QUANTO INTEREST-RATE EXCHANGE OPTIONS IN A CROSS-CURRENCY LIBOR MARKET MODEL

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ABSTRACT

The purpose of this paper is to price quanto interest-rate exchange options (QIREOs) based on a practical and easy-to-use interest-rate model. According to the payoff structure of QIREOs, the cross-currency LIBOR market model (CLMM), in which the initial LIBOR market model (LMM) is extended from a single-currency economy to a cross-currency economy, is suitable to be adopted to price four different types of quanto interest-rate exchange options in this article. Our pricing formulae represent the general formulae in the framework of the CLMM. Hedging strategies are also provided for practical implementation.

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Contribution/ Originality

This study originates new formulas for valuing different types of quanto interest-rate exchange options (QIREOs) under the framework of cross-currency LIBOR market model (CLMM). Our QIREO-pricing formulas are more tractable and feasible for practical implementation.

1. INTRODUCTION

Interest-rate exchange options (IREOs), also known as interest-rate difference options (IRDOs) or options on yield spreads (OOYSs), are written on the underlying difference between two interest rates with different maturities in a single-currency economy and denominated in the same currency, which is analogous to an option to exchange one asset for another studied in Margrabe (1978). For

example, the SYCURVE options introduced by Goldman, Sachs & Co. are calls and puts on the spread between two yields, typically a short-term yield and an intermediate- or long-term yield. When IREOs are written on the difference between two interest rates that are available in different currencies or between two interest rates in one foreign currency, with the final payments made in domestic currency in practice, such IREOs are called quanto interest-rate exchange options (QIREOs). When a cross-currency setting degenerates to a single-currency one, QIREOs will become IREOs. As a result, IREOs are special cases of QIREOs. Interest rate volatility during the past decade has magnified the risk due to an unfavorable shift in the term structure of interest rates, thereby leading to a dramatic increase in the number and types of contingent claims that incorporate options on change in the level of interest rates. These products have been developed to enhance the ability of asset/liability managers to alter their interest-rate exposure. As a result, QIREOs are evolved to exploit interest-rate differentials without directly incurring exchange-rate risk.

The applications of QIREOs are quite extensive and similar to those of differential swaps. However, QIREOs provide more flexibility in certain applications. First, QIREOs provide a mechanism for achieving a payoff based on the differential of interest rates available in two different currencies, which is not directly affected by movements in exchange rate. Second, as compared with differential swaps, the major advantage of QIREOs is that they can be used to fit a very specific strategy since they can be tailored to provide payoffs that depend on whether the spread of two interest rates is above or below a specified level, or within or outside a specified range on a specific date in the future. Third, QIREOs can provide added precision to a strategy involving differential swaps. For example, a portfolio manager might use a differential swap to capitalize on anticipated yield-curve movements while also purchasing an QIREO on the spread in order to limit his downside risk. Moreover, money market investors may use QIREOs to take advantage of a high-yield currency; asset managers may adopt QIREOs to enhance their portfolio return; liability managers and other borrowers can employ QIREOs to reduce their effective borrowing rates. More details regarding the applications of QIREOs can be seen in Schwartz and Smith (1993) and James (2006).

Despite the wide applications of QIREOs, the academic literature has paid little attention to how to price such options. Only few articles were written on the IREOs. Longstaff (1990) derived the pricing formula of YSOs under the Cox-Ingersoll-Ross (CIR) model. Fu (1996) and Miyazaki and Yoshida (1998) derived the pricing formulas of YSOs based on the Heath-Jarrow-Morton (HJM) model. However, there are some problems for these analyses.

First, their formulas all were conducted in a single-currency economy. Their model setting is not consistent with the real economic environment and leads to pricing formulas unsuitable for pricing the QIREOs since the "exchange-rate-effect", long discussed in the finance literature, in a cross-currency economy is not considered. Amin and Jarrow (1991); Schlogl (2002); Musiela and Rutkowski (2005) and Wu and Chen (2007) show that the "exchange-rate-effect" affects the pricing results and should be reflected in valuing cross-currency financial products.

Second, those interest-rate models that have been developed for pricing interest-rate derivatives can be, loosely speaking, divided into two types: traditional interest-rate models and market models. The traditional interest-rate models, such as the Vasicek model (Vasicek, 1997) the CIR model (Cox *et al.*, 1985) and the HJM model (Heath *et al.*, 1992) describe the behavior of interest rates by specifying market-unobservable and abstract interest rates, such as instantaneous short and forward rates. Contrarily, the LIBOR market model (LMM) are constructed by specifying market-observable LIBOR rates. The LMM has been developed by Musiela and Rutkowski (1997); Miltersen *et al.* (1997) and Brace *et al.* (1997) to improve the drawbacks of the traditional interest-rate models. The drawbacks of the traditional interest-rate models and the improvements of the LMM are stated as follows.

The instantaneous short rate in short rate models, such as the Vasicek model and the CIR model, or the instantaneous forward rate in the HJM forward rate model, is abstract and market-unobservable. So the recovery of model parameters from market-observed data is a difficult and complicated task. Since the forward LIBOR rates in the LMM model are market observable, the LMM model circumvents the difficulty of transforming the traded quantities observed in the market into the model parameters.

In addition, the pricing formulas of widely traded interest rate derivatives, such as caps and floors, based on the short rate models or the HJM model are not consistent with market practice. This results in some difficulties in the parameter calibration procedure. With the advantage of pricing interest rate caps and floors consistent with the popular Black formula Black (1976) the LMM model is easier for calibration.

Moreover, the rates in Gaussian term structure models, such as the HJM model, can become negative with a positive probability, which may cause some pricing errors. The forward LIBOR rates in the LMM have a lognormal volatility structure which prevents interest rates from becoming negative with a positive probability.

As a result, the derived pricing formulas under the LMM are more tractable and feasible than those under other models for practitioners.

Therefore, the purpose of this article is to price QIREOs based on a practical and easy-to-use interest rate model, i.e. the LMM. In addition, it is worth noting that the well-known "exchange-rate-effect" has to be considered as dealing with pricing cross-currency-type options. Schlogl (2002) extend the initial LIBOR market model from a single-currency economy to a cross-currency economy. Based on Amin and Jarrow (1991); Wu and Chen (2007) also extend the original LMM model from a single-currency economy to a cross-currency case to incorporate the process of equity-product into the model setting. Consequently, the extended LMM, namely the cross-currency LIBOR market model (CLMM), is suitable to be used for pricing cross-currency-type interest-rate derivatives and is employed in this article to price four different types of QIREOs.

Our article has several contributions to the literature on QIREOs, particularly in the presence of an open cross-currency economic environment and stochastic interest rates.

First, we derive general pricing formulas of QIREOs. Our pricing formulas consider the "exchange-rate-effect" and hence are consistent with the real economic environment. The pricing formulas of QIREOs in this research will be more general and suitable for pricing QIREOs in a real cross-currency environment. If the model setting degenerates to the single-currency case, the pricing formulas of QIREOs become those of IREOs but in the single-currency LMM framework. Therefore, the formulas of Longstaff (1990); Fu (1996) and Miyazaki and Yoshida (1998) are special cases of our results.

Second, our research finds that valuing QIREOs without regard to the effect of the exchange rate (we call it a single-currency framework hereafter) causes an inaccuracy of evaluation of QIREOs. The inaccuracy can be avoided by using our pricing formulas.

Third, our derived formulas under the CLMM are more tractable and feasible than those developed under other interest rate models in previous literatures for practitioners.

Finally, using our pricing formulas for valuing QIREOs is more efficient than adopting time-consuming simulation.

The remainder of this article is organized as follows. Section 2 briefly describes the economic environment and the dynamics of assets for pricing. Section 3 derives the pricing formulae of the four different types of QIREOs based on the CLMM. The hedging strategy of each option is also examined. Section 4 concludes the paper with a brief summary.

2. ECONOMIC MODEL: THE CROSS-CURRENCY LIBOR MARKET MODEL (CLMM)

From the payoff structure of QIREOs illustrated in Section 3, the economic framework for pricing QIREOs should include the dynamics of the domestic interest rates, the foreign interest rates and the exchange rate. An economic model which includes these above dynamics is suitable to be adopted to develop the arbitrage-free pricing formulas of GCSRs. As a result, the CLMM is employed to derive the formulas of QIREOs. In this section, we briefly describe the framework of the cross-currency LIBOR market model.[i]

Assume that trading takes place continuously in time over an interval $[0, \tau], 0 < \tau < \infty$. The uncertainty is described by the filtered probability space $\left(\Omega, F, P, \{F\}_{t \in [0,\tau]} \right)$ where the filtration is generated by independent standard Brownian motions $W(t) = \left(W_1(t), W_2(t), ..., W_m(t) \right)$. Q represents the domestic spot martingale probability measure. The filtration $\{F_t\}_{t \in [0,\tau]}$ denotes the flow of information accruing to all the agents in the economy. The notations are given below with d for domestic and f for foreign:

$f_k(t,T)$ = the kth country's forward interest rate contracted at time t for instantaneous borrowing and lending at time T with $0 \le t \le T \le \tau$, where $k \in \{d, f\}$.

$P_k(t,T)$ = the time t price of the kth country's zero coupon bond (ZCB) paying one dollar at time T.

$r_k(t)$ = the kth country's risk-free short rate at time t.

$\beta_k(t)$ = $\exp\left[\int_0^t r_k(u)\,du\right]$, the kth country's money market account at time t with an initial value $\beta_k(0)=1$.

$X(t)$ = the spot exchange rate at $t \in [0,\tau]$ for one unit of foreign currency expressed in terms of domestic currency.

The Zero-Coupon bond price (ZCB), $P_k(t,T)$, $k \in \{d, f\}$, is defined as:

$$P_k(t,T) = \exp\left[-\int_t^T f_k(t,u)\,du\right]. \qquad (2.1)$$

For some $\delta > 0$, $T \in [0,\tau]$ and $k \in \{d, f\}$, define the forward LIBOR rate process $\{L_k(t,T); 0 \le t \le T\}$ as given by

$$1 + \delta L_k(t,T) = \frac{P_k(t,T)}{P_k(t,T+\delta)} = \exp\left(\int_T^{T+\delta} f_k(t,u)\,du\right) \quad (2.2)$$

The dynamics of the forward LIBOR rates and the exchange rate under the domestic spot martingale measure Q can be given as follows.[ii]

$$\frac{dL_d(t,T)}{L_d(t,T)} = \gamma_{Ld}(t,T)\cdot\sigma_{Pd}(t,T+\delta)\,dt + \gamma_{Ld}(t,T)\cdot dW(t) \quad (2.3)$$

$$\frac{dL_f(t,T)}{L_f(t,T)} = \gamma_{Lf}(t,T)\cdot\left(\sigma_{Pf}(t,T+\delta)-\sigma_X(t)\right)\,dt + \gamma_{Lf}(t,T)\cdot dW(t) \quad (2.4)$$

$$\frac{dX(t)}{X(t)} = \left(r_d(t)-r_f(t)\right)\,dt + \sigma_X(t)\cdot dW(t) \quad (2.5)$$

where $\gamma_{Lk}(t,T)$ is a deterministic, bounded and piecewise continuous volatility function, and $\sigma_{Pk}(t,T)$ is defined as (2.6), and $\sigma_X(t)$ is a deterministic volatility vector function of an exchange rate satisfying the standard regularity conditions .

$$\sigma_{Pk}(t,T) = \begin{cases} \displaystyle\sum_{j=1}^{[\delta^{-1}(T-t)]} \frac{\delta L_k(t,T-j\delta)}{1+\delta L_k(t,T-j\delta)}\gamma_{Lk}(t,T-j\delta) & t \in [0,T-\delta] \\ & \& T-\delta > 0, \\ 0 & otherwise. \end{cases} \quad (2.6)$$

where $\left[\delta^{-1}(T-t)\right]$ denotes the greatest integer that is less than $\delta^{-1}(T-t)$.

To derive the pricing formulas of QIREOs in Section 3 under the framework of CLMM, we need to use the domestic ZCB to be the numeraire such that the domestic forward probability measure Q^T can be induced. The domestic forward measure Q^T can be defined by the Radon-Nikodym derivative $\dfrac{dQ^T}{dQ} = \dfrac{P_d(T,T)/P_d(t,T)}{\beta(T)/\beta(t)}$. From the Radon-Nikodym derivative, the relation of the Brownian motions under different measures can be shown as:

$$dW^Q(t) = dW^T(t) - \sigma_{Pd}(t,T)\,dt. \quad (2.7)$$

Substituting (2.7) into equation (2.3) to (2.5), we can obtain the processes of the forward LIBOR rates and the exchange rate under the domestic forward martingale measure Q^T as follows.

$$\frac{dL_d(t,T)}{L_d(t,T)} = \gamma_{Ld}(t,T)\cdot\big(\sigma_{Pd}(t,T+\delta)-\sigma_{Pd}(t,T)\big)dt+\gamma_{Ld}(t,T)\cdot dW(t) \quad (2.8)$$

$$\frac{dL_f(t,T)}{L_f(t,T)} = \gamma_{Lf}(t,T)\cdot\big(\sigma_{Pf}(t,T+\delta)-\sigma_{Pd}(t,T)-\sigma_X(t)\big)dt+\gamma_{Lf}(t,T)\cdot dW(t) \quad (2.9)$$

$$\frac{dX(t)}{X(t)} = \big(r_d(t)-r_f(t)-\sigma_X(t)\cdot\sigma_{Pd}(t,T)\big)dt+\sigma_X(t)\cdot dW(t) \quad (2.10)$$

where $t\in[0,T]$, $T\in[0,\tau]$ and $\sigma_{Pk}(t,T)$ is defined in (2.6).

There are merits of adopting the LMM. One is that the quotes of interest rates and their derivatives are consistent with market conventions, and thus making the pricing formulas more tractable and feasible for practitioners. In addition, the problems associated with other interest rate models, such as the Vasicek model, the Cox, Ingersoll and Ross (CIR) model, and the HJM model, can be overcome. These problems include: (a) the instantaneous short rate or forward rate is abstract, market-unobservable and continuously compounded. So it is complicated and difficult to recover model parameters from market-observed data; (b) the pricing formulas of extensively traded interest rate derivatives, such as caps, floors, swaptions, etc., based on the short rate models or the Gaussian HJM model are not consistent with market practice. This leads to some difficulties in parameter calibration; (c) as examined in Rogers (1996) the rates under Gaussian term structure models can become negative with a positive probability, which may cause pricing errors.

3. VALUATION OF QUANTO INTEREST-RATE EXCHANGE OPTIONS

In this section, we derive the pricing formulae of four different types of quanto interest-rate exchange options (QIREOs) based on the cross-currency LIBOR market model. Introductions and analyses of each option are presented sequentially as follows.

3.1. Valuation of First-Type QIREOs

Definition 3.1 *A contingent claim with the payoff specified in (3.1.1) is called a First-Type QIREO* (Q$_1$IREO)

$$C_1(T) = N_d\omega\big[L_d^\delta(T,T)-L_f^\eta(T,T)\big]^+, \qquad (3.1.1)$$

where

$L_d^\delta(T,T) =$ *the domestic T-matured LIBOR rates with a compounding period* δ

$L_f^\eta(T,T) =$ *the foreign T-matured LIBOR rates with a compounding period* η, $\eta\neq\delta$

$N_d \ =$ *notional principal of the option, in units of domestic currency*

$T \ =$ *the maturity date of the option*

$(x)^+ \ = Max(x,0)$

$\omega \ =$ *a binary operator (1 for a call option and -1 for a put option).*

An Q$_1$IREO is an option written on the difference between a domestic LIBOR rate with a compounding period δ and a foreign LIBOR rate with a compounding period η, but the final payments are denominated in domestic currency. In addition, an Q$_1$IREO with $\omega=1$ represents a call option on the domestic LIBOR rate with the foreign LIBOR rate serving as the floating strike rate. On the contrary, an Q$_1$IREO with $\omega=-1$ denotes a put option with the foreign LIBOR rate as the underlying rate.

There are several benefits and applications associated with Q$_1$IREOs. First, Q$_1$IREOs provide a mechanism for taking advantage of cross-currency interest-rate differentials without directly incurring exchange rate risk. Second, investors can benefit from utilizing a corresponding Q$_1$IREO with making a correct assessment of the cross-currency interest-rate differential between two underlying LIBOR rates at some particular time point. Third, Q$_1$IREOs also can be used to provide added precision to strategies incorporating differential swaps. For example, a portfolio manager

might use a differential swap to capitalize anticipated yield curve movements while also purchasing an Q_1IREO on the interest-rate differential in order to limit his downside risk. In addition, asset managers whose investments are mainly denominated in domestic currency can utilize Q_1IREOs to enhance portfolio return. A structure of this type can also be employed by liability managers and borrowers to effectively limit interest rate payments to the lower of either the domestic or foreign currency interest rates, without incurring exchange rate risk exposure.

Q_1IREO pricing is expressed in the following theorem, and the proof is provided in Appendix A.

Theorem 3.1 *The pricing formula of Q_1IREOs with the final payoff as specified in (3.1.1) is expressed as follows:*

$$C_1(t) = \omega N_d P_d(t,T) \left[L_d^\delta(t,T) e^{\int_t^T \overline{\mu}_{1_d}^\delta(u,T,T+\delta)du} N(\omega d_{11}) - L_f^\eta(t,T) e^{\int_t^T \overline{\mu}_{1_f}^\eta(u,T,T+\eta)du} N(\omega d_{12}) \right] \quad (3.1.2)$$

where

$$d_{11} = \frac{\ln\left(\frac{L_d^\delta(t,T)}{L_f^\eta(t,T)}\right) + \int_t^T \left[\overline{\mu}_{1_d}^\delta(u,T,T+\delta) - \overline{\mu}_{1_f}^\eta(u,T,T+\eta) \right] du + \frac{1}{2}V_1^2}{V_1}$$

$$d_{12} = d_{11} - V_1$$

$$V_1^2 = \int_t^T \left[\gamma_{Ld}^\delta(u,T) - \gamma_{Lf}^\eta(u,T) \right]^2 du$$

$$\overline{\mu}_{1_d}^\delta(t,T,T+\delta) = \gamma_{Ld}^\delta(t,T) \cdot \left[\overline{\sigma}_{P_d}^s(t,T+\delta) - \overline{\sigma}_{P_d}^s(t,T) \right]$$

$$\overline{\mu}_{1_f}^\eta(t,T,T+\eta) = \gamma_{Lf}^\eta(t,T) \cdot \left[\overline{\sigma}_{P_f}^s(t,T+\eta) - \overline{\sigma}_{P_d}^s(t,T) - \sigma_x(t) \right]$$

$$\omega = 1 \text{ (a call) or -1 (a put).}$$

and $\overline{\sigma}_{Pk}^s(t,\cdot), k \in \{d,f\}$ is defined as (A.7) in Appendix A.

The pricing equation (3.1.2) may be regarded as a generalized representation of Margrabe (1978) in the framework of the cross-currency LMM. Note that when both compounding periods are identical ($\delta = \eta$), the pricing formula (3.1.2) reduces to the pricing model of a regular option on the spread between the domestic and the foreign LIBOR rates in the cross-currency LMM framework.

Theorem 3.1 not only provides the pricing formula for the Q_1IREOs but also reveals a clue to the construction of a hedging (replicating) portfolio for the Q_1IREOs.

For hedging, we rewrite equation (3.1.2) as equation (3.1.3) (the proof is provided in Appendix A) as follows

$$C_1(t) = \Delta_{1t}^{(1)} \left[P_d(t,T) - P_d(t,T+\delta) \right] - \Delta_{2t}^{(1)} \left[P_f(t,T) - P_f(t,T+\eta) \right], \quad (3.1.3)$$

where

$$\Delta_{1t}^{(1)} = \omega N_d \left(1 + \delta L_d^\delta(t,T) \right) \frac{1}{\delta} N(\omega d_{11}) e^{\int_t^T \overline{\mu}_{1_d}^\delta(u,T,T+\delta)du}$$

$$\Delta_{2t}^{(1)} = \omega N_d \left(1 + \eta L_d^\eta(t,T) \right) \frac{1}{\eta} N(\omega d_{12}) QA_1(t,T+\eta)$$

$$QA_1(t,T+\eta) = \frac{P_d(t,T+\eta)}{P_f(t,T+\eta)} \rho_1(t,T)$$

$$\rho_1(t,T) = e^{\int_t^T \overline{\mu}_{1_f}^\eta(u,T,T+\eta)du}.$$

Equation (3.1.3) serves as a guide to the formation of a hedging portfolio $H_t^{(1)}$ for an Q_1IREO. $H_t^{(1)}$ can be completed by a linear combination of four types of assets: holding long $\Delta_{1t}^{(1)}$ units of

$P_d(t,T)$ and $\Delta_{2t}^{(1)}$ units of $P_f(t,T+\eta)$ and selling short $\Delta_{1t}^{(1)}$ units of $P_d(t,T+\delta)$ and $\Delta_{2t}^{(1)}$ units of $P_f(t,T)$.

The term $QA_1(t,T+\eta)$ appearing in (3.1.3) denotes the quanto adjustment due to the hedged risk of the exchange rate. This exchange rate adjustment is induced by the fact that expected foreign cash flow is derived under the domestic martingale measure, and by the compound correlations between all the involved factors (the domestic and foreign bonds and the exchange rate).

It is worth noting that the advantage of adopting the cross-currency LMM model rather than other traditional models is that all the parameters as shown in (3.1.1) and (3.1.2) can be easily obtained from market quotes, which makes the pricing formula more tractable and feasible for practitioners.

3.2. Valuation of Second-Type QIREOs

Definition 3.2 *A contingent claim with the payoff as specified in (3.2.1) is called a Second-Type QIREO* (Q$_2$IREO)

$$C_2(T) = \overline{X} N_f \left[L_f^\delta(T,T) - L_f^\eta(T,T) \right]^+, \qquad (3.2.1)$$

where

N_f = *notional principal of the option, in units of foreign currency*

$\overline{X}$ = *the fixed exchange rate expressed as the domestic currency value of one unit of foreign currency.*

An Q$_2$IREO is an option written on the difference between two foreign LIBOR rates with different compounding periods δ and η, but the final payment is measured in domestic currency. From the viewpoint of domestic investors, holding an Q$_2$IREO acts in much the same way as longing a foreign yield-spread option, whose payoff is based on the difference between the two underlying foreign interest rates, denominated in foreign currency, and converting the foreign-currency payoff via multiplying the fixed exchange rate into the domestic-currency payoff.

Using Q$_2$IREOs has several benefits and applications. Domestic investors can benefit from utilizing a corresponding Q$_2$IREO with making a correct estimate of the differential between two foreign LIBOR rates at some particular time point, thereby avoiding exposure to exchange rate risk. For multinational enterprises or managers of cross-currency financial assets, Q$_2$IREOs can be used to enhance the interest profit of foreign assets or to reduce the interest cost arising from foreign liabilities without incurring exchange rate risk. Furthermore, Q$_2$IREOs can be used to limit the downside risks of some particular payments if a manager of cross-currency financial assets wants to manage the risk of foreign interest rate spread via a long-period foreign basis swap involving the exchange of two series of floating-rate cash flows in the same foreign currency.

To keep the reasonable length of our paper, the pricing formula of Q$_2$IREO is expressed in Theorem 3.2 below and the proof is omitted since the Q$_2$IREO can be priced via the martingale method under the CLMM. The result is available upon request from the authors. [iii]

Theorem 3.2 *The pricing formula of Q$_2$IREOs with the final payoff as specified in (3.2.1) is presented as follows:*

$$C_2(t) = \overline{X} N_f P_d(t,T) \left[L_f^\delta(t,T) e^{\int_t^T \overline{\mu}_{2f}^\delta(u,T,T+\delta)du} N(d_{21}) - L_f^\eta(t,T) e^{\int_t^T \overline{\mu}_{2f}^\eta(u,T,T+\eta)du} N(d_{22}) \right] \quad (3.2.2)$$

where

$$d_{21} = \frac{\ln\left(\dfrac{L_f^\delta(t,T)}{L_f^\eta(t,T)} \right) + \int_t^T \left[\overline{\mu}_{2f}^\delta(u,T,T+\delta) - \overline{\mu}_{2f}^\eta(u,T,T+\eta) \right] du + \dfrac{1}{2} V_2^2}{V_2}$$

$$d_{22} = d_{21} - V_2$$

$$V_2^2 = \int_t^T \square \gamma_{Lf}^\delta(u,T) - \gamma_{Lf}^\eta(u,T) \square^2 \, du$$

$$\overline{\mu}_{2_f}^{*}\left(t,T,T+*\right)=\gamma_{L_f}^{*}\left(t,T\right)\cdot\left[\overline{\sigma}_{P_f}^{s}\left(t,T+*\right)-\overline{\sigma}_{P_d}^{s}\left(t,T\right)-\sigma_x\left(t\right)\right],\quad *\in\{\delta,\eta\}\cdot$$

Longstaff (1990); Fu (1996) and Miyazaki and Yoshida (1998) have derived the pricing formulae for interest rate difference options, which are written on the underlying difference between two domestic interest rates and denominated in domestic currency. In comparison with their pricing formulae, the major differences between Theorem 3.2 and their formulae lie in the fact that not only the "exchange-rate-effect" is considered in Theorem 3.2, but also all parameters appearing in Theorem 3.2 can be extracted from market quotes, which makes our pricing formula more tractable and feasible for practitioners.

Once again, equation (3.2.2) can be written in terms of (3.2.3), and the proof is presented in Appendix B.

$$C_2(t)=\Delta_{1t}^{(2)}\left[P_f\left(t,T\right)-P_f\left(t,T+\delta\right)\right]-\Delta_{2t}^{(2)}\left[P_f\left(t,T\right)-P_f\left(t,T+\eta\right)\right],\quad (3.2.3)$$

where

$$\Delta_{1t}^{(2)}=\overline{X}N_f\left(1+\delta L_d^{\delta}\left(t,T\right)\right)\frac{1}{\delta}N\left(d_{21}\right)QA_2\left(t,T+\delta\right)$$

$$\Delta_{2t}^{(2)}=\overline{X}N_f\left(1+\eta L_d^{\eta}\left(t,T\right)\right)\frac{1}{\eta}N\left(d_{22}\right)QA_2\left(t,T+\eta\right)$$

$$QA_2\left(t,T+*\right)=\frac{P_d\left(t,T+*\right)}{P_f\left(t,T+*\right)}\rho_2^{*}\left(t,T\right),\quad *\in\{\delta,\eta\}$$

$$\rho_2^{*}\left(t,T\right)=e^{\int_t^T\overline{\mu}_{2_f}^{*}\left(u,T,T+*\right)du},\quad *\in\{\delta,\eta\}\cdot$$

Equation (3.2.3) shows the composition of a hedging portfolio $H_t^{(2)}$ for an Q$_2$IREO: it holds long $\Delta_{1t}^{(2)}$ units of $P_f\left(t,T\right)$ and $\Delta_{2t}^{(2)}$ units of $P_f\left(t,T+\eta\right)$ and sells short $\Delta_{1t}^{(2)}$ units of $P_f\left(t,T+\delta\right)$ and $\Delta_{2t}^{(2)}$ units of $P_f\left(t,T\right)$. The implication of the quanto adjustment $QA_2\left(t,\cdot\right)$ is similar to $QA_1\left(t,T+\eta\right)$ as mentioned above.

3.3. Valuation of Third-Type QIREOs

Definition 3.3 *A contingent claim with the payoff as specified in (3.3.1) is called a Third-Type QIREO* (Q$_3$IREO)

$$C_3(T)=X(T)N_f\left[L_f^{\delta}\left(T,T\right)-L_f^{\eta}\left(T,T\right)\right]^{+},\qquad (3.3.1)$$

where

$X(T)=$ *the floating exchange rate expressed as the domestic currency value of one unit of foreign currency at time T.*

An Q$_3$IREO is analogous to the Q$_2$IREO as specified in Subsection 3.2, but with the fixed exchange rate $\overline{X}$ replaced by the floating exchange rate $X(T)$ at maturity *T*. The structure of an Q$_3$IREO is slightly different from that of an Q$_2$IREO in that this option is directly affected by movements in the exchange rate. If the exchange rate moves upward, an investor using this option could enhance profits from the difference between both the foreign interest rates and the exchange rate. And a seller of this option could reduce payments due to downward movements in a foreign currency's value.

Theorem 3.3 below presents the pricing formula of an Q$_3$IREO. The proof is provided in Appendix C.

Theorem 3.3 *The pricing formula of Q$_3$IROs with the final payoff as expressed in (3.3.1) is presented as follows:*

$$C_3(t)=X(t)N_fP_f\left(t,T\right)\left[L_f^{\delta}\left(t,T\right)e^{\int_t^T\left[\overline{\mu}_{3_f}^{\delta}\left(u,T,T+\delta\right)\right]du}N\left(d_{31}\right)-L_f^{\eta}\left(t,T\right)e^{\int_t^T\left[\overline{\mu}_{3_f}^{\eta}\left(u,T,T+\eta\right)\right]du}N\left(d_{32}\right)\right],\qquad (3.3.2)$$

where

$$d_{31} = \frac{\ln\left(\frac{L_f^\delta(t,T)}{L_f^\eta(t,T)}\right) + \int_t^T \left[\overline{\mu}_{3_f}^\delta(u,T,T+\delta) - \overline{\mu}_{3_f}^\eta(u,T,T+\eta)\right]du + \frac{1}{2}V_3^2}{V_3}$$

$$d_{32} = d_{31} - V_3$$

$$V_3^2 = \int_t^T \left|\gamma_{Lf}^\delta(u,T) - \gamma_{Lf}^\eta(u,T)\right|^2 du$$

$$\overline{\mu}_{3_f}^*(t,T,T+*) = \gamma_{Lf}^*(t,T)\cdot\left[\overline{\sigma}_{P_f}^s(t,T+*) - \overline{\sigma}_{P_f}^s(t,T)\right], \quad * \in \{\delta,\eta\}.$$

Similarly, we rewrite (3.3.2) to obtain (3.3.3) as follows

$$C_3(t) = \Delta_{1t}^{(3)}\left[P_f(t,T) - P_f(t,T+\delta)\right] - \Delta_{2t}^{(3)}\left[P_f(t,T) - P_f(t,T+\eta)\right], \quad (3.3.3)$$

where

$$\Delta_{1t}^{(3)} = X(t)N_f\left(1+\delta L_f^\delta(t,T)\right)\frac{1}{\delta}e^{\int_t^T \overline{\mu}_{3_f}^\delta(u,T,T+\delta)du}N(d_{31})$$

$$\Delta_{2t}^{(3)} = X(t)N_f\left(1+\eta L_f^\eta(t,T)\right)\frac{1}{\eta}e^{\int_t^T \overline{\mu}_{3_f}^\eta(u,T,T+\eta)du}N(d_{32}).$$

Equation (3.3.3) also implies a composition for a hedging portfolio $H_t^{(3)}$ similar to that given in the previous theorems. It is worth noting that the quanto adjustment disappears in (3.3.3), since the exchange rate risk in the Q_3IREO is unhedged; this option is directly affected by unanticipated changes in the exchange rate.

3.4 Valuation of Fourth-Type QIREOs

Definition 3.4 *A contingent claim with the payoff as specified in (3.4.1) is called a Fourth-Type QIREO* (Q$_4$IREO)

$$C_4(T) = \omega\left[X(T)N_f L_f^\delta(T,T) - N_d L_d^\eta(T,T)\right]^+. \quad (3.4.1)$$

ω = *a binary operator (1 for a call option and -1 for a put option).*

An Q$_4$IREO is an option written on the difference between a foreign interest payment based on the foreign LIBOR rate with a compounding period δ and a domestic interest payment based on the domestic LIBOR rate with a compounding period η.

This option is slightly different from those options described in the above subsections. It can be considered as an option to exchange domestic-currency-denominated interest payments for foreign-currency-denominated interest payments.

The pricing formula of Q$_4$IREOs is expressed in Theorem 3.4 below and the proof is provided in Appendix D.

Theorem 3.4 *The pricing formula of Q$_4$IREOs with the final payoff as expressed in (3.4.1) is presented as follows:*

$$C_4(t) = \omega X(t)N_f P_f(t,T)L_f^\delta(t,T)e^{\int_t^T\left[\overline{\mu}_{4_g}^\delta(u,T,T+\delta)\right]du}N(\omega d_{41}) \quad (3.4.2)$$

$$-\omega N_d P_d(t,T)L_d^\eta(t,T)e^{\int_t^T\left[\overline{\mu}_{4_d}^\eta(u,T,T+\eta)\right]du}N(\omega d_{42})$$

where

$$d_{41} = \frac{\ln\left(\frac{X(t)N_f P_f(t,T)L_f^\delta(t,T)}{N_d P_d(t,T)L_d^\eta(t,T)}\right) + \int_t^T\left[\overline{\mu}_{4_g}^\delta(u,T,T+\delta) - \overline{\mu}_{4_d}^\eta(u,T,T+\eta)\right]du + \frac{1}{2}V_4^2}{V_4}$$

$$d_{42} = d_{41} - V_4$$

$$V_4^2 = \int_t^T\left|\gamma_g^\delta(u,T) - \gamma_{Ld}^\eta(u,T)\right|^2 du$$

$$\overline{\mu}_{4_g}^\delta(t,T,T+\delta) = \gamma_{Lf}^\delta(t,T)\cdot\left[\overline{\sigma}_{P_f}^s(t,T+\delta) - \overline{\sigma}_{P_f}^s(t,T)\right]$$

$$\overline{\mu}_{4_d}^\eta(t,T,T+\eta) = \gamma_{Ld}^\eta(t,T)\cdot\left[\overline{\sigma}_{P_d}^s(t,T+\eta) - \overline{\sigma}_{P_d}^s(t,T)\right].$$

In order to obtain a hedging portfolio, equation (3.4.2) is rewritten as equation (3.4.3).

$$C_4(t) = \Delta_{1t}^{(4)} \left[P_f(t,T) - P_f(t,T+\delta) \right] - \Delta_{2t}^{(4)} \left[P_d(t,T) - P_d(t,T+\eta) \right], \quad (3.4.3)$$

where

$$\Delta_{1t}^{(4)} = \omega X(t) N_f \left(1 + \delta L_f^{\delta}(t,T) \right) \frac{1}{\delta} e^{\int_t^T \overline{\mu_{4g}^{\delta}}(u,T,T+\delta)du} N(\omega d_{41})$$

$$\Delta_{2t}^{(4)} = \omega N_d \left(1 + \eta L_d^{\eta}(t,T) \right) \frac{1}{\eta} e^{\int_t^T \overline{\mu_{4d}^{\eta}}(u,T,T+\eta)du} N(\omega d_{42}).$$

Equation (3.4.3) shows the composition of a hedging portfolio $H_t^{(4)}$ for an Q_4IREO: holding long $\Delta_{1t}^{(4)}$ units of $P_f(t,T)$ and $\Delta_{2t}^{(4)}$ units of $P_d(t,T+\eta)$ and selling short $\Delta_{1t}^{(4)}$ units of $P_f(t,T+\delta)$ and $\Delta_{2t}^{(4)}$ units of $P_d(t,T)$. Due to the unhedged exchange-rate risk inherent in the Q_4IREO, the quanto adjustment does not exist in equation (3.4.3) as in the case examined in Subsection 3.3; this option is directly affected by exchange-rate movements as well.

4. CONCLUSIONS

We derive the formulas for valuing four different types of QIREOs with four theorems under the framework of CLMM. The derived pricing formulae represent the general formulae of Margrabe (1978) in the framework of the CLMM, and are familiar to practitioners for easy practical implementation. These pricing formulae have been examined to be very accurate as compared with Monte-Carlo simulation.[iv]

Moreover, we have provided the hedging strategies for the QIREOs via the pricing formulae and discussed the calibration procedure in detail. Since the LIBOR rate is market observable and its related derivatives, such as caps and swaptions, are actively traded in the markets, it is easier to calibrate these model parameters than with traditional interest-rate models. Thus, our QIREO-pricing formulas are more tractable and feasible for practical implementation.

Appendix-A. Proof of Theorem 3.1

A.1 Proof of Equation (3.1.2)

By applying the martingale pricing method, the price of an Q_1IRO at time t, $0 \le t \le T$, is derived as follows:

$$C_1(t) = N_d E^Q \left\{ e^{\left(-\int_t^T r_d(s)ds \right)} \omega \left[L_d^{\delta}(T,T) - L_f^{\eta}(T,T) \right]^+ \bigg| F_t \right\} \quad (A.1)$$

$$= N_d E^Q \left\{ \frac{P_d(T,T) / P_d(t,T)}{\beta_d(T) / \beta_d(t)} P_d(t,T) \omega \left[L_d^{\delta}(T,T) - L_f^{\eta}(T,T) \right]^+ \bigg| F_t \right\} \quad (A.2)$$

$$= N_d P_d(t,T) E^T \left\{ \omega \left[L_d^{\delta}(T,T) - L_f^{\eta}(T,T) \right] I_A \big| F_t \right\}, \quad A = \left\{ \omega \left[L_d^{\delta}(T,T) - L_f^{\eta}(T,T) \right] > 0 \right\} \quad (A.3)$$

$$= \omega N_d P_d(t,T) \underbrace{E^T \left\{ L_d^{\delta}(T,T) I_A \big| F_t \right\}}_{(A-I)} - \omega N_d P_d(t,T) \underbrace{E^T \left\{ L_f^{\eta}(T,T) I_A \big| F_t \right\}}_{(A-II)} \quad (A.4)$$

where

$E^Q(\cdot)$ denotes the expectation under the domestic martingale measure Q.

$E^T(\cdot)$ denotes the expectation under the domestic forward martingale measure Q^T defined by the Radon-Nikodym derivative $\dfrac{dQ^T}{dQ} = \dfrac{P_d(T,T) / P_d(t,T)}{\beta_d(T) / \beta_d(t)}$.

ω is a binary operator (1 for a call option and -1 for a put option).

I_A is an indicator function with $\begin{cases} 1, & if \quad \omega \left[L_d^{\delta}(T,T) - L_f^{\eta}(T,T) \right] > 0 \\ 0, & otherwise \end{cases}$.

Part (A-I) and (A-II) are solved, respectively, as follows.

From equation (2.8) to (2.9), the dynamics of $L_d^\delta(t,T)$ and $L_f^\eta(t,T)$ under the domestic forward measure Q^T are shown as follows:

$$\frac{dL_d^\delta(t,T)}{L_d^\delta(t,T)} = \gamma_{Ld}^\delta(t,T) \cdot \left[\sigma_{P_d}(t,T+\delta) - \sigma_{P_d}(t,T)\right] dt + \gamma_{Ld}^\delta(t,T) \cdot dW_t^T, \quad \text{(A.5)}$$

$$\frac{dL_f^\eta(t,T)}{L_f^\eta(t,T)} = \gamma_{Lf}^\eta(t,T) \cdot \left[\sigma_{P_f}(t,T+\eta) - \sigma_{P_d}(t,T) - \sigma_x(t)\right] dt + \gamma_{Lf}^\eta(t,T) \cdot dW_t^T. \quad \text{(A.6)}$$

According to the definition of the bond volatility process $\left\{\sigma_{Pk}(t,T)\right\}_{t\in[s,T]}$ in (2.9), $\left\{\sigma_{Pk}(t,T)\right\}_{t\in[s,T]}$ is not deterministic. Thus, the stochastic differential equations (A.5) and (A.6) are not allowed to solve the distributions of $L_d^\delta(T,T)$ and $L_f^\eta(T,T)$. We can, however, approximate $\sigma_{Pk}(t,T)$ by $\overline{\sigma}_{Pk}^s(t,T)$ which is defined by:

$$\overline{\sigma}_{Pk}^s(t,T) = \begin{cases} \sum_{j=1}^{\left[\delta^{-1}(T-t)\right]} \dfrac{\delta L_k(s,T-j\delta)}{1+\delta L_k(s,T-j\delta)} \gamma_{Lk}(t,T-j\delta) & t\in[0,T-\delta] \\ & \& \ T-\delta > 0, \\ 0 & otherwise. \end{cases} \quad \text{(A.7)}$$

where $0 \le s \le t \le T$ and $k \in \{d,f\}$. Accordingly, the calendar time of the process $\left\{L_k(t,T)\right\}_{t\in[s,T]}$ in (A.7) is frozen at its initial time s, thus the process $\left\{\overline{\sigma}_{Pk}^s(t,T)\right\}_{t\in[s,T]}$ becomes deterministic. This is the Wiener chaos order 0 approximation, which is first used for pricing swaptions by Brace *et al.* (1997). It was further developed in Brace *et al.* (1998) and formalized by Brace and Womersley (2000).

Substituting $\overline{\sigma}_{Pk}^s(t,T)$ for $\sigma_{Pk}(t,T)$ in the drift terms of (A.5) and (A.6), we obtain:

$$\frac{dL_d^\delta(t,T)}{L_d^\delta(t,T)} = \gamma_{Ld}^\delta(t,T) \cdot \left[\overline{\sigma}_{P_d}^s(t,T+\delta) - \overline{\sigma}_{Pd}^s(t,T)\right] dt + \gamma_{Ld}^\delta(t,T) \cdot dW_t^T, \quad \text{(A.8)}$$

$$\frac{dL_f^\eta(t,T)}{L_f^\eta(t,T)} = \gamma_{Lf}^\eta(t,T) \cdot \left[\overline{\sigma}_{P_f}^s(t,T+\eta) - \overline{\sigma}_{P_d}^s(t,T) - \sigma_x(t)\right] dt + \gamma_{Lf}^\eta(t,T) \cdot dW_t^T. \quad \text{(A.9)}$$

In this way, the drift and volatility terms in (A.8) and (A.9) are deterministic. Therefore, we can solve (A.8) and (A.9) and find the approximate distributions of $L_d^\delta(T,T)$ and $L_f^\eta(T,T)$.

Solving the stochastic differential equations (A.8) and (A.9), we obtain:

$$L_d^\delta(T,T) = L_d^\delta(t,T) e^{\int_t^T \left[\overline{\mu}_{1d}^\delta(u,T,T+\delta) - \frac{1}{2}\left|\gamma_{Ld}^\delta(u,T)\right|^2\right] du + \int_t^T \gamma_{Ld}^\delta(u,T) \cdot dW_u^T}, \quad \text{(A.10)}$$

$$L_f^\eta(T,T) = L_f^\eta(t,T) e^{\int_t^T \left[\overline{\mu}_{1f}^\eta(u,T,T+\eta) - \frac{1}{2}\left|\gamma_{Lf}^\eta(u,T)\right|^2\right] du + \int_t^T \gamma_{Lf}^\eta(u,T) dW_u^T} \quad \text{(A.11)}$$

where

$$\overline{\mu}_{1d}^\delta(u,T,T+\delta) = \gamma_{Ld}^\delta(u,T) \cdot \left[\overline{\sigma}_{P_d}^s(u,T+\delta) - \overline{\sigma}_{P_d}^s(u,T)\right], \quad \text{(A.12)}$$

$$\overline{\mu}_{1f}^\eta(u,T,T+\eta) = \gamma_{Lf}^\eta(u,T) \cdot \left[\overline{\sigma}_{P_f}^s(u,T+\eta) - \overline{\sigma}_{P_d}^s(u,T) - \sigma_x(u)\right]. \quad \text{(A.13)}$$

By substituting (A.10) into (A-I), (A-I) can be rewritten as:

$$(A_{-}I) = L_d^\delta(t,T) e^{\int_t^T \overline{\mu}_{1d}^\delta(u,T,T+\delta) du} E^T \left\{ e^{-\frac{1}{2}\int_t^T \left|\gamma_{Ld}^\delta(u,T)\right|^2 du + \int_t^T \gamma_{Ld}^\delta(u,T) \cdot dW_u^T} I_A \middle| F_t \right\} \quad \text{(A.14)}$$

$$= L_d^\delta(t,T) e^{\int_t^T \overline{\mu}_{1d}^\delta(u,T,T+\delta) du} P_r^{R_1} \left\{ \omega\left[L_d^\delta(T,T) - L_f^\eta(T,T)\right] \ge 0 \middle| F_t \right\}. \quad \text{(A.15)}$$

$P_r^{R_1}[\cdot]$ denotes the probability measured in the martingale measure R_1 which is defined by the Radon-Nikodym derivative $\dfrac{dR_1}{dQ^T} = e^{-\frac{1}{2}\int_t^T \llbracket\gamma_{Ld}^\delta(u,T)\rrbracket^2 du + \int_t^T \gamma_{Ld}^\delta(u,T)\cdot dW_u^T}$.

From the Radon Nikodym derivative $\dfrac{dR_1}{dQ^T}$, we know that

$dW_t^{R_1} = dW_t^T - \gamma_{Ld}^\delta(t,T)dt$. (A.16)

Under the measure R_1, we obtain the results by substituting (A.16) into (A.10) and (A.11):

$L_d^\delta(T,T) = L_d^\delta(t,T) e^{\int_t^T \left[\overline{\mu_{1d}^\delta}(u,T,T+\delta) + \frac{1}{2}\gamma_{Ld}^\delta(u,T)\rrbracket^2 \right] du + \int_t^T \gamma_{Ld}^\delta(u,T)\cdot dW_u^{R_1}}$, (A.17)

$L_f^\eta(T,T) = L_f^\eta(t,T) e^{\int_t^T \left[\overline{\mu_f^\eta}(u,T,T+\eta) - \frac{1}{2}\gamma_{Lf}^\eta(u,T)\rrbracket^2 + \gamma_{Ld}^\delta(u,T)\gamma_{Lf}^\eta(u,T) \right] du + \int_t^T \gamma_{Lf}^\eta(u,T)\cdot dW_u^{R_1}}$. (A.18)

By inserting (A.17) and (A.18) into $P_r^{R_1}[\cdot]$, the probability can be obtained after rearrangement as follows:

$P_r^{R_1}\left\{ \omega\left[L_d^\delta(T,T) - L_f^\eta(T,T) \geq 0 \middle| F_t \right] \right\} = N(\omega d_{11})$ (A.19)

where

$N(\cdot)$ represents the cumulative density function of the normal distribution,

$d_{11} = \dfrac{\ln\left(\dfrac{L_d^\delta(t,T)}{L_f^\eta(t,T)} \right) + \int_t^T \left[\overline{\mu}_{1d}^\delta(u,T,T+\delta) - \overline{\mu}_{1f}^\eta(u,T,T+\eta) \right] du + \frac{1}{2}V_1^2}{V_1}$, (A.20)

$V_1^2 = \int_t^T \llbracket \gamma_{Ld}^\delta(u,T) - \gamma_{Lf}^\eta(u,T) \rrbracket^2 du$. (A.21)

The procedures to solve (A-II) are similar to those of (A-I).

By substituting (A.11) into (A-II), (A-II) is derived as follows:

$(A\text{-}II) = L_f^\eta(t,T) e^{\int_t^T \overline{\mu}_{1f}^\eta(u,T,T+\eta) du} E^T\left\{ e^{-\frac{1}{2}\int_t^T \llbracket\gamma_{Lf}^\eta(u,T)\rrbracket^2 du + \int_t^T \gamma_{Lf}^\eta(u,T)\cdot dW_u^T} I_A \middle| F_t \right\}$ (A.22)

$= L_f^\eta(t,T) e^{\int_t^T \overline{\mu}_{1f}^\eta(u,T,T+\eta) du} P_r^{R_2}\left\{ \omega\left[L_d^\delta(T,T) - L_f^\eta(T,T) \geq 0 \middle| F_t \right] \right\}$. (A.23)

$P_r^{R_2}[\cdot]$ denotes the probability measured in the martingale measure R_2 which is defined by the Radon-Nikodym derivative $\dfrac{dR_2}{dQ^T} = e^{-\frac{1}{2}\int_t^T \llbracket\gamma_{Lf}^\eta(u,T)\rrbracket^2 du + \int_t^T \gamma_{Lf}^\eta(u,T)\cdot dW_u^T}$.

From the Radon-Nikodym derivative $\dfrac{dR_2}{dQ^T}$, we find that

$dW_t^{R_2} = dW_t^T - \gamma_{Lf}^\eta(t,T)dt$. (A.24)

Under the measure R_2, we obtain the results by substituting (A.24) into (A.10) and (A.11):

$L_d^\delta(T,T) = L_d^\delta(t,T) e^{\int_t^T \left[\overline{\mu}_{1d}^\delta(u,T,T+\delta) - \frac{1}{2}\llbracket\gamma_{Ld}^\delta(u,T)\rrbracket^2 + \gamma_{Ld}^\delta(u,T)\cdot\gamma_{Lf}^\eta(u,T) \right] du + \int_t^T \gamma_{Ld}^\delta(u,T)\cdot dW_u^{R_2}}$, (A.25)

$L_f^\eta(T,T) = L_f^\eta(t,T) e^{\int_t^T \left[\overline{\mu}_f^\eta(u,T,T+\eta) + \frac{1}{2}\llbracket\gamma_{Lf}^\eta(u,T)\rrbracket^2 \right] du + \int_t^T \gamma_{Lf}^\eta(u,T)\cdot dW_u^{R_2}}$. (A.26)

Inserting (A.25) and (A.26) into $P_r^{R_2}[\cdot]$, we obtain

$P_r^{R_2}\left\{ \omega\left[L_d^\delta(T,T) - L_f^\eta(T,T) \geq 0 \middle| F_t \right] \right\} = N(\omega d_{12})$ (A.27)

$d_{12} = d_{11} - V_1$. (A.28)

By combining A(4), A(15), A(19), A(23) with A(27), equation (3.1.2) of Theorem 3.1 is obtained.

A.2 Proof of Equation (3.1.3)

By definition,

$$L_d^{\delta}\left(t,T\right)=\frac{1}{\delta}\left(\frac{P_d\left(t,T\right)}{P_d\left(t,T+\delta\right)}-1\right) \text{ (A.29)}$$

$$L_f^{\eta}\left(t,T\right)=\frac{1}{\eta}\left(\frac{P_f\left(t,T\right)}{P_f\left(t,T+\eta\right)}-1\right) \text{ (A.30)}$$

By substituting (A.29) and (A.30) into (3.1.2) and rearranging it, equation (3.1.3) is derived.

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Notes

[i] Details can be seen in, Amin and Jarrow (1991). Schlogl (2002) and Wu and Chen (2007).

[ii] More details can be seen in Amin and Jarrow (1991) and Wu and Chen (2007).

[iii] The proofs of Theorem 3.3 and 3.4 are also omitted to keep the reasonable length of our paper. The results are also available upon request from the authors.

[iv] The result is available upon request from the authors