Asian Economic and Financial Review
ISSN(e): 2222-6737/ISSN(p): 2305-2147

URL: www.aessweb.com

Asian Economic and Financial Review

ANALYSIS OF THE CONTAGION EFFECT TO THE CREDIT DERIVATIVE VALUATION

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ABSTRACT

This study explores a credit derivative pricing model with counterparty risk and the contagion effect. To compare with the standard credit derivative pricing model, we analyze the counterparty risk and the contagion effect to a kth-to-default Basket Credit Linked Note (BCLN) valuation by Monte Carlo simulation. Counterparty risk and the contagion effect show significant influence for kth-to-default BCLN valuation. Especially with high k, the BCLN pricing model with the counterparty risk and the contagion effect captures chain defaulting phenomenon successfully. It indicates the higher the kth-to-default BCLN or the lower the correlation degree within reference entities, the more significant the contagion effect becomes to the kth-to-default BCLN valuation. Parameters sensitivity analyses indicate that the coupon rates of the third-to-default BCLN are lower with the higher risk-free rate or the shorter maturity. The higher hazard rate or the lower recovery rate of reference entities results in the higher coupon rates of the third-to-default BCLN. Lastly, the hazard rate and the recovery rate of the counterparty are less sensitive to the BCLN pricing. The major contribution in this study is that the credit derivative pricing model with the contagion effect is developed to capture the default chain reaction, and the valuation performance is significant numerically.

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Contribution/ Originality

This study is one of very few studies which have investigated the credit derivative valuation with the contagion effect to capture the default chain reaction especially during the global financial distress period.

1. INTRODUCTION

The credit crunch of 2007-2008 in the United States is the most substantial financial crisis to become a global phenomenon, enveloping emerging and developed financial and economy markets, with the collapse of Lehman Brothers worldwide and the strategic move of Goldman Sachs and Morgan Stanley to regulated bank status. Due to the collapses of large financial institutions in the subprime mortgage crisis, more and more researchers and practitioners have found that the issuer default risk and the correlations between the issuer and the underlying assets

significantly affect the credit derivatives pricing. And an appropriate modeling and forecasting of inherent credit risk is emphasized by risk managers and analytics. Generally, default probability of the borrower, losses given default, exposures, and correlation between borrowers are considered in credit risk models. Hull and White (2001); Hui and Lo (2002) and Mashal and Naldi (2005) analyze how the possibility of issuer default affects the credit derivatives pricing. Relatively, the counterparty risk model proposed by Wu (2010) is considered directly the default correlation between the reference entities and the issuer. Their numerical results obviously indicate that issuer default risk impacts the Basket Credit Linked Note (BCLN) valuation.

Dungey (2008) measures the contagion effect in the money market and equity market in US, UK, Europe, Japan and Australia from July 2007 to September 2008. Results show that there exists a dominant effect from contagion at 78 percent of observed volatility in the US equity market. In all other markets the contagion effects lie between 4 and 10 percent, with the greatest contribution in the Australian money market and the smallest in the Japanese equity market. In addition, the asymmetric dependency between borrowers in a portfolio has been gradually emphasized and is first applied to the credit pricing model via credit infection or neural network connection between borrowers by Davis and Lo (2001) and Egloff et al. (2007). They find that even a small infection probability has a dramatic impact on the tails of the loss distribution. Neu and Kühn (2004) observe the collective phenomena such as bursts and avalanches by extending the standard Credit Metrics credit risk model based on heterogeneous, and non-symmetric inter firm dependencies. Then Rösch and Winterfeldt (2008) develop a contagion extension of factor models which are used in most credit risk models. Their findings indicate that contagion effects are significant and can seriously affect loss distribution. The structure form approach (Merton, 1974; Black and Cox, 1976; Geske, 1977; Leland, 1994; Longstaff and Schwartz, 1995) and the reduced form approach (Jarrow and Turnbull, 1995; Duffie and Singleton, 1997; Jarrow et al., 1997) are two main credit risk models. They consider default events as the reflection in a company's capital structure and being exogenous, separately. The copula approach for specifying the dependence among default times was developed and extended by Li (2000) and Andersen et al. (2003); Hull and White (2004); Laurent and Gregory (2005) and Sklar (1959). The conception is that default events are independent conditional on the common factor. This study explores a credit derivative pricing model with counterparty risk and the contagion effect. To compare with the naïve credit derivative pricing model, we analyze counterparty risk and the contagion effect to the BCLN valuation by Monte Carlo simulation. A BCLN is a note with coupon linked to credit events of reference entities. The buyer of a BCLN during the contract period receives constant coupon payments periodically until the default event occurs. For example, the default event of a first to default BCLN means that one of the reference entities goes into default before the contract maturity. The buyer will receive the nominal at the maturity date if the default event does not occur, but be redeemed at the recovery value determined initially.

The structure of this paper is organized as follows: In Section 2, we introduce the factor Gaussian copula model, the BCLN valuation model without counterparty risk, the BCLN pricing model with counterparty risk. The simulation results of the BCLN pricing with the issuer default probability and the contagion effect analysis are presented in Section 3. Section 4 is the conclusion.

2. THE MODEL

2.1. The Factor Gaussian Copula

Consider a portfolio with N reference entities, and the default times and hazard rates are $\tau_1, \tau_2, \ldots, \tau_N$ and $\lambda_1, \lambda_2, \ldots, \lambda_N$, respectively. According to the reduced form model, the cumulative default probability of each reference entity before time t is

$$F_i(t) = P(\tau_i \leq t) = 1 - e^{-\lambda_i t}, \quad i = 1, 2, \ldots, N. \qquad (1)$$

Applying the Gaussian copula, the multivariate joint distribution of default times is

$$F(\tau_1, \tau_2, \ldots, \tau_N) = \Phi(\phi^{-1}(F_1(\tau_1)), \phi^{-1}(F_2(\tau_2)), \ldots, \phi^{-1}(F_N(\tau_N))). \qquad (2)$$

Φ and ϕ denote the multivariate and one dimensional cumulative normal distribution, respectively. In the single factor model, assume random variable, X_i, corresponding to the reference entity defaulting time associates a common factor Y and a specific risk factor ε_i,

$$X_i = \rho_{X_i,Y} \cdot Y + \sqrt{1 - \rho_{X_i,Y}^2} \cdot \varepsilon_{X_i}, \quad i = 1,2,\ldots,N. \qquad (3)$$

Y and ε_i are independent standard normal variables and the correlation coefficient between X_i and Y is $\rho_{X_i Y}$. Then the reference entity defaulting time is

$$\tau_i = F_i^{-1}(\phi(X_i)) = \frac{-\ln(1 - \phi(X_i))}{\lambda_i}, \quad i = 1, 2, \ldots, N. \qquad (4)$$

2.2. Pricing BCLN without Counterparty Risk

Assume a kth-to-default BCLN involving N reference entities which the notional principal of each reference entity is one dollar. The coupon rate is c, the payment dates are $t_i, i = 1, 2, \ldots, T$, and the maturity date of the BCLN is t_T. Moreover, Q denotes the risk-neutral probability measure, $I(\cdot)$ is an indicator function, r is the risk-free rate, τ_k is the kth default time, and δ_k is the recovery rate of the kth default reference entity. The value of a kth-to-default BCLN is

$$E^Q[c \times \sum_{i=1}^{T} e^{-rt_i} I(t_i < \tau_k) + \delta_k \times e^{-r\tau_k} I(\tau_k \leq t_T) + e^{-rt_T} I(\tau_k > t_T)] \qquad (5)$$

Then the fair coupon rate c at the initial time can be yield,

$$c = \frac{E^Q[1 - \delta_k \times e^{-r\tau_k} \times I(\tau_k \leq t_T) - e^{-rt_T} \times I(\tau_k > t_T)]}{E^Q[\sum_{i=1}^{T} e^{-rt_i} \times I(\tau_k > t_i)]} \qquad (6)$$

2.3. Pricing BCLN with Counterparty Risk

To base on the standard situation that neither counterparty risk nor the contagion effect is considered, Wu (2010) considers the counterparty risk directly the default correlation between the reference entities and the issuer. The random variable corresponding to the default time of the underlying reference entity is

$$X_i = \rho_{X_i,Z} \cdot Z + \sqrt{1 - \rho_{X_i,Z}^2} \cdot \varepsilon_{X_i}, \quad i = 1,2,\ldots,N. \qquad (7)$$

The common factor in the standard model is replaced by the issuer variable, Z.

The issuer default risk is involved in the BCLN pricing by Wu (2010) he considers whether the issuer defaults before or after the kth default time. Assume $\hat{\tau}$, $\hat{\delta}$, and $\hat{\lambda}$ be the issuer default time, the issuer recovery rate, and hazard rate, respectively. The value of a kth-to-default BCLN is

$$
E^Q[c \times \sum_{i=1}^{T} e^{-rt_i} I(t_i < \min(\tau_k, \hat{\tau})) + \delta_k \times e^{-r\tau_k} I(\tau_k < \min(\hat{\tau}, t_T))
$$
$$
+ \hat{\delta} \times e^{-r\hat{\tau}} I(\hat{\tau} < \min(\tau_k, t_T)) + e^{-rt_T} \times I(t_T < \min(\tau_k, \hat{\tau}))]
$$

(8)

Then the fair coupon rate c at the initial time can be yield,

$$
c = \frac{E^Q \begin{bmatrix} 1 - \delta_k \times e^{-r\tau_k} \times I(\tau_k < \min(\hat{\tau}, t_T)) \\ - \hat{\delta} \times e^{-r\hat{\tau}} \times I(\hat{\tau} < \min(\tau_k, t_T)) - e^{-rt_T} \times I(t_T < \min(\tau_k, \hat{\tau})) \end{bmatrix}}{E^Q[\sum_{i=1}^{T} e^{-rt_i} \times I(t_i < \min(\tau_k, \hat{\tau}))]}
$$

(9)

2.4. Pricing BCLN with Counterparty Risk and the Contagion Effect

In this study, the major point is to examine the BCLN valuation with incorporating the counterparty risk and the contagion effect. The pricing model of the former sector is extended with the contagion idea proposed by Rösch and Winterfeldt (2008). The credit contagion effect in Rösch and Winterfeldt (2008) is estimated by the default rate of the infecting firms. Assume there are two groups of firms, the infecting firms and the infected firms. Default events in the infecting firms group infect the infected firms group by the contagion effect parameter, $\beta < 0$, and the default rate of the infecting firms, D/I. D and I denote the defaulted number of the infecting firms and the number of the infecting firms, separately. The random variable corresponding to the infecting firms and the infected firms are

$$
X_i = \begin{cases} \rho_{X_i,Y} \cdot Y + \sqrt{1 - \rho_{X_i,Y}^2} \cdot \varepsilon_{X_i} & \text{if } i \in \text{infecting firm} \\ \rho_{X_i,Y} \cdot Y + \sqrt{1 - \rho_{X_i,Y}^2} \cdot \varepsilon_{X_i} + \beta \cdot \dfrac{D}{I} & \text{if } i \in \text{infected firm} \end{cases}
$$

(10)

Besides, the fair coupon rate c of the nth to default BCLN at the initial time is the same as equation (9).

3. NUMERICAL RESULTS

3.1. Valuation of a BCLN with Different Models

Suppose that 10 reference entities have notional principal one dollar, hazard rate 5%, recovery rate 30%, the coupon is paid annually, and the risk free rate is 2%. 100,000 runs of Monte Carlo simulation are implemented, and the fair coupon rates of a 5 year kth-to-default BCLN with standard BCLN pricing model are shown in Figure 1. As expected, the fair coupon rate of a BCLN decreases with k and the absolute value of correlation degree within reference entities as a result of the low joint default event possibility. Furthermore, let the hazard rate and recovery rate of the issuer be 1% and 30%, respectively. The fair coupon rates of a 5 year kth-to-default BCLN with the counterparty risk are shown in Figure 2. The BCLN coupon rate curve becomes flat, especially when k=3. It is because that the BCLN coupon rate varies less with the low degree correlation within reference entities, but rises apparently when the reference entities are high positive or negative correlated. Moreover, assume the contagion effect parameter to be -2 and Figure 3 displays the fair coupon rates of a 5 year kth-to-default BCLN both with counterparty risk and the contagion effect. No matter what the correlation degrees within the reference entities are, the fair coupon rate of a kth-to-default BCLN gets higher. And the higher k is, the higher fair coupon rate of a BCLN is. When the

third-to-default BCLN is evaluated, the coupon rate is nearly according to the correlation among reference entities, but it varies significantly if the contagion effect is considered.
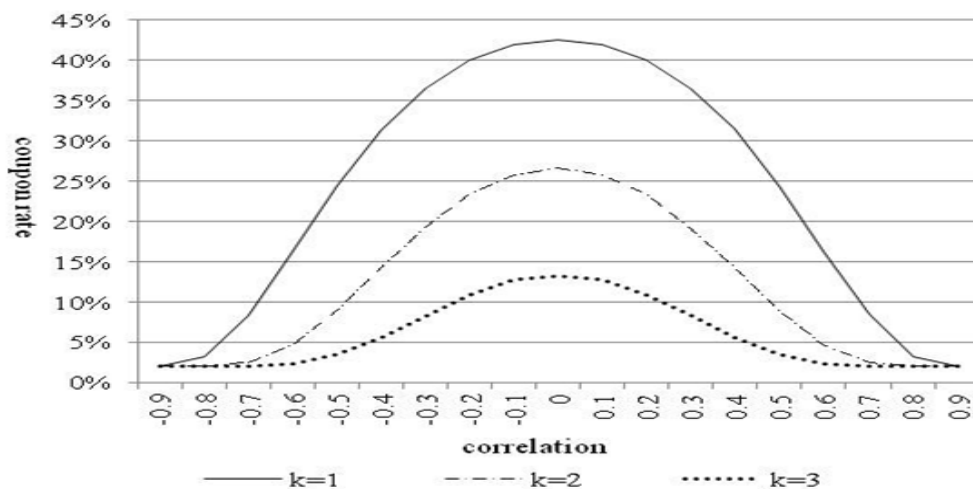

Figure-1. Coupon rates of the kth-to-default BCLN in the standard BCLN pricing model with k=1, k=2, and k=3.
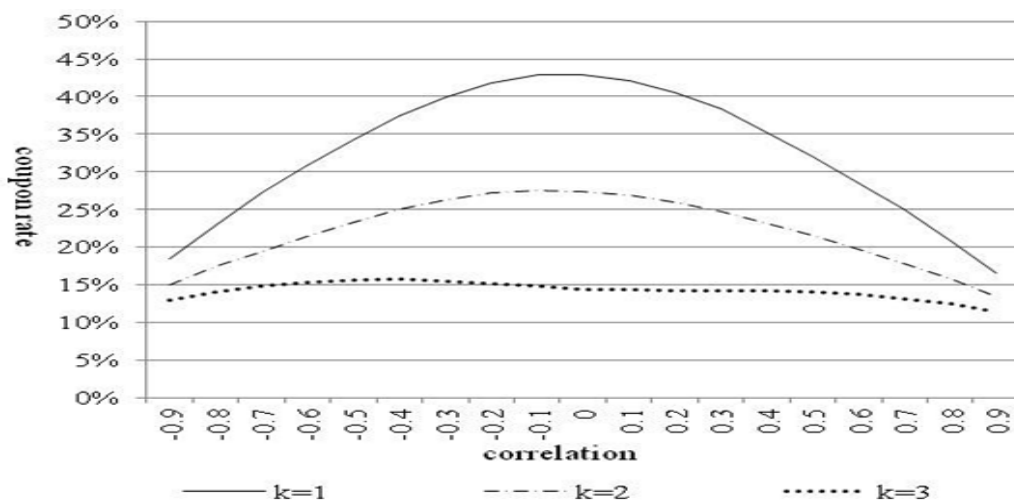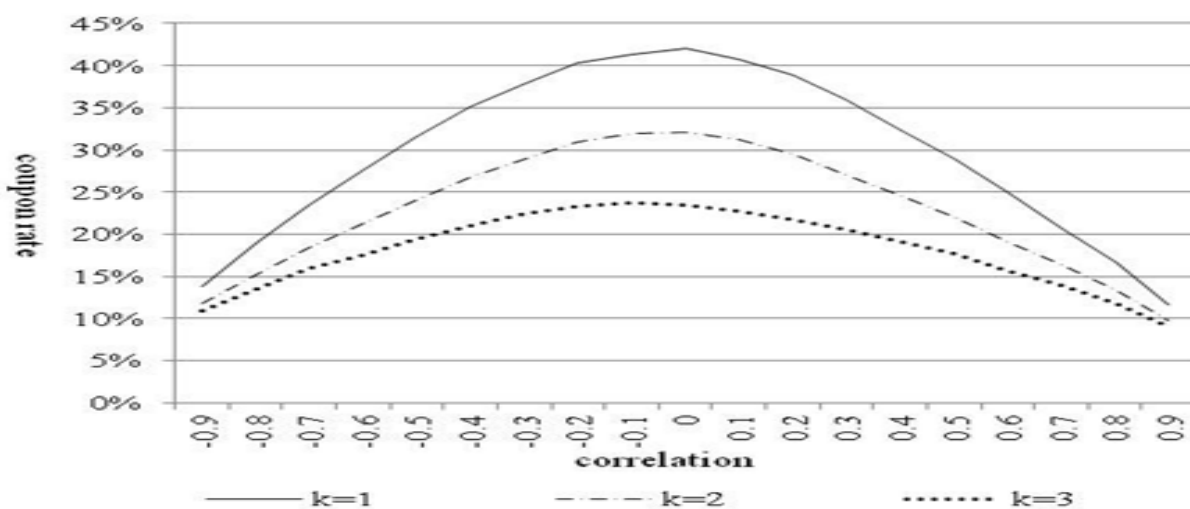**Source:** Simulation results.

Figure-2. Coupon rates of the kth-to-default BCLN in the BCLN pricing model with the counterparty risk (k=1, k=2, and k=3).
Source: Simulation results.

Figure-3. Coupon rates of the kth-to-default BCLN in the BCLN pricing model with counterparty risk and the contagion effect (k=1, k=2, and k=3).
Source: Simulation results.

Figure 4, Figure 5, and Figure 6 display the simulation results of the first-, the second-, and the third-to-default BCLN with different pricing models. Relative to the standard BCLN pricing model, the coupon rate of a BCLN pricing model with counterparty risk is always higher due to the existence of the issuer default possibility. Moreover, if the contagion effect is considered, the fair coupon rate of the BCLN pricing model with counterparty risk and the contagion effect becomes much higher except for the first-to-default BCLN. It likely because that when the first-to-default BCLN is considered, the contract terminated simultaneously with the issuer default or the first reference entity default occurs, so the contagion interaction among the reference entities has less impact on the BCLN pricing. On the other hand, the contagion effect plays actively for the second- or the third-to-default BCLN pricing.
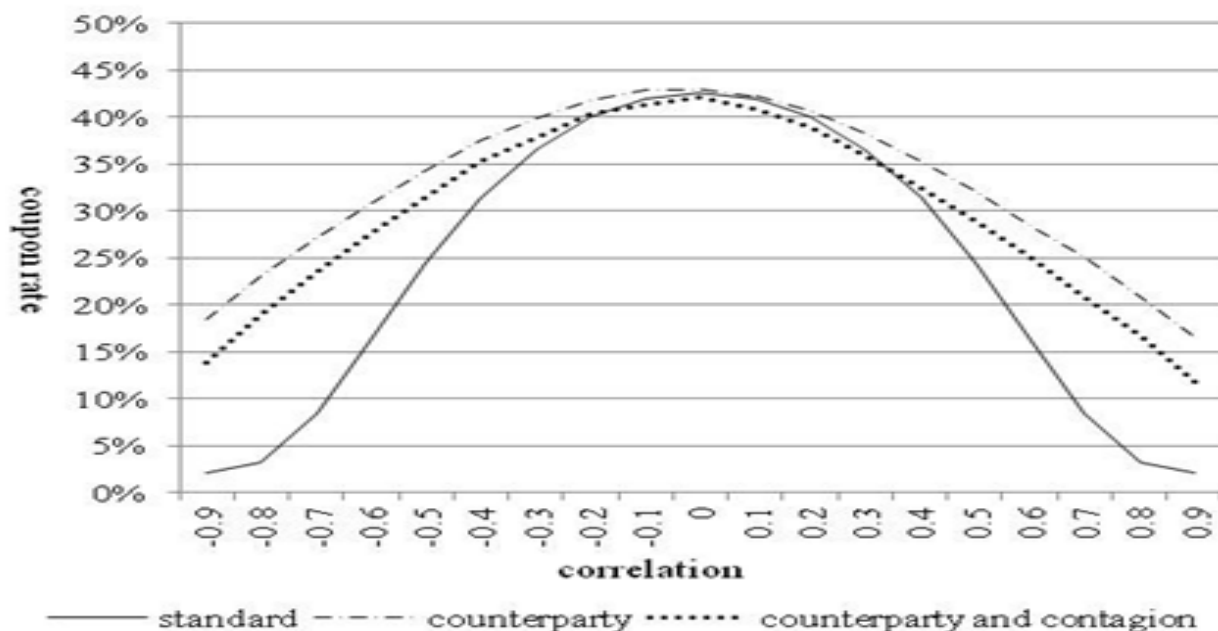
Figure-4. Coupon rates of the first-to-default BCLN in three models, the standard BCLN pricing model, the BCLN pricing model with counterparty risk, and the BCLN pricing model with counterparty risk and the contagion effect.
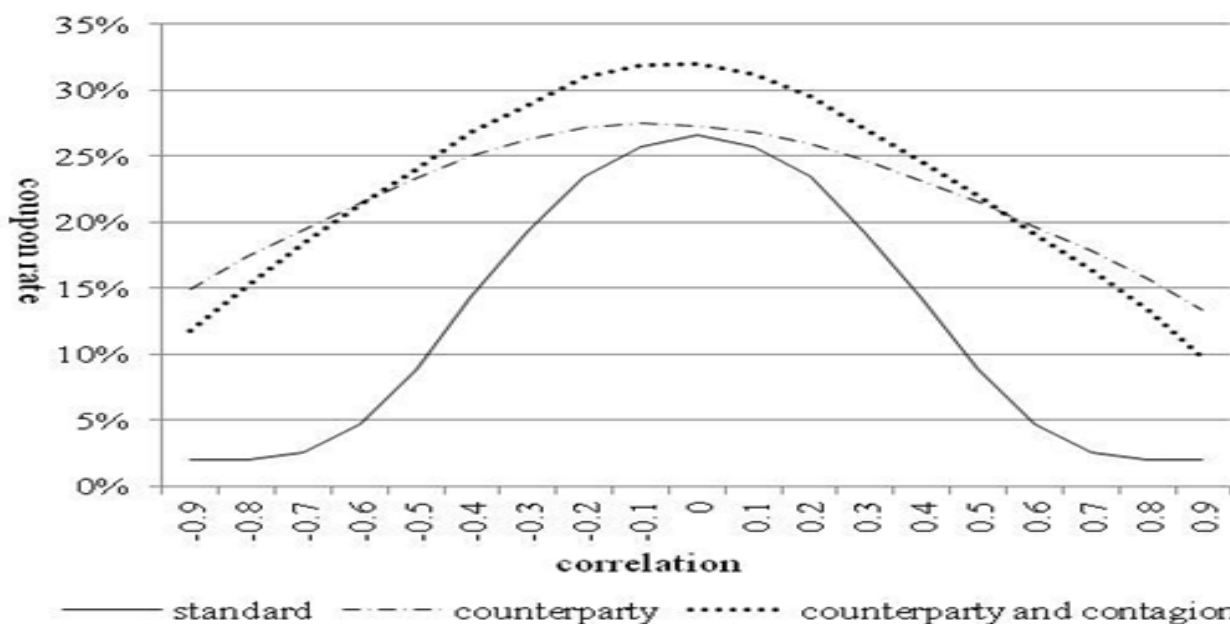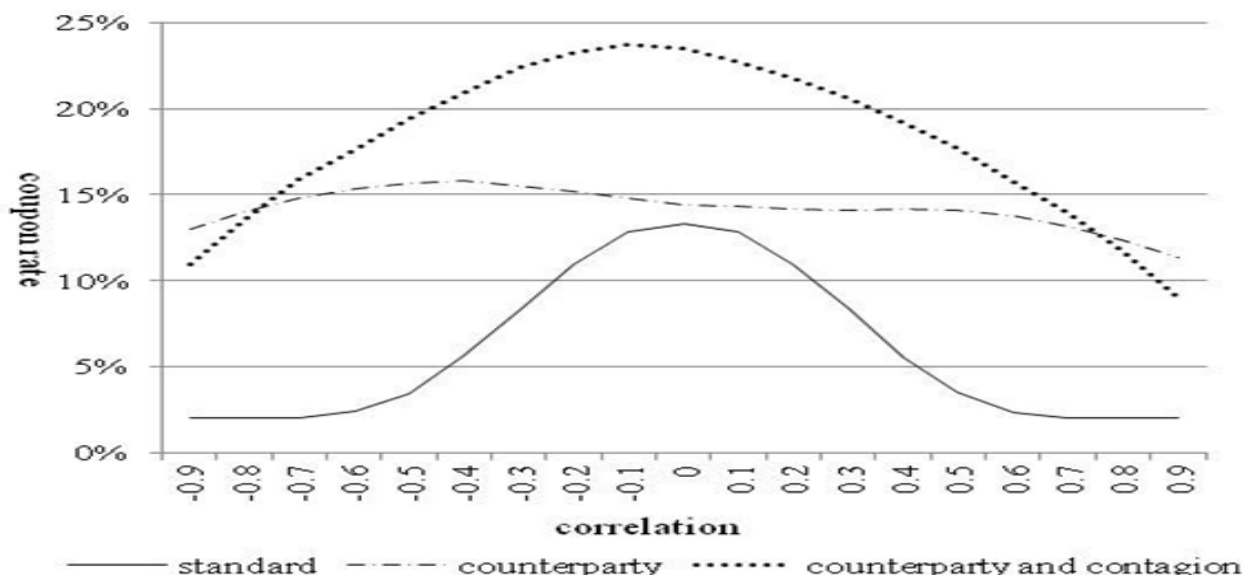**Source:** Simulation results.

Figure-5. Coupon rates of the second-to-default BCLN in three models, the standard BCLN pricing model, the BCLN pricing model with counterparty risk, and the BCLN pricing model with counterparty risk and the contagion effect.
Source: Simulation results.

Figure-6. Coupon rates of the third-to-default BCLN in three models, the standard BCLN pricing model, the BCLN pricing model with counterparty risk, and the BCLN pricing model with counterparty risk and the contagion effect.
Source: Simulation results.

3.2. The Contagion Parameter Analysis

To understand the contagion parameter influence on the BCLN valuation, this study let the contagion parameters, β, be -1, -2, -3, -4, -5, -6, -7, -8, and -9. The higher the absolute value of the contagion parameter is, the larger the contagion effect intensity is. The first-, the second-, and the third-to-default BCLN valuations with different contagion effect intensities are shown in Figure 7, Figure 8, and Figure 9. When the first-to-default BCLN is considered, Figure 7 shows that the coupon rate of a BCLN decreases with the contagion effect intensity increases gradually. But the first-to-default BCLN valuation varies less for the extreme contagion effect parameter (β=-7, -8, and -9).

When the second- or the third-to-default BCLN is considered, the contagion effect displays an interesting phenomenon with different correlation degrees of reference entities. The relative results are displayed in Figure 8 and Figure 9. Obviously, with the highly positive or negative correlated reference entities, the coupon rate of a BCLN decreases with the contagion effect intensity increases. Otherwise, the contagion effect is not explicit with the low degree correlation of reference entities. It indicates that if the higher kth-to-default BCLN or a low correlation degree within reference entities is considered, the contagion effect is more remarkable to the kth-to-default BCLN valuation.
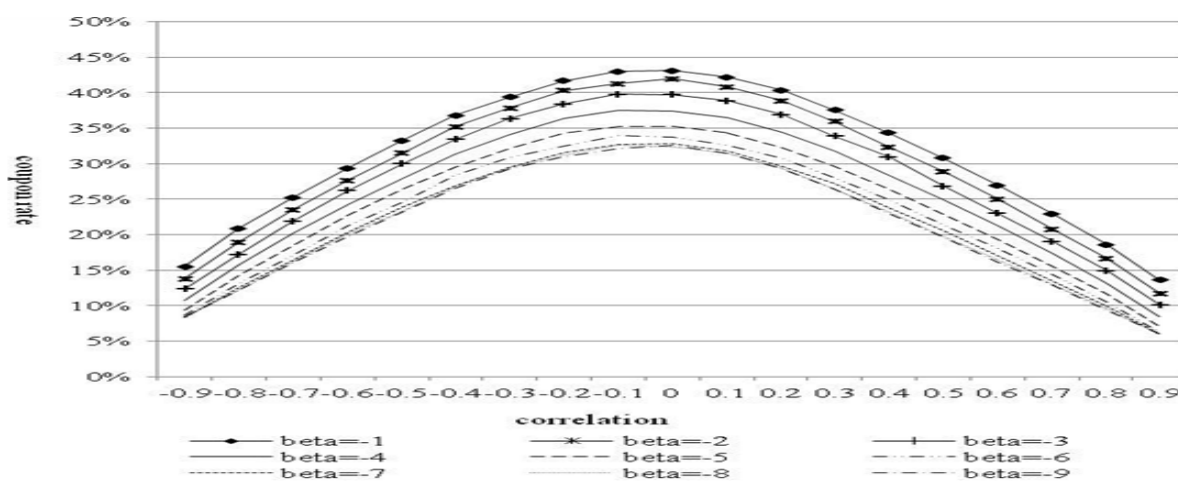

Figure-7. Coupon rates of the first-to-default BCLN with different contagion effect parameters (β=-1, -2, -3, -4, -5, -6, -7, -8, and -9).
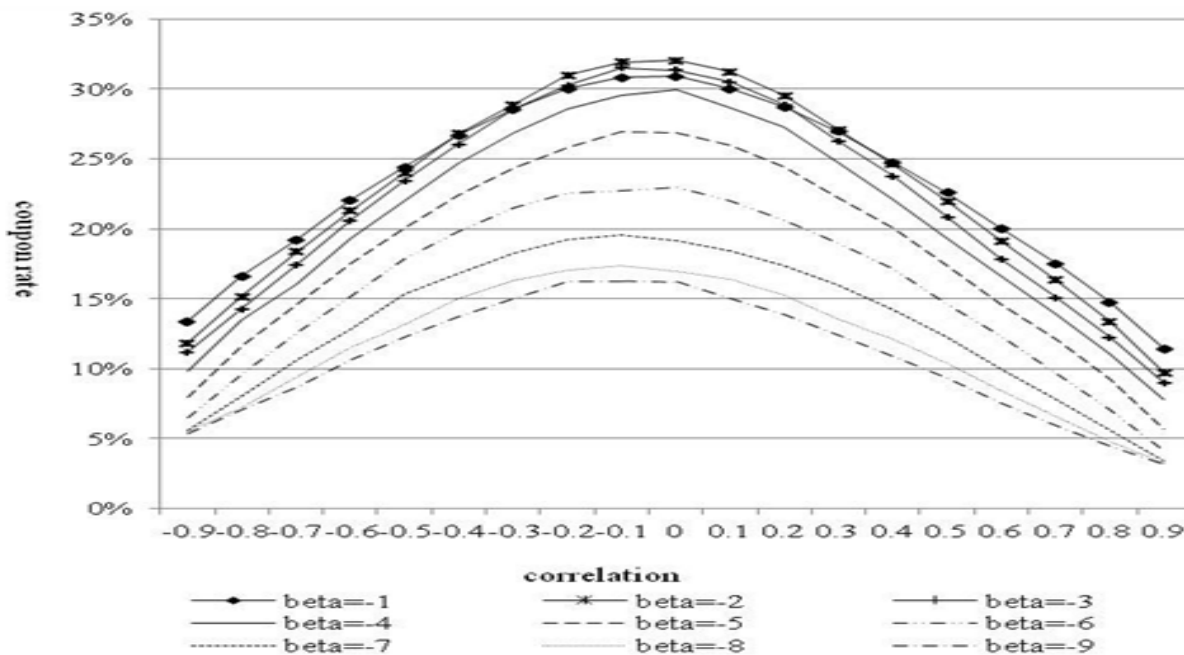**Source:** Simulation results.

Figure-8. Coupon rates of the second-to-default BCLN with different contagion effect parameters (β=-1, -2, -3, -4, -5, -6, -7, -8, and -9).
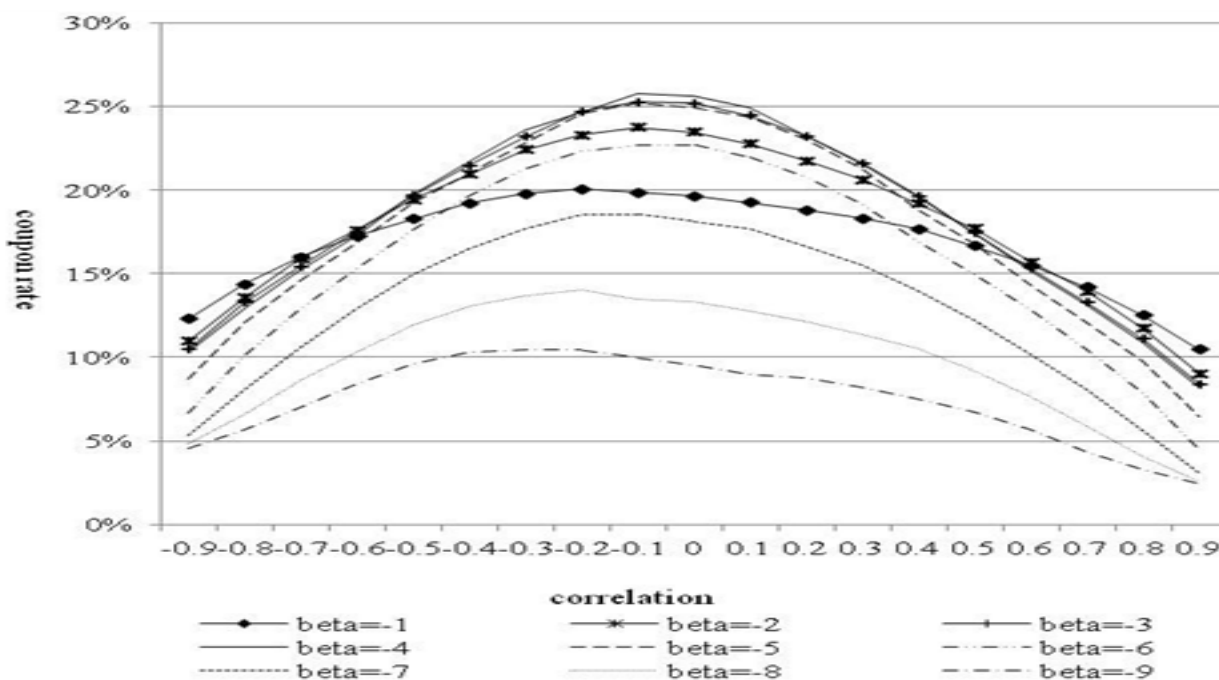**Source:** Simulation results.

Figure-9. Coupon rates of the third-to-default BCLN with different contagion effect parameters (β=-1, -2, -3, -4, -5, -6, -7, -8, and -9).
Source: Simulation results.

3.3. Other Parameters Sensitivity Analyses

Both counterparty risk and the contagion effect are considered, we study the sensitivity analyses with the risk-free rate, the maturity, the hazard rate of reference entities, the recovery rate of reference entities, the hazard rate of the counterparty, and the recovery rate of the counterparty. Figure 10 shows that the coupon rates of the third-to-default BCLN are lower with the higher risk-free rate, since the higher discount effect. Due to the higher occurrence possibility of default event, the coupon rates are higher with the longer maturity as the results of Figure 11.
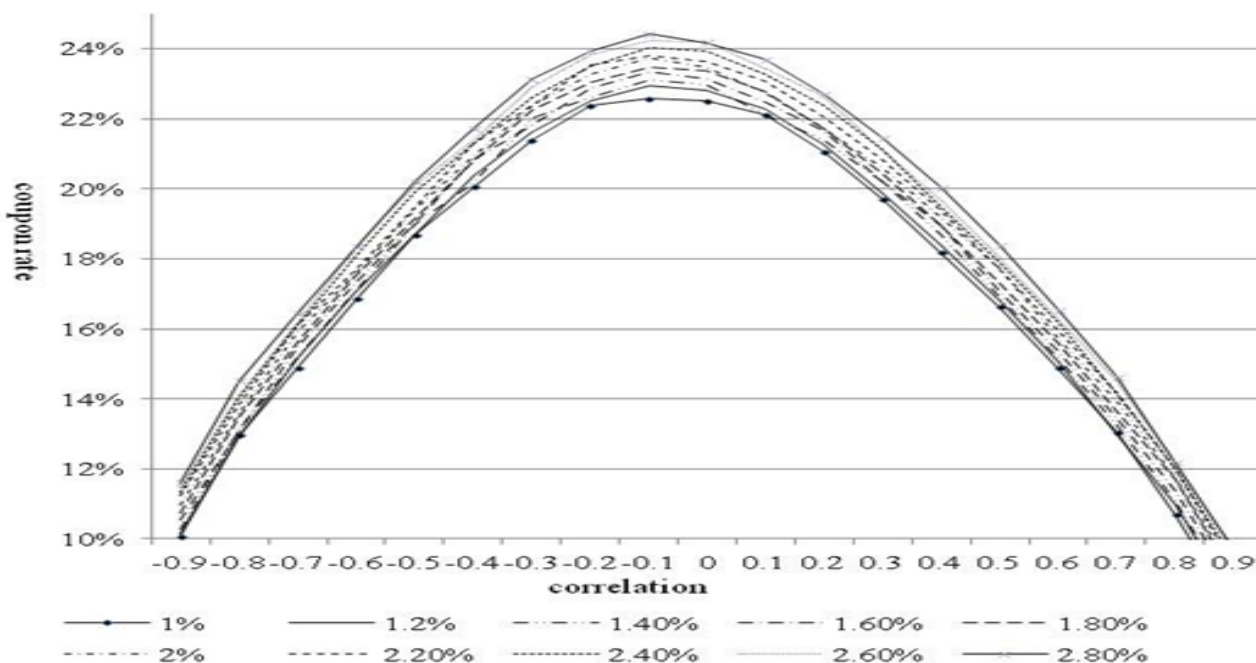
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Figure-10. Coupon rates of the third-to-default BCLN with different risk-free rate (r=1%, 1.2%, 1.4%, 1.6%, 1.8%, 2%, 2.2%, 2.4%, 2.6%, and 2.8%).
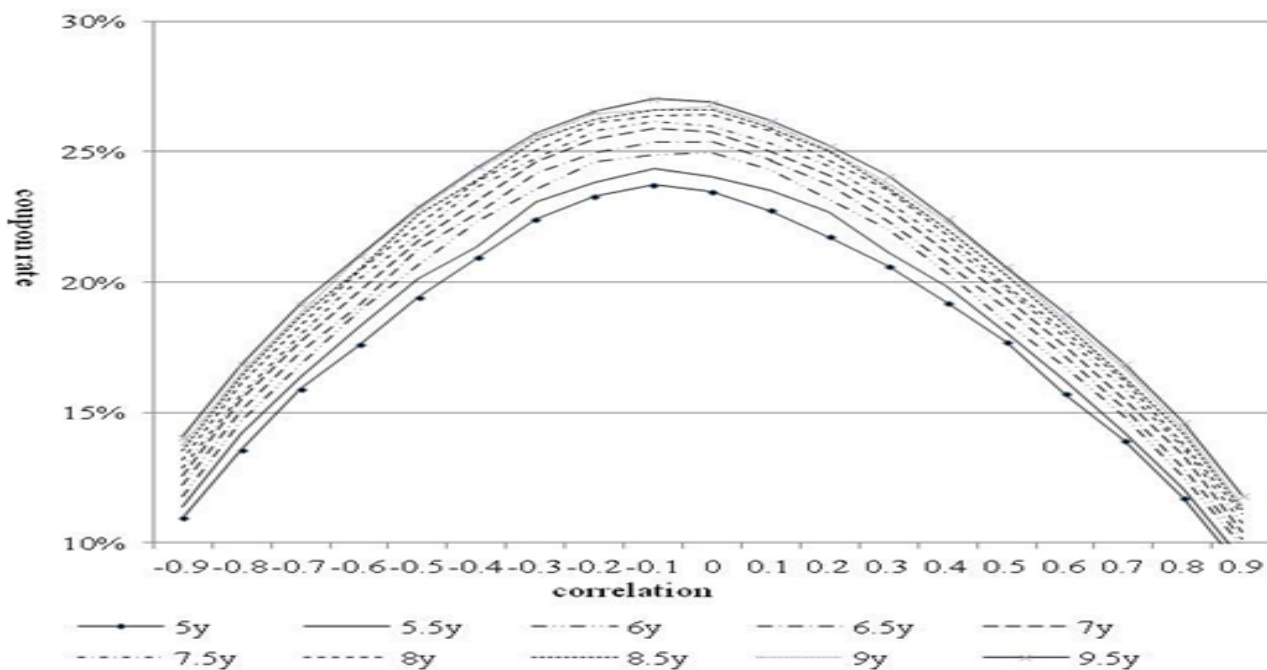**Source:** Simulation results.

Figure-11. Coupon rates of the third-to-default BCLN with different maturity (t_T =5y, 5.5y, 6y, 6.5y, 7y, 7.5y, 8y, 8.5y, 9y, and 9.5y).
Source: Simulation results.

Figure 12 and Figure 13 show that the higher hazard rate or the lower recovery rate of reference entities results in the higher coupon rates of the third-to-default BCLN. Explicitly, the higher hazard rate or the lower recovery rate causes the severer expected loss.

Lastly, the hazard rate and the recovery rate of the counterparty are less sensitive to the BCLN pricing. Figure 14 and Figure 15 display that the BCLN valuation converges especially with the positive correlation coefficient.
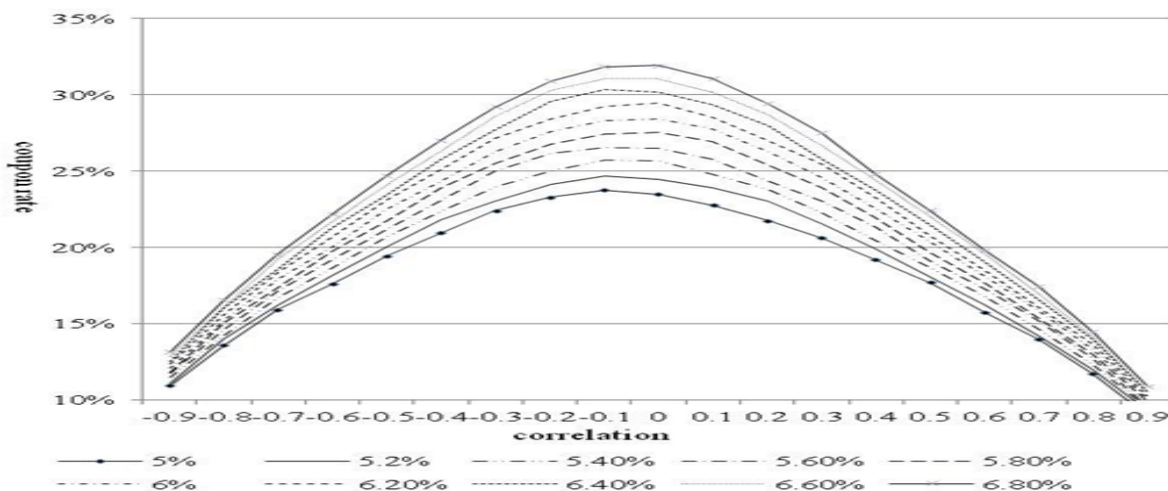
Figure-12. Coupon rates of the third-to-default BCLN with different hazard rate of reference entities (λ_i =5%, 5.2%, 5.4%, 5.6%, 5.8%, 6%, 6.2%, 6.4%, 6.6%, and 6.8%, $i = 1,...,10$).
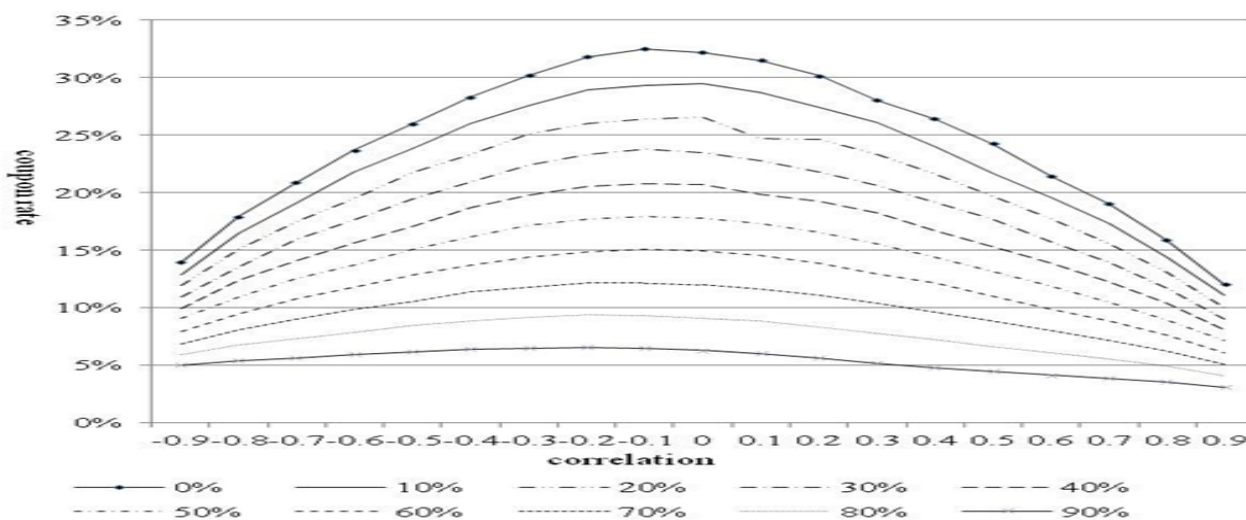
Source: Simulation results.

Figure-13. Coupon rates of the third-to-default BCLN with different recovery rate of reference entities (δ_k =0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, and 90%, $k = 1,...,10$).
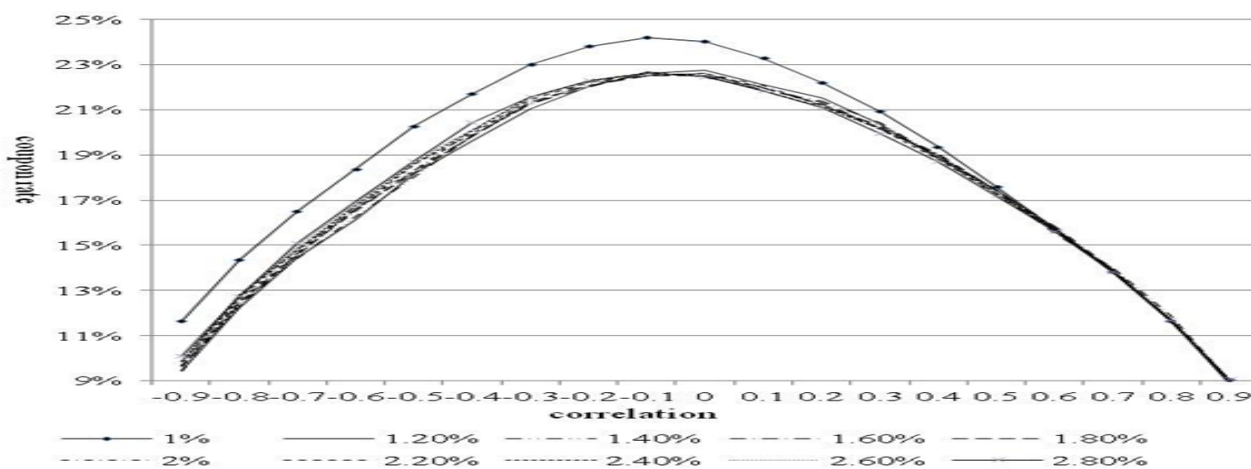
Source: Simulation results.

Figure-14. Coupon rates of the third-to-default BCLN with different hazard rate of the counterparty ($\hat{\lambda}$ =1%, 1.2%, 1.4%, 1.6%, 1.8%, 2%, 2.2%, 2.4%, 2.6%, and 2.8%).

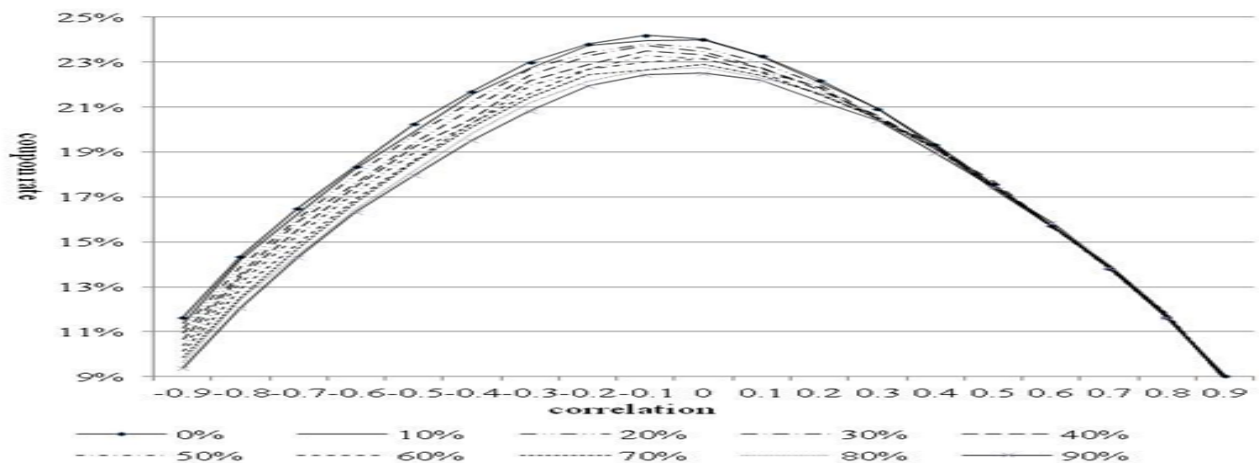**Source:** Simulation results.

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Figure-15. Coupon rates of the third-to-default BCLN with different recovery rate of the counterparty ($\widehat{\delta}$ =0%, 10%, 20%, 30%, 40%, 50%, 60%, 70%, 80%, and 90%).
Source: Simulation results.

4. CONCLUSIONS

This study explores a credit derivative pricing model with counterparty risk and the contagion effect. To compare with the standard credit derivative pricing model, we analyze counterparty risk and the contagion effect to a kth-to-default BCLN valuation by Monte Carlo simulation.

If the counterparty risk or the contagion effect is considered, the fair coupon rate of a kth-to-default BCLN decreases with the absolute value of the correlation coefficient within the reference entities. Relative to the standard BCLN pricing model, when the counterparty risk and the contagion effect are both considered, the fair coupon rate of the BCLN becomes much higher except for the first-to-default BCLN. Especially with the highly positive or negative correlated reference entities, the coupon rate of a BCLN decreases with the contagion effect intensity increases.

The counterparty risk and the contagion effect have substantial influence on kth-to-default BCLN valuation. Especially with high k, the BCLN pricing model with counterparty risk and the contagion effect captures the chain defaulting phenomenon successfully. It indicates that if the higher kth-to-default BCLN or a low correlation degree within reference entities is considered, the contagion effect is more significant to the kth-to-default BCLN valuation.

Other parameters sensitivity analyses are also studied. The coupon rates of the third-to-default BCLN are lower with the higher risk-free rate or the shorter maturity. The higher hazard rate or the lower recovery rate of reference entities results in the higher coupon rates of the third-to-default BCLN. Lastly, the hazard rate and the recovery rate of the counterparty are less sensitive to the BCLN pricing.

The major contribution in this study is that the credit derivative pricing model with the contagion effect is developed to capture the default chain reaction, and the valuation performance is significant numerically.

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