Asian Economic and Financial Review, 2016, 6(6): 336-351

Asian Economic and Financial Review
ISSN(e): 2222-6737/ISSN(p): 2305-2147

URL: www.aessweb.com

IMPOSITION OF R&D SUBSIDY IN A PRODUCT DIFFERENTIATED DUOPOLISTIC INDUSTRY

Ram Kumar Phuyal[1]
[1]*Associate Professor, Centre for Economic Development and Administration (CEDA), Tribhuvan University, Kirtipur, Kathmandu, Nepal*

ABSTRACT

This study analyzes an imposition of government R&D investment subsidy in the standard model of vertical product differentiation with two products in a duopolistic competition. We show that the imposition of distinct subsidy to low quality firm increases its profit, but decreases the high quality firm's profits, whereas subsidy to high quality firm increases both firms' profits. It can be concluded that the subsidization effects of government are socially beneficial whether it is uniform or distinct R&D subsidy policies. Therefore, it can be suggested to the emerging economies that industrial transformation and enhancement of social welfare are possible at a given period of time through the best adoption of moderate R&D investments subsidy in the firms under vertical product differentiation setting.

Keywords: Bertrand price competition, Fixed-costs, Government R&D policy, Nash-Equilibrium, Product quality, Vertical product, Differentiation.

JEL Classification: G18, L11, L15.

Received: 18 February 2016/ **Revised:** 13 April 2016/ **Accepted:** 19 April 2016/ **Published:** 25 April 2016

Contribution/ Originality

This study contributes to the existing literatures of product quality differentiated industries in which firms compute with strategic relationship on their quality decisions under the game theoretical framework in economic methodology. This paper deals with an assessment of two private firms where government imposes R&D quality improvement subsidy which supports for the enhancement of firms' quality vs. profits and the social welfare scale in an economy.

1. INTRODUCTION

A large number of countries have been offering special fiscal incentives for industries to induce spending on R&D and increase their level of innovation for the last two decades. A survey of the European Commission (1995) on state aid reports that its members spent over $1 billion per annum on R&D tax incentives during the early 1990's. In Canada, because of R&D tax incentives, the after-tax cost of R&D expenditure ranges between 35 and 50 cents per dollar spent depending on the type of firm and the province in which the R&D activity is conducted. In many other industrial and OECD countries, governments are trying to stimulate and encourage the creation of new technical knowledge. Actually, in high-technology industries, such as automobiles, computers, consumer electronics, and

DOI: 10.18488/journal.aefr/2016.6.6/102.6.336.351
ISSN(e): 2222-6737/ISSN(p): 2305-2147

others, the firms engage in R&D activities to develop new products and improve their product qualities, i.e. product R&D. The strategic interaction between firms also plays a crucial role in firms' decisions about their output and R&D investment levels. In that case, socially optimal product qualities are not necessarily chosen by an individual firm. But there are many cases where governments have used various policy measures to affect R&D activities.

Despite the debate in the empirical literature over the effectiveness of R&D subsidy/tax incentives, public policy regarding R&D investment has not received much attention from applied micro economists. Although there are many theoretical models addressing process R&D activities, it is known that a very few studies analyzing the economic implications of product R&D activities under imperfect competition, e.g. Symeonidis (2003) the present approach, among a few others, is an attempt to fill this gap in the literature. This study set up a standard model of Vertical Product Differentiation with R&D investment in the presence of strategic interaction among firms and it shows that the widely held idea that subsidizing R&D is beneficial may and may not in fact hold. The objective of the analysis of this study is to present a framework that can be used for micro economic analysis of firm's performance in high-tech industries in which the usual story is in support of subsidies that R&D investment has some of the characteristics of a public good. It further examines to what extent the assumptions of this setting of the model are consistent with the findings that have emerged from theoretical studies of R&D subsidy and patenting at the firm level.

In series of works, governments in this world may have strong interests in a strategicuse of their R&D policies that can help their domestic firms to capture a larger share of profits in domestic and the international markets. These R&D policies are strategic in the sense that they are designed to promote their quality in domestic market and affect foreign firms' behavior in order to improve domestic firms' positions in competition for the world market. There are many theoretical models addressing process R&D activities, but a few studies have analyzed the economic implications of product R&D activities under imperfect competition. Brander and Spencer (1985) analyzed the strategic R&D policy when two firms, each from a different country, play a two-stage game: firms simultaneously decide their R&D investment levels to reduce the production costs process *R&D*, then they compete with homogeneous or horizontally differentiated products by setting Cournot competition in a third country. They concluded that a government can reduce the foreign firm's R&D investment by subsidizing the domestic firm's R&D activities because the foreign firm will reduce its R&D activities in response to the increased R&D of the domestic firm.

Bagwell and Staiger (1994) explore the robustness of this strategic R&D subsidy result against various assumptions about oligopolistic industries: different forms of competition, various types of uncertainty in R&D investments, and different numbers of firms. Their analysis shows that the negative externality of R&D activities and the negatively sloped investment reaction curves are still valid under various assumptions, thus implying the robustness of strategic R&D subsidy policy. White (1996) used a regulated mixed oligopoly model for a closed economy with a linear inverse demand function and an identical quadratic cost function across the firms where government imposes production subsidy while Phuyal (2014) examines welfare difference of product differentiated banking industries in a mixed market oligopolies.

Later, Zhou *et al.* (2002) have examined the implications of a 'strategic trade policy' targeted at investments in quality improvements of exported products. It is assumed in their model that the firm producing a higher quality product locates in a developed country and lower quality firm in less developed country, and that the two firms compete in a third country's market. Symeonidis (2003) pointed out that the product R&D directly affects the consumer's surplus, whereas process R&D affects it only indirectly. Toshimitsu (2003) have discussed the implications of subsidy / tax policies targeted at R&D investments to improve product qualities in the cases of Bertrand and Cournot duopoly. They have shown that the effects of R&D subsidies on qualities and quantities demanded depend on the firm's strategic relationship in the quality decision.

Jinji and Toshimitsu (2006) analyze the case of asymmetric cost of product R&D with a small technology gap between firms. Although symmetric duopoly (Aoki and Prusa, 1997) and asymmetric duopoly with a large technology gap (Zhou *et al.*, 2002) have been examined, the intermediate case has received less attention, despite its relevance in the real world. Recently, Shin and Kim (2010) analyze the effect of government subsidy policies on creating an incentive for domestic firms to improve their product quality before exporting to an outside market. They simulate a dynamic profit maximization problem of the firm and derive the optimal path of the product quality development, then test the efficiency of the different types of subsidy methods.

In sum up, it has been studied that there are very limited papers closely related to the present study: Lahiri and Ono (1999); Zhou *et al.* (2002) and Toshimitsu (2003). They all discuss the subsidy policies applied to R&D investments for quality improvements in concentrated industries using the standard VPD models where Bertrand product–market competition prevails. Toshimitsu (2003) has discussed the optimal R&D policy and its effects on qualities where quantities demanded depend on the firm's strategic relationship in the quality decision. Moreover, authors have presented R&D policies : (i) to maximize net consumer surplus; is to subsidize the two firms, if the government's burden of the subsidies is sufficiently small, (ii) to maximize net producer surplus; is to subsidize the higher quality firm and tax the lower quality firm with Bertrand competition. On the other hand, under Cournot competition, R&D taxation upon the two firms increases net producer surplus and (iii) under Bertrand competition, subsidizing the two firms is socially optimal. Under the Bertrand competition, Lee and Phuyal (2013) also contributed about how the limiting quality device of high-quality firm in regulatory mechanism of minimum quality standards of a product differentiated industry works effectively to enlarge the profits of the firms and entire social welfare in the economy.

Similarly, Zhou *et al.* (2002) have discussed the optimal R&D policy in the context of international rivalry. That context implies that the purpose of a government is to maximize the net profit of each country's firm. In other words, they mainly focused on the international distribution of profits in order to interpret the implications of a *'strategic trade policy'*. In the same line, examined strategic research and development(R&D) policy for quality differentiated product in a third market trade model. They extended previous studies by including a third exporting firm/country in their model. Firms export their entire production to a fourth country. They have different R&D capabilities, but their R&D cost functions are identical as long as their products qualities are below their R&D capabilities. Contrary to earlier studies, authors find that the optimal strategic R&D policy is influenced by the nature of market competition only in the case of the high-quality exporter. The governments of the middle and low-quality exporter would respectively tax and subsidize their domestic firm's R&D under both price and quantity competition. If firms coordinate, the joint optimal R&D policies differ depending on the countries' coordination pairs and competition mode.

Ishii (2014) develops a theoretical third-country trade model of price competition with less stringent demand and cost functions. As opposed to his predecessor, Ishii (2014) finds that the optimal R&D policy does not necessarily depend only on the competition mode, given that in certain situations, both governments' optimal policy involves a product R&D subsidy even when firms compete in a Bertrand fashion. Finally, Taba (2016) derives non-cooperative and cooperative optimal product research and development (R&D) policies of a country with a high-quality firm and a country with a low-quality firm in the presence of technology spillover under Cournot and Bertrand competitions in an international duopoly. He shows that the non-cooperative optimal product R&D Policy is tax for a wider range of spillover effects under Cournot competition, compared to the case of Bertrand competition.

In the same spirit of the above theories and well exposed research papers, this study has made an attempt to analyze all the possible effects of R&D subsidy policies and examines the degree of quality differentiation, and derives the maximum social welfare level in both distinct and uniform R&D subsidy policy cases. This is one of the new approaches to discourse the government R&D investment under game theoretical framework. The model considers a three stage game in which firms compete in two stages and prior to firms' decision, government imposes quality improvement R&D subsidy to maximize social welfare and Bertrand price competition happens at the final stage.

The major findings of this paper are as follows. *First,* government R&D subsidy to lower quality firm increases the quantity demanded and market shares in which firms compete in qualities at the final stage. *Second,* we find that subsidy to higher quality firm increases both firms' profits.*Third,* this result exhibits a sharpcontrast to the outcome in the case where subsidy on both qualities improves social welfare. *Fourth,* imposition of uniform R&D subsidy policies always increases the firms' profits and social welfare.

The rest of the paper is organized as follows. Section two deals with basic frame work model. Section three derives benchmark equilibrium, section four consists with effects of R&D subsidy policy, and section five concludes the results.

2. THE BASIC MODEL

In an extension of the standard model of vertical product differentiation with R&D subsidy, this paper initially considers a domestic duopoly market of industrial countries where there are two identical firms which produce vertically differentiated goods for the domestic consumers. Later, this concept can be extended to more than one country case, too.

On the demand side of the study, it is assumed that there is a continuum of consumers indexed by θ, which is uniformly distributed on the index $\theta \in [0,1]$ with density one. Each consumer is supposed to buy at most one unit of the quality differentiated product produced by the firms.

The basic model of this study is taken from Choi and Shin (1992) followed by the original papers of Mussa and Rosen (1978) and Gabszewicz and Thisse (1979) which explains about an indirect utility function of a consumer θ given by;

$$U = \theta q_i - p_i \text{ where i=1,2} \quad (1)$$

If consumer buys one unit of a product of quality $q_i \in [0,\infty)$ at price $p_i \in [0,\infty)$ .His utility is zero if he buys nothing, his utility is explained as follows;

$$\theta q_i - p_i = 0 \quad (2)$$

Similarly, a consumer will be willing to buy the product of any one of the firm, only if;

$$\theta q_i > p_i \quad (3)$$

On the supply side of the model, the quality of each product is a consequence of R&D investment; however, this expenditure becomes a fixed cost in the production process so, quality $c(q_i)$ is a fixed cost which is quadratic in quality and written as;

$$c(q_i) = \frac{1}{2} q_i^2 \quad (4)$$

Based on the established literatures on vertical product differentiation i.e. Shaked and Sutton (1982); Ronnen (1991); Aoki and Prusa (1997); Zhou *et al.* (2002); Toshimitsu (2003) this study assumes that firms face identical cost structure which depends on quality. Each firm engages in product R&D to improve product quality. The cost of quality improvement for producing quality q_i is given byby $c(q_i)$, with $0 < c'(q_i)$ and $0 < c''(q_i)$ for all feasible quality $q_i \in [0, \infty)$ holds. Throughout the analysis it shall also maintain the regularity in assumptions such that $c(0) = c'(0) = 0$ and that $\lim_{q_i \to \infty} c'(q_i) = \infty$. Marginal cost of production is assumed to be constant and without loss of generality they are to be zero. Whereas government seeks to maximize social welfare and set the quality improvement subsidy (S) to their domestic firms, prior to the game played by firms' i.e.

$$(S) = \frac{1}{2} s_i q^2 \text{, where } 0 < s_i < 1 \tag{5}$$

In this setting, firms compete in a two-stage game in which they simultaneously choose the quality of their products in the first stage, and they compete in prices at the specified markets in the second stage.

3. BENCHMARK: EQUILIBRIUM ANALYSIS

In a setting of duopolistic model of an imperfect competition market, initially two identical firms produce q_i and sell at price p_i where $i = L, H$, and $0 < q_L < q_H$. It is considered that the market is *ex ante* assumed to be uncovered where the total quantity demanded to the high and low quality firms is supposed to be $D_L + D_H < 1$ in which $D_i(p) > 0$.

For all θ, the consumer indifferences between buying the low quality and not buying any unit of the good in the market are;

$$\theta_1 = \frac{p_L}{q_L} \tag{6}$$

For all θ, the consumer indifferences between buying one unit of the good of any firm is;

$$\theta_2 = \frac{p_H - p_L}{q_H - q_L} \tag{7}$$

The demand and revenue functions of low and high quality firms are characterized as follows;

$$D_L(p;q) = \left[\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L} \right] \& \, D_H(p;q) = \left[1 - \frac{p_H - p_L}{q_H - q_L} \right] \tag{8}$$

$$R_L(q_L, q_H) = D_L p_L \,\&\, R_H(q_L, q_H) = D_H p_H \tag{9}$$

Whereas, the profit functions of the firms are expressed as follows;

$$\pi_L = D_L p_L - \frac{1}{2} q_L^2 \,\&\, \pi_H = D_H p_H - \frac{1}{2} q_H^2 \tag{10}$$

At this stage, both firms choose price to maximize their profits (10) then, we have the following first order conditions;

$$\frac{\partial \pi_H}{\partial p_H} = \left[(1 - \frac{(2p_H - p_L)}{(q_H - q_L)} \right] = 0 \;\&\; \frac{\partial \pi_L}{\partial p_L} = \left[\frac{(p_H - 2p_L)}{(q_H - q_L)} - \frac{2p_L}{q_L} \right] = 0 \quad (11)$$

Profit maximizing private firms result equilibrium prices in the second stage and they are obtained as follows from the first order condition;

$$p_l = \frac{1}{2}\left[\frac{p_h}{q_h}\right] q_l \qquad (12)$$

$$p_H = \frac{1}{2}[p_L + (q_H - q_L)] \qquad (13)$$

Solving (12) and (13)

Price functions;

$$p_L = \left[\frac{(q_H - q_L)}{(4q_H - q_L)} \right] q_L \;\&\; p_H = \left[\frac{2(q_H - q_L}{(4q_H - q_L)} \right] q_H \qquad (14)$$

With the equilibrium prices in the second stage, it has the corresponding demand and profits for the high-quality and low-quality firms respectively;

Demand functions;

$$D_H(p;q) = 2\left[\frac{q_H}{4q_H - q_L} \right] \;\&\; D_L(p;q) = \left[\frac{q_H}{4q_H - q_L} \right] \qquad (15)$$

Revenue functions;

$$R_H = \left[\frac{4(q_H - q_L)}{(4q_H - q_L)^2} \right] q_H^2 \;\&\; R_L = \left[\frac{(q_H - q_L)}{(4q_H - q_L)^2} \right] q_L q_H \qquad (16)$$

Many of the works including Aoki and Prusa (1997); Zhou *et al.* (2002) etc have shown the various properties of the revenue functions. In the same way, the different signs of the revenue functions in the first order differentiation from equation (8) are obtained as follows;

$$\frac{\partial R_H}{\partial q_H} > 0, \frac{\partial R_H}{\partial q_L} < 0, \frac{\partial R_L}{\partial q_H} > 0 \text{ and } \frac{\partial R_L}{\partial q_L} > (\le) 0 \leftrightarrow q_H > (\le) \frac{7}{4} q_L \qquad (16.1)$$

From expression (16.1), it is assumed that $q_H > \frac{7}{4} q_L$ for the further analysis of this paper.

The signs of second order conditions obtained here are;

$$\frac{\partial^2 R_H}{\partial q_H^2} < 0, \text{ and } \frac{\partial^2 R_L}{\partial q_L^2} < 0 \qquad (16.2)$$

Similarly, cross partial differentiation are found to be;

$$\frac{\partial^2 R_H}{\partial q_H q_L} = -\frac{q_H(\partial^2 R_H / \partial q_H^2)}{q_L} > 0 \quad \frac{\partial^2 R_L}{\partial q_L q_H} = -\frac{q_L(\partial^2 R_L / \partial q_L^2)}{q_H} > 0 \qquad (16.3)$$

The positive cross partial derivative shows that qualities are strategic complements.

Profit functions;

Let us assume that government of industrial countries impose *R&D* subsidy S (to firms i), which is the portion of the cost of investments in quality and their profit functions i.e.

$$\pi_i = R_i - c(q_i) + S \text{ , where } c(q_i) = \frac{1}{2} q_i^2 \text{ and } S = \frac{1}{2} s_i q_i^2 \quad (17)$$

While specifying them in two firms;

$$\pi_H(s) = \left[\frac{4(q_H - q_L)}{(4q_H - q_L)^2} \right] q_H^2 - \frac{1}{2}(1 - s_H) q_H^2 \quad (18)$$

$$\pi_L(s) = \left[\frac{(q_H - q_L)}{(4q_H - q_L)^2} \right] q_L q_H - \frac{1}{2}(1 - s_L) q_L^2 \quad (19)$$

In the second stage, firms maximize (18) and (19) with respect to q_H and q_L respectively then, we obtain the best replies functions for the high-quality and low-quality firms in the first order conditionwhere;

$$\frac{\partial \pi_H}{\partial q_H} = 4q_H \left[\frac{4q_H^2 - 3q_L q_H + 2q_L^2}{(4q_H - q_L)^3} \right] - (1 - s_H) q_H = 0$$

$$\text{Or, } 4q_H \left[\frac{4q_H^2 - 3q_L q_H + 2q_L^2}{(4q_H - q_L)^3 (1 - s_H)} \right] = q_H \quad (18.1)$$

To simply the above equation, let us assume that $\frac{q_H}{q_L} = w > \frac{7}{4}$ then, (10.1) is expressed as;

$$q_H^* = \frac{4(4w^2 - 3w + 2)}{(4w - 1)^3 (1 - s_H)} \quad (18.2)$$

$$\text{Now, } \frac{\partial \pi_L}{\partial q_L} = q_H^2 \left[\frac{(4q_H - 7q_L)}{(4q_H - q_L)^3} \right] - (1 - s_L) q_L = 0$$

$$\text{Or, } q_H^2 \left[\frac{(4q_H - 7q_L)}{(4q_H - q_L)^3 (1 - s_L)} \right] = q_L \quad (19.1)$$

Expression (11.1) also can be expressed as; $q_L^* = \frac{w^2(4w - 7)}{(4w - 1)^3 (1 - s_L)}$ (19.2)

Firms face two basic considerations upon decision on their qualities here. The first is the profitability of the location in quality space based on revenues and the cost of investment in quality for a given distance from the rival's quality as measured by the degree of quality differentiation, $(q_H / q_L) = w$. The second is the effect of a change in the quality ratio, which determines the degree of price competition. For low quality firm, since an increase in q_L reduces the gap between the products keeping q_H fixed and an associated increase in price competition tends to limit the gain from an increase in quality. Nevertheless, low quality firm has an incentive to set $q_L > 0$ for any q_H because the assumptions $c(0) = c'(0) = 0$ ensure that its marginal profit from a very low quality is always strictly positive. By contrast, the prospect of reduced price competition favors an increase in quality by high quality firm, but the extent of the increase is limited by rising marginal cost of investment in quality.

Since each firm's marginal revenue from own quality is increasing in the other firm's quality, the reaction functions are; $q_H^* = \frac{4(4w^2 - 3w + 2)}{(4w-1)^3(1-s_H)}$ and $q_L^* = \frac{w^2(4w-7)}{(4w-1)^3(1-s_L)}$ for firms H and L respectively. They have positive slopes as seen in figure-1 which is already proved in equation (8.3), making the products strategic complements in quality space.

The properties of the revenue functions and the fixed costs discussed in this paper has proved that the second order condition and the fixed cost are satisfied which makes it sure that the lower quality firm has a steeper reaction curve than that of higher quality firm. Thus, they have a unique Nash equilibrium point in quality space at $N(q_L^*, q_H^*)$. Since $w > 1$, the reaction functions of both qualities lie above the 45° line and reaction curves of two firms are upward sloping in quality space.

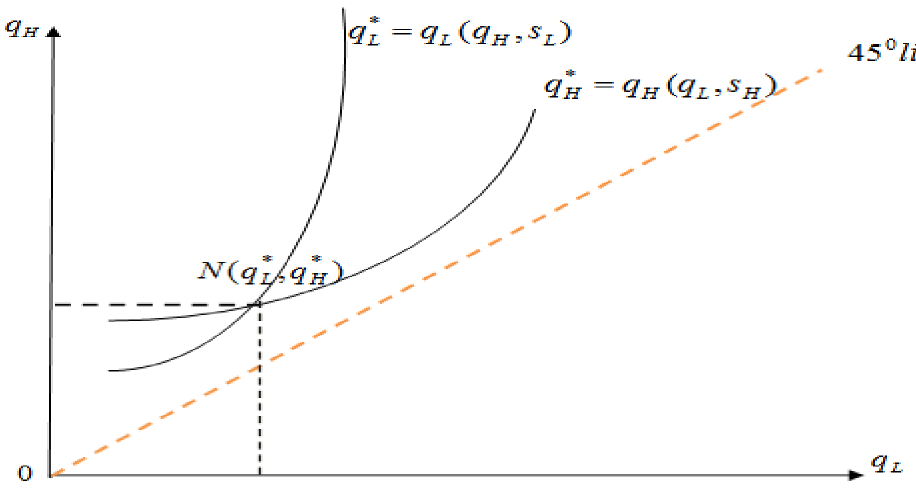

Figure-1. Conceptual framework of quality reaction functions

4. GOVERNMENT R&D SUBSIDY POLICY

This section examines the effects of R&D subsidy policy in a duopolistic trading market where it explores the R&D behavior of the domestic firms under the given competition described earlier and investigates its effects on quality differentiation, and market shares with the aim of achieving maximum social welfare. It also seeks to extend the cases into optimal uniform R&D subsidy policy effect. The explanations of each of different cases are discussed briefly in the following sub-sections.

4.1. Imposition of Distinct R&D Subsidy Policy

The case when government imposes a distinct level of R&D subsidy ($0 < s_L < s_H < 1$) to both firms in an industry then, its possible effects on different sectors are examined as follows;

4.1.2. Effects of R&D Subsidy on Quality

First, this study examines the effect of government *R&D* subsidy on quality improvements of the firms. For that, dividing (18.1) by (19.1) the basic expression (20) is obtained as follows;

$$\left[\frac{4(4q_H^2 - 3q_L q_H + 2q_L^2)q_L}{(4q_H - 7q_L)q_H^2}\right] = \frac{(1-s_H)}{(1-s_L)} \qquad (20)$$

Or, $(16q_L q_H^2 - 12q_L^2 q_H + 8q_L^3)(1-s_L) = (4q_H^3 - 7q_l q_H^2)(1-s_H)$ (20.1)

Then now, considering (20.1) as a function of q_H then, differentiating it with respect to s_H ;

Then, we get;

$$(1-s_L)(32q_L q_H - 12q_L^2)\frac{\partial q_H}{\partial s_H} = (-12q_H^2 s_H + 14q_L q_H s_H + 12q_H^2 - 14q_L q_H)\frac{\partial q_H}{\partial s_H} + (-4q_H^3 + 7q_L q_H^2)$$

or, $$\frac{\partial q_H}{\partial s_H} = \frac{q_H^2(7q_L - 4q_H)}{-12(1-s_H)q_H^2 - 12(1-s_L)q_L^2 + 46q_L q_H - (32s_L + 14s_H)q_L q_H}$$

Solving above expression, we get $, \frac{\partial q_H}{\partial s_H} > 0$ (20.2)

Again, as a function of q_H , differentiating (20.1) with respect to s_L we get;

$$\frac{\partial q_H}{\partial s_L} = \frac{4q_L(4q_H^2 - 3q_L q_H + 2q_L^2)}{4q_L(1-s_L)(8q_H - 3q_L) + 2q_H(1-s_H)(7q_L - q_H)} > 0 \qquad (20.3)$$

Considering (12.1) as a function of q_L then; differentiating with respect to s_L ;

We get; $$\frac{\partial q_L}{\partial s_L} = \frac{4q_L(4q_H^2 - 3q_L q_H + 2q_L^2)}{8(1-s_L)(2q_H^2 - 3q_L q_H + 3q_L^2) + 7(1-s_H)q_H^2} > 0 \qquad (20.4)$$

Again, differentiating (20.1) with respect to s_H ;

$$\frac{\partial q_L}{\partial s_H} = \frac{(7q_L - 4q_H)q_H^2}{8(1-s_L)(2q_H^2 - 3q_L q_H + 3q_L^2) + 7(1-s_H)q_H^2} > 0 \qquad (20.5)$$

4.1.3. Effects of R&D Subsidy on Market Shares

The market shares of firms are explained with adopting simple calculation methods. For the exact explanation of the results, the notations assumed here to define (18.2) and (19.2) are;

$a = (4w^2 - 3w + 2) > 0$, $b = w^2(4w - 7) > 0$, and $c = (4w - 1)^3 > 0$ and substituting them into equations (15) to simplify the demand functions of both firms.

$$D_H(s) = \left[\frac{8a(1-s_L)}{16a(1-s_L) - b(1-s_H)}\right] \& D_L(s) = \left[\frac{4a(1-s_L)}{16a(1-s_L) - b(1-s_H)}\right] \quad (21)$$

Differentiating (21) with respect to s_L & s_H ; we get

$$\frac{\partial}{\partial s_H} D_H(s) = \left[\frac{-8ab(1-s_L)}{[16a(1-s_L) - b(1-s_H)]^2}\right] < 0 \quad (21.1)$$

$$\frac{\partial}{\partial s_L} D_H(s) = \left[\frac{8ab(1-s_H)}{[16a(1-s_L) - b(1-s_H)]^2}\right] > 0 \quad (21.2)$$

$$\frac{\partial}{\partial s_L} D_L(s) = \left[\frac{4ab(1-s_H)}{[16a(1-s_L) - b(1-s_H)]^2}\right] > 0 \quad (21.3)$$

$$\frac{\partial}{\partial s_H} D_L(s) = \left[\frac{-4ab(1-s_L)}{[16a(1-s_L) - b(1-s_H)]^2}\right] < 0 \quad (21.4)$$

Proposition 1: *The imposition of government R&D subsidies to firms increases both low and high qualities; in contrast, subsidies to high quality firm decreases the quantity demanded and market shares but subsidy to lower quality firm increases the quantity demand and market shares.*

4.1.4. Effects of R&D Subsidy on Firms' Profits

It is obtained by substituting the optimal qualities of both firms. Using expressions (18.1) and (19.1) into their profit functions in expressions (18) and (19), the expression (22) and (23) are derived.

$$\pi_L(s) = \left[\frac{4ab[4a(1-s_L) - b(1-s_H)]}{c[16a(1-s_L) - b(1-s_H)]^2} - \frac{b^2}{2c^2(1-s_L)}\right] \quad (22)$$

$$\pi_H(s) = \left[\frac{64a^2[4a(1-s_L) - b(1-s_H)](1-s_L)}{c[16a(1-s_L) - b(1-s_H)]^2(1-s_H)} - \frac{8a^2}{c^2(1-s_H)}\right] \quad (23)$$

It is to be noted that, $c > a > b > 0$ for all $w > \frac{7}{4}$

Now, examining the effect of an R&D subsidy on the firms' profits as follows;

Differentiating (22) with respect to s_L & s_H is expressed as;

$$\frac{\partial \pi_L}{\partial s_L} = \left[\frac{256a^3 b(1-s_L)}{c[16a(1-s_L)-b(1-s_H)]^3} - \frac{b^2}{2c^2(1-s_L)^2} \right] > 0 \qquad (22.1)$$

$$\frac{\partial \pi_L}{\partial s_H} = \left[\frac{4ab^2[24a(1-s_L)-3b(1-s_H)]}{c[16a(1-s_L)-b(1-s_H)]^3} \right] > 0 \qquad (22.2)$$

Differentiating (23) with respect to s_L & s_H and expressed as;

$$\frac{\partial \pi_H}{\partial s_L} = \left[\frac{64a^2 t(1-s_H)[t\{b(1-s_H)-4a\} + 32a(1-s_H)\{4a(1-s_L)-b(1-s_H)]}{ct^2} \right] < 0 \quad (23.1)$$

$$\frac{\partial \pi_H}{\partial s_H} = \left[\frac{64a^2(1-s_L)(1-s_H)[8a(1-s_L)+b(1-s_H)]}{ct^3(1-s_H)^2} - \frac{t(1+8a^2t^2)}{ct^3(1-s_H)^2} \right] > 0 \quad (23.2)$$

Where, $t = 16a(1-s_L) - b(1-s_H) > 0$

It is seen that an increase the R&D subsidy of the lower quality firm reduces the higher quality firm's revenue. But, increasing R&D subsidy of higher quality firm increases the revenues of both firms. This is because the higher quality firm has an incentive to raise its quality since maintaining quality differentiation prevents price competition from intensifying.

4.1.5. Effects of R&D Subsidy on Social Welfare

This subsection deals to show an effect of R&D policy to maximize social welfare function. The government imposed subsidy to the firms in order to maximize the social welfare and it is obtained by reducing the government subsidy from the sum of consumer's surplus and firms' profits given by;

$$W^d = CS + PS - S \qquad (24)$$

Where, PS=producers' surplus and S=Subsidies

$$W^d = \int_{\theta_1}^{\theta_2} (\theta q_L - p_L) d\theta + \int_{\theta_2}^{1} (\theta q_H - p_H) d\theta + \pi_L + \pi_H - \frac{1}{2}(s_L q_L^2 + \frac{1}{2} s_H q_H^2)$$

$$= \frac{1}{2}\left[q_H - \frac{(p_H - p_L)^2}{(q_H - q_L)} - \frac{p_L^2}{q_L} \right] - \frac{1}{2}(q_L^2 + q_H^2)$$

$$= \frac{1}{2}\left[\frac{(12q_H^2 - q_L q_H - 2q_L^2)}{(4q_H - q_L)^2} \right] q_H - \frac{1}{2}(q_L^2 + q_H^2) \qquad (25)$$

Substituting the optimal values of $q_H^* = \frac{4a}{c(1-s_H)}$ and $q_L^* = \frac{b}{c(1-s_L)}$ in (25)

We get;

$$W^d = \left[\frac{384a^3(1-s_L)^2 - 8a^2b(1-s_H)(1-s_L) - 4ab^2(1-s_H)^2}{c[16a(1-s_L) - b(1-s_H)]^2(1-s_H)} - \frac{b^2}{2c^2(1-s_L)^2} - \frac{8a^2}{c^2(1-s_H)^2} \right] \quad (26)$$

In expression (25), the direct effect of subsidy on welfare is zero but equilibrium outcomes is the form of subsidy obtained in (26) influences on firms' outcomes. Thus, the government ought to subsidize the both firms in order to promote the R&D investments to improve product qualities and maximize the social welfare.

Differentiating (26) on both sides with respect to s_L & s_H respectively, are written as follows;

$$\frac{\partial W^d}{\partial s_L} = \frac{8a^2[16b(1-2a)(1-s_L)(1-s_H) - 33b(1-s_L)^2 - 192a(12-8a)(1-s_L)^3 + 144ab(1-s_H)(1-s_L)^2]}{c\{16a(1-s_L) - b(1-s_H)\}^3(1-s_H)} - \frac{b^2}{c^2(1-s_L)^3} > 0 \quad (26.1)$$

$$\frac{\partial W^d}{\partial s_h} = \frac{\begin{array}{l} 8ab(1-s_H)[256a^3(1-s_L)^3 - 224a^2b(1-s_L)^2(1-s_H) - 31ab^2(1-s_H)^2(1-s_L) \\ + b^3(1-s_H)^3 + 196608a^4(1-s_L)^4 - 4096a^3b(1-s_L)^2(1-s_H) - 12288a^3b(1-s_L)^3(1-s_H) \\ -1792a^2b^2(1-s_L)(1-s_H)^2 + 128ab^2(1-s_H)^3 \end{array}}{c^2\{16a(1-s_L) - b(1-s_H)\}^4(1-s_H)^2} - \frac{16a^2}{c^2(1-s_H)^3} > 0 \quad (26.2)$$

Expressions (26.1) and (26.2) prove that the effect of R&D subsidy on social welfare is positive in both cases. Hence, subsidizing both firms is socially optimal.

Proposition 2: *Government's R&D subsidy to lower quality firm increases its own profit but decreases the high quality firm's profits whereas subsidy to higher quality firm increases both firms' profits. Eventually, subsidy on both qualities improves social welfare.*

4.1.6. Effects of R&D Subsidy on Quality Differentiation

For the effect of government R&D subsidy on quality differentiation, expression (20) can be written as;

$$\frac{(16w^2 - 12w + 8)}{(4w-7)w^2} = \frac{(1-s_H)}{(1-s_L)}$$

$$\text{Or, } (16w^2 - 12w + 8)(1-s_L) = (4w^3 - 7w^2)(1-s_H) \quad (27)$$

Now, differentiating (27) on both sides with respect to s_L and s_H respectively, then, we have;

$$\frac{\partial}{\partial s_H}(16w^2 - 12w + 8)(1-s_L) = \frac{\partial}{\partial s_H}(4w^3 - 7w^2)(1-s_H)$$

$$\text{Or, } \frac{\partial w}{\partial s_H} = \frac{w^2(4w-7)}{12(1-s_H)w^2 + 12(1-s_L) - 46w + (32s_L + 14s_H)w}$$

Since $w > \frac{7}{4}$ then $w^2(4w-7) > 0$,

And then, $12(1-s_H)w^2 + 12(1-s_L) - 46w + (32s_L + 14s_H)\omega < 0$

$$\text{Hence, } \frac{\partial w}{\partial s_H} < 0 \quad (27.1)$$

Similarly,

$$\frac{\partial}{\partial s_L}(16w^2-12w+8)(1-s_L)=\frac{\partial}{\partial s_L}(4w^3-7w^2)(1-s_H)$$

$$\frac{\partial w}{\partial s_L}=\frac{4(4w^2-3w+2)}{-12(1-s_H)w^2-12(1-s_L)+46w-(32s_L+14s_H)w}$$

Here, we obtained two cases;

(i) We obtained $16w^2-12w+8>0$

(ii) Since $0<s_i<1$ then, $-(12(1-s_H)w^2-12(1-s_L)+46w-(32s_L+14s_H)w>0$

$$\text{Hence, } \frac{\partial w}{\partial s_L}>0 \qquad (27.2)$$

Lemma 1: *The government R&D subsidy to lower quality firm increases the degree of quality differentiation whereas subsidy to higher quality firm decreases the degree of quality differentiation.*

4.2 Imposition of Uniform R&D Subsidy Policy

Let us assume that government in some cases imposes uniform scale of subsidy (i.e. $s_L=s_H=s$ say) on both qualities of two firms. Then, equilibrium quality outcomes and the welfare effects of optimal subsidy are calculated using the basic model from expression (20) i.e. $\frac{(16w^2-12w+8)}{(4w-7)w^2}=1$,

$$\text{then, } 4w^3-23w^2+12w-8=0 \qquad (28)$$

By algebraic solution, equation (28) produces an ideal level of the degree of quality differentiation i.e. $w\approx 5$ .251233.Now, from eq. (18.2) and (19.2), we get;

$$q_H^*=4\frac{(4w^2-3w+2)}{(4w-1)^3(1-s)} \text{ Then, } q_H^*=\frac{1}{(1-s)}(0.253311) \qquad (29)$$

$$\text{And, } q_L^*=\frac{w^2(4w-7)}{(4w-1)^3(1-s)} \text{ Then, } q_L^*=\frac{1}{(1-s)}(0.048238) \qquad (30)$$

Similarly, it is obtained that $\frac{\partial q_H}{\partial s}>0$ and $\frac{\partial q_L}{\partial s}>0$ for all $0<s<1$.

Above equations (29) and (30) represent the Nash equilibrium since $\frac{1}{(1-s)}>0$ and the other facts for the equilibrium outcomes are same as distinct subsidy case of the previous section.

Then, profits from (18) and (19) are explained as;

$$\pi_L(s) = \frac{(w-1)}{(4w-1)^2} wq_L - \frac{1}{2}q_L{}^2 + \frac{1}{2}sq_L^2 = \frac{1}{(1-s)}(0.001527) \qquad (31)$$

$$\pi_H(s) = 4\frac{(w-1)}{(4w-1)^2} wq_H - \frac{1}{2}q_H^2 + \frac{1}{2}sq_H^2 = \frac{1}{(1-s)}(0.024439) \qquad (32)$$

The social welfare obtained in this setting from equation (25) is written as;

$$W(s) = \frac{1}{2}\left[\frac{(12w^2 - w - 2)}{(4w-1)^2}\right]q_H - \frac{1}{2}(q_L^2 + q_H^2) = \frac{1}{(1-s)^2}(0.069184) \qquad (33)$$

Government maximizes social welfare from expression (27) with respect to subsidy (s), then we get;

$$\frac{\partial W^d}{\partial s} = \frac{0.138368}{(1-s)^3} > 0\,. \qquad (34)$$

Since the welfare function is increasing in subsidy, the government chooses the highest subsidy level for the quality improvement of both firms i.e. $s^* = \bar{s}$ (Notice that the government can provides optimum level of subsidy $\bar{s}$ such that $\bar{s} < 1$). Then, equilibrium outcomes of qualities with optimal subsidy are;

$$q_H^*(s) = \frac{1}{(1-\bar{s})}(0.253311)\,;\, q_L^*(s) = \frac{1}{(1-\bar{s})}(0.048238) \qquad (35)$$

Then,optimal profits from expressions (31) and (32) are explained as follows;

$$\pi_L^*(s) = \frac{1}{(1-\bar{s})}(0.001527) \qquad (36)$$

$$\pi_H^*(s) = \frac{1}{(1-\bar{s})}(0.024439) \qquad (37)$$

Socially Optimal welfare is;

$$W^d(s^*) = \frac{1}{(1-\bar{s})^2}(0.069184) \qquad (38)$$

Proposition 3: *When a government imposes R&D subsidies uniformly, quality equilibrium outcomes, market shares, firms' profits and welfare in subsidized market are always increasing.*

5. CONCLUDING REMARKS

The implications of government R&D subsidy investments improve the product qualities of the firms. The study has drawn some key remarks from the entire analysis such as; *First,* it has proved that R&D distinct subsidies on quality demanded depends on the firms' strategic relationship on their quality decisions where effects of distinct and uniform R&D subsidy on (i) quality differentiation, (ii) producer's profit, and (iii) social welfare, concludes that an R&D subsidy to lower quality firm increases its own profit but decreases the high quality firm's profits. However, subsidy to higher quality firm increases both firms' profit. Eventually, subsidy on both qualities improves the social welfare. *Second,* in the extended model with uniform subsidy case; producers receives more profits in which welfare maximizing strategy shifts the focus of government policy to enhance welfare by modifying the behavior of the firms.

This study has thus highlighted two polar forms of the government's objective function: either quality differentiation or social welfare is maintained with whatever distribution of subsidy, or vice-versa. Obviously, the more realistic case with a trade-off between the two objectives is also interesting. It has been indeed tried in finding the accurate optimal subsidy level imposed by the governments of industrial countries to carry through the analysis with a more general social welfare function. But this was not very successful, since the problem became surprisingly difficult and vague to solve at this moment.

Nonetheless, this study has provided a genuine explanation of R&D subsidy phenomenon in a domestic market of a duopolistic industry. In the focal point, the result obtained in here predicts that subsidizing of two firms by a government is socially optimal. More importantly, uniform R&D subsidization policy is found quite effective in both social and economic viewpoints. Thus, it is concluded that the government of industrial economies may be obliged to apply this policy in the quality differentiated industries.

For further research, it will be better if we consider other regulatory instruments to compare the relative performance of policies toward quality improvement. For example, (i) we can compare the R&D subsidy effect with or without minimum quality standard (MQS), regarding subsidy size or welfare.(ii) Or, we can also compare the welfare effect between R&D subsidy and output subsidy.

Funding: This study received no specific financial support.

Competing Interests: The author declare that there are no conflict of interests regarding the publication of this paper.

REFERENCES

Aoki, R. and T.J. Prusa, 1997. Sequential versus simultaneous choice with endogenous quality. International Journal of Industrial Organization, 15(1): 103–121.

Bagwell, K. and R.W. Staiger, 1994. The sensitivity of strategic and corrective R&D policy in oligopolistic industries. Journal of International Economics, 36(1-2): 133-150.

Brander, J.A. and B.J. Spencer, 1985. Export subsidies and international market share rivalry. Journal of International Economics, 18(1-2): 83–100.

Choi, J.C. and H.S. Shin, 1992. A comment on a model of vertical product differentiation. Journal of Industrial Economics, 40(2): 229-231.

European Commission, 1995. Fourth survey on state aid in the European Union in the manufacturing and certain other sectors. Brussels: European Commission.

Gabszewicz, J. and J.F. Thisse, 1979. Price competition, quality and income disparities. Journal of Economic Theory, 20(3): 340–359.

Ishii, Y., 2014. Quality-price competition and product R&D investment policies in developing and developed countries. Economic Record, 90(289): 197-206.

Jinji, N. and T. Toshimitsu, 2006. Optimal policy for product R&D with endogenous quality ordering: Asymmetric duopoly. Australian Economic Papers, 45(2): 127-140.

Lahiri, S. and Y. Ono, 1999. R&D subsidies under asymmetric duopoly: A note. Japanese Economic Review, 50(1): 104–111.

Lee, S.H. and R.K. Phuyal, 2013. Strategic entry deterrence with limiting qualities under minimum quality standards. Japanese Economic Review, 64(4): 50-64.

Mussa, M. and S. Rosen, 1978. Monopoly and product quality. Journal of Economic Theory, 18(2): 301-317.

Phuyal, R.K., 2014. Mixed market competition in a quality differentiated duopoly model—an approach to banking sector. Chinese Business Review, 13(1): 19-27.

Ronnen, U., 1991. Minimum quality standards, fixed costs and competition. RAND Journal of Economics, 22(4): 490–504.

Shaked, A. and J. Sutton, 1982. Relaxing price competition through product differentiation. Review of Economic Studies, 49(1): 3–13.

Shin, I.Y. and H.H. Kim, 2010. The effect of subsidy policies on the product quality improvement. Economic Modeling, 27(3): 687-696.

Symeonidis, G., 2003. Comparing cournot and Bertrand equilibrium in a differentiated duopoly with product R&D. International Journal of Industrial Organization, 21(1): 39–55.

Taba, Y., 2016. Optimal product R&D policies with endogenous quality choices and unilateral spillover, the B.E. Journal of Economic Analysis and Policy, 16(1): 365-391.

Toshimitsu, T., 2003. Optimal R&D policy and endogenous quality choice. International Journal of Industrial Organization, 21(8): 1159–1178.

White, M., 1996. Mixed oligopoly, privatization and subsidization. Economics Letters, 53(2): 189-195.

Zhou, D., B.J. Spencer and I. Vertinsky, 2002. Strategic trade policy with endogenous choice of quality and asymmetric costs. Journal of International Economics, 56(1): 205–332.