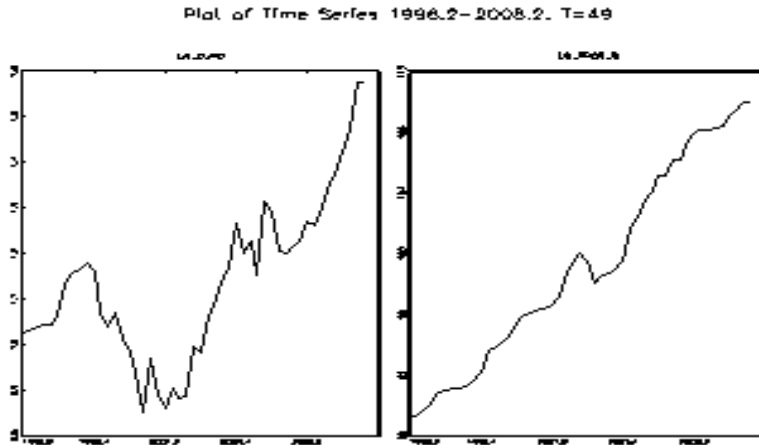


Figure-6. Euro GDP > USD/EURO Exchange Rate

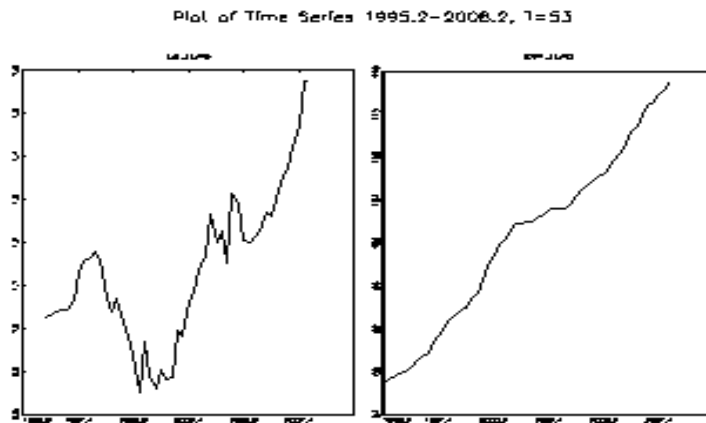


Figure-7. US GDP > USD/EURO Exchange Rate

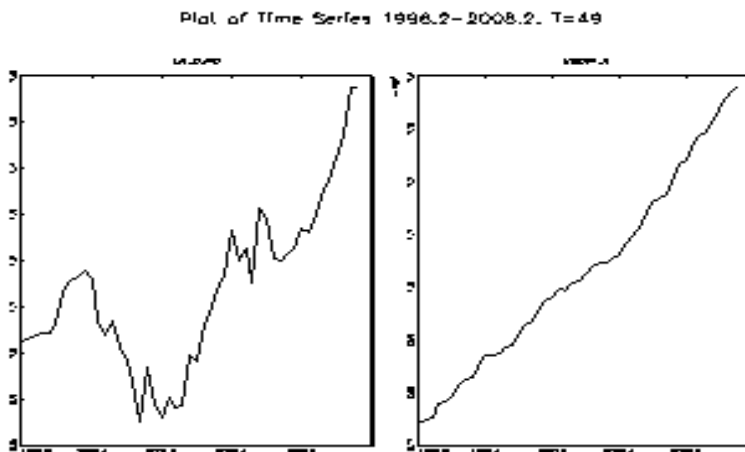


Figure-8. US Prod > Dollar/Euro Exchange Rate

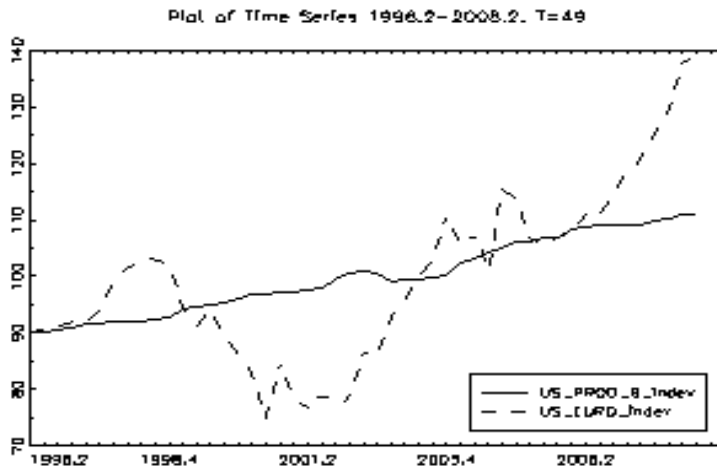


Figure-9. US Productivity → US/EURO Exchange Rate

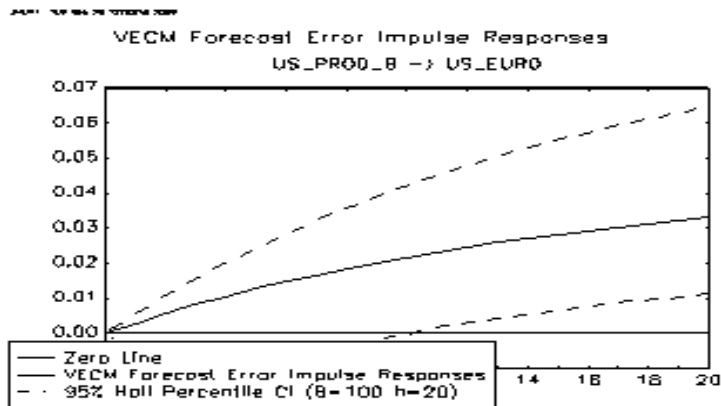


Figure-10. Oil Prices → US/EURO Exchange Rate

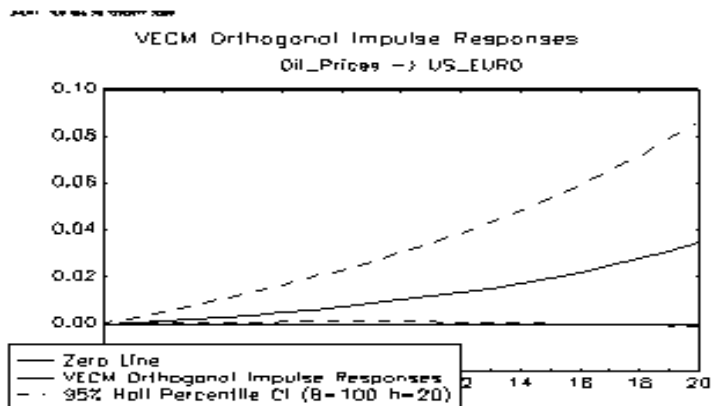


Chart 1

*** Fri, 30 Oct 2009 10:11:31 ***

VECM FORECAST ERROR VARIANCE DECOMPOSITION

Proportions of forecast error in "bUS_EURO"

accounted for by:

forecast horizon	aUS_PROD_B	bUS_EURO	cOil_prices	dm2	g_spend_q
1	0.10	0.90	0.00	0.00	0.00
2	0.11	0.89	0.00	0.00	0.00
3	0.11	0.89	0.00	0.00	0.00
4	0.12	0.88	0.00	0.00	0.00
5	0.13	0.87	0.00	0.00	0.00
6	0.13	0.87	0.00	0.00	0.00
7	0.14	0.86	0.00	0.00	0.00
8	0.14	0.85	0.00	0.00	0.00
9	0.15	0.84	0.00	0.00	0.00
10	0.16	0.83	0.00	0.00	0.00
11	0.16	0.82	0.01	0.01	0.01
12	0.17	0.81	0.01	0.01	0.01
13	0.18	0.80	0.01	0.01	0.01
14	0.19	0.79	0.01	0.01	0.01
15	0.19	0.78	0.01	0.01	0.01
16	0.20	0.76	0.01	0.01	0.01
17	0.21	0.75	0.01	0.01	0.01
18	0.22	0.74	0.02	0.02	0.02
19	0.22	0.72	0.02	0.02	0.02
20	0.23	0.71	0.02	0.02	0.02

Table 5

Luthepohl (2004)

*** Tue, 18 Aug 2009 11:28:21 ***

TEST FOR GRANGER-CAUSALITY: H0: "US_PROD Differentials" do not Granger-cause "US_EURO"

Test statistic $l = 0.9604$ $pval-F(l; 1, 86) = 0.3298$

TEST FOR INSTANTANEOUS CAUSALITY:

H0: No instantaneous causality between "US_PROD Differentials" and "US_EURO"

Test statistic: $c = 3.3221$ $pval-Chi(c; 1) = 0.0684$

Table 6

*** Sun, 26 Jul 2009 07:38:32 ***

PORTMANTEAU TEST (H0: $Rh=(r1, \dots, rh)=0$)

tested order: 16

test statistic: 419.1197

p-value: 1.0000

adjusted test statistic: 505.9513

p-value: 0.9746

degrees of freedom: 570.0000

*** Sun, 26 Jul 2009 07:38:33 ***

LM-TYPE TEST FOR AUTOCORRELATION with 5 lags

LM statistic: 301.5520

p-value: 0.0000

df: 180.0000

*** Sun, 26 Jul 2009 07:38:33 ***

TESTS FOR NONNORMALITY

Reference: Doornik & Hansen (1994)

joint test statistic: 89.2009

p-value: 0.0000

degrees of freedom: 12.0000

skewness only: 42.7256
p-value: 0.0000
kurtosis only: 46.4753
p-value: 0.0000

Reference: Lütkepohl (1993),
Introduction to Multiple Time
Series Analysis, 2ed, p. 153

joint test statistic: 59.1903
p-value: 0.0000
degrees of freedom: 12.0000
skewness only: 27.2345
p-value: 0.0001
kurtosis only: 31.9558
p-value: 0.0000

*** Sun, 26 Jul 2009 07:38:33 ***

JARQUE-BERA TEST

variable	teststat	p-Value(
u1	1.3867	0.4999
u2	0.6571	0.7200
u3	1.7748	0.4117
u4	35.4963	0.0000
u5	8.6994	0.0129
u6	33.7747	0.0000

*** Sun, 26 Jul 2009 07:38:33 ***

MULTIVARIATE ARCH-LM TEST with 2 lags

VARCHLM test statistic: 908.0688
p-value(chi²): 0.2642
degrees of freedom: 882.0000

Table 7

*** Sun, 26 Jul 2009 07:10:23 ***

CHOW TEST FOR STRUCTURAL BREAK

On the reliability of Chow-type tests.

..., B. Candelon, H. Lütkepohl, Economic Letters 73 (2001), 155-160

sample range: [1996 Q3,
2008 Q2], T = 48
tested break date: 1999 Q4
(13 observations before break)

break point Chow test: 83.7823
bootstrapped p-value: 0.0000
asymptotic χ^2 p-value: 0.0000
degrees of freedom: 27

sample split Chow test: 9.3234
bootstrapped p-value: 0.2500
asymptotic χ^2 p-value: 0.1562
degrees of freedom: 6

Chow forecast test: 1.3188
bootstrapped p-value: 0.0000
asymptotic F p-value: 0.2388
degrees of freedom: 210, 20

Table 8

*** Mon, 2 Nov 2009 11:22:23 ***

VECM Orthogonal Impulse Responses

Selected Confidence Interval (CI):

a) 95% Hall Percentile CI (B=100 h=20)

Selected Impulse Responses: "impulse variable -> response variable"

time aUS_PROD_B
 ->bUS_EURO

point estimate -0.0174
 CI a) [-0.0310, -0.0021]

1 point estimate -0.0185
 CI a) [-0.0336, -0.0037]

2 point estimate -0.0197
 CI a) [-0.0356, -0.0040]

3 point estimate -0.0209
 CI a) [-0.0381, -0.0044]

4 point estimate -0.0221
 CI a) [-0.0412, -0.0041]

5 point estimate -0.0234
 CI a) [-0.0446, -0.0035]

6 point estimate -0.0248
 CI a) [-0.0482, -0.0027]

7 point estimate -0.0263
 CI a) [-0.0519, -0.0029]

8 point estimate -0.0278
 CI a) [-0.0556, -0.0031]

9 point estimate -0.0294
 CI a) [-0.0594, -0.0036]

10 point estimate -0.0310
 CI a) [-0.0634, -0.0042]

11 point estimate	-0.0327
CI a)	[-0.0676, -0.0050]
12 point estimate	-0.0345
CI a)	[-0.0720, -0.0059]
13 point estimate	-0.0364
CI a)	[-0.0765, -0.0070]
14 point estimate	-0.0384
CI a)	[-0.0812, -0.0083]
15 point estimate	-0.0405
CI a)	[-0.0862, -0.0085]
16 point estimate	-0.0426
CI a)	[-0.0915, -0.0083]
17 point estimate	-0.0449
CI a)	[-0.0973, -0.0076]
18 point estimate	-0.0472
CI a)	[-0.1034, -0.0069]
19 point estimate	-0.0497
CI a)	[-0.1103, -0.0060]
20 point estimate	-0.0523
CI a)	[-0.1175, -0.0051]

Table 11

Time Series Euro Productivity and US Productivity Dollar/Euro Real Exchange Rate

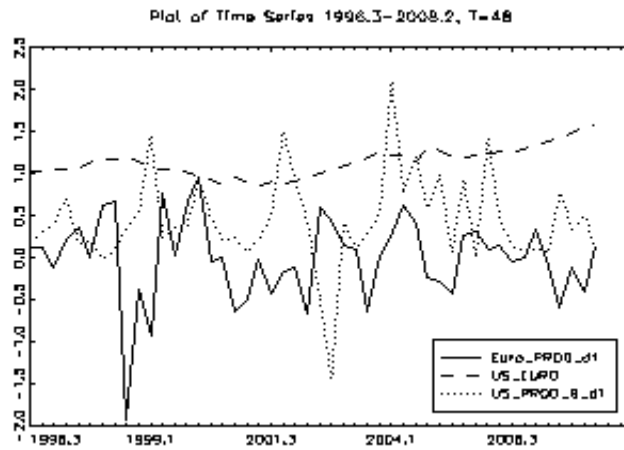
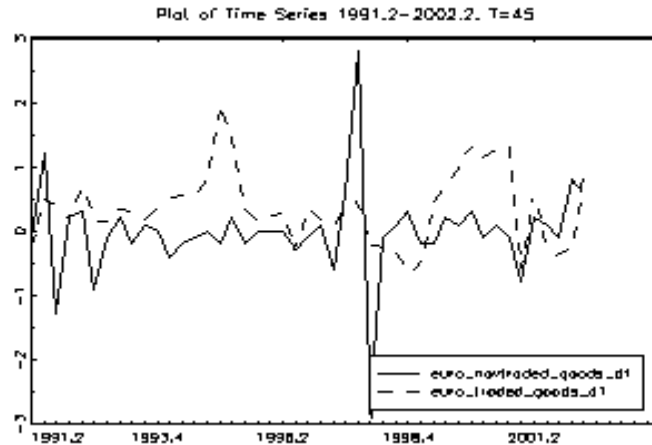


Figure-12. Time Series Euro Traded and Nontraded Goods



¹Refers to the strength of the pegged to currency relative to the major industrial countries that are major trading partners of the regime or any country that is a trading partner.

²Reference is made to economic equilibrium defined by John von Neumann's "A Model of Economic Equilibrium" (1937). We assume that if the exchange rate of the domestic country is not in economic equilibrium (i.e. John von Neumann's model) it is in disequilibria

THE ROLE OF PRODUCTIVITY IN ECONOMIC GROWTH AND EQUILIBRIUM

A graphical illustration of economic growth and equilibrium employing the von Neumann Model of General Economic Equilibrium

