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VOLATILITY MODELLING AND PARAMETRIC VALUE-AT-RISK FORECAST ACCURACY: EVIDENCE FROM METAL PRODUCTS

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ABSTRACT

In this paper, we investigate the one-day-ahead VaR and ES accuracy of four metal daily return series including Aluminium, Copper, Nickel and Zinc. Since, all sample presents volatility clustering, volatility asymmetry, and volatility persistence, we have assessed five GARCH-type models including three fractionary integrated models assuming three alternative distributions (normal, Student-t and skewed Student-t distributions). Estimates results reveal the performance of AR (1) - FIAPARCH model under a skewed Student-t distribution. We have computed one-day ahead VaR and (ES) for both short and long trading positions. Backtesting results show very clearly that the skewed Student-t FIAPARCH model provides the best results for both short and long VaR estimations. These results present several potential implications for metal markets risk quantifications and hedging strategies.

Keywords: Long-range memory, Value at risk, Asymmetry, Fat tail, GARCH. Volatility forecast
JEL Classification: G11, G12, G15; C53.

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1. INTRODUCTION

Commodity markets have been buoyant for the last few years and commodity spot prices continue to reach new record highs, driven by a strong world demand and a tight supply. More precisely, the increase of prices in the last years has been even more dramatic, with a huge demand coming from China, India, and developing countries. Therefore, risk management has become a central issue for both managers and financial deciders. Indeed, taking the best decision in the suitable time needs more information about the commodity market price dynamics. So, investors have to know the metal products' prices evolution, market risk factors and try to measure risk then be covered and hedged against it. A correct risk measure model must take into account stylized facts of commodity assets.

Value-at-Risk (VaR) has become known as a widely common financial risk measure. It represents the amount of money can be loosed over a definite holding period. VaR's simplicity methodology gives managers and investors possibilities to control their portfolio risk level. It helps them to take the best decision and the adequate risk management policy. So, VaR is being very interesting for investors, portfolio managers and financial institution supervisors (see (Cordell and King, 1995; Gjerde and Semmen, 1995; Dimson and Marsh, 1995;1997; Basle Committee on Banking Supervision, 1996): in fact that VaR allows them to compute the amount of resources which institution needs to immobilize as a guarantee against their risk exposure level (Cabedo and Moya, 2003). In

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literature, many VaR methods are developed. Indeed, it is crucial to evaluate them and select the most suitable one for their particular position characteristics. A correct risk measure model must take into account stylized facts of commodity return series. Many empirical studies have argued that financial and commodity data present volatility clustering, are fat tailed, skewed, governed by a long range memory phenomenon...etc. Since 1982, when Engle introduced the ARCH model (which takes into account volatility clustering phenomenon) many GARCH-type models were been developed. These models give us the ability to forecast future variance values by combining past squared deviations and past variance values. Furthermore, recent articles(see (Srinanthakumar and Silvapulle, 2003; Angelidis *et al.*, 2004; Degiannakis, 2004; So and Philip, 2006; Tang and Shieh, 2006; Bali and Panayiotis, 2007; Marzo and Zagalia, 2007; Wu and Shieh, 2007; Aloui, 2008; Kang and Yoon, 2008; Mabrouk and Aloui, 2010) affirm that financial time series data are governed by long range memory in the variance behaviour, fat tailed and skewed. Therefore, an adequate VaR model needs a correct specification of the chosen GARCH-type model. Those finding recommend us to use these models. In our paper, we look to study the price dynamics for four metal products via computing the VaR and ES based on GARCH-type models. More precisely, we assess five alternatives GARCH-type models including three fractionary integrated models to know whether we can better assess the one-day aheadVaR and ES when time series exhibit long memory in the variance dynamics. In line with some previous empirical studies, we based our study on three alternative distributions: the normal, the Student-t and the skewed Student-t distributions. Indeed, the Student-t and the skewed Student-t distributions consider for some stylized facts on financial time series data behavior such as excess kurtosis, heavy tails and excess of skewness. Finally, we computed the VaR and ES for both short and long trading positions then we assess their performance for both in-sample and out-of-sample periods.

The remainder of our paper is structured as follows. In Section 2, we present five GARCH-type models (used in our study) and the error's density models including normal, Student-t and skewed Student-t distributions. In Section 3, we introduce the VaR model and show how can it be computed with these GARCH-type models and we present the statistical accuracy of model-based VaR estimations. Empirical results are provided in sections 4 and 5 while section 6 concludes.

2. THE GARCH-TYPE MODELS

2.1. Riskmetrics Model

Thanks to its management team the U.S. bank JP Morgan in 1994 announced its own risk management methodology for measuring its internal risk. This methodology is called RiskMetrics. Indeed, due to its simplicity this method is rapidly becoming a reference for risk managers. In reality, the *RiskMetrics* model is an Integrated GARCH (1.1) where ARCH and GARCH parameters are prefixed.

RiskMetrics model can be written as follow:

$$\sigma_t^2 = \omega + (1-\lambda)\varepsilon_t^2 + \lambda\sigma_{t-1}^2 \quad (1)$$

where$\omega = 0$ and λ fixed at 0.94 for daily data. While, for weekly data λ is equal to 0.97.

2.2. GARCH Model

The ARCH model of Engle (1982) was generalized by Bollerslev (1986). The generalized ARCH (GARCH) model is specified as an infinite ARCH. Indeed, it allows the reduction of parameters included in the model ARCH. The GARCH (p,q) model can be expressed as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_j\, \sigma_{t-j}^2 \quad (2)$$

The lag operator allows us to specify GARCH model as:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2 \quad (3)$$

where

$$\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \cdots + \alpha_q L^q \ \text{ et } \ \beta(L) = \beta_1 L + \beta_2 L^2 + \cdots + \beta_p L^p$$

Bollerslev (1986) has shown that the GARCH model is a short memory model since its autocorrelation function decay slowly with a hyperbolic rate.

2.3. The Fractional Integrated GARCH Model

Since, financial time series are usually governed by long memory process in the variance dynamics, Baillie *et al.* (1996) have proposed a new GARCH-type model which has the ability to model this fact. The FIGARCH model can distinguish between short memory and infinite long memory thanks to the fractionary parameter *d*. The FIGARCH process is able to distinguish between short memory and long memory in the conditional variance behavior. Formally, the FIGARCH(p. d. q) process is specified as follows:

$$[\varphi(L)(1-L)^d]\varepsilon_t^2 = \omega + [1-\beta(L)](\varepsilon_t^2 - \sigma_t^2) \quad (4)$$

or

$$\sigma_t^2 = \omega + \beta(L)\sigma_t^2 + [1-\beta(L)]\varepsilon_t^2 - \varphi(L)(1-L)^d\varepsilon_t^2 \quad (5)$$
$$= \omega[1-L]^{-1} + \lambda(L)\varepsilon_t^2$$

where(L) is the lag-operator, $\lambda(L) = \sum_{i=1}^{\infty}\lambda_i L^i$ and $0 \le d \le 1$. $\lambda(L)$ is an infinite summation which, in practice, has to be truncated. According to Baillie *et al.* (1996)$\lambda(L)$ should be truncated at 1000 lags. $(1-L)^d$is the fractional differencing operator. It can be defined as follows:

$$(1-L)^d = \sum_{k=0}^{\infty}\frac{\Gamma(d+1)L^k}{\Gamma(k+1)\Gamma(d-k+1)} = 1 - dL - \frac{1}{2}d(1-L)L^2 - \frac{1}{6}d(1-d)(2-d)L^3 - \cdots \quad (6)$$
$$= 1 - \sum_{k=1}^{\infty}c_k(d)L^k$$

where, $c_1(d) = d, c_2(d) = \frac{1}{2}d(1-d)$, etc.

2.4. The Fractional Integrated Asymmetric Power ARCH Model

Tse (1998) have extended the FIGARCH (p,d,q) model in order to consider jointly for volatility asymmetry and volatility persistence (long memory). Indeed, the function $(|\varepsilon_t| - \gamma\varepsilon_t)^\delta$ of the APARCH process has been added to the FIGARCH process. The FIAPARCH (p, d, q) can be expressed as follows:

$$\sigma_t^\delta = \omega[1-\beta(L)]^{-1} + \{1 - [1-\beta(L)]^{-1}\rho(L)(1-L)^d\}(|\varepsilon_t| - \gamma\varepsilon_t)^\delta \quad (7)$$

whereδ, γ and λ are the model parameters. The FIAPARCH process nests the FIGARCH process when $\gamma = 0$and $\delta = 2$. Thus, the FIGARCH process is sample case of the FIAPARCH model.

2.5.The Hyperbolic GARCH Model

Davidson (2004) has created a new GARCH-type model namely hyperbolic GARCH model. This model is built to test whether the non-stationarity of the FIGARCH model holds. The HYGARCH model extends the conditional variance of the FIGARCH model by the fact that weights are introduced in the difference operator. The HYGARCH model can be written as follows:

$$\sigma_t^2 = \omega[1-\beta(L)]^{-1} + \{1 - [1-\beta(L)]^{-1}\rho(L)[1 + \alpha\{(1-L)^d\}]\}\varepsilon_t^2 \quad (8)$$

The HYGARCH is Generalized FIGARCH since it nests to GARCH when $\alpha = 0$ and to FIGARCH model when $\alpha = 1$.

2.6. The Error's Density Models

Assuming the assumption that the random variable is $z \sim N(0,1)$, the log-likelihood of normal distribution ( ) Norm L can be written as follows

$$L_{Norm} = -\frac{1}{2}\sum_{t=1}^{T}[\ln(2\pi) + \ln(\sigma_t^2) + z_t^2] \quad (9)$$

whereT is the number of observations. In practice it's difficult to consider that economic time series are normally distributed. Indeed, previous empirical studies have shown that residuals are fat tailed. In order to take into account the excess of kurtosis, the Student-t distribution is included in our study. If the random variable is $z \sim ST(0,1,\nu)$, the log-likelihood function of the Student-t distribution ( ) Stud L will be defined as follows:

$$L_{Stud} = T\left\{ln\Gamma\left(\frac{\upsilon+1}{2}\right) - ln\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}ln[\pi(\nu-2)]\right\} - \frac{1}{2}\sum_{t=1}^{T}\left[ln(\sigma_t^2) + (1+\nu)\left[ln(1+\frac{z_t^2}{\sigma_t^2(\nu-2)})\right]\right] \quad (10)$$

where$2 < \nu \leq \infty$ and $\Gamma(.)$is the gamma function. In contrast to the normal distribution, the Student-t distribution is estimated with an additional parameter ν, which stands for the number of degrees of freedom measuring the degree of fat-tails in the density.

To consider jointly for the excess skewness and kurtosis, we include the skewed Student-t distribution proposed by Lambert and Laurent (2001)in our study. If $z \sim SKST(0,1,k,\nu)$, the log-likelihood of the skewed Student-t distribution ( ) SkSt L is as follows:

$$L_{Skst} = T\left\{ln\Gamma\left(\frac{\upsilon+1}{2}\right) - ln\Gamma\left(\frac{\upsilon}{2}\right) - \frac{1}{2}ln[\pi(\nu-2)] + \ln\left(\frac{2}{k+\frac{1}{k}}\right) + \ln(s)\right\}$$

$$-\frac{1}{2}\sum_{t=1}^{T}\left[\ln(\sigma_t^2) + (1+\nu)ln\left[1+\left(1+\frac{(sz_t+m)^2}{(\nu-2)}\right)k^{-2I_t}\right]\right] \quad (11)$$

where $I_t = 1$ if $z_t \geq {}^{m}/_{s}$ or $I_t = -1$ si $z_t < {}^{m}/_{s}$, k is an asymmetry parameter. The constants $m = m(k,\nu)$ and $s = \sqrt{s^2(k,\nu)}$ are the mean and standard deviations of the skewed Student-t-t distribution:

$$m(k,\nu) = \frac{\Gamma(\frac{\nu-1}{2})\sqrt{\nu-2}}{\sqrt{\pi}\Gamma(\frac{\nu}{2})}\left(k-\frac{1}{k}\right) \quad (12)$$

$$s^2(k,\nu) = \left(k^2+\frac{1}{k^2}-1\right) - m^2 \quad (13)$$

The value of $\ln(k)$ can also represent the degree of asymmetry in the residual distribution. We note that when $\ln(k) = 0$, the skewed Student-t-t distribution equals the general Student-t-t distribution, $z \sim ST(0,1,\nu)$.

3. THE VALUE-AT-RISK

We present in this sub-section the VaR's values using a FIAPARCH model with skewed Student-t distribution innovation. We consider that:

$$r_t = \mu_t + \varepsilon_t \quad (14)$$

$$\mu_t = \mu + \sum_{i=1}^{m}\xi_i r_{t-i} + \sum_{j=1}^{n}\theta_j \varepsilon_{t-j} \quad (15)$$

The $\epsilon_t = z_t \sigma_t$ is governed by a FIAPARCH (p, d, q) process and the innovations are said to be described by the skewed Student-t distribution if:

$$f(z_t|k,v) = \begin{cases} \frac{2}{k+\frac{1}{k}} sg(k(sz_t+m)|v) \\ \frac{2}{k+\frac{1}{k}} sg(k(sz_t+m)/k|v) \end{cases}, \quad \text{if} \quad \begin{matrix} z_t < -m/s \\ z_t \geq -m/s \end{matrix} \quad (16)$$

In the above equation, $g(.|v)$ denotes the symmetrical Student-t density and k is the asymmetry parameter. The estimated VaR for the long and short trading positions can be expressed as follows:

$$\alpha = P(r_t < VaR_{t,L}) = P\left(\frac{r_t - \mu_t}{\sigma_t} < \frac{VaR_{t,L} - \mu_t}{\sigma_t}\right) \quad (17)$$

$$\alpha = P(r_t > VaR_{t,s}) = P\left(\frac{r_t - \mu_t}{\sigma_t} > \frac{VaR_{t,s} - \mu_t}{\sigma_t}\right) \quad (18)$$

In the Eqs. 17 and 18, $VaR_{t,L}$ and $VaR_{t,s}$ are the VaR for, respectively, the long and the short trading positions. They are written as follows:

$$VaR_{t,L} = \mu_t + st_\alpha(v,k)\sigma_t \quad (19)$$

$$VaR_{t,s} = \mu_t + st_{1-\alpha}(v,k)\sigma_t \quad (20)$$

Where $st_\alpha(v,k)$ is the left quantile at the α% of the skewed Student-t distribution innovation. Correspondingly, $st_{1-\alpha}(v,k)$ is the right quantile of the skewed Student-t distribution[1]. According to Lambert and Laurent (2001) and Wu and Shieh (2007) we can compute the one-day-ahead VaR estimated at time $(t-1)$ for the long and the short trading positions. Under the hypothesis of skewed Student-t distribution, these VaRs are given by:

$$\widehat{VaR_{t,L}} = \widehat{\mu_t} + st_\alpha(v,k)\widehat{\sigma_t} \quad (21)$$

$$\widehat{VaR_{t,s}} = \widehat{\mu_t} + st_{1-\alpha}(v,k)\widehat{\sigma_t} \quad (22)$$

3.1. Test of VaR Model Accuracy

Backtesting the accuracy for the estimated VaR is crucial. The VaR quality estimation depends on the methodology of computation of VaR. Therefore, to investigate the VaR performance we have computed the empirical failure rates for both short and long trading positions. The prescribed probability is ranging from 0.25% to 5%. In reality, the failure rate is the number of times in which returns exceed (in absolute value) the forecasted VaR. If the model is said to be correctly specified, when the failure rate is equal to the specified VaR's level. In our study, the backtestingVaR is based on Kupiec (1995) test. In order to test the accuracy and to evaluate the performance of the model-based VaR estimates, Kupiec (1995) provided a likelihood ratio test (LR_{UC}) for testing whether the failure rate of the model is statistically equal to the expected one (unconditional coverage). Consider that $N = \sum_{t=1}^{T} I_t$ is the number of exceptions in the sample size T. Then,

$$I_{t+1} = \begin{cases} 1 & \text{if} \quad r_{t+1} < VaR_{t+1|t(\alpha)} \\ 0 & \text{if} \quad r_{t+1} \geq VaR_{t+1|t(\alpha)} \end{cases} \quad (23)$$

follows a binomial distribution,$N \sim B(T, \alpha)$. If $p = E(\frac{N}{T})$ is the expected exception frequency (i.e. the expected ratio of violations), then the hypothesis for testing whether the failure rate of the model is equal to the expected one is expressed as follows: $H_0: \alpha = \alpha_0$. $\alpha_0$is the prescribed VaR level. Thus, the appropriate likelihood ratio statistic in the presence of the null hypothesis is given by:

[1]For more details, see Lambert and Laurent (2001), Giot and Laurent (2003) and Wu and Shieh (2007).

$$LR_{uc} = -2\log\{\alpha_0^N(1-\alpha_0)^{T-N}\} + 2\log\left\{(\frac{N}{T})^N(1-(\frac{N}{T}))^{T-N}\right\} \quad (24)$$

Under the null hypothesis, LR_{uc} has a $\chi^2(1)$ as an asymptotical distribution. Thus, a preferred model for VaR prediction should provide the property that the unconditional coverage measured by $p = E(\frac{N}{T})$ equals the desired coverage level p_0.

4. DATA AND PRELIMINARY ANALYSIS

The data consist of daily closing prices for four metal assets including Aluminium, Copper, Nickel and Zinc (in US dollars per ton) on the London Metal Exchange from January 1989 to December 2005. The period of study and the number of observation are indicated in the table below.

Table-1. Data

Assets	Sample period	Observations
Aluminium	01/03/1989 − 11/30/2005	4413
Copper	01/03/1989 − 11/30/2005	4413
Nickel	01/03/1989 − 11/30/2005	4413
Zinc	01/03/1989 − 11/30/2005	4413

Source: London Metal Exchange

For each series, the log-returns is expressed (in %) as,

$$r_t = 100 * \log(\frac{S_t}{S_{t-1}})$$

where St denotes the daily closing price.

Table-2.descriptivestatistics of daily returns

	Aluminium	***Copper***	***Nickel***	***Zinc***
Mean	−0.0042137	0.00506652	−0.00906287	0.000387313
Maximum	7.57487	15.6733	10.6565	8.70114
Minimum	−9.78264	−10.3029	−14.1198	−14.7158
SD	1.2275	1.51525	1.9538	1.45043
Skewness	−0.141145	−0.0137069	−0.18231	−0.724583
Kurtosis	7.75353	9.07106	7.36573	10.876
Jarque − Béra	4167.6	6774.28	3527.43	11786.9
Q20)	48.3517	79.9366	19.5396	61.3718
$Q^2(20)$	1221.83	1435.08	409.086	486.933
Observations	4412	4412	4412	4412

Notes: S.D. is the standard deviation. For all the time series, the descriptive statistics for cash daily returns are expressed in percentage. $Q^2(20)$ and Q(20) are respectively the Ljung-Box Q-statistic of order 20 on the squared returns and the returns.

As it's shown on the table above, the aluminum and the nickel returns have a negative mean, however, the copper and the zinc have a positive one. Furthermore, those time series data are not normally distributed, in fact that the 3rd and the 4th moment respectively are different from zero and three. More precisely, the return series are negatively skewed and heavy tailed. The same conclusion is confirmed by the Jarque – Bera statistic which indicates the non normality of our time series2. Using the Ljung-Box Q statistic of order 20 based on the returns and the squared returns, we can also reject the null hypothesis of white noise and affirm that all the time series are autocorrelated.

4.1. Graphical Analysis

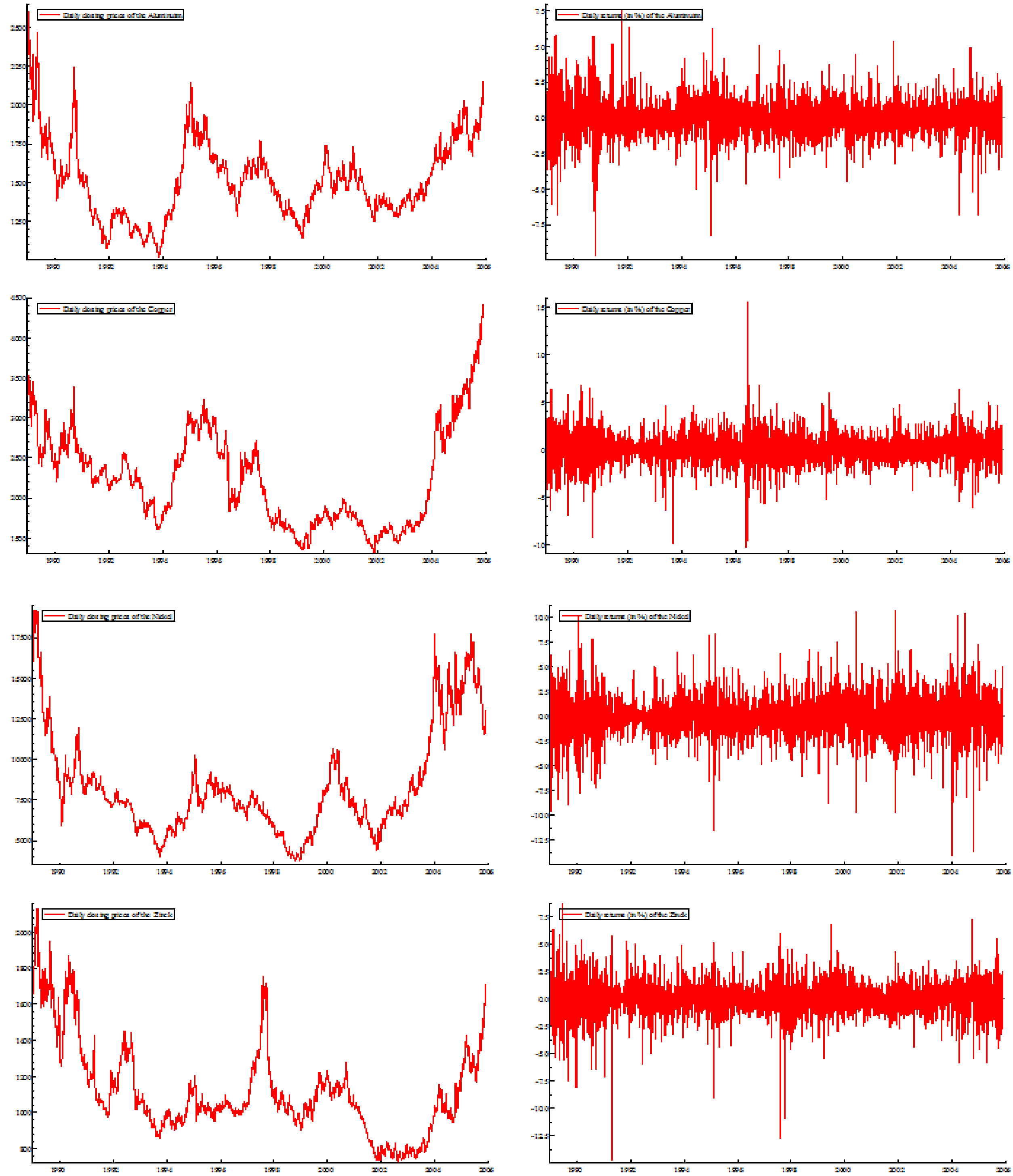

Figure-1. Indicates that all sample return series exhibit volatility clustering as periods of low volatility followed by periods of high volatility. Indeed, this confirms the presence of ARCH effect in the series.
Source: OxMetrics outputs

2For the normal distribution the skewness coefficient equals 0 while the kurtosis coefficient equals 3

4.2. Unit Root and Stationarity Tests

Checking the presence or not of unit root and testing stationarity is crucial. Indeed, we have employed the Augmented Dickey and Fuller (1979) (ADF), Phillips and Perron (1988) (PP) unit root tests and Kwiatkowski *et al.* (1992)(KPSS) stationarity test.

Table-3. ADF, PP and KPSS tests for daily returns

	Aluminium	*Copper*	*Nickel*	*Zinc*
ADF test statistic	−46.91	−48.81	−50.127	−38.96
PP test statistic	−66.24	−68.53	−71.4	−64.96
KPSS test statistic	0.2586	0.3997	0.3470	0.1224
−	{< 1}	{< 1}	{< 1}	{< 1}
Observations	4412	4412	4412	4412

Notes: MacKinnon's 1% critical value is –3.435 for the ADF and PP tests. The KPSS critical value is 0.739 at the 1% significance level. ADF is the Augmented Dickey and Fuller (1979) unit-root test statistic. PP is the Phillips and Perron (1988) unit-root test statistic. KPSS is the Kwiatkowski *et al.* (1992)stationarity test statistic.

As it is given by the above, the results indicate that for all the time series the null hypothesis of presence of unit root is absolutely rejected by the ADF and PP tests. The KPSS test indicates that the metal returns time series are stationary at a 1% significant level.

4.3. Long Memory Tests

Testing the presence of long range memory is important. Indeed, like many previous studies we use the absolute returns and the daily squared volatility returns as two proxies of daily volatility. To test long memory we employed three long-range memory tests: Lo (1991) test, the log-periodogram regression (GPH) of Geweke (1983) and the Gaussian semi-parametric estimate (GSP) of Robinson and Henry (1998).

Table-4. Long range memory tests

Test	$\lvert r_t \rvert$				r_t^2			
	Aluminium	*Copper*	*Nickel*	*Zinc*	*Aluminium*	*Copper*	*Nickel*	*Zinc*
GPH Test								
$m = T^{0.5}$	0.61	0.47	0.48	0.24	0.58	0.36	0.44	0.25
$m = T^{0.6}$	0.51	0.43	0.46	0.34	0.46	0.36	0.39	0.26
$m = T^{0.7}$	0.46	0.44	0.37	0.30	0.40	0.38	0.37	0.24
Lo's RS Test								
	2.646	1.917	2.925	0.932	3.581	2.622	3.899	1.606
	{< 0.005}	{< 0.025}	{< 0.005}	{< 0.95}	{< 0.005}	{< 0.005}	{< 0.005}	{< 0.02}
GSP Test								
$m = \frac{T}{2}$	0.302	0.272	0.283	0.277	0.263	0.235	0.239	0.196
$m = \frac{T}{4}$	0.396	0.362	0.368	0.374	0.318	0.281	0.286	0.279
$m = \frac{T}{8}$	0.504	0.483	0.449	0.451	0.422	0.388	0.354	0.320

Notes: (r_t), (r_t^2), and $\lvert r_t \rvert$ are respectively log return, squared log return and absolute log return. (m)denotes the bandwidth.

The table above displays results of long memory tests including three tests which are Lo'sR/S , GPH test for three BANDWITH $m = T^{0.5}$; $m = T^{0.6}$ et $m = T^{0.7}$ and GSP for three BANDWITH $\mathrm{m} = \frac{\mathrm{T}}{2}$; $\mathrm{m} = \frac{\mathrm{T}}{4}$ et $\mathrm{m} = \frac{\mathrm{T}}{8}$. as it is shown, the Lo's R/S which tests in null hypothesis H_0 presence of short memory versus long memory indicates the presence of long memory in both absolute log return and squared log return. Furthermore, the GPH and GSP tests reject the null hypothesis of short memory. Indeed, the two time series are governed by long memory process. Thus, we are motivated to look for fractionally integrated models.

5. EMPIRICAL RESULTS

5.1. Estimates GARCH-Type Models

Results of RiskMetrics, GARCH, FIGARCH, FIAPARCH and HYGARCH models under normal, Student-t and skewed Student-t distributions are provided in Tables 5–10

Tables-5. AR (1)-RiskMetrics model estimation

$Panel1-a$	$AR(1)-RiskMetrics$			
	$Aluminium$	$Copper$	$Nickel$	$Zinc$
α_1	0.06	0.06	0.06	0.06
β_1	0.94	0.94	0.94	0.94
$ln(\ell)$	−6894	−7720	−8949	−7587
AIC	3.13	3.50	4.058	3.440
$Q(20)$	17.83	21.40	12.21	34.28
$Q^2(20)$	6.16	19.63	37.90	20.85
$RBD(10)$	10.39	21.28	31.09	16.82
$ARCH(10)$	0.42	1.43	2.58	0.71
$P(60)$	137.42***	197***	360***	367***

Notes: $\ln(\ell)$ is the value of the maximized log-likelihood. AIC is the Akaike (1974) Information criterion. Q(20) and $Q^2(20)$ are the Box-Pierce statistics for remaining serial correlation for respectively standardized and squared standardized residuals. RBD(10) is the residual based diagnostic for conditional heteroscedasticity, using 10lags. The ARCH(10) is LM-ARCH test of Engle (1982) using 10lags. P(60) is the Pearson goodness-of-fit statistic for 60 cells. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

As it is founded by JP Morgan, the RiskMetrics model is a pre-specified IGARCH (1,1) model. Indeed, the GARCH coefficient(β_1) is equal to 0.94with daily data while with weekly data the β_1 is set to 0.97. Since our study is focused on daily time series data, we have fixed β_1 at 0.94.

To check the flexibility of this GARCH-type models, we are referred to the output of the Box-Pierce test on standardized residuals and squared standardized residuals, the RBD test, the ARCH-LM test and the Pearson goodness-of-fit statistic applied after the estimation of the *RiskMetrics* model (including an AR (1) term). Those tests clearly indicate that the *RiskMetrics* specification is not appropriate.

Table 6 provides the estimation results of GARCH model under three alternative distributions (normal, Student-t and skewed Student-t). As we see both ARCH and GARCH coefficients are positive for all our time series data. Furthermore, the condition of existence of conditional variance is justified since for all return series $\alpha_1 + \beta_1 < 1$. As it is shown by Bollerslev (1986) for a GARCH (1, 1) the autocorrelations decline exponentially with a decay factor of $\alpha_1 + \beta_1$. Indeed, this sign clearly indicates that the GARCH is a short memory model. In reality, financial and commodity returns series are usually not normally distributed. Therefore, the normal distribution fails to consider same stylized facts such the excess of kurtosis (fat tail) and asymmetry measured by skewness. To take into account those facts, we estimate GARCH-Types[3] models under three different innovation distributions. The Student-t distribution can model fat tails and the skewed Student-t distribution is used to assess both the fat tails and excess of skewness in the returns. As it is provided in table below we note that the GARCH models can models the dynamics of our time series data. The log-likelihood values for the GARCH models indicate its superiority compared to the RiskMetrics model. Furthermore, the (same) misspecification tests reported below suggest that the ability of GARCH

[3] *RiskMetrics* could be only assessed under a normal distribution. For more detail visit JP Morgan website

(1,1) in modeling our stock returns series is proved. Well, both The log-likelihood and the Akaike information criterion reveal that GARCH models under a skewed Student-t distribution provide the best results compared to other distributions (normal and Student-t).

Table-6. AR (1)-GARCH (1-1) model estimation

$AR(1)-GARCH(1-1)$												
	$Aluminium$			$Copper$			$Nickel$			$Zinc$		
	N	t	SKt	N	t	SKt	N	t	SKt	N	t	SKt
α_1	0.06^{***}	0.06^{***}	0.06^{***}	0.07^{***}	0.06^{***}	0.06^{***}	0.04^{***}	0.05^{***}	0.05^{***}	0.05^{***}	0.04^{***}	0.04^{***}
	(0.01)	(0.009)	(0.09)	(0.03)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
β_1	0.90^{***}	0.91^{***}	0.91^{***}	0.93^{***}	0.94^{***}	0.94^{***}	0.95^{***}	0.94^{***}	0.94^{***}	0.95^{***}	0.95^{***}	0.95^{***}
	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.08)	(0.01)	(0.01)	(0.01)	(0.01)	(0.009)	(0.009)
ν	−	5.19^{***}	5.21^{***}	−	5.64^{***}	5.64^{***}	−	4.36^{***}	4.36^{***}	−	4.17^{***}	4.17^{***}
	−	(0.40)	(0.41)	−	(0.46)	(0.47)	−	(0.29)	(0.29)	−	(0.27)	(0.27)
ξ	−	−	0.02^{***}	−	−	0.05^{**}	−	−	0.007^{*}	−	−	0.02^{*}
	−	−	(0.01)	−	−	(0.01)	−	−	(0.01)	−	−	(0.01)
$ln(\ell)$	−6825	− 6639	− 6638	−7708	− 7567	− 7563	−8925	− 8708	− 8708	−7565	− 7311	− 7311
AIC	3.09	3.01	3.01	3.49	3.432	3.431	4.048	3.950	3.950	4.431	3.317	3.317
$Q(20)$	29.54	29.22	17.21	47.72	24.66	24.59	13.18	27.27	27.35	35.54	38.39	38.47
$Q^2(20)$	16.05	17.35	36.83	16.9	21.16	20.54	56.34	46.03	46.01	29.08	33.52	33.4
$RBD(10)$	7.91	5.3	8.3	10.78	9.20	8.71	11.49	25.64	25.71	0.96	2.73	2.78
$ARCH(10)$	0.84	0.92	0.91	1.24	1.51	1.47	4.00	3.15	3.14	1.65	2.12	2.11
$P(60)$	681.02^{***}	581^{***}	583^{***}	211^{***}	68^{***}	61^{***}	432^{***}	229^{***}	222^{***}	403^{***}	135^{***}	117^{***}

Notes: $\ln(\ell)$ is the value of the maximized log-likelihood. AIC is the Akaike (1974) Information criterion. $Q(20)$ and $Q^2(20)$ are the Box-Pierce statistics for remaining serial correlation for respectively standardized and squared standardized residuals. RBD(10) is the residual based diagnostic for conditional heteroscedasticity, using 10lags. The ARCH(10) is LM- ARCH test of Engle (1982) using 10lags. P(60) is the Pearson goodness-of-fit statistic for 60 cells. Figures between parentheses are the standard errors. N , t and SKt are respectively normal, Student-t and the skewed Student-t distribution. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

In reality, return volatility changes quite slowly over time. Indeed, the autocorrelation function decays hyperbolically as shown in Ding *et al.* (1993) among others. Therefore, the effects of a shock can take a considerable time to decay. So, when we consider a stationary process, the propagation of shocks decays very quickly (at an exponential rate). But when the process is a unit root the shocks effect is infinite. Thus, a factionary integrated model can be a good solution to take into account the long memory (long-run dependence.) in the return volatility. Estimates results of long memory GARCH-type models are given in table 7-9

The table 7 provides the estimation results of FIGARCH (1.d.1) under three alternative innovation distributions. Results show that all times series are governed by long memory process. Indeed, the fractionary integrated parameter d ranges from 0.30 to 0.46. Therefore, this models which takes into account volatility clustering and long range memory in the variance outperforms GARCH and RiskMetrics models. This superiority is confirmed by the misspecification tests. Furthermore, the log-likelihood and the Akaike information criterion confirmed these results. As we see the skewed Student-t distribution provides the most adequate FIGARCH model for the seven time series. In the fact this distribution considers heavy tails and skewed distribution.

Table-7. AR (1)-FIGARCH (1-d-1) model estimation

	$AR(1)-FIGARCH(1,d,1)$											
	Aluminium			*Copper*			*Nickel*			*Zinc*		
	N	t	SKt	N	t	SKt	N	t	SKt	N	t	SKt
d	0.30^{***} (0.06)	0.42^{***} (0.12)	0.42^{***} (0.11)	0.46^{***} (0.03)	0.46^{***} (0.03)	0.46^{***} (0.03)	0.44^{***} (0.03)	0.46^{***} (0.03)	0.46^{***} (0.03)	0.41^{***} (0.02)	0.43^{***} (0.02)	0.43^{***} (0.09)
φ_1	0.20^{***} (0.20)	0.27^{***} (0.07)	0.27^{***} (0.07)	0.33^{***} (0.05)	0.31^{***} (0.04)	0.30^{***} (0.04)	0.52^{***} (0.06)	0.46^{***} (0.05)	0.46^{***} (0.05)	0.31^{***} (0.06)	0.36^{***} (0.04)	0.37^{***} (0.04)
β_1	0.41^{***} (0.21)	0.60^{***} (0.15)	0.60^{***} (0.14)	0.68^{***} (0.05)	0.68^{***} (0.04)	0.68^{***} (0.04)	0.79^{***} (0.04)	0.76^{***} (0.04)	0.76^{***} (0.04)	0.64^{***} (0.05)	0.70^{***} (0.04)	0.70^{***} (0.03)
ν	–	4.92^{***} (0.39)	4.94^{***} (0.4)	–	6.84^{***} (0.51)	6.83^{***} (0.51)	–	5.48^{***} (0.32)	5.48^{***} (0.32)	–	5.21^{***} (0.28)	5.20^{***} (0.28)
ξ	–	–	0.03^{***} (0.01)	–	–	0.04^{***} (0.01)	–	–	0.005^{*} (0.01)	–	–	0.02^{*} (0.01)
$ln(\ell)$	−6828	−6646	−6645	−7694	−7560	−7557	−8901	−8702	−8702	−7553	−7311	−7309
AIC	3.09	3.01	3.01	3.49	3.430	3.429	4.037	3.947	3.948	3.426	3.317	3.317
$Q(20)$	27.9	20.53	30.62	21.30	23.18	23.06	12.70	21.60	21.63	32.17	34.86	34.96
$Q^2(20)$	16.3	17.88	17.6	16.38	18.53	18.53	27.60	21.95	21.91	14.37	18.71	18.92
$RBD(10)$	9.38	14.07	9.3	29.48	22.55	22.27	20.47	−17.7	−15.8	−11.6	111.05	104.6
$ARCH(10)$	0.75	1.05	1.03	1.12	1.26	1.25	2.55	1.85	1.48	0.43	0.80	0.82
$P(60)$	326^{***}	467^{***}	543^{***}	169^{***}	98^{***}	88^{***}	369^{***}	255^{***}	265^{***}	373^{***}	180^{***}	156^{***}

Notes: $\ln(\ell)$ is the value of the maximized log-likelihood. AIC is the Akaike (1974) information criterion. Q(20) and $Q^2(20)$ are the Box-Pierce statistics for remaining serial correlation for respectively standardized and squared standardized residuals. RBD(10) is the residual based diagnostic for conditional heteroscedasticity, using 10lags. The ARCH(10) is LM -ARCH test of Engle (1982) using 10lags. P(60) is the Pearson goodness-of-fit statistic for 60 cells. Figures between parentheses are the standard errors. N , t and SKt are respectively normal, Student-t and the skewed Student-t distribution. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

Table 8 reports the FIAPARCH model estimates results for the four times series under the same three distributions (normal, Student-t and skewed Student-t). In reality, this model considers for long range memory in volatility, clustering volatility and asymmetry. Those stylized facts are very important since all our time series dynamics support those facts. The results given above showed very clearly that the FIAPARCH model performs very well compared to the other models (RiskMetrics, GARCH and FIGARCH). Furthermore, under a skewed Student-t distribution, the FIAPARCH provides the best adequate model for all our time series.

Table-8.AR (1)-FIAPARCH (1-d-1) model estimation

	$AR(1)-FIAPARCH(1,d,1)$											
	Aluminium			***Copper***			***Nickel***			***Zinc***		
	N	t	SKt	N	t	SKt	N	t	SKt	N	t	SKt
d	0.38^{***} (0.07)	0.49^{***} (0.12)	0.49^{***} (0.11)	0.38^{***} (0.05)	0.40^{***} (0.05)	0.41^{***} (0.05)	0.29^{***} (0.07)	0.43^{***} (0.06)	0.43^{***} (0.06)	0.30^{***} (0.06)	0.34^{***} (0.04)	0.34^{***} (0.04)
φ_1	0.27^{***} (0.11)	0.27^{***} (0.05)	0.26^{***} (0.05)	0.33^{***} (0.09)	0.32^{***} (0.05)	0.31^{***} (0.05)	0.68^{***} (0.11)	0.5^{***} (0.09)	0.49^{***} (0.09)	0.33^{***} (0.07)	0.38^{***} (0.05)	0.38^{***} (0.05)
β_1	0.57^{***} (0.15)	0.66^{***} (0.13)	0.66^{***} (0.12)	0.59^{***} (0.10)	0.64^{***} (0.05)	0.64^{***} (0.05)	0.79^{***} (0.07)	0.73^{***} (0.07)	0.73^{***} (0.07)	0.57^{***} (0.08)	0.65^{***} (0.04)	0.65^{***} (0.04)
γ_1	-0.1^{***} (0.08)	-0.1^{***} (0.05)	-0.1^{***} (0.05)	0.006^{*} (0.06)	-0.07^{*} (0.05)	-0.06^{*} (0.13)	-0.04^{*} (0.04)	-0.02^{*} (0.04)	-0.02^{*} (0.04)	-0.29^{*} (0.10)	-0.33^{**} (0.10)	-0.33^{**} (0.10)
δ	1.60^{***} (0.18)	1.53^{***} (0.13)	1.53^{***} (0.13)	2.25^{***} (0.08)	2.16^{***} (0.07)	2.18^{***} (0.07)	2.36^{***} (0.12)	2.13^{***} (0.17)	2.13^{***} (0.18)	2.18^{***} (0.12)	2.11^{***} (0.09)	2.11^{***} (0.09)
ν	$-$	5.18^{***} (0.4)	5.19^{***} (0.4)	$-$	6.11^{***} (0.51)	6.10^{***} (0.51)	$-$	4.72^{***} (0.32)	4.72^{***} (0.32)	$-$	4.59^{***} (0.29)	4.59^{***} (0.22)
ξ	$-$	$-$	-0.03^{*} (0.01)	$-$	$-$	-0.04^{**} (0.01)	$-$	$-$	-0.008^{*} (0.01)	$-$	$-$	-0.02^{**} (0.01)
$ln(\ell)$	-6819	-6639	-6636	-7679	-7553	-7547	-8875	-8692	-8689	-7504	-7289	-7268
AIC	3.09	3.019	3.012	3.484	3.428	3.424	4.026	3.944	3.941	3.405	3.318	3.308
$Q(20)$	28.55	31.08	58.84	21.06	22.55	22.44	12.74	21.40	21.46	33.14	36.22	36.17
$Q^2(20)$	17.48	19.15	18.91	15.12	20.18	19.77	15.51	17.46	17.41	11.43	13.43	19.81
$RBD(10)$	6.98	7.38	6.96	20.89	-41.7	-35.4	3.75	14.89	14.94	-0.86	-4.68	-4.76
$ARCH(10)$	0.81	1.09	0.3	1.85	1.2	1.19	1.26	1.47	1.47	0.52	0.69	0.69
$P(60)$	285^{***}	321^{***}	482^{***}	192^{***}	99^{***}	69^{***}	425^{***}	244^{***}	274^{***}	370^{***}	128^{***}	135^{***}

Notes: $\ln(\ell)$ is the value of the maximized log-likelihood. AIC is the Akaike (1974) Information criterion. Q(20) and $Q^2(20)$ are the Box-Pierce statistics for remaining serial correlation for respectively standardized and squared standardized residuals. RBD(10) is the residual based diagnostic for conditional heteroscedasticity, using 10lags. The ARCH(10) is LM -ARCH test of Engle (1982) using 10lags. P(60) is the Pearson goodness-of-fit statistic for 60 cells. Figures between parentheses are the standard errors. N , t and SKt are respectively normal, Student-t and the skewed Student-t distribution. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

Table 9 provides the estimation of HYGARCH models under three alternative distributions. Basically this model is a generalization of FIGARCH model. Indeed, the HYGARCH nest to a FIGARCH model when $\alpha = 1$. Thanks to it is specification, this model takes into consideration long range dependence which is measured by the fractionary integrated parameter d. this parameter ranges from 0.20 to 0.47. Indeed, the long memory phenomenon is proved since the *d* value is ranging between zero and one. Results show that HYGARCH is a good model but it is not the best. Since, the FIAPARCH model outperforms all the other models including HYGARCH. Those results are justified by referring to the log-likelihood and the Akaike information criterion values.

5.2. The VaR Analysis

5.2.1. The in-Sample VaR Estimation Results

In this sub-section, we estimate the one-day-ahead VaR and ES for the AR (1)-FIAPARCH model under the three alternative innovation's distributions (normal, Student-t and skewed Student-t) for the four metal return series.

Indeed, we have computed the Kupiec (1995) LR tests. The VaR levels range (α) from 0.05 to 0.01 for short and it ranges from 0.95 to 0.99 for the long trading positions. In addition, the Expected Shortfall (ES) is computed for both short and long trading positions for the mentioned levels. As we knew, the failure rate for the short trading position denotes the percentage of positive returns larger than the VaR prediction. However for the long trading positions, the failure rate is the percentage of negative returns smaller than the VaR prediction.

Those results are reported in the following Tables.

Table-9. AR (1)-HYGARCH (1-d-1) model estimation

$AR(1)-HYGARCH(1,d,1)$												
	Aluminium			**Copper**			**Nickel**			**Zinc**		
	$NtSKt$			$Nt\ SKt$			$NtSKt$			$Nt\ SKt$		
d	0.41^{***}	0.47^{***}	0.47^{***}	0.33^{***}	0.36^{***}	0.37^{***}	0.30^{***}	0.36^{***}	0.36^{***}	0.20^{***}	0.28^{***}	0.29^{***}
	(0.12)	(0.05)	(0.07)	(0.07)	(0.06)	(0.06)	(0.07)	(0.06)	(0.06)	(0.09)	(0.07)	(0.06)
φ_1	0.22^{***}	0.02^{***}	0.01^{***}	0.35^{***}	0.32^{***}	0.31^{***}	0.63^{***}	0.56^{***}	0.56^{***}	0.27^{***}	0.41^{***}	0.42^{***}
	(0.14)	(0.04)	(0.04)	(0.09)	(0.06)	(0.06)	(0.11)	(0.10)	(0.10)	(0.19)	(0.08)	(0.08)
β_1	0.5^{***}	0.93^{***}	0.93^{***}	0.59^{***}	0.62^{***}	0.62^{***}	0.77^{***}	0.62^{***}	0.75^{***}	0.43^{***}	0.63^{***}	0.63^{***}
	(0.2)	(0.01)	(0.01)	(0.10)	(0.06)	(0.06)	(0.07)	(0.07)	(0.07)	(0.22)	(0.08)	(0.07)
$Log(\hat{\alpha})$	-0.1^{***}	-0.1^{***}	-0.1^{***}	0.1^{**}	0.08^{*}	0.04^{**}	0.06^{*}	0.11^{**}	0.11^{**}	0.26^{*}	0.16^{***}	0.16^{***}
	(0.06)	(0.01)	(0.00)	(0.05)	(0.05)	(0.02)	(0.07)	(0.05)	(0.02)	(0.19)	(0.07)	(0.01)
ν	–	5.19^{***}	5.20^{***}	–	5.89^{***}	5.89^{***}	–	4.52^{***}	4.52^{***}	–	4.29^{***}	4.29^{***}
	–	(0.41)	(0.41)	–	(0.50)	(0.51)	–	(0.32)	(0.32)	–	(0.29)	(0.28)
ξ	–	–	0.02^{**}	–	–	0.04^{***}	–	–	0.008^{*}	–	–	0.16^{**}
	–	–	(0.01)	–	–	(0.01)	–	–	(0.01)	–	–	(0.07)
$ln(\ell)$	−6826	−6638	−6637	−7676	−7552	−7548	−8875	−8690	−8690	−7518	−7297	−7269
AIC	3.09	3.019	3.013	3.483	3.426	3.425	4.026	3.943	3.943	3.411	3.311	3.311
$Q(20)$	28.27	31.04	31.12	21.28	23.12	23.00	12.89	22.01	22.06	33.44	35.50	35.58
$Q^2(20)$	16.8	15.60	15.53	17.73	18.21	18.24	17.39	15.48	15.46	9.46	11.39	11.57
$RBD(10)$	7.3	7.52	7.4	9.99	35.49	57.6	36.53	17.77	17.90	3.96	4.24	4.31
$ARCH(10)$	0.79	0.8	0.8	0.92	1.08	1.08	1.47	1.26	1.26	0.29	0.35	0.36
$P(60)$	321^{***}	436^{***}	511^{***}	199^{***}	79^{***}	64^{***}	430^{***}	249^{***}	272^{***}	375^{***}	156^{***}	149^{***}

Notes: $\ln(\ell)$ is the value of the maximized log-likelihood. AIC is the Akaike (1974) Information criterion. $Q(20)$ and $Q^2(20)$ are the Box-Pierce statistics for remaining serial correlation for respectively standardized and squared standardized residuals. $RBD(10)$ is the residual based diagnostic for conditional heteroscedasticity, using 10lags. The $ARCH(10)$ is LM- ARCH test of Engle (1982) using 10lags. $P(60)$ is the Pearson goodness-of-fit statistic for 60 cells. Figures between parentheses are the standard errors. N , t and SKt are respectively normal, Student-t and the skewed Student-t distribution. *, ** and *** the significant level of 10%; 5% and 1%, respectively.

Table-10. In-sample VaR estimation results

	Panel a : Aluminium									
	Short trading position					**Long trading position**				
	Quantile	Failure rate	Kupiec LRT	P-value	ES	Quantile	Failure rate	Kupiec LRT	P-value	ES
Normal	0.95	0.960	14.711	0.0001	0.97	0.05	0.055	4.182	0.041	4.179
	0.975	0.981	11.232	0.0009	0.96	0.025	0.034	11.51	0.0005	11.51
	0.99	0.990	1.779	0.183	0.99	0.01	0.017	14.52	0.0001	14.52
	0.995	0.994	0.253	0.616	0.99	0.005	0.011	19.90	8.1e-005	19.94
	0.9975	0.998	0.041	0.839	0.99	0.0025	0.008	30.23	3.2e-009	30.25
										Continue

Student-t	0.95	0.958	8.812	0.002	8.81	0.05	0.059	10.61	0.001	10.59
	0.975	0.984	17.65	2.9e-6	17.5	0.025	0.029	5.157	0.024	5.16
	0.99	0.995	12.32	0.003	12.4	0.01	0.012	0.566	0.452	0.57
	0.995	0.996	8.216	0.003	8.22	0.005	0.005	0.532	0.466	0.54
	0.9975	0.999	6.82	0.008	6.83	0.0025	0.005	3.841	0.05	3.85
skSt	0.95	0.951	0.036	0.846	0.02	0.05	0.052	0.114	0.732	-3.02
	0.975	0.979	2.975	0.085	2.98	0.025	0.026	0.009	0.920	-3.58
	0.99	0.990	0.751	0.384	0.76	0.01	0.008	1.386	0.238	-4.60
	0.995	0.997	3.465	0.059	3.47	0.005	0.005	0.085	0.770	-5.67
	0.9975	0.999	1.316	0.252	1.32	0.0025	0.003	0.143	0.708	-6.25
	Panel b : Copper									
Normal	0.95	0.962	10.570	0.001	2.67	0.05	0.052	2.023	0.153	-2.76
	0.975	0.979	8.078	0.003	3.14	0.025	0.032	7.85	0.004	-2.93
	0.99	0.995	5.708	0.015	3.20	0.01	0.016	12.92	0.000	-3.24
	0.995	0.998	0.573	0.449	3.38	0.005	0.009	9.978	0.001	-3.82
	0.9975	0.997	0.006	0.942	3.59	0.0025	0.005	14.125	0.001	-4.45
Student-t	0.95	0.958	8.328	0.002	2.59	0.05	0.061	9.759	0.001	-2.58
	0.975	0.982	15.308	0.000	2.92	0.025	0.030	7.850	0.004	-2.76
	0.99	0.994	14.612	0.001	3.31	0.01	0.012	4.02	0.042	-3.23
	0.995	0.997	10.537	0.000	3.36	0.005	0.005	0.522	0.468	-4.27
	0.9975	0.998	4.392	0.037	3.47	0.0025	0.002	0.262	0.611	-5.22
skSt	0.95	0.950	0.040	0.843	2.43	0.050	0.051	0.910	0.340	-2.65
	0.975	0.977	0.516	0.472	2.81	0.025	0.025	0.020	0.880	-2.89
	0.99	0.991	4.207	0.039	3.26	0.010	0.011	0.088	0.763	-3.54
	0.995	0.996	1.470	0.226	3.20	0.005	0.005	0.286	0.590	-4.64
	0.9975	0.999	1.856	0.173	3.40	0.0025	0.002	0.141	0.703	-6.01
	Panel c : Nickel									
Normal	0.95	0.957	4.085	0.042	1.99	0.05	0.045	0.140	0.703	-2.11
	0.975	0.978	3.434	0.060	2.31	0.025	0.024	0.785	0.370	-2.49
	0.99	0.989	0.903	0.340	2.60	0.01	0.011	9.547	0.001	-2.97
	0.995	0.997	0.020	0.890	2.85	0.005	0.008	17.930	0.001	-3.22
	0.9975	0.995	0.373	0.541	2.82	0.0025	0.006	29.682	0.000	-3.40
Student-t	0.95	0.953	1.770	0.172	1.98	0.05	0.052	1.019	0.310	-2.06
	0.975	0.979	6.509	0.011	2.35	0.025	0.026	0.184	0.662	-2.54
	0.99	0.992	10.441	0.000	2.79	0.01	0.013	1.512	0.215	-3.03
	0.995	0.997	11.979	0.000	2.78	0.005	0.007	5.559	0.019	-3.37
	0.9975	0.998	12.575	0.001	4.13	0.0025	0.003	4.674	0.034	-3.90
skSt	0.95	0.946	0.770	0.380	1.91	0.050	0.051	0.013	0.910	-2.09
	0.975	0.974	0.352	0.551	2.22	0.025	0.026	0.617	0.432	-2.62
	0.99	0.991	0.039	0.845	2.53	0.010	0.010	0.415	0.521	-3.28
	0.9950	0.995	3.670	0.057	2.77	0.005	0.006	1.568	0.212	-3.43
	0.9975	0.997	2.321	0.128	2.86	0.0025	0.004	2.772	0.099	-3.77
	Panel d : Zinc									
Normal	0.95	0.956	10.219	0.001	2.17	0.05	0.050	0.061	0.810	-2.31
	0.975	0.978	4.139	0.042	2.39	0.025	0.031	8.114	0.000	-2.50
	0.99	0.992	0.239	0.629	2.98	0.01	0.013	6.012	0.012	-3.09
	0.995	0.996	0.233	0.632	3.22	0.005	0.009	14.046	0.000	-3.05
	0.9975	0.996	7.008	0.009	3.31	0.0025	0.005	14.312	0.000	-3.29
Student-t	0.95	0.959	5.194	0.023	2.19	0.05	0.053	1.069	0.305	-2.27
	0.975	0.979	6.430	0.010	2.48	0.025	0.030	4.632	0.032	-2.55
	0.99	0.992	8.275	0.003	3.25	0.01	0.011	0.677	0.409	-2.99
	0.995	0.996	0.308	0.580	3.35	0.005	0.005	0.765	0.379	-3.22
	0.9975	0.997	0.735	0.390	4.78	0.0025	0.003	0.790	0.375	-3.71
skSt	0.95	0.951	0.811	0.369	2.14	0.05	0.051	0.013	0.910	-2.31
	0.975	0.976	1.560	0.213	2.42	0.025	0.024	0.000	0.987	-2.68
	0.99	0.991	0.231	0.630	2.98	0.01	0.011	0.035	0.853	-3.11
	0.995	0.995	0.018	0.897	3.15	0.005	0.004	1.472	0.227	-3.68
	0.9975	0.996	0.115	0.733	4.05	0.0025	0.002	0.115	0.735	-3.54

Source: OxMetrics outputs

In-sample VaR and ES results for the three models are reported in the table (10) above. Results indicate that VaR models based on the normal AR (1)-FIAPARCH model fail in modelling large positive and negative returns. Indeed, the hypothesis of correct model is fully rejected since there is a considerable difference between the prefixed level (α) and the failure rate. These results are the consequence of excess of kurtosis and skewness which are not taken into account in the case of a normal distribution. While, results given in the case of the symmetric Student-t FIAPARCH model are quietly better than normal distribution in the fact that the Student-t distribution may models fat tail in returns. Therefore, VaR based on Student-t AR (1)-FIAPARCH model improves those given in the normal case. More precisely, VaR performance is improved but is still not satisfactory in all cases. VaR model based on the skewed-Student-t FIAPARCH model outperforms all other models for both short and long trading position. Results indicate the skewed-Student-t distribution which has the ability to model both fat tailed and skewed returns performs very well for both short and long VaR.

5.2.2. The out-of-SampleVaR Estimation Results

As we know Value-at-Risk target is to quantify the potential losses in a definite horizon. Indeed, VaR model is based on forecasting risk which has to be made for a holding period forecast h. In our study we have tested the short and long VaR out-of-sample for one day horizon. Therefore, the skewed-Student-t FIAPARCH model under three alternative innovations' distribution was assessed to predict the one-day-ahead VaR. Indeed, we considered 1000 observations of the out-of-sample (last five years). Our forecast updated the FIAPARCH model parameters every 50 observations in the out-of-sample period.

Table-11. Out-of-sample VaR estimation results

	Panel a : Aluminium									
	Short trading position					Long trading position				
	Quantile	Failure rate	Kupiec LRT	P-value	ES	Quantile	Failure rate	Kupiec LRT	P-value	ES
Normal	0.95	0.962	3.889	0.049	2.23	0.05	0.066	4.916	0.028	-2.81
	0.975	0.988	7.144	0.005	3.42	0.025	0.037	5.998	0.016	-3.50
	0.99	0.997	6.826	0.009	4.01	0.01	0.018	6.474	0.011	-4.12
	0.995	0.998	4.796	0.026	4.77	0.005	0.009	3.886	0.049	-5.47
	0.9975	0.999	1.168	0.280	4.78	0.0025	0.008	5.437	0.018	-6.32
Student-t	0.95	0.956	0.542	0.464	2.25	0.05	0.060	1.620	0.204	-2.30
	0.975	0.980	2.225	0.131	3.03	0.025	0.034	1.851	0.171	-3.68
	0.99	0.994	3.091	0.072	3.40	0.01	0.012	0.832	0.360	-4.78
	0.995	0.998	2.341	0.126	3.75	0.005	0.007	1.531	0.218	-6.05
	0.9975	0.999	1.170	0.280	4.71	0.0025	0.003	0.764	0.385	-6.12
skSt	0.95	0.966	5.269	0.022	2.52	0.05	0.060	4.920	0.019	-2.87
	0.975	0.988	7.146	0.008	3.50	0.025	0.035	7.830	0.004	-3.40
	0.99	0.992	1.017	0.312	3.65	0.01	0.019	12.484	0.000	-4.29
	0.995	0.996	2.342	0.127	3.77	0.005	0.015	17.756	0.001	-4.52
	0.9975	0.998	0.108	0.744	3.79	0.0025	0.017	21.978	0.000	-5.00
	Panel b : Copper									
Normal	0.95	0.965	6.881	0.009	2.00	0.05	0.071	6.830	0.007	-2.25
	0.975	0.986	3.804	0.050	2.25	0.025	0.040	6.887	0.007	-2.63
	0.99	0.994	4.705	0.029	2.80	0.01	0.020	9.284	0.002	-3.31
	0.995	0.994	2.341	0.124	2.46	0.005	0.014	8.907	0.001	-4.06
	0.9975	1.001	NaN	1.000	NaN	0.0025	0.011	12.782	0.000	-4.59
Student-t	0.95	0.965	6.039	0.012	1.96	0.05	0.071	9.815	0.000	-2.16
	0.975	0.984	2.950	0.084	2.21	0.025	0.042	10.976	0.000	-2.51
	0.99	0.998	6.823	0.009	2.55	0.01	0.023	9.285	0.003	-3.26
	0.995	1.002	NaN	1.003	NaN	0.005	0.015	3.890	0.051	-4.59
	0.9975	1.000	NaN	1.000	NaN	0.0025	0.005	3.516	0.062	-5.73
sk St	0.95	0.960	1.811	0.180	1.86	0.050	0.066	4.344	0.039	-2.21
										Continue

	0.975	0.981	0.691	0.402	2.11	0.025	0.034	2.390	0.124	-2.76
	0.99	0.997	3.095	0.080	2.60	0.010	0.015	3.078	0.081	-3.64
	0.995	0.998	2.342	0.124	2.47	0.005	0.010	2.597	0.108	-4.75
	0.9975	1.001	NaN	1.002	NaN	0.0025	0.002	0.760	0.385	-6.50
	Panel c : Nickel									
Normal	0.95	0.960	2.745	0.098	1.89	0.05	0.061	2.389	0.120	-2.20
	0.975	0.987	5.886	0.016	2.16	0.025	0.036	4.380	0.034	-2.49
	0.99	0.991	0.433	0.511	2.51	0.01	0.023	17.944	0.000	-2.81
	0.995	0.997	0.216	0.644	2.42	0.005	0.013	15.342	0.000	-2.84
	0.9975	0.998	1.168	0.280	2.03	0.0025	0.011	10.10	0.001	-3.23
Student-t	0.95	0.960	1.422	0.234	1.87	0.05	0.061	3.299	0.070	-2.12
	0.975	0.990	8.559	0.004	2.08	0.025	0.032	2.390	0.123	-2.60
	0.99	0.995	3.091	0.080	2.30	0.01	0.016	4.090	0.041	-2.78
	0.995	0.998	4.790	0.026	2.03	0.005	0.011	2.598	0.105	-3.21
	0.9975	1.000	NaN	1.000	NaN	0.0025	0.002	0.096	0.760	-4.40
skSt	0.95	0.953	0.347	0.555	1.82	0.050	0.059	2.389	0.120	-2.16
	0.975	0.982	4.776	0.027	2.09	0.025	0.031	0.962	0.326	-2.72
	0.99	0.991	0.432	0.511	2.50	0.010	0.014	0.829	0.364	-2.87
	0.9950	0.998	4.798	0.029	2.04	0.005	0.010	2.594	0.109	-3.25
	0.9975	0.999	1.171	0.281	2.03	0.0025	0.003	0.108	0.743	-3.01
	Panel d : Zinc									
Normal	0.95	0.966	7.778	0.004	2.09	0.05	0.056	0.989	0.319	-2.40
	0.975	0.982	8.559	0.002	2.28	0.025	0.029	1.374	0.240	-2.86
	0.99	0.997	9.628	0.001	1.67	0.01	0.019	9.285	0.001	-3.29
	0.995	0.999	4.799	0.029	1.91	0.005	0.015	10.912	0.000	-3.67
	0.9975	0.999	1.171	0.281	1.93	0.0025	0.007	7.642	0.004	-3.61
Student-t	0.95	0.965	7.778	0.004	2.11	0.05	0.059	1.986	0.160	-2.32
	0.975	0.989	11.901	0.000	2.52	0.025	0.029	0.966	0.324	-2.89
	0.99	0.998	13.475	0.001	1.93	0.01	0.018	6.470	0.009	-3.40
	0.995	0.998	4.795	0.027	1.91	0.005	0.007	1.531	0.217	-3.61
	0.9975	1.000	NaN	1.000	NaN	0.0025	0.004	0.760	0.380	-4.75
skSt	0.95	0.964	5.267	0.022	2.06	0.05	0.055	0.731	0.391	-2.40
	0.975	0.986	7.144	0.006	2.23	0.025	0.028	0.625	0.431	-2.92
	0.99	0.997	9.623	0.002	1.70	0.01	0.012	1.439	0.232	-3.70
	0.995	0.998	4.798	0.026	1.90	0.005	0.005	0.187	0.661	-4.09
	0.9975	1.000	NaN	1.000	NaN	0.0025	0.003	0.760	0.380	-4.79

Source: OxMetrics outputs

As it's given in the table (11) above, estimates results of VaR out-of-sample are quietly similar to those of the In-sample VaR. More precisely, the skewed Student-t FIAPARCH model has a great ability to improve the VaR estimation quality in fact that the null hypothesis of correct model is not rejected. However, unlike in-sample VaRresults, the out-of-sample VaR under a normal distribution outperforms the symmetry Student-t FIAPARCH because the Student-t distribution is still very conservative.

6. CONCLUSION

In this paper we have computed VaR and ES for four metal return series. Since all sample return series are characterized by volatility clustering, we have assessed five GARCH-type models including three fractionary integrated models for the sake of taking into account volatility persistence phenomenon. GARCH-type models are assessed under three alternative distributions which are normal, Student-t and skewed Student-t distributions (only RiskMetrics model was been assessed under a normal distribution). Our findings show that the skewed Student-t FIAPARCH model performs others thanks to its ability to consider jointly for asymmetry, long memory. We have computed the VaR for one day ahead for both short and long trading position. Backtesting results reveal the performance of this model (skewed Student-t FIAPARCH). These results may help manager in measuring and hedging market risk.

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