Asian Economic and Financial Review
ISSN(e): 2222-6737/ISSN(p): 2305-2147
URL: www.aessweb.com

COASIAN THEOREM, PUBLIC DOMAIN, AND PROPERTY RIGHTS PROTECTION

Yiming He[1] --- Shaohui Gao[2†]
[1]*Division of Resource Management, West Virginia University, Morgantown, WV 26506, USA; College of Economics & Management, South China Agricultural University, Guangzhou, Guangdong, China*
[2]*College of Economics & Management, South China Agricultural University, Guangzhou, Guangdong, China; Project of Border Econometrics, University of Texas at El Paso, El Paso, USA*

ABSTRACT

In this paper, the expressions of Coasian Theorem and the concept of public domain are discussed at the beginning. Based on it, this paper derived a mathematic logic about Coasian Theorem. At last, it proves that if the number of owners who protect the public property is more than one, and then rent dissipation will occur. In addition, the enthusiasm for property rights protector in the information symmetry is greater than in the case of asymmetric information. Therefore, in order to protect the property rights more effective, we need to give the property rights to only man, so that asymmetric information structure can be transformed into the symmetric information structure. At this point, the public domain has become the private domain, so rent dissipation disappears.

Keywords: Coasian theorem, Public domain, Property rights protection, Rent dissipation, Non-cooperative game, Information structure.
JEL Classification: C72, K11, Q15.

Received: 1 December 2016/ **Revised:** 10 January 2017/ **Accepted:** 19 January 2017/ **Published:** 30 January 2017

Contribution/ Originality

This paper originates new game logic about Coasian Theorem. It theoretically finds that if the number of owners who protect the public property is more than one, then rent will dissipate. When property rights are given to only man, the public domain transfers into the private domain and rent dissipation disappears.

1. INTRODUCTION

If the economics was understood as a science on optimal allocation of resources according to the classic definition of history of economic thought Master Robbins of the British London School of Economics (Robbins, 1932) then the land right economics can be defined as a subdiscipline of delimitation, allocation and performance evaluation of property rights about land resources. Furthermore, the land right economics can be regarded as the specific application of economic theory paradigm of property rights in land resources and exactly originated in the Coasian Theorem (Coase, 1960). The founder of the New Institutional Economics professor Coase proposed the Coasian Theorem (Coase, 1960). Thus, the Coasian theorem first is briefly reviewed in the study before the basic

† Corresponding author
DOI: 10.18488/journal.aefr/2017.7.5/102.5.470.485
ISSN(e): 2222-6737/ISSN(p): 2305-2147

principles of land right economics are discussed in detailed. And the relation between the Coasian theorem and the allocative efficiency of land property rights is described on the basis.

The word "Coasian theorem" was not put forward by Coase himself, but was first proposed by the master of history of economic thought of American Chicago School Professor Stigler in the book of "Price Theory" (Stigler, 1966). The study of "The Problem of Social Cost" published in the Law and Economics journal (Coase, 1960) was summarized. One of Stigler's conclusions is as follows: "the property rights endowed initially can not affect on the efficiency of resource allocation when the transaction costs are zero and the definition of property right is clear." In fact the conclusion is the "Coase (first) theorem and is eulogized by today academia". However, the statement was not agreed by Coase, then Coase published a study entitled "Explanation on the Problem of Social Cost" to clarify all misunderstandings to his thought (Coase, 1988) after taking office in the chief editor of Law and Economics journal. Rents and transaction costs two basic concepts were used by Coase in the study. The "Coasian (second) theorem" was restated and was accepted by Coase himself. He pointed out, the study of the Problem of Social Cost would like to emphasize that "the property rights endowed initially will have a decisive impact on the resource allocation when the transaction cost is greater than zero". As a matter of fact, "Federal Communications Commission" published in 1959 by Coase had expressed the prototype of Coasian second theorem according to Coase student's interpretation. A simple conclusion obtained by Coase in the study was that: "the delimitation of rights is the necessary precondition of market transaction" (Coase, 1959). The theorem had a standard significance, although was not proposed by Coase. Besides Edgeworth et al had previously come up with the exchange theorem. However, all the constraints they described were incomplete while the Coase's contribution just rested with that the constraints were described in detailed. The exchange theorem played a role that must be subject to the constraints-the transaction costs. Namely only under conditions of zero transaction costs is the theorem established.

Thus it can be seen that Coase and his students all emphasized the initial given object of property rights and the effect on the efficiency of resource allocation in the real world containing transaction costs. In addition, the property rights of one resource could be divided into private property right, collective property right and state-owned property right three types in accordance with the classification criteria of property rights economics to categories of property right owners (Alchian, 1965). So, the land property rights can be also divided into private land right, collective land right and state land right three types by this standard. However, the Coasian second theorem can be known, who the land right initially given becomes fairly significant when the transaction costs are greater than zero in the real world. Because the initial land right determines the final allocation efficiency of land resources. However, Coase himself did not indicate which owner property right was given was the most efficient and only stressed that the materiality of the initial delimitation of property rights and the transaction costs in resource allocation. Because comparative advantages and transaction costs faced of each property right owner (herein including private, collective and state) are not the same. A uniform and exclusive conclusion can not be gotten when the private property right or other non-private property rights are assumed as the optimal. In reality, the Coasian second theorem wanted to the world that there was no absolute optimal property right institution in the real world. People must determine and compare the efficiencies of resource allocation according to the results obtained by the property right owners with different active abilities under constraints of different transaction costs after the property right owners carried out the rational calculation of costs and benefits. Thus, the basic principles of land right economics presented are unfolded hereunder on the basis of Coase's idea.

Actually, economics has a long history to study the governance and protection of public resources and the relevant rights. However, no matter in Hardin's "tragedy of the commons" (Hardin, 1968) Nash's "prisoner's dilemma" (Nash, 1951) Olson's "collective action logic" (Olson, 1965) or Coase's "social costs" (Coase, 1960) it is always indicated that the management of public resources is full of difficulties. In addition, Ostrom's polycentric

governance model is also focused on the property rights in a common pool. The so-called common-pool resources are as well referred to as common property resources featuring both non-exclusivity (or a high exclusive cost) and competitiveness (Ostrom, 1990). When taking advantage of public resources, people share the entire resource system together but individually utilize resource units. Different from the non-exclusive and non-competitive pure public goods, these resources may come into being as a result of the government's price control behavior. This is because, under the constraints of government regulation, the most valuable usage of resources helps to limit the transaction price below the equilibrium market prices. Owing to this, the divergence between the highest evaluation and the regulated prices of resources emerges and this additional revenue is actually placed in the "public domain" and thus turned into public rent. Those potential rational actors will enter the domain where resources of value are grabbed through queuing, violence, transactions or other means.

In the field of property economics, Barzel (1989) was the first to adopt the concept of "public domain" to analyze the delimitation and distribution of property rights of public resources. His introduction of the concept mainly starts from the resource attributes of diversity and variability. According to him, "…Commodities have many attributes whose levels vary from one specimen to another. Measuring these levels is too costly to be comprehensive or entirely accurate. How difficult it is to obtain full information in the face of variability fundamentally determines how difficult it is to delineate rights. Because it is costly to measure commodities fully, the potential of wealth capture is present in every exchange. The opportunity for wealth capture is equivalent to finding property in the public domain…Some wealth spills over into the public domain in every exchange, and individuals spend resources to capture it…Whereas people always expect to gain from exchange, they also always spend resources on capture……Restrictions may serve to separate rights…Incomplete separation, however, makes attributes common property, relinquishing them to the public domain." In other words, in Barzel's view, the public domain is the unclearly defined part of the bundle of rights and the expensive evaluation cost caused by the diversity of commodity attributes and their variations is daunting. So the public domain is filled with attributes not accurately understood and resources in "absence of owner". Consequently, anyone who is willing to spend resources or to pay for the capture is allowed to enter the domain and no individual has the right to exclude others from coming in. Obviously, the "public domain" is equivalent to "the rights not completely and clearly defined by the contract" in the GHM model (Grossman and Hart, 1986; Hart and Moore, 1990; Hart, 1995).

Indeed in this study, Coasian Theorem – the basic law of New Institutional Economics – will be regarded as the starting point. With the help of the conceptual tools in the public domain, the article will continue to analyze the problems of rent dissipation and protection efficiency concerning public property.

First, as it is known to all, "Coasian Theorem" was put forward by Stigler, master of history of economic thoughts, in his textbook *Price Theory* (Stigler, 1966) while Coase himself did not claim to have proposed this term. Later, faced with numerous fault or tautology studies on Coasian Theorem, Coase's student Steven N.S. Cheung argues that all the falsifications on the theorem are likely to be made out of context. But if the property economic thought of Coase must be summarized, there might exist three varied expressions (2000):

The first statement was in Coase's paper on "the Federal Communications Commission" in 1959. In this paper, Coase came to the simple conclusion that "the definition of rights is an essential prerequisite for market transactions". This is a standard definition, although it was not put forward by Coase (Edgeworth and others had proposed "exchange theorem". However, the constraint conditions they described were incomplete). Coase's contribution lies in a detailed description that exchange theorem works under constraints – transaction costs. That is to say, only when the transaction cost is zero will the law be tenable.

The second statement, which is also known as "invariance theorem", is inferred from Coase's article *The Problem of Social Cost*. According to the theorem, if property rights are clearly defined and all transaction costs are

zero, then no matter who owns the property, resources operate in the same way. Obviously, the precise invariance is not Coase's concern. Here, what draw his attention are the constraint conditions in which the value of the resources utilized maximizes. Though it is not Coase's original intention, this statement becomes today's Coasian Theorem.

The third one is expressed as follows: if property rights are clearly defined and the transaction cost is zero, Pareto Optimality will be satisfied.

Thus, Coase's study prompts us to expand (information or transaction costs) options under constraints and apply them in the comparative analysis of the system. Firstly, this study gives the mathematical proof of Coasian Theorem. Then the theorem is employed to discuss the issue of property rights protection with complete information in the public domain. On this basis, the paper further eases constraint conditions and examines the efficiency of property protection with complete or incomplete information in public domain in symmetric information and asymmetric information games.

2. FROM TWO COASIAN THEOREMS TO PROPERTY RIGHTS ALLOCATION EFFICIENCY

In the absence of calculation and comparison of transaction costs, rights is the most efficient. If the evaluation of public resource utilization efficiency is set as the starting point, the sorting results of different types of right efficiencies could be converted into size comparison of transaction costs in line with this logic. In the past, why neoclassical economics were criticized that could not explain the real economic phenomenon, the reason was that the paradigm did not investigate seriously the transaction cost constraints or simply assumed that the transaction costs were zero, conclusions or hypotheses educed were only a set of "blackboard economics". The key issue is that the "idealization" process of transaction costs will result in the "efficiency evaluation failure". Because the differences of economic efficiencies are assessed by comparing institutional arrangements between the imagine world and the real world, the conclusion is obtained inevitably that the latter will always is "inefficient". The specific contents are expressed as the following theorems:

Theorem 1: Compared to the ideal world (transaction costs are zero), the same kind of land property right institutions must be inefficient in the real world (transaction costs are positive numbers).

The economic interpretation of theorem is, if perfectly rational people make public resources free flow towards the place with the highest rating or make public resources use change via free information search activities in the ideal world, the land maximum value can be achieved. So, the end results of public resources allocations will be the embodiment of Pareto optimal efficiency levels. But there is uncertainty everywhere in the real world and the land resources have contiguous management scale, geographical location specificity, soil fertility heterogeneity and heterogeneity of public resource type spatial distribution and other special attributes. Therefore delimitation and implementation of public resource property rights need all necessarily an information cost incurred when the public resource characteristics are understood, measured and controlled. Only when public resources rights are clearly defined can optimal allocation and use of public resources be achieved. Thus, when the maximum public resources useful value is achieved, public resources need inevitably the public resources right owner to bear a determination right cost. In other words, compared to the ideal world without transaction costs, when the optimal allocation of public resources gets the ultimate value in the real world, the ultimate value equals to the difference of the maximum value of public resources minus the determination right cost. While the optimal allocation of public resources achieves the maximum value in the ideal world the maximum value does not have to deduct the determination right cost. Thereby the ultimate value of public resources obtained must be greater than that of the real world. The determination right cost is the transaction costs according to the logic. In comparison with the ideal world of free determination right cost, the transaction costs are bound to be seen as an economic waste or efficiency loss, so the ideal world without transaction costs is efficient certainly than the real world with transaction costs.

It is thus clear that such a comparison is worthless. Only comparisons between institutional arrangements in the real world is testable scientific implication included. More importantly, after the transaction costs are taken into account, the sorting results of land right efficiencies in the ideal world will change. Herein, the sorting theorem of land right efficiencies first is gained in the ideal world:

Theorem 2: The efficiency of collective land right is lower than that of private land right but higher than that of state land right in the ideal world.

The theorem can be demonstrated via a simple geometric model of land right rent (see particularly Figure 1). In Figure 1, there are only two factors of production, homogeneous labor force e and land resources l. Then the two production factors are combined by the marginal factor cost mfc in applications. mfc equals the wage rate w. Various alternative supply and demand relations in external market determine the wage rate w. vap and vmp denote the value curve of land average output and that of land marginal output respectively.

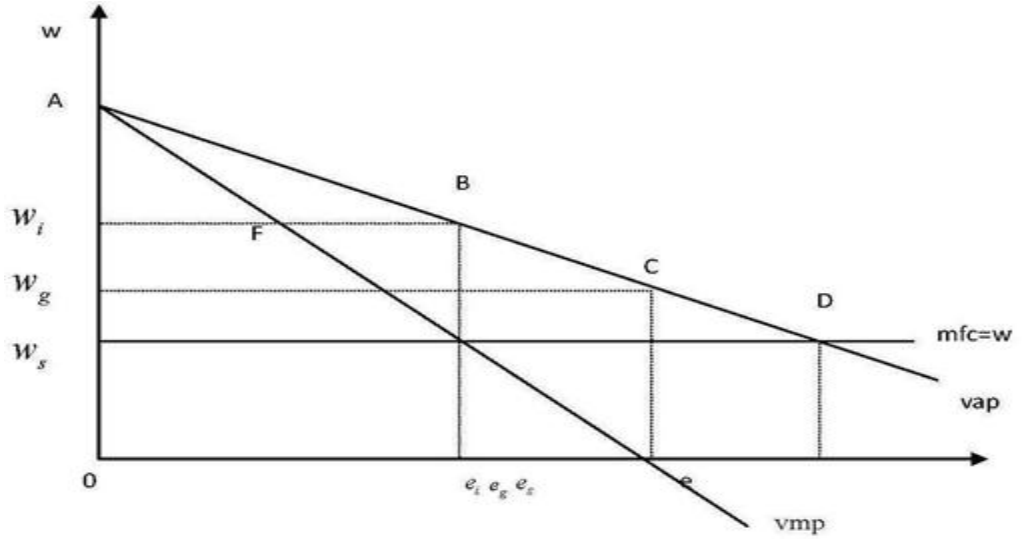

Figure-1. Rent comparison geometric diagram in different right institutions

Source: Compiled by this study

Figure 1 can be shown that the land rent is the maximum under the private land right, the collective land right takes second place and the state property right is zero in the ideal world: (1) When the land resources l belongs to private ownership, the private property right owner will determine the labor input e_i according to the marginal condition (vmp = w). Thus the rent $R_i = s_{w_s w_i BE}$ is obtained. (2) The state is the nominal owner of land rights under the state property right. But the people are entitled to get the rent of state land right within the "public sector" as long as the people belong to the citizens of this country. No one is entitled to reject others for using land resources due to the absence of actual land right owner. Then people enter continuously the public sector and contest with "public rent" each other until all rents are dissipated and used up ($R_s = 0$), so as to achieve an equilibrium state (vap = w). At the moment the labor input is e_s. (3) When the land resource l is share among the members of collective land rights, the labor input is between the private land rights and the state land rights. $e_g[\in (e_i, e_s)]$ might as well is set. The members of collective land rights are entitled to reject non-collective members for using collective lands. But any one is not entitled to hinder the opposite side for using collective land resource use and getting revenue among members. Moreover the less the decision-making units, the more the each unit obtained rents in the public sector.

Since the number of decision-making individuals of collective land right is more than that (only one) of private land right, but is less than the number of all citizens of state land right (close to infinite) in line with the logic. The rent is gotten as $R_g = s_{w_s w_g CG} < R_i$ and $R_g > R_s = 0$. If the rent sizes represent the efficiency levels of all types of land right institutions, then the private land right thus is the most efficient because the rent is maximum. The rent of state land right is the minimum and therefore the efficiency is the lowest. And the collective land right is in the middle.

Indeed, there is the transaction cost and the costs can not be ignored in the real world because the additional transaction costs are caused by the special properties of land resources. If the transaction costs are fully thought about, the sorting results of land right efficiency in the theorem 2 may be reversed. For example, non-proprietary of land resources may be more efficient than privatization under a given condition. In other words, originally the most efficient private land rights may become inefficient after transaction costs are thought over. Concretely, when the determination right costs of private land right (i.e. transaction costs) tc are larger than the monopoly rents, the net rent of private land rights is negative, that of state land rights may be zero and that of collective land rights may be greater than zero. Thus another major viewpoint is achieved in the study:

Theorem 3: The efficiency of collective land right may be the maximum in the real world when the transaction costs are larger than the monopoly rents.

The theorem demonstration is quite simple as long as the geometric analysis in Figure 1 is combined. But now the efficiency is represented by the net rent (the difference between rental income and transaction cost). For example, the net rent of private land right is $NR_i = s_{w_s w_i BE} - tc < 0$, that of state land right is $NR_s = \lim_{n\to\infty}(0 - \frac{tc}{n}) = 0$ and n is the number of nominal owners under the $s_{w_s w_i BE} < tc$ condition. And the net rent of collective land right is $NR_g = s_{w_s w_g CG} - \frac{tc}{n} > 0$ (The principle of integer continuity shows that there is a necessary n^*, so that $s_{w_s w_g CG} > \frac{tc}{n^*}$ i.e. $NR_g > 0$). As a result, $NR_i < 0 = NR_s < NR_g$ is educed.

In short, the efficiency of state land right is not necessarily the lowest, that of private land right is also not necessarily the highest while the collective land right may also be the most efficient after the transaction costs are taken into account in the real world.

Theorems 1-3 are summarized and then the fundamental principles of land right economics are achieved in the study:

Theorem 4: The efficiency of land rights depends on the comparison between rental size and transaction cost size in the real world. Irrespective of the transaction costs is unrealistic in the path of ideal land right transformation.

That is to say, the collective land right is higher than the state land right under the constraints of established land resource attributes. Therefore, the transition of the state land rights towards the collective land rights is a path of rational choice with maximized efficiency as orientation. At the same time the constraints of transaction cost faced by the behavioral owners are changed therewith when the attribute of land resources changes. Possibly the efficiency of private land right is lower than that of collective land right. Right now if the collective land right is segmented and assigned to each collective member and the land resources are privatized, the optimization rational behavior criterion under constraints is not met. The transformation of continual land right collective ownership is actually a rational choice rather than nationalization or privatization in line with the logic after the calculation of costs and benefits.

3. MATHEMATICAL MODEL OF COASIAN THEOREM

Here, the study attempts to demonstrate that the optimal allocation of resources will ultimately be realized if there is no information cost. Whether the ownership is in the hands of the state or possessed by decentralized decision-making individuals, the above fact will always exist.

First, assume that in the economic system, only the state S and decentralized decision-making individual i will conduct resource allocation activities and that their factors of production are land l and employee e. Among them, the production function of S is $Y_S = F_S(l_S, e_S)$. Let S have an external effect on i's resource usage. Then the production function of i will be $y_i = f_i(l_i, e_i, l_S, e_S)$. In this case, the resource constraints of elements are $\bar{l} \geq l_S + l_i$ and $\bar{e} \geq e_S + e_i$. Consequently, the problem of social optimal configuration can be expressed as:

$$\max_{l,e} F_S(l_S, e_S) + f_i(l_i, e_i, l_S, e_S)$$
$$s.t. \quad \bar{l} \geq l_S + l_i$$
$$\bar{e} \geq e_S + e_i$$

This is a non-linear programming problem. Construct Lagrange function in the following way:

$$L = F_S(l_S, e_S) + f_i(l_i, e_i, l_S, e_S) - \mu_1(\bar{l} - l_S - l_i) - \mu_2(\bar{e} - e_S - e_i)$$

Apply Kuhn-Tucker Theorem, and the following result can be obtained:

$$\begin{cases} \frac{\partial L}{\partial l_S} = \frac{\partial F_S}{\partial l_S} + \frac{\partial f_i}{\partial l_S} - \mu_1 = 0 \\ \frac{\partial L}{\partial l_i} = \frac{\partial f_i}{\partial l_i} - \mu_1 = 0 \\ \frac{\partial L}{\partial e_S} = \frac{\partial F_S}{\partial e_S} + \frac{\partial f_i}{\partial e_S} - \mu_2 = 0 \\ \frac{\partial L}{\partial e_i} = \frac{\partial f_i}{\partial e_i} - \mu_2 = 0 \\ \frac{\partial L}{\partial \mu_1} = -(\bar{l} - l_S - l_i) = 0 \\ \frac{\partial L}{\partial \mu_2} = -(\bar{e} - e_S - e_i) = 0 \end{cases}$$

Therefore, the optimal conditions for an economic system is;

$$\begin{cases} \frac{\partial f_i}{\partial l_i} = \frac{\partial F_S}{\partial l_S} + \frac{\partial f_i}{\partial l_S} \\ \frac{\partial f_i}{\partial e_i} = \frac{\partial F_S}{\partial e_S} + \frac{\partial f_i}{\partial e_S} \end{cases}$$

3.1. Property Allocated by the State

Now, the optimal input choice of decentralized decision-maker i depends on the action of S, i.e. $l_i = l_i(l_S)$ and $e_i = e_i(e_S)$. So i's net rent should be $NR_i(l_S, e_S) = f_i[\, l_i(l_S)\ , l_S, e_i(e_S)\ , e_S] - we_i(e_S)\ - \mathrm{r}\, l_i(l_S)$.

When these individuals transfer their property rights into the control of the state, net rents of the former will drop to $NR_i(0,0)$. This causes individual rent dissipation $NR_i(0,0)-NR_i(l_S,e_S)=\Delta NR_i$. Now assume that after private bargaining, S will be responsible for the rent loss. Then the optimal selection of S becomes $\max_{l_S,e_S} F_S(l_S,e_S)-rl_S-we_S-\Delta NR_i$.

Therefore: $\begin{cases}\dfrac{\partial F_S}{\partial l_S}-w+\dfrac{\partial NR_i}{\partial l_S}=0\\ \dfrac{\partial F_S}{\partial e_S}-r+\dfrac{\partial NR_i}{\partial e_S}=0\end{cases}$.

Meanwhile, the optimal selection of the individual decision-makers is $\max_{l_i,e_i} f_i(l_i,l_S,e_i,e_S)-we_i-\mathrm{r}\,l_i+\Delta NR_i$. Hence, $\begin{cases}\dfrac{\partial f_i}{\partial l_i}-\mathrm{r}=0\\ \dfrac{\partial f_i}{\partial e_i}-w=0\end{cases}$.

Since $\frac{\partial NR_i}{\partial l_S}=\frac{\partial f_i}{\partial l_S}+\frac{\partial f_i}{\partial l_i}\frac{dl_i}{dl_S}-r\frac{dl_i}{dl_S}=\frac{\partial f_i}{\partial l_S}+\frac{dl_i}{dl_S}(\frac{\partial f_i}{\partial l_i}-r)=\frac{\partial f_i}{\partial l_S}$, $\frac{\partial NR_i}{\partial e_S}=\frac{\partial f_i}{\partial e_S}$ is obtained for the same reason.

Equilibrium is achieved when $\begin{cases}\dfrac{\partial F_S}{\partial l_S}+\dfrac{\partial f_i}{\partial l_S}=r=\dfrac{\partial f_i}{\partial l_i}\\ \dfrac{\partial F_S}{\partial e_S}+\dfrac{\partial f_i}{\partial e_S}=w=\dfrac{\partial f_i}{\partial e_i}\end{cases}$.

As it can be seen, even if the state controls the property, as long as the former is willing to pay for the rent dissipation, optimal allocation of social resources will still be realized under the condition of zero negotiating cost.

3.2. Property Dominated by Decentralized Decision-Making Individuals

In contrast to the situation above, if the property is controlled by decentralized individual i on the condition that i pays S $\Delta NR_i'=NR_i(l_S,e_S)-NR_i(l_S^0,e_S^0)$, S's optimal choice after receiving rent compensation will become $\max_{l_S,e_S} F_S(l_S,e_S)-rl_S-we_S+\Delta NR_i'$.

Hence, $\begin{cases}\dfrac{\partial F_S}{\partial l_S}-r+\dfrac{\partial NR_i'}{\partial l_S}=0\\ \dfrac{\partial F_S}{\partial e_S}-w+\dfrac{\partial NR_i'}{\partial e_S}=0\end{cases}$.

In this case, the optimal choice of decentralized decision-making individual i becomes $\max_{l_i,e_i} f_i(l_i,l_S,e_i,e_S)-we_i-\mathrm{r}\,l_i-\Delta NR_i'$.

Therefore, $\begin{cases} \dfrac{\partial f_i}{\partial l_i} - \mathrm{r} = 0 \\ \dfrac{\partial f_i}{\partial e_i} - w = 0 \end{cases}$

As it can be seen, even if the state controls the property, as long as the former is willing to pay for the rent dissipation, optimal allocation of social resources will still be realized under the condition of zero negotiating cost.

Obviously, in equilibrium the result is the same as that in state-dominated property.

4. COASIAN THEOREM IN PUBLIC DOMAIN: EXCESSIVE RENT CAPTURE AND INADEQUATE PROPERTY PROTECTION

4.1 Excessive Consumption of Rent and Coasian Theorem

First of all, in the public domain, there exists the non-exclusive economic rent. Since the unit capture amount of the rent by rational individual i is d_i (i=1, …, n), the total amount equals to $\mathrm{D}=\sum_{i=1}^{n} d_i$. Individual i has a subjective evaluation of the value of public rent v (D) and $v'(D)<0$, $v''(D)<0$. Rent-hastening cost is w (D), $w'(D)>0$ and $w''(D)>0$.

Then, the equilibrium condition of rent capture in which all the n rational actors without exclusive right pursue maximum net rent NR_i is discussed. Assume that under this circumstance the initial property rights are given to n rational actors (Situation Ⅰ):

$$\max_{d_i} NR_i = d_i[v(D)-w(D)] \qquad (\mathrm{i}=1,...,\mathrm{n})$$

$$\text{s.t. } \mathrm{D}=\sum_{i=1}^{n} d_i \qquad (\mathrm{i}=1,...,\mathrm{n})$$

.

In order to maximize the value of the individual net rental income NR_i, let $\partial NR_i / \partial d_i = 0$. That is, the following first-order condition is satisfied: $\mathrm{v}(\sum_{i=1}^{n} d_i)-\mathrm{w}(\sum_{i=1}^{n} d_i)+d_j\ [\mathrm{v}'(\sum_{i=1}^{n} d_i)-\mathrm{w}'(\sum_{i=1}^{n} d_i)]=0 \quad (\mathrm{i}=1,...,\mathrm{n})$.

Hence the only stagnation point $d_i = d_i^*$.

Sine $v'(D)-w'(D)<0$ and $v''(D)-w''(D)<0$, the following equation is obtained:

$$\partial^2 NR_i / \partial d_i^2 = 2[v'(\sum_{i=1}^{n} d_i)-\mathrm{w}'(\sum_{i=1}^{n} d_i)]+d_j\ [v''(\sum_{i=1}^{n} d_i)-w''(\sum_{i=1}^{n} d_i)]<0$$

.

So d_i^* is the extreme maximal value point, the only stagnation point and thus the point of the maximum value. Add up n first-order conditions, then:

$k[v(D_1)-w(D_1)]+D_1[v'(D_1)-w'(D_1)]=0$ and $D_1=\sum_{i=1}^{n} d_i^* (i=1,\ldots,n)$.

That is, $v(D_1)-w(D_1)+\frac{D_1}{n}[v'(D_1)-w'(D_1)]=0$ and

$NR_1=\sum_{i=1}^{n} NR_i^*=D_1[v(D_1)-w(D_1)]=0(i=1,\ldots,n)$.

Next, the discussion comes to the situation where there is a unique rational actor with the exclusive rights of the whole public rent, i.e., he is in possession of the right to monopolize the public domain. Assume that initially the property is given to only one rational actor. In other words, the "public domain" has become his "private sphere" and the amount of rent capture of the single individual is equivalent to that of the sum of n actors. Then the optimization problem (Situation Ⅱ) is as follows:

$\underset{D}{\text{Max}}\ NR=D[v(D)-w(D)]$.

Calculate the derivative of the above-mentioned arithmetic expression and the equation $v(D)-w(D)+D[v'(D)-w'(D)]=0$ is obtained.

Thus, the only stagnation point is $D=D_2$. Since $v'(D)-w'(D)<0$ and $v''(D)-w''(D)<0$, the conclusion is that

$\frac{d^2NR}{dD^2}=2[v'(D)-w'(D)]+D[v''(D)-w''(D)]<0$.

so D_2 is the extreme maximal value point, the only stagnation point and thus the point of the maximum value. In this way, the optimal value of the whole public rent is calculated: $NR_2=D_2[v(D_2)-w(D_2)]$.

At last, reduction to absurdity is employed to compare Situation Ⅰ and Situation Ⅱ. Assume that $D_1 \le D_2$.

Since $v''(D)<0$, hence $v'(D_1)\ge v'(D_2)$.

Since $w''(D)>0$, hence: $w'(D_1)\le w'(D_2)$.

Consequently, $v'(D_1)-w'(D_1)\ge v'(D_2)-w'(D_2)$.

Since $D_2>0$, hence $D_2[v'(D_1)-w'(D_1)]\ge D_2[v'(D_2)-w'(D_2)]$.

Equations of $\frac{D_1}{n}[v'(D_1)-w'(D_1)]=w(D_1)-v(D_1)$ and $D_2[v'(D_2)-w'(D_2)]=w(D_2)-v(D_2)$ haven been proved.

Since $v'(D)<0$, hence: $v(D_1) \geq v(D_2)$.

Since $w'(D)>0$, hence: $w(D_1) \leq w(D_2)$.

Therefore, $w(D_2)-v(D_2) \geq w(D_1)-v(D_1)$.

So: $D_2[v'(D_2)-w'(D_2)] \geq \frac{D_1}{n}[v'(D_1)-w'(D_1)]$.

From the fundamental assumption, it is obtained that $v'(D)-w'(D)<0$ and that n>1. So:

$0>\frac{D_1}{n}[v'(D_1)-w'(D_1)]>D_1[v'(D_1)-w'(D_1)]=0$.

Combine the expressions above, there is $D_2[v'(D_1)-w'(D_1)]>D_1[v'(D_1)-w'(D_1)]=0$.

Since $v'(D_1)-w'(D_1)<0$, hence: $D_1>D_2$. This is in contradiction with the original assumptions, so the hypothesis is incorrect but $D_1>D_2$ is true.

Since $\frac{dNR}{dD_2}=0$ and $\frac{d^2NR}{dD_2^2}<0$, hence: $NR(D_2)>NR(D)$ ($D_2<D<+\infty$). Since $D_1>D_2$, hence: $NR(D_2)>NR(D_1)$.

$D_1>D_2$ and $NR(D_2)>NR(D_1)$ indicate that when all rational behavior subjects do not own exclusive rights, they will be driven by opportunistic motives, leading to the excessive and inefficient consumption of the economic rent in the public domain (This is usually not the case when a single individual is with exclusive property rights). In other words, to reduce rent dissipation, property rights in the public domain must be endowed to a certain decentralized decision-maker. However, the subject chosen should not be the state. This is because the state has to entrust other agents to indirectly exercise the exclusive rights, which will generate new problems of rent dissipation. The only solution to improve the efficiency of resource allocation is to re-distribute the property rights from the state to decentralized decision-makers. These individuals will, according to their own comparative advantage, apply resource rights directly to where the highest evaluation is offered. In light of this logic, if the initial property is given to a number of decentralized individuals, one of them will eventually obtain the corresponding right after the bargaining game of zero transaction cost. In this way, the efficiency of resource allocation will be improved. It is obvious that this conclusion is consistent with Coasian Theorem.

4.2. Inadequate Protection of Public Property and Coasian Theorem

The analysis above may be considered to have been conducted from the perspective of the demand of the subjects without ownership to infringe public property rights. Then the discussion of this section is one basing research on the property protection services provided by the owner of public resources. Similarly, the protection amount provided by the public resource owners i (i = 1, ..., n) for public property is g_i. Thus, the total amount of protection is $G=\sum_{i=1}^{n} g_i$

. They protect public resources in order to utilize them to gain individual benefits v_i. The consumption of other resources is referred to as $other_i$, the costs of property protection etc and other expenditures unrelated to public property protection as_i. Then the function of i's personal rent is $R_i = f(G, other_i)$. Here, $\partial^2 f / \partial other_i^2 < 0$, $\partial^2 f / \partial G^2 < 0$ and $d(\frac{\partial f / \partial G}{\partial f / \partial other_i}) / dG < 0$.

First of all, assume that the property rights are initially given to n rational behavior subjects and that all these public resource owners seek to maximize the rent (Situation Ⅲ):

$$\underset{G, other_i}{\text{Max}} R_i = f(G, other_i) \qquad (i=1,...,n)$$

$$\text{s.t.} \quad v_i = as_i other_i + etcg_i$$

$$G = \sum_{i=1}^{n} g_i$$

.

From Lagrange Function $L(other_i, G; \iota) = f(G, other_i) + \iota(as_i other_i + etcg_i - v_i)$, the first-order condition is inferred:

$$\frac{\partial f}{\partial G} - \iota etc = 0; \frac{\partial f}{\partial other_i} - \iota as_i = 0; as_i other_i + etcg_i - v_i = 0$$

.

Therefore, $\frac{\partial f / \partial G}{\partial f / \partial as_i} = \frac{etc}{as_i}, (i=1,...,n)$.

Because $\frac{\partial f / \partial G}{\partial f / \partial other_i}$ is the decreasing function of G, there must exist such a point G_1 that fulfills the expression above, i.e. $\frac{\partial f / \partial G}{\partial f / \partial other_i}(G_1) = \frac{etc}{as_i}, (i=1,...,n)$.

Next, assume that one of the individuals acquires the property of the entire public resources through bargaining and thus the "agglomeration" and "integration" of public property rights are realized. Then here comes the optimization problem regarding the only public property owner (Situation Ⅳ). Because the single individual is the sole owner of public property, it can be assumed that his rent function is equivalent to the total rent functions of n subjects.

$$\text{Max R} = \sum_{i=1}^{n} \beta_i R_i \qquad (i=1,...,n)$$

$$\text{s.t.} \sum_{i=1}^{n} v_i = as_i \sum_{i=1}^{n} n_i + \text{etcG}$$

.

From the first-order condition of Lagrange Function $L(G, \beta_i; \mu) = \sum_{i=1}^{n} \beta_i R_i - \mu(\sum_{i=1}^{n} v_i - as_i \sum_{i=1}^{n} other_i - \text{etcG})$, it can be inferred that

$$\sum_{i=1}^{n} \frac{\partial f / \partial G}{\partial f / \partial other_i} = \frac{etc}{as_i}.$$

Furthermore, $\frac{\partial f / \partial G}{\partial f / \partial other_i} = \frac{etc}{as_i} - \sum_{j \neq i} \frac{\partial f / \partial G}{\partial f / \partial other_j}, (i=1,...,n)$.

Because $\frac{\partial f/\partial G}{\partial f/\partial other_i}$ is the decreasing function of G, there must exist such a point G_2 that fulfills the following expression: $\frac{\partial f/\partial G}{\partial f/\partial other_i}(G_2)=\frac{etc}{as_i}-\sum_{j\neq i}\frac{\partial f/\partial G}{\partial f/\partial other_i}$,(i=1,...,n).

At last, compare Situation Ⅲ and Situation Ⅳ. Since $\sum_{j\neq i}\frac{\partial f/\partial G}{\partial f/\partial other_j}>0$, hence:

$$\frac{\partial f/\partial G}{\partial f/\partial other_i}(G_2)=\frac{etc}{as_i}-\sum_{j\neq i}\frac{\partial f/\partial G}{\partial f/\partial other_j}<\frac{etc}{as_i}-\frac{\partial f/\partial G}{\partial f/\partial other_i}(G_1)$$

Because $\frac{\partial f/\partial G}{\partial f/\partial other_i}$ is the decreasing function of G, the relationship between G_1 and G_2 fulfills: $G_1<G_2$.

Obviously, when public property rights are in possession of a number of behavior subjects rather than in the hands of an exclusive owner, the protection offered to the property is far from adequate. Coase Theorem states that when the transaction cost is zero, free contracts will help to optimize resources allocation, regardless of the receiver of the initial property. Therefore, if initially the property of public resources is given to the state, the state must entrust agents with the indirect use of exclusive rights, relinquishing the actual property to administrative organs and making them the real property owners of public resources. On the contrary, if a decentralized decision-maker gains, through free bargaining games, the whole property right of public resources, the protection efficiency concerned will be enhanced. Therefore, to realize effective protection, the property rights of public resources must be endowed to a certain decentralized decision-maker and the subject chosen should not be the state. This is because the state has to entrust other agents to indirectly exercise the exclusive rights, which will generate new problems of agency efficiency loss. Only when the state re-assigns its property rights to decentralized decision-making individuals to enable them to exercise exclusive rights in accordance with their own value preference, can the resource protection efficiency be improved.

5. EXPANSION OF COASE THEOREM: PROPERTY RIGHT PROTECTION GAME IN INCOMPLETE INFORMATION CONSTRAINTS

The Coasian theorem believes that the property right given initially will affect hardly the efficiency of resource allocation when the transaction (information) costs are zero. The problem belongs to the property right protection in the public sector under the complete information. At present, the study further narrows down constraints, the efficiency of property right protection in the public sector is investigated in either complete or incomplete information, and either symmetric information or asymmetric information game pattern.

5.1. Property Right Protection Game in Symmetric Information

The property right owner i and the property right owner j are assumed to carry out the property right protection activities under the conditions of symmetric information. The probabilities are K_i and K_j respectively. The specific game structure is shown in as following (Table 1).

Table-1. Game payoff matrix in symmetric information

		Property right owner j	
Property right owner i	Protection (K_i)	Protection (K_j)	No protection ($1-K_j$)
		v−etc, v−etc	v−etc, v
	No protection ($1-K_i$)	v, v−etc	0, 0

Source: Compiled by this study

it was shown that the protection property right expectation net rent function of property right owner j is (Table 1):

$E NR_j(K_i, K_j) = K_j[K_i(v-etc)+(1-K_i)(v-etc)]+(1-K_j)[K_i v+(1-K_i)0]$ and the expectation function of property right owner i is:

$E NR_i(K_i, K_j) = K_i[K_j(v-etc)+(1-\alpha)(v-etc)]+(1-K_i)[K_j v+(1-K_j)0]$.

After the derivation of above two formulas, the optimal first-order conditions of j and i are:

$\frac{\partial ENR_j}{\partial K_j} = v-etc\text{-}K_i v=0$ and $\frac{\partial ENR_i}{\partial K_i} = v-etc-K_j v=0$ respectively.

Thereby the Nash equilibrium solution is obtained: $K_j^* = K_i^* = \frac{v-etc}{v}$.

5.2. Property Right Protection Game in Asymmetric Information

Different from the above situations, ξ_i is assumed as the regret value of participant i in the asymmetric information when the participant i learns about others who do not participate except for himself. In addition, ξ_i satisfies the uniform distribution in the interval [0, h] (h≤2). Distribution function is common knowledge.

Table-2. Game payoff matrix in asymmetric information

		Property right owner j	
Property right owner i		Protection	No protection
	Protection	v－etc, v－etc	$v-etc-\xi_i$, v
	No protection	v, $v-etc-\xi_i$	0, 0

Source: Compiled by this study

Since ξ_i satisfies the uniform distribution in the interval [0, h], the probabilities of j participation and non-participation property right protection are $1\text{-}\frac{\xi_j}{h}$ and $\frac{\xi_j}{h}$ respectively.

The j strategy now is given, i chooses that the expectation profit (net rent) of either "protection" or "no protection" is as follows: $E NR_1 = (1-\frac{\xi_j}{h})(v-etc)+\frac{\xi_j}{h}(v-etc-\xi_i)$ or $E\pi_2 = (1-\frac{\xi_j}{u})v+\frac{\xi_j}{u}0$. Hence, ξ_i meets that:

$$(1-\frac{\xi_j}{h})(v-etc)+\frac{\xi_j}{h}(v-etc-\xi_i) = (1-\frac{\xi_j}{h})v+\frac{\xi_j}{h}0.$$

Then the above formula is simplified into: $\xi_j v=etcu+\xi_i\xi_j$,

because the game is symmetric $\xi_j=\xi_i$ in equilibrium.

And because the game is symmetric, the Nash equilibrium solution is $\xi_j = \xi_i = \xi^* = \frac{v-\sqrt{v^2-4etch}}{2}$.

It is worth noting that: the other root $\frac{v+\sqrt{v^2-4etch}}{2}$ is rounded herein, because $h \to 0$, $\xi^* \to 0$, but $0 \le \xi^* \le h \to 0$ is contradicted. Thus, the probability of j participation property right protection is $1-\frac{v-\sqrt{v^2-4etch}}{2h}$.

i.e. $f(\mathrm{h})=1-\frac{v-\sqrt{v^2-4etch}}{2h}$ and is continuous in the definitional domain. It is shown that $f(0)=\lim_{h\to 0} f(\mathrm{h})=\lim_{h\to 0}(1-\frac{v-\sqrt{v^2-4etch}}{2h})=\frac{v-etc}{v}$ when $h \to 0$ via observation.

Because $f'(h)=\frac{v\sqrt{v^2-4etch}-v^2+4etch-2etc}{2h^2\sqrt{v^2-4etch}}<\frac{v^2-v^2+4etch-2etc}{2h^2\sqrt{v^2-4etch}}\le 0(0\le \mathrm{h}\le 2)$,

f=f (h) obtained is a decreasing function namely $1-\frac{v-\sqrt{v^2-4etch}}{2h}=f(\mathrm{h})<f(0)=\frac{v-etc}{v}$ in the interval [0, h].

The above equation manifests that: the participating into property right protection enthusiasm of property right owner of symmetric information should be greater than that of the asymmetric information. If the property protection becomes effective, it is necessary to change asymmetric information into symmetric information. However, information is always incomplete and asymmetrical in the real world, or when the property right owner collects the relevant information, only by a certain amount of resources consumed can the asymmetric information change into symmetric information. Even though the resource allocation effect can be achieved under complete information, considerable information costs should also be paid. If the property right protection owners are not more nothing but one, the problem of asymmetric information does not exist. In other words, the property right is endowed a unique owner, the information costs can be internalized. At this time the public sector also becomes the private sector, rent dissipation also disappears therewith. Therefore, if the state allocates the disposable property rights to the individual of each specific decentralized decision-making, and endows their clear right again, so that each of them all becomes the sole owner of a resource and plays dual role of pursuing (private sector) rent and protection (private sector) property right. But the right overlapping phenomenon does not appear. The information cost achieves internalization. The private property right thus is born in the public sector in endogenous. Ultimately the phenomena of rent convergence (inverse dissipation) and wealth accumulation improve the institutional performance.

6. CONCLUSION

In this paper, the expressions of Coasian Theorem and the concept of public domain are discussed at the beginning. Based on it, this paper derived a mathematic logic about Coasian Theorem. At last, it proves that if the number of owners who protect the public property is more than one, and then rent dissipation will occur. In addition, the enthusiasm for property rights protector in the information symmetry is greater than in the case of asymmetric information. Therefore, in order to protect the property rights more effective, we need to give the property rights to only man, so that asymmetric information structure can be transformed into the symmetric information structure. At this point, the public domain has become the private domain, so rent dissipation disappears.

Funding: The authors acknowledge financial support from the USDA research grant "Evaluating irrigation water quality regulation in fresh produce production: the case of Treasure Valley in Eastern Oregon and Southwest Idaho", Guangdong complementary fund of National Ten Thousand Outstanding Young Scholar Program "The Paradigm of Agricultural Deregulation", the Project of Guangdong College Excellent Young Teacher Training Plan [Yq2014032], and the Key Project of National Natural Science Foundation in China [71333004].

Competing Interests: The authors declare that they have no competing interests.

Contributors/Acknowledgement: All authors contributed equally to the conception and design of the study.

REFERENCES

Alchian, A., 1965. Some economics of property rights. Il Politico, 30(4): 816-829. *View at Google Scholar*

Barzel, Y., 1989. Economic analysis of property rights. Cambridge: Cambridge University Press.

Coase, R., 1959. The federal communications commission. Journal of Law and Economics, 2(1): 1-40.

Coase, R., 1960. The problem of social cost. Journal of Law and Economics, 3(3): 1-44.

Coase, R., 1988. The firm, the market, and the law. Chicago: University of Chicago Press.

Grossman, S. and O. Hart, 1986. The costs and benefits of ownership: A theory of vertical and lateral integration. Journal of Political Economy, 94(4): 691-719. *View at Google Scholar | View at Publisher*

Hardin, G., 1968. The tragedy of the commons. Science, 162(3859): 1243-1248. *View at Publisher*

Hart, O., 1995. Firms, contracts, and financial structure. Oxford: Clarendon Press.

Hart, O. and J. Moore, 1990. Property rights and the nature of the firm. Journal of Political Economy, 98(6): 1119-1158. *View at Google Scholar | View at Publisher*

Nash, J., 1951. Non-cooperative games. Annals of Mathematics, 54(2): 286-295. *View at Google Scholar*

Olson, M., 1965. The logic of collective action: Public goods and the theory of groups. New York: Schocken Books.

Ostrom, E., 1990. Governing the commons. Cambridge: Cambridge University Press.

Robbins, L., 1932. An essay on the nature and significance of economic science. London: Macmillan.

Stigler, G., 1966. The theory of price. Basingstoke: Macmillan.