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DEPENDENCE OF REAL ESTATE AND EQUITY MARKETS IN CHINA WITH THE APPLICATION OF COPULA

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ABSTRACT

This paper examines the dynamic dependence and extreme co-movements between real estate and equity markets in China. We illustrate these ideas in simple empirical settings, implementing the relatively techniques from copulas. When comparing the real estate indices and equity market indices in China, our results show that series in both Shanghai Exchange and Shenzhen Exchange exhibit tail dependence with their respective equity indices. Time-varying SJC copula is the optimal dependence structure while illustrate the extreme co-movement between real estate and equity markets in China.

Keywords: Copula, Dependence structure, Equity, Extreme co-movement, Real estate, Tail dependence.

Contribution/ Originality

This study is one of very few studies which have investigated the dependence structure between real estate and equity markets in both Shanghai Exchange and Shenzhen Exchange with the application of the copula. Moreover, we illustrate the extreme co-movement effect between China's real estate and equity markets.

1. INTRODUCTION

China has made a huge progress forward since it opened its door to the outside world in late 1970's and embraced a market-oriented economy after being ruled under the central controlled system for almost 30 years. China's economic growth has a positive impact on China's real estate market. China's real estate market has continued to boom alarmingly. Investors create strong diversification gains in real estate market. In this paper, we seek to examine the relationship between commercial real estate stocks and the general equity market in the tails of their respective return distributions. Hoesli and Reka (2015) conducted an empirical investigation of the channels

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underlying the risk of contagion between real estate and stocks in the United States. Leung and Tang (2015) studied the stock market and the real estate market in Hong Kong and found that improve the prediction of the China's real estate asset markets.

Copulas are functions that connect multivariate distribution functions to their marginal distributions of any dimension. They have all the relevant information about the dependence structure among the variables. This allows greater flexibility in modeling the multivariate distributions and their margins. Firstly, because there are more details in the full set of copula specifications than there are in the traditional multivariate setting by itself. Secondly, the underlying methodology allows joint distributions to be derived from their marginal distributions even when the latter are not normal distributions. And finally, it is possible to separate the characteristics of each marginal from the dependence parameter.

In the tests used in this paper, the univariate return series are assumed to be independent and identically distributed. On the other hand, it is well known that financial time series tend to exhibit volatility clustering. This problem can be solved by filtering the data using, for example, with an autoregressive conditional model, i.e., a GARCH/EGARCH process. Poon *et al.* (2004) observed that the correlations for the returns of equities are just lightly lower if filtered and that external dependence among volatility-filtered residuals is much weaker than for non-filtered residuals. Thus, we can expect that filtering the series for heteroscedasticity will reduce the tail dependence, such as in Patton (2006b).

The rest of the paper is comprised as follows. In Section 2 we present a summary of the theory of conditional copulas and their estimation via maximum likelihood also describing the parametric forms used in this work. In Section 3 we describe our empirical methodology. In Section 4 we describe our data, model the indexes margins and fit the parametric copulas along with their dependence parameters. Lastly, some closing comments are presented.

2. METHODOLOGY

Copula functions permit flexible modeling of the dependence between random variables, by enabling the construction of multivariate densities that are consistent with the univariate marginal densities. This separation enables researchers to construct multivariate distribution functions, starting from given marginal distributions which avoid the common assumption of normality, for either the marginal distributions or their joint distribution function. In this paper, we employ single-parameter conditional copulas to represent the dependence between two index returns, conditional upon the historical information provided by previous pairs of index returns. The parameter of the conditional copula, like the marginal densities of the separate index returns, depends upon the conditioning information. The general theory of copulas is covered in the books by Joe (1997) and Nelsen (2006). Important conditional theory has been developed and applied to financial market data by Patton (2006b).

Patton (2006a) seems to be the first to extend the theorem of Sklar (1959) on copulas for a conditional version. Copulas provide a convenient way to express multivariate distributions.

Copulas are often defined in literature as distribution functions whose marginal distributions are uniform in the interval [0, 1]. That is, for an n-dimensional vector U in the unit cube, a copula can be informally defined as

$$c\left(u_1,\ \dots,\ u_n\right) = \Pr\left(U_1 \le u_1,\ \dots,\ U_n \le u_n\right) \qquad (1)$$

Where U_i is a random variable with uniform distribution in [0, 1] and u_i is a realization of U_i, $i = 1,\ 2,\ \dots,\ n$. The bivariate dynamics of the returns X_t and Y_t are determined by the three functions $f_t(x_t|\phi_{t-1})$, $g_t(x_t|\phi_{t-1})$ and $c_t(u_t, v_t|\phi_{t-1})$. Parameter estimation is straightforward when separate parameters are used in the functions f_t, g_t and c_t, which we denote respectively by the vectors θ_x, θ_y and θ_c. The contribution to the log-likelihood of all the data made by the two observations at time *t* is then

$$\log h_t(x_t, y_t|\phi_{t-1}, \theta) = \log c_t(u_t, v_t|\phi_{t-1}, \theta_c) + \log f_t(x_t|\phi_{t-1}, \theta_x) + \log g_t(y_t|\phi_{t-1}, \theta_y) \qquad (2)$$

With $\theta = \left[\theta_x; \theta_y; \theta_c\right]$. To sum these contributions across a set of times gives the log-likelihood of an observed time series of *n* pairs of returns $\{x_t, y_t, 1 \le t \le n\}$, which can be stated as

$$L_{x,y}(\theta) = L_{u,v}(\theta_c) + L_x(\theta_x) + L_y(\theta_y) \qquad (3)$$

With L_k denoting the sum of the log-likelihood function values across observations of the variable(s) *k*. Patton (2006a) proposes a two-stage estimation procedure that is appropriate for large samples when the dependence vector θ_c does not have any impact upon the marginal distributions. In the first stage, the parameters of marginal distributions are estimated from univariate time series as

$$\hat{\theta}_x = \arg\max \sum_{t=1}^{n} \log f_t(x_t|\phi_{t-1}, \theta_x),$$
$$\hat{\theta}_y = \arg\max \sum_{t=1}^{n} \log g_t(y_t|\phi_{t-1}, \theta_y). \qquad (4)$$

The second stage then estimates the dependence parameter(s) as

$$\hat{\theta}_c = \arg\max \sum_{t=1}^{n} \log c_t(u_t, v_t|\phi_{t-1}, \theta_c, \hat{\theta}_x, \hat{\theta}_y) \qquad (5)$$

The two-stage ML estimates $\theta = \left[\theta_x; \theta_y \theta_c\right]$ are asymptotically as efficient as one-stage ML estimates. The variance–covariance matrix of θ has to be obtained from numerical derivatives. We have only been able to obtain satisfactory first derivatives, from which the fully efficient two-stage estimator $(n\hat{I})^{-1}$ of the variance– covariance matrix is given by $\hat{I} = n^{-1}\sum_{t=1}^{n} \hat{s}_t \hat{s}_t^{\mathrm{T}}$, where the score vector $\hat{s}_t = \frac{\partial \log h_t}{\partial \theta}$ is evaluated at $\theta = \theta$.

2.1. Estimating Marginal Distributions

The conditional densities of equity index returns are leptokurtic and have variances that are asymmetric functions of previous returns (Nelson, 1991; Engle and Ng, 1993; Glosten *et al.*, 1993). We study several models for the margins, both for the conditional mean and for the conditional variance. For this univariate setting we assume student-t errors, with the mean equations following autoregressive processes, and variance equations following TGARCH model (Glosten *et al.*, 1993) which allows the conditional variance to respond differently to the past negative and positive innovations. Defining $R_{ri,t}$ as the log-return of stock index *i* in r country at time *t*, the AR (1) -t-TGARCH(1,1) model can be written as :

$$\begin{aligned} R_{ri,t} &= a_{ri,0} + a_{ri,j} R_{ri,t-1} + \varepsilon_{ri,t} \\ \sigma_t^2 &= \omega + \alpha \varepsilon_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I(\varepsilon_{t-1} < 0) + \beta \sigma_{t-1}^2 \\ \varepsilon_{ri,t} &\sim i.i.d\ t(\upsilon_{ri}) \end{aligned} \tag{6}$$

Where $I(\cdot)$ denotes the indicator function, υ_{ri}: degree of freedom of *t*-distribution. When estimating the TGARCH model with equity index returns, γ is typically found to be positive, so that the volatility increases proportionally more following negative than positive shocks. This asymmetry is sometimes referred to in the literature as a “leverage effect,” although it is now widely agreed that it has little to do with actual financial leverage (Bollerslev, 2008).

2.2. Copula Specifications and Dependence Measures

We employ various copula functions that are frequently applied in finance, offering different dependence structures. Trivially speaking, tail dependence expresses the probability of having a high (low) extreme value of Y given that a high (low) extreme value of X has occurred. The analytic form for the coefficient of upper tail dependence (λ_{U}) is $\lim_{u\to 1} P\left\{\mathrm{Y} > F_y^{-1}(u) \middle| \mathrm{X} > F_x^{-1}(u)\right\} = \lambda_{\mathrm{U}}$ provided that the random variables X and Y are asymptotically dependent while $\lambda_{\mathrm{U}} \in (0,1]$, on the contrary, if $\lambda_{\mathrm{U}} = 0$, then X and Y are asymptotically independent. The coefficient of lower tail dependence can be defined in a similar way: $\lim_{u\to 0} P\left\{\mathrm{Y} < F_y^{-1}(u) \middle| \mathrm{X} < F_x^{-1}(u)\right\} = \lambda_{\mathrm{L}}$, $\lambda_{\mathrm{L}} \in (0,1]$. Table 1 shows the function form of each copula and tail dependence. We perform a set of goodness-of-fit tests and use two information criteria for selecting the optimal copula model based on the maximized log-likelihood function(Ln) are the Akaike information criterion(AIC) and the Bayesian information criterion (BIC) where n is sample size and q is number of parameters.

$$\begin{aligned} AIC &= -2\ln\left[L_{x,y}(\theta)\right] + 2q \\ BIC &= -2\ln\left[L_{x,y}(\theta)\right] + q\ln(n) \end{aligned} \quad (7)$$

Table-1. Copula function form and tail dependences

Copula Family	function form	lower tail λ_L	upper tail λ_U
Gaussian	$C(u,v)=\int_{-\infty}^{\Phi^{-1}(u)}dx\int_{-\infty}^{\Phi^{-1}(v)}dy\frac{1}{2\pi\sqrt{1-\delta^2}}\exp\left\{-\frac{x^2-2\delta xy+y^2}{2(1-\delta^2)}\right\}$	0	0
Clayton	$C^C(u,v)=\left(u^{-\delta}+v^{-\delta}-1\right)^{-\frac{1}{\delta}}$	$2^{-\frac{1}{\delta}}$	0
Gumbel	$C^G(u,v)=\exp\left\{-\left[(-\log u)^{\delta}+(-\log v)^{\delta}\right]^{\frac{1}{\delta}}\right\}$	0	$2-2^{\frac{1}{\delta}}$
Rotated Clayton	$C^{RC}(u,v)=u+v-1+C^C(1-u,1-v;\delta)$	0	$2^{-\frac{1}{\delta}}$
SJC	$C_{SJC}(u,v\vert\delta,\theta)=\frac{1}{2}\left[C_{JC}(u,v\vert\delta,\theta)+C_{JC}(1-u,1-v\vert\delta,\theta)+(u+v-1)\right]$ $C_{JC}(u,v)=1-\left\{1-\left(\left[1-(1-u)^{\theta}\right]^{-\delta}+\left[1-(1-v)^{\theta}\right]^{-\delta}-1\right)^{-\frac{1}{\delta}}\right\}^{\frac{1}{\theta}}$	$2^{-\frac{1}{\delta}}$	$2-2^{\frac{1}{\theta}}$

Note: are parameters of copula functions. λ_L andλ_U represents the lower tail and upper tail dependences, respectively.

3. DATA AND EMPIRICAL RESULTS

3.1. Data

The analysis of series pairs requires a highly detailed dataset spanning a long time period. Daily stock equity returns for the real estate and market indices of Shanghai Exchange and Shenzhen Exchange from July 2, 2001[1] to June 30, 2015, including 3,393 trading days are obtained from Taiwan Economic Journal Database (TEJ). Table II shows some descriptive statistics for the log-returns of such data. The non-normality of the data is apparent from the coefficients of skewness and kurtosis. We can see that most of the return series shows signals of positive asymmetry except Shenzhen market return. The kurtosis excess which shows positive asymmetry and leptokurtosis. Also, the Jarque-Bera test strongly rejects normality. Meanwhile, estimated mean and standard deviation are visually very similar.

[1] The sample period starts from July 2, 2001 due to the stock indices of China being rebalanced on July 1, 2001.

Table-2. Descriptive and serial dependence test statistics

	Shanghai		Shenzhen	
	real estate	**stock**	**real estate**	**stock**
Mean	0.057	0.031	0.071	0.037
Median	0.017	-0.014	0.047	0.129
Maximum	10.021	10.581	9.973	9.192
Minimum	-9.478	-9.044	-9.615	-8.958
Std. Dev.	2.265	1.769	2.250	1.974
Skewness	-0.069	0.194	-0.063	-0.410
Kurtosis	5.205	7.084	5.218	4.991
Jarque-Bera	689.8452***	2378.904***	697.597***	655.7413***
$Q^2(30)$	769.09***	876.46***	107.53***	712.18***

Note: Q(20) represent Ljung-Box statistics for serial correlation up to the 20th order for the standardized residuals. This empirical study is based on the daily equity returns. The asterisks 0** represents significance level at 1%.

3.2. Marginal Models

We first estimate the marginal models for each return series. We determined the extensions in both marginal models based on the model specification test results in Table 3. For all return series, the estimated coefficients β are statistically significant, indicating that volatilities have high persistence. In other words, large (small) changes in the conditional variance are followed by other large (small) changes. The coefficients γ of stock returns in Shenzhen Exchange is positive and significant, indicating that the volatility of the stock market increases more after a negative shock compared to a positive shock.

Table-3. Marginal distributions

	Shanghai		Shenzhen	
	real estate	**stock**	**real estate**	**stock**
a_0	-0.003	-0.565	-0.324	0.033
	(0.030)	(1.195)	(0.329)	(0.027)
a_1	0.031*	1.000***	1.001***	0.082***
	(0.018)	(0.001)	(0.001)	(0.017)
ω	0.041**	4.455***	0.240	0.057***
	(0.014)	(1.539)	(0.156)	(0.015)
α	0.059***	0.072***	0.0734***	0.070***
	(0.010)	(0.010)	(0.009)	(0.013)
β	0.930***	0.931***	0.945***	0.901***
	(0.009)	(0.008)	(0.006)	(0.011)
γ	0.013	0.002	-0.027**	0.0378***
	(0.012)	(0.013)	(0.011)	(0.016)
υ	6.811***	4.714***	6.431***	6.600***
	(0.835)	(0.435)	(0.773)	(0.742)

Note: The table reports the marginal estimation as equation (6). The brackets report standard errors .This empirical study is based on the daily equity returns. The asterisks ***,** and * represents significance level at 1%, 5% and 10%, respectively.

3.3. Results for Copula Parameter and Dependence Estimates

We estimate the copula models discussed in section 3 by applying the IFM estimation procedure. Table 4 reports the estimated copula with the value of log-likelihood (lnL) and the diagnostic statistics AIC and BIC are reported. The optimal dependence structure using these information criteria is time-varying SJC copula for both Shanghai and Shenzhen indices pairs, SJC and Gaussian are the run-ups. In Figure 1 we plot the estimated conditional tail dependence from the constant and time-varying SJC copula models. As in the conditional correlations, we see substantial time variation in tail dependence in Shangai, the upper dependence ranging from 0.066 to 0.799, and the lower dependence ranging from 0.243 to 0.796. As in the conditional correlations, we see substantial time variation in tail dependence in Shenzhen, the upper dependence ranging from 0.090 to 0.861, and the lower dependence ranging from 0.038 to 0.936. The time-varying dependences of SJC copula enable risk managers to measure the risk effectively and avoid underestimating the likelihood of a joint crash. The systematically higher dependence between real estate and stock markets by the left tails in China implies the higher downturn co-movement of these two assets. Investors who would like to invest in the real estate and stock markets in China shall take this phenomenon into account.

Table-4. Dependence measures and comparing structures using information criteria

	Shanghai				
	lnL	**AIC**	**BIC**	λ_L	λ_U
Gaussian	1369.865	-2739.730	-2739.729	0.000	0.000
Clayton	1236.027	-2472.054	-2472.052	0.674	0.000
Gumbel	1304.438	-2608.875	-2608.873	0.000	0.624
Rotated Clayton	1004.219	-2008.438	-2008.437	0.000	0.605
SJC	1426.654	-2853.308	-2853.303	0.624	0.494
time-varying SJC	1545.804	-3091.608	-3091.603	0.625	0.511
	Shenzhen				
	lnL	AIC	BIC	λ_L	λ_U
Gaussian	1374.689	-2749.378	-2749.376	0.000	0.000
Clayton	1248.849	-2497.697	-2497.695	0.674	0.000
Gumbel	1272.493	-2544.986	-2544.984	0.000	0.615
Rotated Clayton	969.914	-1939.827	-1939.825	0.000	0.599
SJC	1412.254	-2824.508	-2824.503	0.668	0.596
time-varying SJC	1557.254	-3114.508	-3114.502	0.661	0.603

Note: and represent the right and left tail dependences respectively. lnL, AIC and BIC represent values of log-likelihood, Akaike information criterion respectively. The values of , of time-varying SJC is the mean of the time-varying dependences.

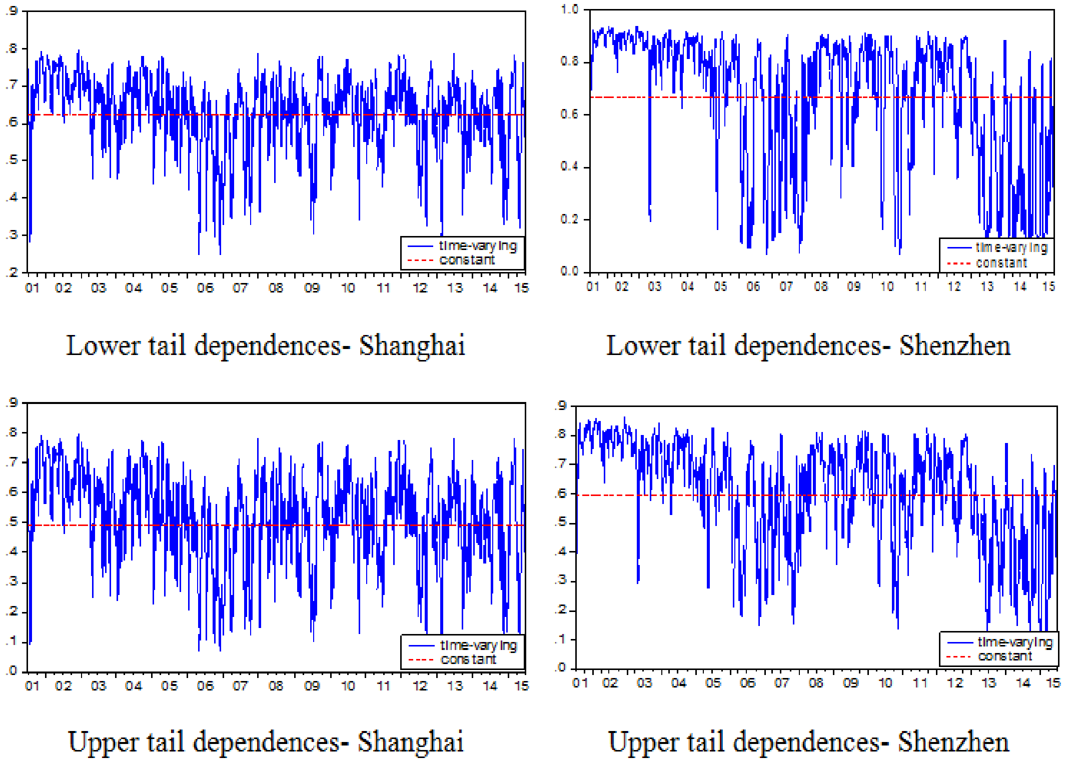

Figure-1. Tail dependences of time-varying SJC copula

Source: All data was obtained from Authors' calculations

4. CONCLUSION

Dependence is quiet important in modern financial economics. We naturally try to understand dependence in modern economic life. There has been a recent flurry of research seeking to understand dependence in economic settings. We illustrate these ideas in simple empirical settings, implementing the relatively techniques from copulas. This paper examines whether the pairwise co-movement between real estate and equity markets in China can be correctly quantified by copulas. Copulas allow us to construct models that go beyond the standard ones at the level of dependence. When comparing the real estate indices and equity market indices in China, our results show that series in both Shanghai Exchange and Shenzhen Exchange exhibit strong tail dependence with their respective equity indices. The optimal dependence structure using these information criteria is time-varying SJC copula for both Shanghai and Shenzhen indices pairs, SJC and Gaussian copula are the run-ups.

Our contribution provides the ability to estimate the diversification effects to international investors that are attributed to the real estate market in China. This has important implications for risk management and asset allocations during extreme events as the significant tail dependence implies higher than normal joint risk. Furthermore, the finding is also important for international asset pricing since the exposure to the joint tail risk should be compensated and thus should be included in pricing international assets.

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