



A FRAMEWORK TO CHARGE FOR UNIT-LINKED CONTRACTS WHEN CONSIDERING GUARANTEED RISK

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ABSTRACT

Risk management for investment guarantees with unit-linked contracts is very critical for the insurer. This paper proposes a framework to charge for guaranteed risk when the insurer reserves for the guaranteed risk. This framework can facilitate the calculation of risk reserves and charge for the investment guarantees. In this framework the charge is determined by two criteria of meeting the insurance company's target internal rate of return and simultaneously the reserving standard. The framework is built on the stochastic cash-flow analysis. For illustrative purposes, we set up quantile reserves as the actuarial reserving standard in our framework. In this framework, the procedure to work out the charges is in reverse. In our numerical illustration, we investigate a unit-linked policy with maturity guarantees. Our framework would also apply to other types of contacts, guarantees and reserving standard.

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Keywords: Unit-linked policy, Variable insurance, Charge, Stochastic cash flows analysis, Guaranteed benefit, Reserve.

Contribution/ Originality

This paper contributes in the existing literature on the pricing and charging of unit-linked contracts by an alternative framework, where the charge is determined by meeting the target internal rate of return and the reserving standard simultaneously.

1. INTRODUCTION

Recently, unit-linked insurance (unit-linked, variable life, or segregated fund) has become very popular in the Taiwan insurance market. In the United Kingdom, unit-linked insurance rose in popularity in the late 1960s. In the United States, variable life insurance started in the 1980s, while

in Canada segregated fund contracts became popular in the late 1990s. The functions of unit-linked insurance are to provide the insured both the life protection and investment opportunity. The insurer invests the premiums received less deductions in a separate fund.

The investment component of unit-linked insurance is similar to mutual funds. Thus, in order to compete with mutual funds, it is common for the insurer to design guaranteed benefits on unit-linked products. There are several categories of guaranteed benefits, such as guaranteed minimum maturity benefit, guaranteed minimum death benefit and guaranteed minimum surrender benefit.

However, these guarantees are financial guarantees and cannot be treated with traditional deterministic actuarial approaches, which emphasize the law of large numbers and rely on diversification. The guaranteed risk on unit-linked is non-diversifiable. In other words, in the event of a highly adverse investment performance, the insurer may require additional funds to cover the guarantee at the same time.

In last few years, many researchers started paying attention in the risk management to financial guarantees with unit-linked policies. In general, there are three main risk management methods. First, there is the traditional actuarial reserving method whereby the insurer sets aside additional capital to ensure that the liabilities under the guarantee will be covered with a high probability. The second approach is to reinsure the liability under the guarantee. The third approach is to hedge the guarantee liability.

Maturity Guarantees Working Party (MGWP) (1980) and Boyle and Hardy (1997) used the stochastic simulation method to calculate the prudent reserves for maturity guaranteed for equity-linked contracts. Brennan and Schwartz (1976), Boyle and Schwartz (1977), and Boyle and Hardy (1997) have applied financial economics theory to calculate the value of the maturity guarantee. Hardy (2000) makes comparisons of the three risk management methods for maturity guarantee with segregated fund contracts using cash flow testing.

Persson and Aase (1997) presented a model for the valuation of life insurance contracts including a guaranteed minimum return. Grosen and Jørgensen (2000) presented a dynamic model for valuing such guarantees with participating policies based on the well-developed theory of contingent claim valuation. They decomposed the guaranteed liability into a risk-free bond, a bonus, and a surrender option.

Miltersen and Persson (1999) priced rate of return guarantees in a Heath-Jarrow-Merton Framework. Hansen and Miltersen (2002) studied the minimum rate of return guarantees for the Danish case. They analysed the guarantees for life insurance contracts and pension plans with a smoothing surplus distribution mechanism.

In addition, there were some literatures about guaranteed annuity options recently. Yang (2001) and Wilkie *et al.* (2003) investigated the GAO using the actuarial approach and the hedging approach. The market information for both approaches is based on the Wilkie investment model (Wilkie, 1995; 1984). Pelsser (2002) analysed a static hedging strategy based on buying long-dated receiver swaptions. Boyle and Hardy (2003) explored the pricing and risk management of these guarantees.

The research based on risk management issue has been discussed widely. The main purpose of this paper is to investigate how the policyholder should be charged for the capital supporting the guarantees. We want to reflect the guaranteed risk to the charges. Currently, there is no clear method to charge for guaranteed risk.

Wilkie (1978) showed a way to charge for maturity guarantees, which was the expected claim costs plus h of the money put up by the shareholders per year. The shareholders hope to earn an extra h per annum more on their invested funds than they will earn from the normal investment proceeds.

Hare *et al.* (1999) priced with-profits guarantees with traditional actuarial reserving method and then used the hedging method to provide a similar level of security. In their paper, they discussed how the with-profit policyholder should be charged for capital support. Wilkie *et al.* (2003) calculated the charges for the policies with guaranteed annuity options.

In this paper, in order to reflect the guaranteed risk in the charges, we propose a framework. This framework can facilitate the calculation of risk reserves and charge for the investment guarantee. The framework work is based on the cash flow projection. We assume the policyholder should be charged a fixed percentage of fund value in order to compensate the shareholders for setting additional capital for the guarantee risk.

The charge is determined by two criteria of meeting the company's target internal rate of return and reserving standard simultaneously. Internal rate of return is one of the profit measures used by life insurance companies. In order to assess the internal rate of return, we model the cash flow for unit-linked contracts by means of simulation. In the cash flow model, we incorporate the reserves required for guaranteed risk. Thus, the cash flows emerging each year from a given contract are estimated. Hardy (2000) used cash-flow analysis to assess which risk management methods in dealing with maturity guarantee is most profitable.

The rest of the paper is organized as follows. In section 2 the profit testing of unit-linked contracts are described. We depict the general cash flow notion of a unit-linked contract. In section 3 we introduce the framework and the procedure that we estimate the reasonable management charge rates. In section 4 we give a numerical example. Based on this example, the sensitivity of the charging rates to assumptions regarding the investment return rates of the sterling fund, the cost of capital (that is the projected IRR) and the term of policy period are discussed in section 5. Section 6 provides a summary and conclusion.

2. PROFIT TESTING OF UNIT-LINKED CONTRACTS

The cash flows related to a unit-linked policy include the unit fund (or separate account) and the sterling fund (or general account). In general, the premiums received less deductions are invested in a fund separate from the insurance funds of the office. This fund is called unit fund. The money in the unit fund belongs to policyholders. For a unit-linked policy offering guarantees, sterling fund incomes come from the policyholder. The items of the incomes arise not only from the expense, management charges and surrender charges but also the charges arising from offering

guarantees. In this research, we investigate how the policyholder should be charged for the capital supporting the guarantees using cash flow analysis. We describe the profit testing methodology in this section.

2.1. Assumptions and Notation

Suppose a unit-linked policy is issued to an insured at age x . We use the following notation:

P_t = the premium paid at the start of the year t ($t = 1, 2, \dots, n$)

a_t = the allocation proportion in year t

= the proportion of the premium P_t which is allocated to buy units

q_{x+t} = the probability of death before age $x+t+1$ given survival to age $x+t$

p_{x+t-1} = the probability of survival to age $x+t$ given survival to age $x+t-1$

km_t = the fund management charge rate in year t , which is charged to support the business (such as the expenses, commission, ...etc.), except the guarantee cost.

h_t = the fund management charge rate in year t , which is charged due to the guarantee cost.

c_t = the total fund management charge in year t . We assume it is charged at the start of year after the premium is paid, and which is a percentage of the value of the unit fund. Thus,
 $c_t = (F_{t-1} + a_t P_t)(km_t + h_t)$.

$i_{u,t}$ = the rate of growth of the unit fund during the year $t-1$ to t .

$i_{s,t}$ = the rate of interest of the sterling fund during the year $t-1$ to t .

F_t = the value of the unit fund at the end of year t (before payment of any premium then due and deduction of the fund management charge) assuming that the policy is still in force.

F_{t^+} = the value of the unit fund at the start of year $t+1$, which is after payment of premium and deduction of the management charge,
 $= (F_t + a_{t+1} P_{t+1})(1 - (km_{t+1} + h_{t+1}))$, $t = 0, 1, \dots, n-1$

e_t = projected expenses in year t , assuming deducted in advance.

${}_tV$ = the reserves held by the insurer at the end of the policy year t , where ${}_0V$ is the initial reserves at the start of the policy.

G_t = the guarantee liability at time t .

2.2. Cash Flows

According to the preceding notation, we will explain the cash flows of unit fund and sterling fund in the following. The insured pay premium at the beginning of each policy year t . The proportion of the premium is allocated into the unit fund and to the sterling fund. Thus, the fund level of the unit fund at the end of year t is as follows:

$$F_t = F_{(t-1)^+} \cdot (1 + i_{u,t}) \quad (1)$$

Let $M(t) = h_t \cdot (F_{t-1} + a_t P_t)$, the portion of the fund management charges due to guarantee in the year t . The initial fund level of the unit fund is zero, i.e. $F_0 = 0$ and $F_{0^+} = (a_1 P_1)(1 - (km_1 + h_1))$. The charging rate h_t will be zero if the insurer doesn't promise

any guarantee, since the charge that comes from h_t is to support the guaranteed cost.

We assume the benefit is paid at the end of each year. Let $(GC)_t$ be the expected guaranteed cost in year t and ${}_tV$ be the reserves required at the end of t according to the reserving standard. We assume the profit is computed annually at the end of each year. Thus, the cash flows of the sterling fund at the end of each year t are:

$$(CF)_t = ((1 - a_t)P_t - e_t + c_t)(1 + i_{s,t}) - (GC)_t + (1 + i_{s,t})_{t-1}V - p_{x+t-1} \cdot {}_tV, \quad t = 1, 2, \dots, n \quad (2)$$

The initial cash flows of the sterling fund is equal to $(CF)_0 = -{}_0V$. ${}_0V$ is the initial capital requirement for providing the guarantee. Note that the term of h_t , $(GC)_t$ and ${}_tV$ will be zero if there is no guaranteed benefits promise.

2.3. Profit Measures

There are many profit measures used by most life insurance companies today. The most common profit measure is internal rate of return (IRR). The IRR is a solved-for discount rate that causes the net present value of profits to equal zero. In this research, we use IRR as one of the basis for deciding the charge for guarantees. Suppose the risk discount rate is j , the net present value of the profits is

$$NPV = (CF)_0 + \sum_{t=1}^n \left(\frac{1}{1+j}\right)^t {}_{t-1}p_x (CF)_t \quad (3)$$

3. A FRAMEWORK OF CHARGING FOR UNIT-LINKED CONTRACTS WHEN CONSIDERING GUARANTEED RISK

In this section, we introduce the framework and the process that we estimate the optimal management charge rates, km and h . We assume that the management charge rate is a fixed proportion of the unit fund each year.

3.1. Optimal Charging Criteria

In our framework, the charge is determined by two criteria of meeting the insurance company's target internal rate of return and simultaneously the risk reserving standard. These two criteria are defined in the following.

3.1.1. Criteria One

Criteria one is to meet the insurance company's target internal rate, say j . Thus, we are seeking a charging rate to meet that the expectation of the simulated NPV equals zero under discounting rate equals j . The relationships of management charging rate (km) and investment guarantee charging rate (h) with $E(NPV)$ are shown in Figure 1 separately. The left graph shows the higher the rate km the higher the expectation of NPV . It is very intuitive. The right graph shows the relationship between h and $E(NPV)$ given a constant management charging rate of km . In general, the investment guarantee charging rate of h increases as the $E(NPV)$. However, as the company is involved in more guaranteed risk, the $E(NPV)$ might decrease later on.

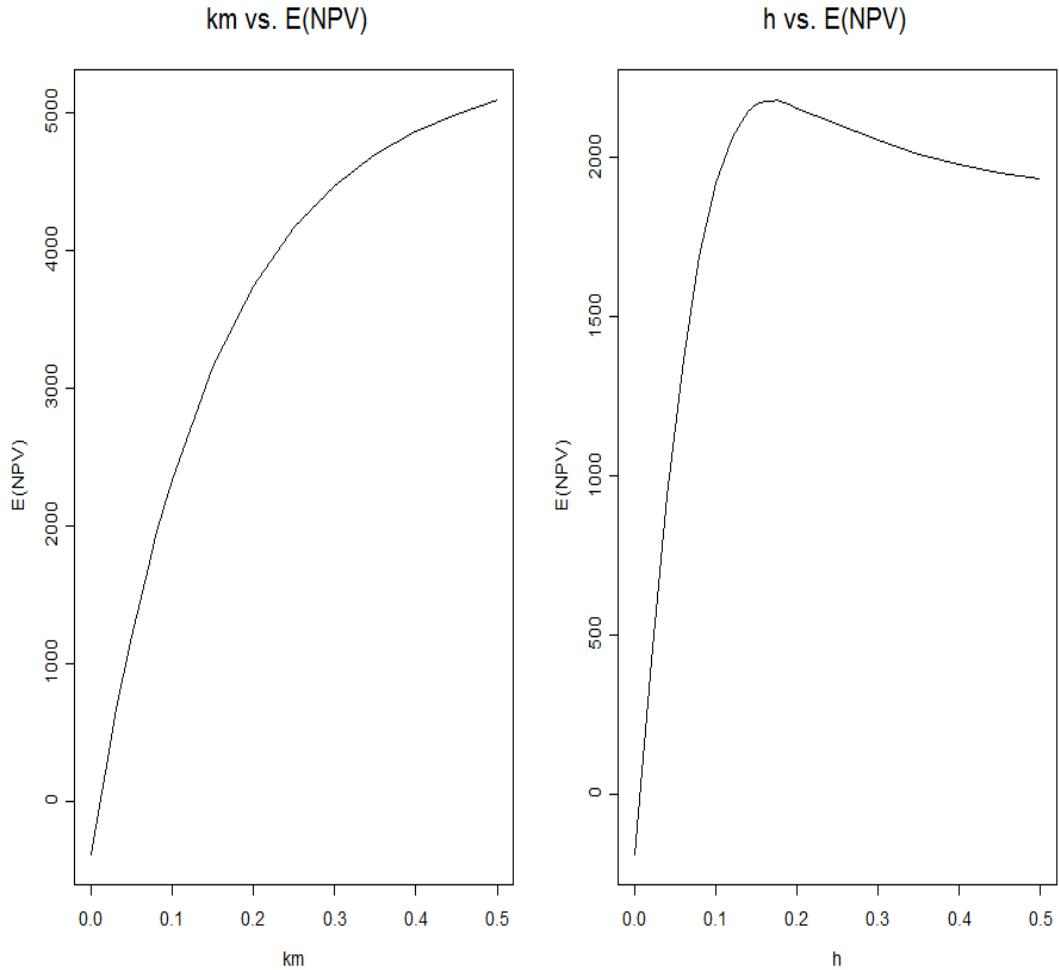


Figure-1. The relationships between $E(NPV)$, km and h . (Assume the insurance company's target internal rate equals to 0.12.)

Notes: km = the fund management charge rate, which is charged to support the business (such as the expenses, commission, ...etc.), except the guarantee cost. h = the investment guarantee charge rate, which is charged due to the guarantee cost. $E(NPV)$ = the expectation of net present value.

3.1.2 Criteria Two

We set up our second criteria so that the guarantee charge and the capital support can meet out reserving standard in each period. For a unit-linked policy that doesn't offer any guarantee, the future investment experience involves no risk to the insurer, which merely acts as a steward of the common funds. In this case, since there is no liability due to the policy, no reserves need to be set. However, for a unit-linked contract that offers guarantees, many researches [Maturity Guarantees Working Party \(MGWP\) \(1980\)](#), [Boyle and Hardy \(1997\)](#) have showed that there is a need for the insurance company to set up reserves. [Boyle and Hardy \(1997\)](#) and [Hardy \(2000\)](#) used quantile risk measure to set up reserves for guarantee. The concept of quantile reserves is that setting up the reserves under the required probability of having sufficient funds to meet the cost of the guarantee. In their work, they calculated the reserves for a single premium contract with guaranteed minimum

maturity benefit. They ignored mortality and the management charges of guarantee in their research. In terms of our notation, their methodology can be written in equation (4).

$$Pr[(F_n + {}_0V \prod_{t=1}^n (1+i_{s,t})) > G_n] \geq p \tag{4}$$

In this research we extend Hardy (2000) 's methodology of reserving to build up a framework that can provide optimal charges for the insurance contracts that offer guarantees. In this framework, we consider mortality but in a deterministic environment. We can also incorporate a stochastic mortality model in our framework.

When considering mortality rates, the reserving methodology is shown in the following. The first reserving standard p_1 and second reserving standard p_2 are shown in equation (5).

$$\begin{cases} Pr[(M(n) + {}_{n-1}V)(1+i_{s,n}) > (G_n - F_n)_+ \cdot p_{x+n-1}] \geq p_1 & \text{if } t = n \\ Pr[(M(t) + {}_{t-1}V)(1+i_{s,t}) > V \cdot p_{x+t-1}] \geq p_1 & \text{if } t \neq n \end{cases} \tag{5}$$

where p_1 is the first reserving standard.

The reserving standard p_1 is used to control the reserves, such that $({}_{t-1}V)$ will be sufficient to meet the guaranteed cost plus the required reserving at the end of each year t with the given probability p_1 . If the investment returns are favorable, the insurer could release reserves, otherwise more capital should be added. Thus the reserving standard p_1 would always be met during each period of the contract in our framework.

We use a second standard, p_2 , such that with probability p_2 the insurer will receive at least the management charges assumed. That is, we use the $100(1-p_2)$ percentile of the possible guarantee charges in the year t , denoted by $M^{(p_2)}(t)$, to estimate the required reserves. In this framework, we assume km and h are both in the form of a percentage of the accumulated unit funds. We project unit funds using simulation. While the investment scenarios of each funds have been projected and the management charge rate has been given, all the $M^{(p_2)}(t)$ of each t can be computed and then the required reserves can be derived from the policy year n back to the policy year 1.

In our framework, we first simulate the future investment return rate, $i_{u,t}$ and $i_{s,t}$. Then, the distribution of $M(t)$ at different time t can be figured out and then the $M^{(p_2)}(t)$, by given a specific km_t and h_t . In this research, we investigate unit-linked contracts with maturity guarantees. The framework for this type of guarantee is described more detail in the following. The insurer needs to hold the amount of the reserves ${}_{n-1}V^*$ at the start of the last policy year such that

$$Pr[(M^{(p_2)}(n) + {}_{n-1}V^*)(1+i_{s,n}) > (G_n - F_n)_+ \cdot p_{x+n-1}] \geq p_1 \tag{6}$$

This implies that the necessary reserves need to be kept at the beginning of the policy year n , under the reserving standard p_1 and p_2 is at least

$${}_{n-1}V^* = \left[\frac{(G_n - F_n)_+ \cdot p_{x+n-1}}{(1+i_{s,n})} - M^{(p_2)}(n) \right]^{(p_1)}, \tag{7}$$

where the right upper symbol (p_1) means the $100p_1$ percentile of the distribution of

$\left[\frac{(G_n - F_n)_+ \cdot p_{x+n-1}}{(1+i_{s,n})} - M^{(p_2)}(n) \right]$. Similar with the last year of the policy period, ${}_{t-1}V^*$ under

p_1 and p_2 must satisfied

$$Pr[(M^{(p_2)}(t) + {}_{t-1}V^*)(1+i_{s,t}) > {}_tV^* \cdot p_{x+t-1}] \geq p_1, \quad (8)$$

which implies

$${}_{t-1}V^* = \left[\frac{{}_tV^* \cdot p_{x+t-1}}{1+i_{s,t}} - M^{(p_2)}(t) \right]^{(p_1)} \quad \forall t = n-1, n-2, \dots, 1 \quad (9)$$

3.2. The Procedure of Our Framework

In our framework, we use the Wilkie investment model to generate a number of scenarios, each considered equally likely. The unit fund value, the cost of the guarantee can be estimated under each scenario. The distribution of the simulated values of NPV can be obtained. According to the two criteria, we can estimate the charge for the policy without guarantee first. Then, we use this information to derive the charge for the policy with guarantee. In order to make sure that the insurance company can meet their guarantee liability, we work out the charge in reverse. The steps to search for the optimal charge are described in Figure 2.

We use backward method to find the charge for unit-linked contracts in this research. The idea is different than the traditional actuarial technique to price for traditional life insurance policy. For a traditional life contract, while the mortality rates, the lapse rates, the projected interest rates and the expenses assumptions are determined; the net premiums under the equivalence principle can be derived. As the loading assumptions are added in, one knows the gross premiums and the projected profitability. For unit-linked policies, the dynamics of the rates of investment return cannot be predicted deterministically. Thus, in our framework, we incorporate an investment model to predict the rates of future investment return stochastically and work out the charges in reverse.

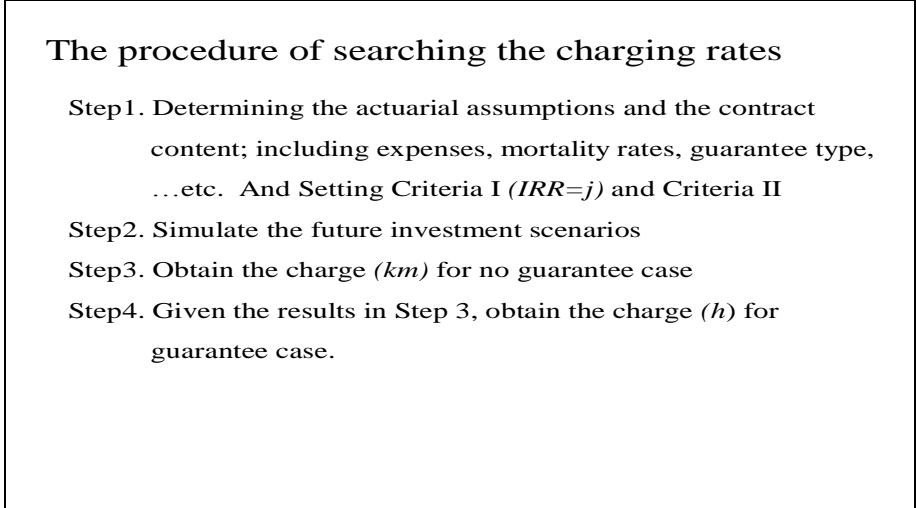


Figure-2. The procedure of searching the fund management charge rates.

4. NUMERICAL ILLUSTRATION

Assume a 10-year unit-linked policy with maturity guarantee is issued to a person aged x . We assume the fund of the unit account will be all invested in the stock market. Using the notations introduced in section 2 and 3, all the related assumptions on the policy are projected as follows.

$$P_t = 1000 \quad \forall t = 1, 2, \dots, 10 \tag{10}$$

$$a_t = \begin{cases} 0.8 & t = 1 \\ 1 & t = 2, 3, \dots, 10 \end{cases} \tag{11}$$

$$e_t = \begin{cases} 500 & t = 1 \\ 20 & t = 2, 3, \dots, 10 \end{cases} \tag{12}$$

$$G_t = \begin{cases} 0 & t = 1, 2, \dots, n-1 \\ \sum_{i=1}^n P_i & t = n \text{ and the insured is survival at } (x+n) \end{cases} \tag{13}$$

Furthermore, in order to estimate the distribution of the guarantee costs, the future investment scenarios are projected using the Wilkie model (Wilkie, 1995). In the investment outcome of the sterling fund, we assume the annual return rate is a constant, say $i_{s,t} = 0.075$. The two reserving standards, (p_1, p_2) , are set to be $(0.975, 0.95)$ respectively.

The objective profitability should be decided first. We use the IRR to be the profitability measurement in this paper. Now, suppose the target profitability is $IRR = 0.12$. Under the stochastic cash-flow model constructed in section 2, the corresponding km is the numerical root of the equation,

$$E(NPV) = 0 \Big|_{j=0.12, G_t=0} \tag{14}$$

Let the root of equation (14) be km^* , then the corresponding h , say h^* , is the root of the equation,

$$E(NPV) = 0 \Big|_{j=0.12, G_t=G_t, km^*} \tag{15}$$

Based on our assumptions, the numerical results are $km^* = 0.0109$ and $h^* = 0.0059$. Thus the fair fund management charge rate corresponding to IRR equal to 0.12 is $km^* + h^* = 0.0168$. In addition, the related consequence, such as $Var^*(NPV)$ and ${}_0V^*$ (* means corresponding to km^* and h^*) can be derived subject to km^* and h^* . Actuaries may study the information of every scenario under different IRR assumptions, and then determine a feasible charge.

In our example, the variance of simulated NPV is 27,313 and the initial reserve required is 470.85. Figure 3 illustrates the histogram of simulated NPV. It is left skewed due to the maturity guarantee cost being right skewed.

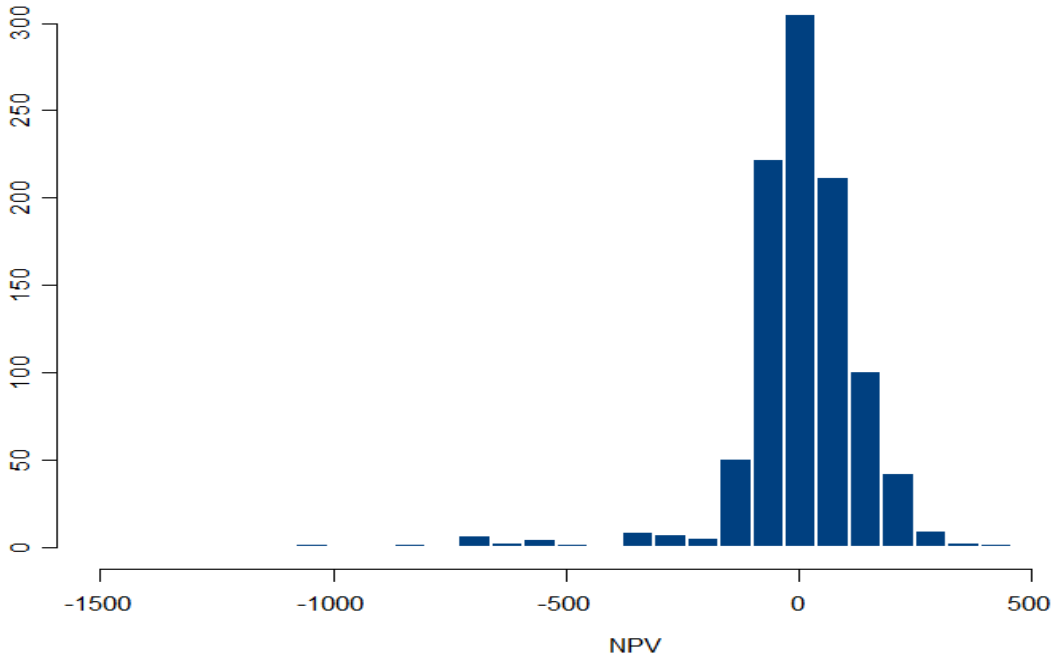


Figure-3. The histogram of simulated NPV.

Although the km^* and h^* are derived from the expected internal rate of return equal to 0.12, it is important to understand the possible expected value of the profit emerging each year from this contract. Figure 4 gives four simulated results of cash flows from the policy inception to maturity.

The cash flow at time 0 is negative and equal to $-{}_0V^*$. This is the initial capital requirement for the insurer to support the business. The rest of the cash flows are ${}_{t-1}p_x(CF)_t^*$ in each term.

The cash flow in term 1 is negative due to the expenses assumption, a certain amount of initial expenses in the first policy year. In general, the remainder of the cash flows would increase stepwise as the increasing fund management charge, but they would variant because of the different investment outcomes. The cash flow in the last term is significant since the release of the reserving. It is positive and vast if the maturity guaranteed cost is zero, and it is a significant negative as the maturity guaranteed occurs. Figure 4-(a) is the cash flows with respect to the NPV nearest zero. Figure 4-(b) is the cash flows with respect to maximum NPV. The bars at $t = 5$ to 9 in this diagram are higher than the others due to the excellent investment performance. Figure 4-(c) is the minimum NPV case. The enormous negative cash flow is as a consequence of huge maturity guaranteed cost. Figure 4-(d) is a random case of the 1000 simulations.

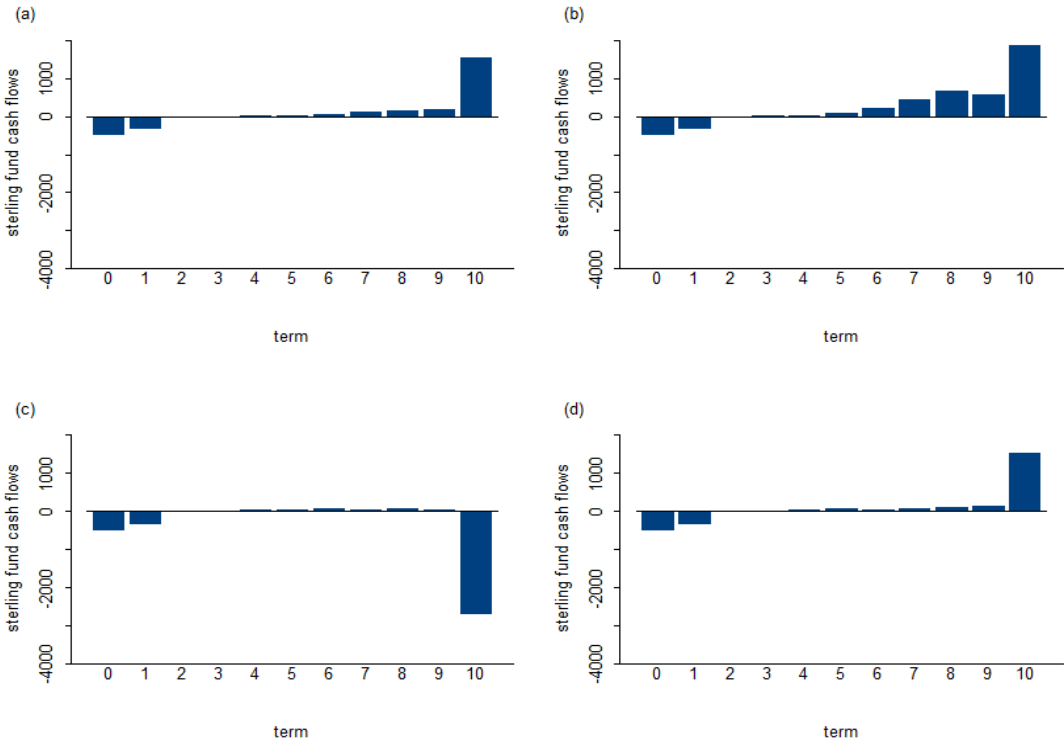


Figure-4. Examples of simulated cash flows based on our example.

In our numerical illustration, the IRR we used to obtain km^* and h^* are both equal to 0.12. In reality, the shareholder may ask for a lower IRR in the process of getting km^* since issuing a unit-linked contract without guarantee is a risk free investment. On the other hand, they may request a higher IRR when deriving h^* , for the risk of issuing the guarantee. In this paper, we don't distinguish the distinction of risk reward. We focus on the matter of the framework.

5. SENSITIVITY ANALYSIS

In this section, we examine the sensitivity of the fund management charge rate, km^* and h^* , to changes in some basic assumptions.

5.1. The Sensitivity of Reserving Standards

The reserving standard p_1 is a notion of Value at Risk (VaR). Based on the example in section 4, we estimate the h^* under different (p_1, p_2) . The reserving standards are applied to measure the necessary reserves, so the fund management charge rate, km^* , without guarantee is a constant over different values of (p_1, p_2) .

The contour plot in Figure 5 tells us the relationships among h , p_1 and p_2 . The left blank space reveals that the charging rate h^* is low sensitivity if the reserving standard p_1 is less than 0.95. In fact, the charging rate is the same if the p_1 percentile of the guarantee cost is zero. This is a vital problem associated with the VaR shown in Wirch and Hardy (1999). We do not investigate the problem of VaR in this paper. The selected reserving standard level affects the result seriously. It

may not be a reasonable risk measurement for such a long tail distribution. In accordance with the example and the corresponded km^* and h^* in section 4, only 62 over the 1,000 simulated maturity guarantee costs are greater than zero.

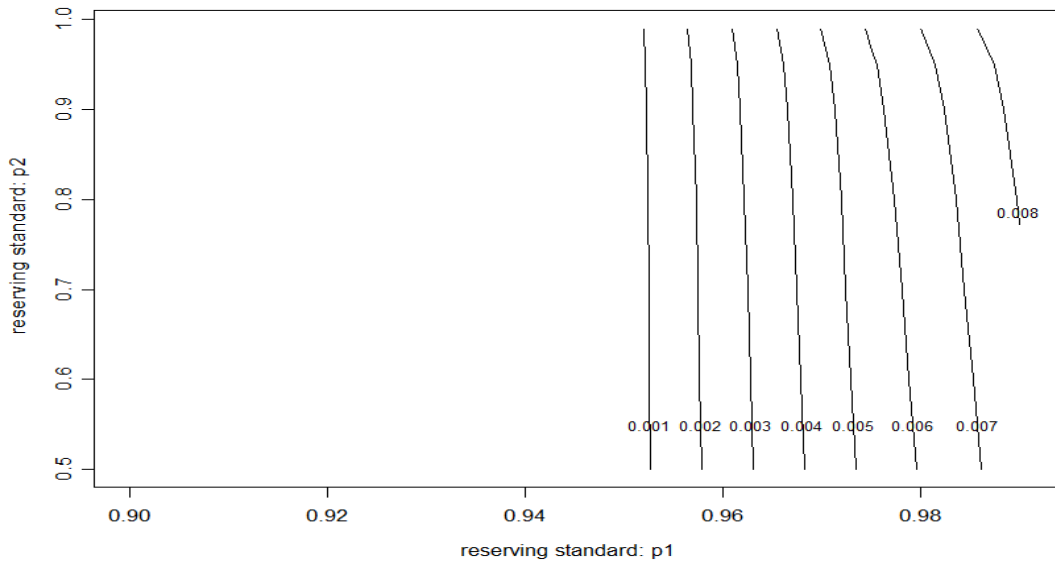


Figure-5. The contour plot of p_1 and p_2 .

Notes: Reserving standard p_1 is used to control the reserves, such that $({}_{t-1}V)$ will be sufficient to meet the guaranteed cost plus the required reserving at the end of each year t with the given probability p_1 . Reserving standard p_2 means that with probability p_2 the insurer will receive at least the management charges assumed.

The conditional tail exception (CTE), which is also known as tail-VaR, is a popular alternative to the VaR. Given α , the CTE is defined as the expected value of the guarantee cost given that the guarantee cost falls in the upper $(1-\alpha)$ tail of the distribution. Thus the reserving standard under

$CTE = \alpha$ means that the maintaining reserves will sufficient to meet $E[(GC)_t] \Big|_{(GC)_t > (GC)_t^{(\alpha)}}$ at the end of each policy year. We change the reserving standard p_1 (a VaR metric) to CTE in the following subsections.

5.2. The Investment Return Rates of Sterling Fund and the Target Profitability

According to the example in section 4, we examine the sensitivity of the investment return rates of sterling fund and the projected IRR under the reserving standard $CTE = 0.9$ and $p_2 = 0.95$. We assume that the investment return rates of sterling fund are constant but vary from 0.06 to 0.09. We find that the fund management rate without guarantee, km , is independent of return rates of sterling fund.

The reason is provided by the following. In the process of searching km^* , we ignore the guarantee first. So the net present value becomes

$$NPV = \sum_{t=1}^n \left(\frac{1}{1+j}\right)^t {}_{t-1}p_x [(1-a_t)P_t - e_t + (km) \cdot (F_{t-1} + a_t P_t)] (1+i_{s,t}) \quad (16)$$

Under the constant $i_{s,t}$ assumptions, the $E(NPV) = 0$ turns out to be

$$(1+i_s) \cdot E \left\{ \sum_{t=1}^n \left(\frac{1}{1+j}\right)^t {}_{t-1}p_x [(1-a_t)P_t - e_t + (km) \cdot (F_{t-1} + a_t P_t)] \right\} = 0 \quad (17)$$

This implies the root, km^* , in equation (17) is independent of i_s . Table 1 illustrates the numerical result of km^* under different projected IRR. It is intuitive that as the target IRR increases, the km^* increases.

Table-1. The km^* under different projected IRRs

Projected IRR	0.08	0.09	0.10	0.11	0.12
km^*	0.0091	0.0096	0.0100	0.0104	0.0109
Projected IRR	0.13	0.14	0.15	0.16	
km^*	0.0114	0.0119	0.0124	0.0129	

Notes: km^* = the optimal fund management charge rate, which is charged to support the business (such as the expenses, commission, ...etc.), except the guarantee cost. IRR = internal rate of return.

Figure 6 is the contour plot of the charging rate due to guarantees, h^* , relating to the constant return rate of sterling fund and the projected IRR. It is intuitive that the charging rate h^* decreases when the investment return rate of sterling fund increases and increases when the projected IRR increases.

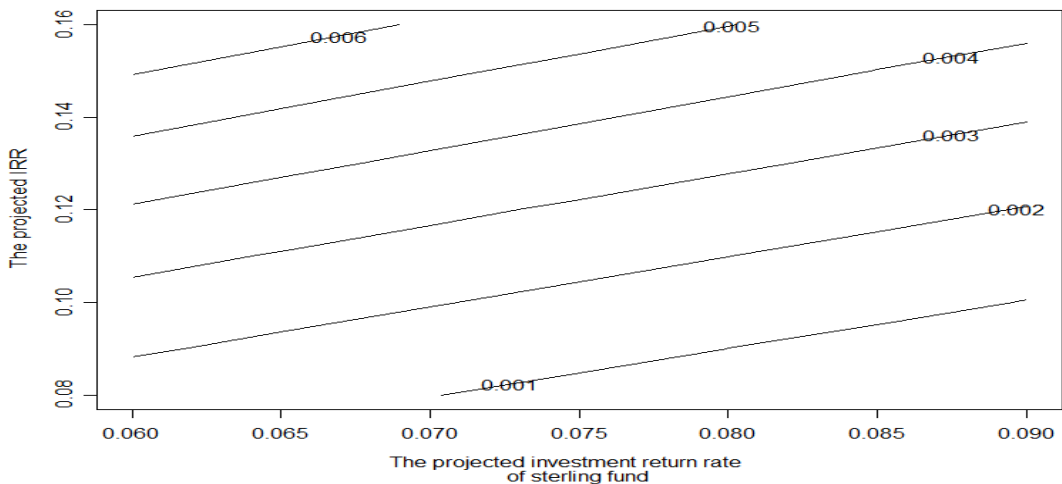


Figure-6. The contour plot of h^* relates to the projected investment return rate of sterling fund and projected IRR.
Notes: h^* = the optimal fund management charge rate due to the guarantee cost. IRR = internal rate of return

5.3. The Impact of Policy Period

The impact of the policy period is examined based on the example in section 4 ($CTE = 0.9$). We use the same assumptions except for the lengths of the contract. We find from Table 2 that the fund management charge rates, both km^* and h^* , are very expensive if the policy period is short. The former can be explained by two reasons. One is due to the certain amount of initial expense. The longer the period of the policy, the lower the annual share of the initial expense cost. The other is due to the proportional fund management charge assumption, thus the attainable charge increases as the increasing of the policy period. The charging rate h^* is also decreasing speedily with the policy period. This is due to the positive drift of the long-term rate of return of the stock market.

The variance of the investment result will increase, but the probability of losing money will decrease as time goes by.

Table-2. The numerical results of different policy periods.

policy periods	5	10	15	20
km^*	0.0320	0.0109	0.0058	0.0038
h^*	0.0293	0.0029	0.0005	0.000034

Notes: km^* = the optimal fund management charge rate, which is charged to support the business (such as the expenses, commission, ...etc.), except the guarantee cost. h^* = the optimal fund management charge rate due to the guarantee cost.

6. CONCLUSION

Risk management for unit-linked contracts with guarantees is very important to the solvency of a life insurer. The studies to date have developed the reserving method for investment guarantees. However, there is less research to discover the method to charge for guaranteed risk. This study proposes a framework to charge for a unit-linked contract when considering guarantee risk. This framework allows the insurance company to measure the charge under a projected profitability. In addition, the future liability is estimated and the required reserves are computed in the framework. Thus, the insurance company can reflect the guaranteed risk in the charge.

The framework and the procedure to search for the optimal charges are explored numerically. In our numerical work, we illustrate a unit-linked contract with maturity guarantee. Other types of guarantee are also applied to our framework. We examine the parameter sensitivity of reserving standards, the investment return rate of sterling fund, the projected IRR and the policy period sequentially. There are several natural extensions of this study. For example, we could use a range of stochastic investment models to project future liabilities of the guarantees. Another example is to compare the capital efficiency of meeting the guaranteed liability between the hedging method and this traditional actuarial approach.

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