Asian Economic and Financial Review, 2013, 3(12):1609-1619

Asian Economic and Financial Review

journal homepage: http://aessweb.com/journal-detail.php?id=5002

THE CORRELATION AND CONTAGION EFFECT BETWEEN US REITS AND JAPAN REITS - BASED ON THE ARMAX-GJR-GARCH-COPULA MODEL

Miin-Yu Peng

Ph.D student ,School of Economics, Nankai University,China

Wo-Chiang Lee

Department of Banking and Finance, Tamkang University, Yin-Chuan Road, Tamsui, New Taipei City, Taiwan, ROC

ABSTRACT

The article discuss the relationship between US REITs and Japan REITs. In empirical study, we apply five static ARMAX-GJR-GARCH copula models and two time-varying dynamic copula models. The results show that the kendall tau is lower before the submortgage crisis. The contagion effect test exhibits the US submortgate crisis will affect Japan REITs. Last, no matter the large, middle or small scale positive and negative shock, the contagion probability during the crisis is larger than before the submortgage crisis.

Keywords: Submortgage crisis, Copula model, Contagion effect, ARMAX-GJR-GARCH.
JEL classification: G20, C12, C13

1. INTRODUCTION

United State is the indicator of REITs. However, Japan opened the REITs market in September 2001 which is the earliest country in Asia with REITs legislation. From a top-down perspective, REITs can be affected by anything that impacts the supply of and demand for property. Population and job growth tend to be favorable for all REITs types. In early studies, Devaney and MAI (2001) employed GARCH-M model examine REITs risk premium and discovered different REITs own different risk premium. Glascock *et al.* (2000) used REITS total return index, federal reserve interest, one month treasury bill rate, total industrial produce index and seasonal adjust CPI, using Vector Error Correction Model (VECM) model to versified the causal relationship of above variables, the result showed that the change of monetary policies will affect inflation rate and REITs return appear negative relationship. Liow (2004)employed multiple factor model investigate the behaviors of business real estate market access return discovered access return were affected by GDP change rate, industry produce rate, short term interest rate, unexpected inflation rate, and market invest portfolio. The above variables are all time varying. Many studies focus on the sensitivities of REITs returns to major risk factors. For example, He (1997) provided evidence that price changes in one sub-market have significant impacts on other three sub-markets. Therefore, US long-term interest rates may be a risk factor for real estate investment companies in Asia. He *et al.* (2003)found that both Hong Kong and South Korea stock markets are very sensitive to change in US long-term interest rates. Moreover, in a study about the price discovery in the Hong Kong security markets. Ling *et al.* (2003) analyzed seven different interest rate proxies that have been widely used in the REITs literature. Lee (2009)examined the volatility spillover in Australian REITs futures. The results also illustrates that the equity market is more influential than REITs in affecting the volatility of REITs futures.

The motivation for this study is three folds: First, we want to understand the dynamic relationship between US REITs and Japan REITs before and during the US submortgage crisis.

Second, we want to test the contagion effect of US submortgage. Third, we try to calculate the probability of positive and negative shock before and during the crisis.

In empirical study, we apply the ARMAX-GJR-GARCH copula model to investigate the correlation. Copula function was widely used in financial econometrics and risk management. These related studies like as Palaro and Hotta (2006) used conditional copula to estimate VaR. Junker *et al.* (2006) discussed the nonlinear term structure dependence and risk implication based on copula function.Hu (2006)proposed a mixed copula model that it can capture various patterns of dependence structures. Rodriguez (2007) modeled dependence with switching-parameter copulas to study financial contagion. Chiou and Tsay (2008) addressed a copula-based approach to option pricing and risk assessment. Hsu *et al.* (2008) proposed copula-based GARCH models for the estimation of the futures optimal hedge ratio. Manner and Reznikova (2009) used copula models with time-varying dependence structure. Lai Y. H. *et al.* (2009) exploited copula methodology, with two threshold GARCH models as marginals, to construct a bivariate copula-threshold-GARCH model. They found that the optimal dynamic hedge model for spot and futures market. Lee and Fang (2010) applied copula function in the pair event of operation risk based on Taiwan's commercial banks. Lee (2010) investigated the dynamic correlation between NASDAQ and Toronto Stock index through Copula-AR-GARCH Model. Wei *et al.* (2011) proposed a new hedging model combining the newly introduced multifractal volatility (MFV) model and the dynamic copula functions. They found that the multifractal analysis may offer a new way of quantitative hedging model design using financial futures.

The study investigates the relationship between US REITs and Japan-REITs. The period of time chosen is from March 31,2005 to December 31, 2011. The data is obtained from Bloomberg and the variables contain MSCI US REITs index, DJ Composite all REITs index, DJ Real Estate Index, Japan REITs index and Japan real estate index.

Empirical results show that the kendall's tau is low before the sub-mortgage crisis. The contagion effect test exhibits the US submortgate crisis will affect Japan REITs. Last, no matter the large, middle or small scale positive and negative shock, the contagion probability during the crisis is more larger than before the mortgage crisis.

The paper is organized as follows. Section 2 presents a brief review of the research methodology. Section 3 contains our empirical results and analyse followed by a few concluding remarks and ideas on future works.

2. RESEARCH METHODOLOGY

ARMAX-GJR-GARCH (1, 1) model assume two return series $r_{1,t}$, $r_{2,t}$ following the Gaussian residuals.[1]

$$r_{i,t} = c + \phi X_{i,t} + \varepsilon_{i,t}\text{, i=1, 2 ; t=1,2,...,T} \tag{1a}$$

$$\varepsilon_{i,t} = \sqrt{h}_{i,t} z_{i,t}\,,\quad z_{i,t} \sim N(0,1) \tag{1b}$$

$$h_{i,t} = \sigma_{i,t}^2 = \omega + \alpha\varepsilon_{i,t-1}^2 + \beta h_{i,t-1} + \gamma\varepsilon_{i,t-1}^2 d_{i,t-1} \tag{1c}$$

where

$$d_{i,t} = \begin{cases} 1, if & \varepsilon_{i,t} < 0 \\ 0, if & \varepsilon_{i,t} \geq 0 \end{cases} \tag{1d}$$

$$(z_{1t}, z_{2t}) \sim C_t(F(z_{1t}), F(z_{2t})) \tag{1e}$$

Where $X_{i,t}$is an explanatory regression matrix. [2] $z_{i,t} = \varepsilon_{i,t} / \sqrt{h_{i,t}}$ is the conditional distribution of standardized innovations. In this study,we set i=1, 2. The distribution of the

[1] Based on the min: AIC(Akaike information criterion), we set optimal order of ARMAX (0,0)-GJR-GARCH (1,1).This specification is able to solve both the autocorrelation and heteroscedasticity and asymmetric problems.

innovation vector $z_t = (z_{1t}, z_{2t})$ is modeled by copula. $C_t(\ldots\ldots,.)$. Here, C was modeled by Normal, student-t, Clayton-Copula, Gumbel Copula and Frank Copula function and time varying copula (time varying normal copula and Joe-Gumbel copula), respectively.[3]

Normal copula is the copula of multivariate normal distribution. It is defined as follows: Assuming $X = (X_1, X_2, ..., X_n)$ is multivariate normal, if and only if (a) its margins $F_1, ..., F_n$ are normally distribution, and (b) a unique copula function[4] exists, such that

$$C_R^N(u_1, ..., u_n) = \Phi_R(\phi^{-1}(u_1), ..., \phi^{-1}(u_n)) \tag{2}$$

where Φ_R denotes the standard multivariate normal distribution with correlation matrix R and ϕ^{-1} is the inverse function of standard univariate normal distribution. When n=2, we can obtain the copula function as follows:

$$C_R^N(u,v) = \int_{-\infty}^{\phi^{-1}(u)} \int_{-\infty}^{\phi^{-1}(v)} \frac{1}{2\pi(1-R_{12}^2)^{1/2}} \exp\{-\frac{s^2 - 2R_{12}st + t^2}{2(1-R_{12}^2)}\} dsdt \tag{3}$$

By the same concept, t-copula is the copula function of multivariate Student's t distribution. Assuming $X = (X_1, X_2, ..., X_n)$ observes standard multivariate normal distribution with correlation matrix R, Y is the random variable of χ^2 distribution with v degree of freedom, then t-copula function is:

$$C_{v,R}^t(u_1, ..., u_n) = t_{v,R}(t_v^{-1}(u_1), ..., t_v^{-1}(u_n)) \tag{4}$$

where $u_i = \frac{\sqrt{v}}{\sqrt{Y}} X_i,\ i = 1, ..., n$

When n=2, we can obtain the t-copula as follows:

$$C_{v,R}^t(u,v) = \int_{-\infty}^{t_v^{-1}(u)} \int_{-\infty}^{t_v^{-1}(v)} \frac{1}{2\pi(1-R_{12}^2)^{1/2}} \{1 + \frac{s^2 - 2R_{12}st + t^2}{v(1-R_{12}^2)}\}^{-(v+2)/2} dsdt \tag{5}$$

Another important class of copulas is known as Archimedean copulas. These copulas find a wide range of applications. A n-dimension copula function,

---

[2]In the model, we consider the variables US REITs index and Japan REITs index.

[3] To save space, copula functions will not be shown here. The books of Joe, H., 1997. and Nelsen, R.B., 1999. presented a good introduction to the copula theory.

[4]i.e. the normal copula.

$$C\left(x_1,\cdots x_n\right)=\Psi^{-1}\left(\sum_{i=1}^{n}\Psi\left(F_i\left(x_i\right)\right)\right) \tag{6}$$

where Ψ : generator function and satisfies $\Psi(1)=0$; $\lim_{x\to 0}\Psi(x)=\infty$; $\Psi'(x)<0$; $\Psi''(x)>0$

then there are three types of Archimedean copulas functions, namely Clayton-n-Copula, Gumbel-n-Copula and Frank-n-Copula function, respectively.
Clayton-n-Copula function: when $\alpha>0$，

$$C(u_1,...,u_n)=\left[\sum_{i=1}^{n}u_i^{-\alpha}-n+1\right]^{-1/\alpha} \tag{7}$$

Gumbel-n-Copula function: when $\alpha>1$

$$C(u_1,...,u_n)=\exp\left[-\left(\sum_{i=1}^{n}(-\ln u_i)^{\alpha}\right)^{1/\alpha}\right] \tag{8}$$

Frank-n-Copula function: when $\alpha>0$，n > 3

$$C(u_1...u_n)=-\frac{1}{\alpha}\ln\left\{1+\frac{\prod_{i=1}^{n}\left(e^{-\alpha u_i}-1\right)}{\left(e^{-\alpha_i}-1\right)^{n-1}}\right\} \tag{9}$$

We further use the Kendall tau (τ) coefficient to calculate the rank correlation coefficient of operation events-pair. It is a non-parametric statistic used to measure the association or statistical dependence between two measured quantities. For a pair (X, Y), we can construct a two-dimension copula C and obtain the Kendall tau as equation (10),

$$\tau=4\iint C\left(u,v\right)dC\left(u,v\right)-1 \tag{10}$$

The time-varying normal copula tau function is given:

$$\rho_{1,2,t}=\tilde{L}[\omega_\rho+\beta_\rho\rho_{1,2,t-1}+\alpha_\rho\frac{1}{10}\sum_{j=1}^{10}\Phi^{-1}(u_{t-j}).\Phi^{-1}(v_{t-j})]$$
(11)

Where ρ is normal kendall'tau, $\tilde{L}(x)=\dfrac{1-e^{-x}}{1+e^{-x}}$, the modified logistic function ; Φ^{-1} is the inverse of the standard normal CDF.

The symmetric Joe-Clayton copula (Patton, 2006) is given by:

$$C_{JC}(u,v\mid\tau_U,\tau_V)$$
$$=1-(\{[1-(1-u)^k]^{-\gamma}+[1-(1-v)^k]^{-\gamma}-1\}^{-(1/\gamma)})^{(1/k)}$$
(12)

Where $k = 1/\log_2(2-\tau_U)$, $\gamma = -1/\log_2(\tau_L)$, $\tau_U \in (0,1),$, $\tau_L \in (0,1)$

τ_U and τ_L are the coefficients of upper and low tail dependence, respectively.

Upper tail dependence

$$\tau_t^u = L[\omega_u + \beta_u \tau_{t-1}^u + \alpha_u \frac{1}{10}\sum_{j=1}^{10} | u_{t-j} - v_{t-j} |] \quad (13)$$

Lower tail dependence

$$\tau_t^L = L[\omega_L + \beta_L \tau_{t-1}^L + \alpha_L \frac{1}{10}\sum_{j=1}^{10} | u_{t-j} - v_{t-j} |] \quad (14)$$

$L(x) = \dfrac{1}{1+e^{-x}}$ is the logistic function.

3. EMPIRICAL RESULTS AND ANALYSIS

3.1. Data description and Descriptive Statistics

The study investigate the dynamic relationship between US REITs and Japan-REITs. The period of time chosen is from March 31,2005 to December 31, 2011. The data is obtained from Bloomberg databank and the variables contain MSCI US REITs index, DJ Composite all REITs index, DJ Real Estate Index, Japan REITs index and Japan real estate index.

Table-1 reports the summary statistics of US REITs and Japan REITs returns. Both of the mean returns are negative and high kurtosis, left skewness. In addition, In addition, all of the Jarque-Berra (J-B) statistics reject the null hypotheses of normality distribution. (also see the Figure-1 and 2). The Figure-3 exhibits the time series of US REITs and Japan REITs. The two series are almost the same trend. So, the scatter plot of Figure-4 also indicates a high correlation.

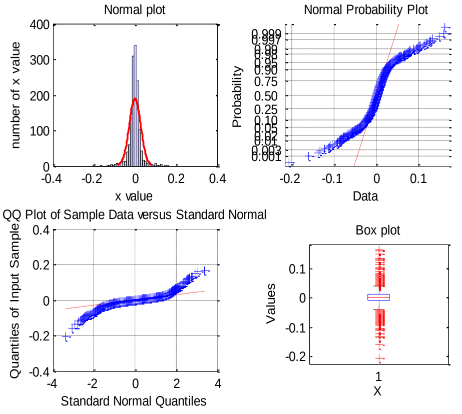
Table-1. Summary statistics for US REITs and Japan REITs returns

Models	**Mean**	**Std**	**Max**	**min**	**skewness**	**kurtosis**	**J-B**
USD REITs	-0.0001	0.0279	0.1625	-0.2065	-0.0570	11.8236	4535.8521 ***
Japan REITs	-0.0002	0.0186	0.1064	-0.1278	-0.3211	10.0099	2886.3184 ***

Note: 1.Std means the standard deviation, J-B stat is obtained from Jarque -Berra normality test.

2.* indicates the statistical significance and the rejection of null hypothesis at 1% significance level.

Figures-1. Statistics Descriptive of US REITs Return



Figures-2. Statistics Descriptive of Japan REITs Return

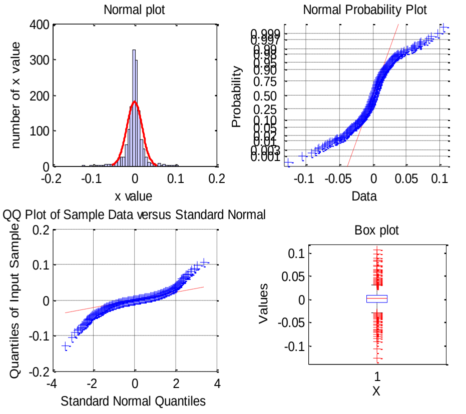


Figure-3. The time series of Japan and US REITs Net Value

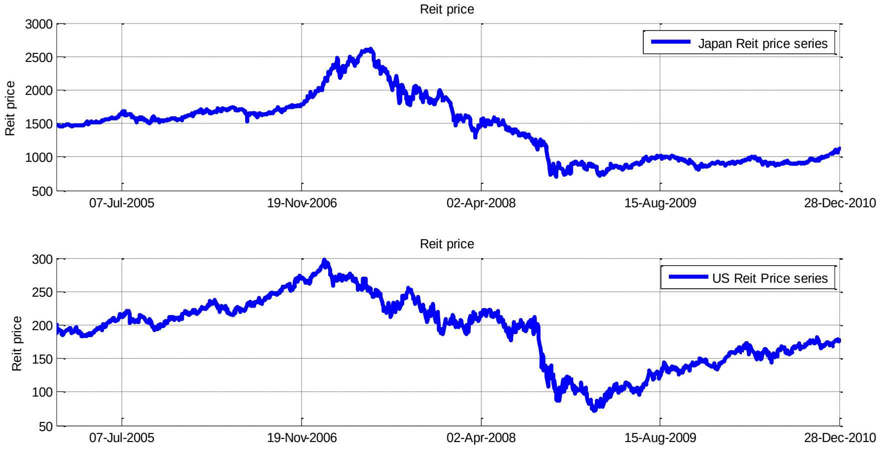
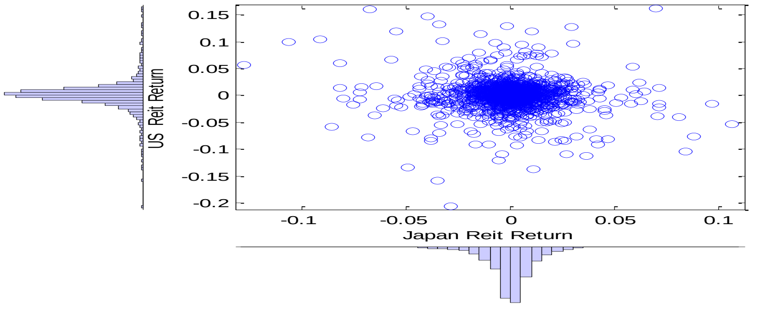


Figure-4. The Scatter plot of Japan and US REITs Return



3.2. Empirical Results Analysis

For comparison purposes, we spilt the data into two subsample, namely the before subsample and during the subsample, respectively.

Table 2 exhibits the ARMAX–GJR-GARCH (1, 1) result of before Submortgage crisis. The two REITs index are significant in the conditional mean equation. What is more, the parameters of conditional variance equation are also significant. Especially, the leverage effect is significant which shows that the volatility asymmetric effect. However, the correlation is low before the submortgage crisis according to the Figure-5. Whereas Table-3 exhibits the five static couple results including their AIC, BIC and kendall's tau. The best model is normal copula via the minimum AIC criteria. The kendall tau is -0.0035 which means that the US REITs return and Japan REITs return has a negative relationship before the crisis.

Table-2. Results from the ARMAX–GJR-GARCH (1, 1) model (before)

	US REITs	Japan REITs
Panel A: Conditional mean equation		
C	0.00053027***	-0.0057997
	(0.001444)	(0.0066206)
Φ	-6.8465e-008***	5.8963e-005***
	(2.7093e-008)	(2.7752e-005)
Panel B: Conditional variance equation		
ω	5.7263e-005***	3.8872e-006***
	(1.3403e-005)	(1.2479e-006)
β	0.89684***	0.67244***
	(0.14323)	(0.045022)
α	0.1015858	0.28667***
	(0.044196)	(0.063471)
γ	0.44217***	0.081778***
	(0.12234)	(0.047687)
LL	1.6087e+003	1.7628e+003

Note: 1. The estimated parameters correspond to equations (1a) and (1c). LL corresponds to the log - likelihood function value.
2. The t values are in the parenthesis.
3. The * **, ** stand for 10%, 5%, 1%, respectively.
4. Model states as (1a), (1b), (1c), and (1d).

Figure-5. The Scatter plot of Japan REITs and US REITs Return (before)

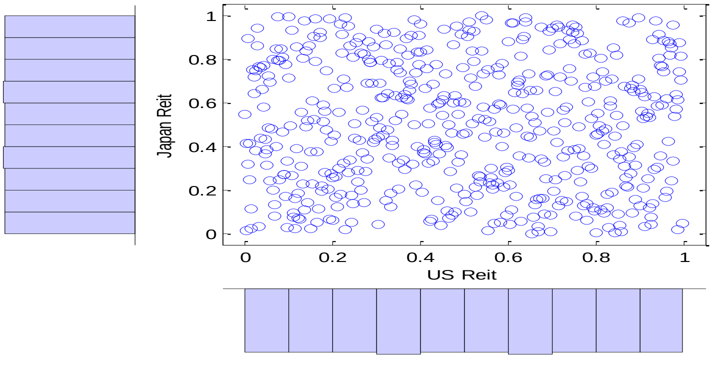


Table-3. Copula between Japan REITs and Japan REITs (before)

	AIC	BIC	Kendall tau
Normal Copula	-0.0000	-0.0001	-0.0035
Student T Copula	0.3515	0.3515	-0.0039
Clayton Copula	0.0194	0.0195	0.0031
Gumbel Copula	0.2468	0.2550	-0.0127
Frank Copula	0.00019	0.00019	0.000112

Table-4 presents the ARMAX–GJR-GARCH (1, 1) result of during the submortgage crisis. The parameters in the conditional mean equation are significant. While the REITs index are negative. The parameters of conditional variance equation are also significant. In addition, the leverage effect is also significant which represents the volatility asymmetric effect during the crisis. However, the correlation is higher than before the submortgage crisis according the Figure-5. Table 5 represents the five static copula results including their AIC, BIC and kendall tau. The best model is Gumbel copula via the minimum AIC criteria. The kendall tau is 0.0319 which implies the US REITs return and Japan REITs return has a positive relationship during the submortgage crisis. Table 6 shows the results of contagion effect test. The model before crisis is time-varying normal copula model (Figure-7) and the during crisis is time varying Gumbel Copula model (Figure-8). It is significant via t-test, which implies that the US submortgage crisis has a contagion effect to US REITs. We further discuss the extreme positive and negative shock contagion probability of US REITs to Japan REITs. For example, once occur the large negative shock, the probability of US REITs net value will down 10% , then the Japan REITs will decrease 10% is 1.3046% before the crisis. However, the probability will increase to 3.2982% during the crisis. In contrast, in the large positive shock , the probability of US REITs net value grows 10% , then the Japan REITs will also grows 10% is 1.2305% before the crisis and 3.1943% during the crisis, respectively.

Figure-6. The Scatter plot of Japan REITs and US REITs Return (during)

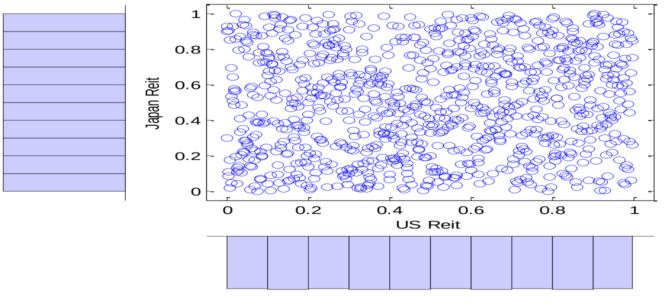


Table-4. Results from the ARMAX–GJR-GARCH (1, 1) model (during)

	US REITs	**Japan REITs**
Panel A: Conditional mean equation		
C	0.0018788* (0.001342)	0.0057112* (0.0032342)
Φ	-1.7941e-006** (1.0984e-006)	-6.087e-005* (3.4253e-005)
Panel B: Conditional variance equation		
ω	7.579e-006*** (2.7713e-006)	6.2792e-006*** (1.8254e-006)
β	0.88831*** (0.018235)	0.84842*** (0.018768)
α	0.049029*** (0.021255)	0.08501*** (0.025456)
γ	0.11473*** (0.035846)	0.10966*** (0.030924)
LL	2.0192e+003	2.3274e+003

Note: 1. The estimated parameters correspond to equations (1a) and (1c). LL corresponds to the log - likelihood function value.
2. The t values are in the parenthesis.
3. The * **, ** stand for 10%, 5%, 1%, respectively.
4. Model states as (1a), (1b), (1c), and (1d)

Table-5. Copula kendall tau between Japan REITs and Japan REITs (during)

	AIC	BIC	Kendall tau
Normal Copula	4.1815	4.1819	0.0437
Student T Copula	6.3300	6.3304	0.0484
Clayton Copula	3.9557	3.9561	0.0368
Gumbel Copula	2.2235	2.2291	0.0319
Frank Copula	6.0459	6.0487	0.0557

Table-6. Contagion test of US REITs to Japan REITs

before	during	Diff rho	t-test	Contagion
-0.0018	0.0459	0.0477	-83.2805***	YES

Note:1. Before crisis is time varying normal copula model; during crisis is time varying Gumbel copula model.
2.The *** stand for 1%, respectively.

Figure-7. The time-varying normal copula kendall tau

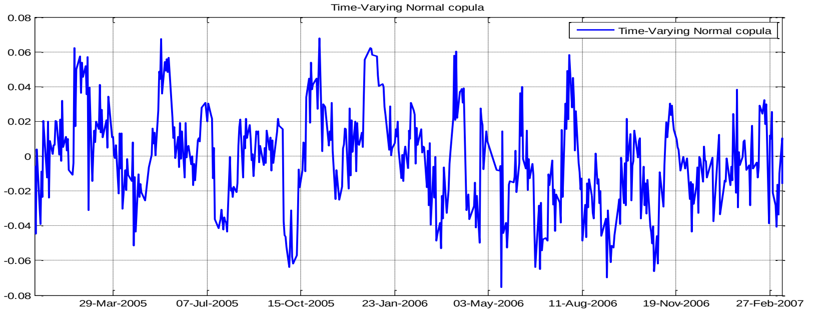


Figure-8. The time-varying Gumbel copula kendall's tau

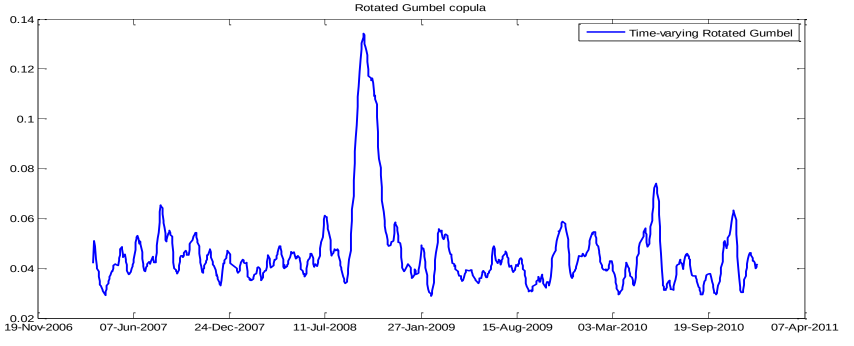


Table-7. The Contagion Probability of US REITs to Japan REITs

size / Period	Extreme Negative Shock			Extreme Positive Shock		
	Large (10%)	Middle (5%)	Small (1%)	Large (10%)	Middle (5%)	Small (1%)
before	1.3046%	4.2332%	8.116%	1.2305%	4.323%	8.3948%
during	3.2982%	5.896%	10.6665%	3.1943%	5.5887%	10.8977%

4. CONCLUSION AND RESULTS

As describe above, the article discuss the relationship between US REITs and Japan REITs. In empirical study, we apply five static ARMAX-GJR-GARCH copula models and two time-varying dynamic copula models. The results show that the kendall tau is lower before the sbumortgage crisis. That is to say, the US REITs will not affect the Japan REITs.

Whereas the contagion effect test exhibits the US submortgate crisis will affect Japan REITs. Last, no matter the large, middle or in small scale positive and negative shock, the contagion probability during the crisis is more larger than before the submortgage crisis.

Due to the results, we recommend that the model will be helpful to the risk management of REITs portfolio. The future works maybe consider the mixed copula model and calculate the Value at Risk of REITs portfolio.

REFERENCES

Chiou, S.C. and R.S. Tsay, 2008. A copula-based approach to option pricing and risk assessment. Journal of Data Science, 6(3): 273-301.

Devaney, M. 2001. Time varying risk remain for real estate investment trust: A garch-m model. The Quarterly Review of Economics and Finance, 41(3): 335-346.

Glascock, J., L. Liu, C., and R. So, 2000. Further evidence on the integration of reit, bond and stock returns. Journal of Real Estate Finance and Economics, 20(2): 177-194.

He, L.T., 1997. Price discovery in hong kong security markets: Evidence from cointegration tests, international financial markets. Institutions and Money, 7(2): 157-169.

He, L.T., J. Webb and N. Myer, 2003. Interest rate sensitivity of reits returns. International Real Estate Review, 6(1): 1-21.

Hsu, C.C., C.P. Tseng and Y.H. Wang, 2008. Dynamic hedging with futures: A copula-based garch model. Journal of Futures Markets, 28(11): 1095-1116.

Hu, L., 2006. Dependence patterns across financial markets: A mixed copula approach. Applied Financial Economics, 116(10): 717-729.

Joe, H., 1997. Multivariate models and dependence concepts. London: Chapman & Hall.

Junker, M., A. Szimayer and N. Wagner, 2006. Nonlinear term structure dependence: Copula functions, empirics, and risk implications. Journal of Banking and Finance, 30(4): 1171-1199.

Lai Y. H., W.S. Chen and R. Gerlach, 2009. Optimal dynamic hedging via copula-threshold-garch models. Mathematics and Computers in Simulation, 79(8): 2609-2624.

Lee, C.L., 2009. Volatility transmission in australian reit futures. Journal of Real Estate Portfolio Management, 15(3): 221-238.

Lee, W.C., 2010. The dynamic correlation between nasdaq and toronto stock index - copula-ar-garch model The Empirical Economics Letters, 9(10): 993-1002.

Lee, W.C. and C.J. Fang, 2010. The measurement of capital for operational risk of taiwanese commercial banks. The Journal of Operational Risk, 5(2): 79-102.

Ling, T.J., R.W. He and F.C.N. Myer, 2003. Interest rate sensitivities of reit return. International Real Estate Review, 6(1): 1-21.

Liow, K.H., 2004. Time varying macroeconmic risk and commercial real estate : An asset pricing perspective. Journal of Real Estate Portfolio Management 10(1): 47-57.

Manner, H. and O. Reznikova, 2009. Time-varying copulas: A survey. Universite catholique de louvain, institut de statistique. Institut de statistique, Discussion Paper: DP0917.

Nelsen, R.B., 1999. Introduction to copulas. Springer Verlag, New York.

Palaro, H.P. and L.K. Hotta, 2006. Using conditional copula to estimate value at risk. Journal of Data Science, 4(1): 93-115.

Rodriguez, J.C., 2007. Measuring financial contagion: A copula approach. Journal of Empirical Finance, 14(3): 401-423.

Wei, Y., Y.D. Wang and D. Huanga, 2011. A copula–multifractal volatility hedging model for csi 300 index futures. Physica A, 390(23): 4260-4272.