



BUSINESS CYCLES WITH PERIODIC SHOCKS IN A MULTI-COUNTRY AND MULTI-REGIONAL NEOCLASSICAL GROWTH MODEL



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ABSTRACT

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This paper generalizes the global economic growth model with any number of countries and each country with any number of regions recently proposed by Zhang (2016). Zhang's model extends Uzawa's two-sector growth model to a global economy for examining dynamic interactions between international trade, national and global growth, interregional migration, wealth accumulation and regional amenities. This study generalizes Zhang's model by allowing all the time-independent parameters to be time-dependent. The generalization makes it possible to examine effects of any types of exogenous time-dependent shocks on the dynamic system cross regions and countries over time. We simulate the model with three countries and each country with two regions. We demonstrate the existence of equilibrium point and confirm (local) stability of the equilibrium point when all the parameters are time-independent. We conduct comparative dynamic analysis with regard to exogenous periodic shocks in the total factor productivity of regions' capital good sectors, the total factor productivities of the service sectors, the propensity to save, the amenity parameters, and the propensity to consume housing. Our comparative analysis shows how business cycles are generated by periodic exogenous shocks.

Contribution/ Originality: This study contributes to the existing literature of nonlinear dynamic economics by generalizing the global economic growth model proposed by Zhang (2016). The study is one of very few studies that deal with business cycles of multi-country multi-region economic growth with microeconomic foundation.

1. INTRODUCTION

This paper generalizes the global economic growth model with any number of countries and each country with any number of regions recently proposed by Zhang (2016). Zhang's model extends Uzawa's two-sector growth model to a global economy for examining dynamic interactions between international trade, national and global growth, interregional migration, wealth accumulation and regional amenities. This study generalizes Zhang's model by allowing all the time-independent parameters to be time-dependent. The generalization makes it possible to examine effects of any types of exogenous time-dependent shocks on the dynamic system cross regions and countries over time. We specially show how business cycles can be generated by different exogenous periodic shocks.

Zhang's model provides insights into economic mechanisms of national spatial agglomeration and globalization which are taking place simultaneously in a well-connected world economy. There are only a few theoretical economic models which address these issues within a single comprehensive framework. The neoclassical economic growth model studies a global economy with multiple (any number of) countries and multiple (any number of) regions in each country. With regard to capital mobility and international and interregional trade, the model is based on the neoclassical growth trade model. The model is specially influenced by Oniki and Uzawa (1965) which studies trade patterns between two economies in the Heckscher-Ohlin modeling framework with fixed savings rates. Deardorff and Hanson (1978) propose a two country trade mode with different saving rates across countries. There are some other growth models with international trade (e.g., (Brecher *et al.*, 2002; Nishimura and Shimomura, 2002; Bond *et al.*, 2003; Ono and Shibata, 2005)). The study demonstrates the motion of global economies with multiple countries and multiple regions with the help of computer. Although spatial economists propose interregional dynamic models based on different factors of nonlinear dynamics (e.g., (Fujita *et al.*, 1999; Forslid and Ottaviano, 2003)) most of the formal models in economic geography do not include capital/wealth accumulation as endogenous processes of industrialization and agglomeration. Tabuchi (2014) observes, "The scopes of most of the theoretical studies published thus far have been limited to two regions in order for researchers to reach meaningful analytical results. The two-region NEG models tend to demonstrate that spatial distribution is dispersed in the early period (high trade costs or low manufacturing share) and agglomerated in one of the two regions in the late period (low trade costs or high manufacturing share). However, it is no doubt that the two-region NEG models are too simple to describe the spatial distribution of economic activities in real-world economies. Since there are only two regions, their geographical locations are necessarily symmetric, and thus diverse spatial distributions cannot occur." Not only does Zhang's model include wealth accumulation as endogenous processes of the global growth, but also it takes account of environmental dynamics. This paper generalizes Zhang's model by allowing all the time-independent parameters to be time-dependent. The paper is organized as follows. Section 2 defines the multi-country and multi-regional model with capital accumulation and economic structure. Section 3 identifies the differential equations which are applied to simulate the model of the global economy, plots the motion of the economic dynamics, demonstrates the existence of a unique equilibrium point, and proves the stability of the equilibrium point. Section 4 carries out comparative dynamic analysis with regard to exogenous perturbations in the total factor productivity of region's industrial sectors, total factor productivity of region's service sectors, propensity to save, regional amenity parameters, and propensity to consume housing. Section 5 concludes the study. The main analytical results of section 3 are proved in the appendix.

2. THE MULTI-COUNTRY AND MULTI-REGIONAL GROWTH MODEL

This study is to generalize Zhang's dynamic multi-region model (Zhang, 2016). The generalization is carried out by allowing all the time-dependent variables to be time-dependent. This allows us to examine any exogenous change at any time. Zhang's original model is influenced by the neoclassical trade theory with capital accumulation (Uzawa, 1961; Oniki and Uzawa, 1965; Brecher *et al.*, 2002; Sorger, 2002). The model deals with a global economy with multiple national and multiple regional economies. The economy of each region is described as in the Uzawa two-sector model. Each region produces one capital good and one consumer good (services) and has one consumer good sector and capital good sector. The entire economic system produces only one capital good. The capital good can be used for capital accumulation and for consumption. Households own assets of the economy. All markets are perfectly competitive. Input factors are inelastically supplied and are fully utilized at every moment. Saving is undertaken only by households.

There are J countries, indexed by $j = 1, \dots, J$. Country j consists of Q_j regions, indexed by $q = 1, \dots, Q_j$.

The capital good is traded without any barriers. We measure prices in terms of the commodity and the price of the commodity is unity. The assumption of zero transportation cost of the commodity implies price equality for the

commodity in the global economy. We denote wage and interest rates by $w_{jq}(t)$ and $r_{jq}(t)$, respectively, of region q in country j . The interest rate is equalized throughout the global economy, i.e., $r(t) = r_{jq}(t)$. Each country has a homogeneous population, denoted by $N_j(t)$. This is no international migration. People in each country are free to choose their residential location. We assume that people work and reside in the same region. Each region has homogenous land $L_{jq}(t)$ which is only for residential use. Let $\tilde{F}_{jq}(t)$ and $\hat{F}_{jq}(t)$ stand for the output levels of the capital good sector and service sector in region (j, q) .

2.1. The Capital Good Sectors

We denote the two productive factors, capital by $\tilde{K}_{jq}(t)$ and labor by $\tilde{N}_{jq}(t)$, at each point in time t . The production functions are

$$\tilde{F}_{jq}(t) = \tilde{A}_{jq}(t) \tilde{K}_{jq}^{\tilde{\alpha}_{jq}(t)}(t) \tilde{N}_{jq}^{\tilde{\beta}_{jq}(t)}(t), \quad \tilde{\alpha}_{jq}(t), \tilde{\beta}_{jq}(t) > 0, \quad \tilde{\alpha}_{jq}(t) + \tilde{\beta}_{jq}(t) = 1, \\ j = 1, \dots, J, \quad q = 1, \dots, Q_j, \quad (1)$$

where $\tilde{\alpha}_{jq}(t)$, $\tilde{\beta}_{jq}(t)$ and $\tilde{A}_{jq}(t)$ are parameters. The marginal conditions imply

$$r(t) + \delta_{jq}(t) = \frac{\tilde{\alpha}_{jq}(t) \tilde{F}_{jq}(t)}{\tilde{K}_{jq}(t)}, \quad w_{jq}(t) = \frac{\tilde{\beta}_{jq}(t) \tilde{F}_{jq}(t)}{\tilde{N}_{jq}(t)}, \quad (2)$$

where $\delta_{jq}(t)$ is the depreciation rate of physical capital in region (j, q) .

2.2. The Service Sectors

We denote the two productive factors, capital by $\hat{K}_{jq}(t)$ and labor by $\hat{N}_{jq}(t)$, at each point in time t . The production functions are

$$\hat{F}_{jq}(t) = \hat{A}_{jq}(t) \hat{K}_{jq}^{\hat{\alpha}_{jq}(t)}(t) \hat{N}_{jq}^{\hat{\beta}_{jq}(t)}(t), \quad \hat{\alpha}_{jq}(t), \hat{\beta}_{jq}(t) > 0, \quad \hat{\alpha}_{jq}(t) + \hat{\beta}_{jq}(t) = 1, \quad j = 1, \dots, J, \quad q = 1, \dots, Q_j. \quad (3)$$

We use $p_{jq}(t)$ to represent region (j, q) 's service price. The marginal conditions imply

$$r(t) + \delta_{jq}(t) = \frac{\hat{\alpha}_{jq}(t) p_{jq}(t) \hat{F}_{jq}(t)}{\hat{K}_{jq}(t)}, \quad w_{jq}(t) = \frac{\hat{\beta}_{jq}(t) p_{jq}(t) \hat{F}_{jq}(t)}{\hat{N}_{jq}(t)}. \quad (4)$$

2.3. Behavior of Consumers

We assume absent landownership, which means that the income of land rent is spent outside the economic system. A possible reasoning for this is that as the land is owned by the government, people can rent the land in competitive market, and the government uses the income for military or other public purposes. Consumers choose lot size,

consumption levels of services and the commodity, and saving. Let $\bar{k}_{jq}(t)$ stand for the per household wealth in region (j, q) . Consumer (j, q) obtains the current income $y_{jq}(t)$ from the interest payment and the wage payment as follows

$$y_{jq}(t) = r(t)\bar{k}_{jq}(t) + w_{jq}(t). \quad (5)$$

The disposable income $\hat{y}_{jq}(t)$ is the sum of the current come and the total value of wealth

$$\hat{y}_{jq}(t) = y_{jq}(t) + \bar{k}_{jq}(t). \quad (6)$$

The disposable income is used for saving and consumption. At each point in time, consumer (j, q) distributes the total available budget among the lot size $l_{jq}(t)$, the saving $s_{jq}(t)$, the consumption of the capital good $c_{jq}(t)$, and consumption of services $\hat{c}_{jq}(t)$. The budget constraint implies

$$R_{jq}(t)l_{jq}(t) + c_{jq}(t) + p_{jq}(t)\hat{c}_{jq}(t) + s_{jq}(t) = \hat{y}_{jq}(t), \quad (7)$$

where $R_{jq}(t)$ is region (j, q) 's land rent. The utility function $U_{jq}(t)$ is

$$U_{jq}(t) = \theta_{jq}(t)l_{jq}^{\eta_{j0}(t)}(t)c_{jq}^{\xi_{j0}(t)}(t)\hat{c}_{jq}^{\gamma_{j0}(t)}(t)s_{jq}^{\lambda_{j0}(t)}(t), \quad \eta_{j0}(t), \gamma_{j0}(t), \xi_{j0}(t), \lambda_{j0}(t) > 0, \quad (8)$$

in which $\eta_{j0}(t)$, $\xi_{j0}(t)$, $\gamma_{j0}(t)$, and $\lambda_{j0}(t)$ are the elasticities of utility of the consumer in country j with regard to the lot size, capital good, services, and savings in region (j, q) . We call $\eta_{j0}(t)$, $\xi_{j0}(t)$, $\gamma_{j0}(t)$, and $\lambda_{j0}(t)$ the propensities to consume lot size, capital good, and services, and to hold wealth (save), respectively. In (8), $\theta_{jq}(t)$ is called region (j, q) 's amenity level. We specify $\theta_{jq}(t)$ as follows

$$\theta_{jq}(t) = \bar{\theta}_{jq}(t)N_{jq}^{d_j(t)}(t), \quad (9)$$

where $\bar{\theta}_{jq}(t) (> 0)$ and $d_j(t)$ are parameters and $N_{jq}(t)$ is region (j, q) 's population. We don't specify signs of $d_j(t)$ as the population may have either positive or negative effects on regional attractiveness. Maximizing $U_{jq}(t)$ subject to the budget constraint yields

$$l_{jq}(t)R_{jq}(t) = \eta_j(t)\hat{y}_{jq}(t), \quad c_{jq}(t) = \xi_j(t)\hat{y}_{jq}(t), \quad p_{jq}(t)\hat{c}_{jq}(t) = \gamma_j(t)\hat{y}_{jq}(t), \quad s_{jq}(t) = \lambda_j(t)\hat{y}_{jq}(t), \quad (10)$$

where

$$\eta_j(t) \equiv \eta_{j0}(t)\rho_j(t), \quad \xi_j(t) \equiv \xi_{j0}(t)\rho_j(t), \quad \gamma_j(t) \equiv \gamma_{j0}(t)\rho_j(t), \quad \lambda_j(t) \equiv \lambda_{j0}(t)\rho_j(t),$$

$$\rho_j(t) \equiv \frac{1}{\eta_{j0}(t) + \xi_{j0}(t) + \gamma_{j0}(t) + \lambda_{j0}(t)}.$$

2.4. Wealth Accumulation

According to the definition of $s_{jq}(t)$, the wealth accumulation of the representative person (j, q) is given by

$$\dot{\bar{k}}_{jq}(t) = s_{jq}(t) - \bar{k}_{jq}(t). \quad (11)$$

The equation implies that the change in wealth is the saving minus the missaving.

2.5. Equalization of Utility Levels between Regions

As households are freely mobile between regions, the utility level of people in the same country should be equal, irrespective of in which region they live, i.e.

$$U_{jq}(t) = U_{jm}(t), \quad q, m = 1, \dots, Q_j. \quad (12)$$

2.6. Capital and Wealth

We use $K_{jq}(t)$ and $\bar{K}_{jq}(t)$ to denote the capital stocks employed by and the value of wealth owned by the population in region (j, q) . We have

$$K_{jq}(t) = \tilde{K}_{jq}(t) + \hat{K}_{jq}(t), \quad \bar{K}_{jq}(t) = \bar{k}_{jq}(t)N_{jq}(t). \quad (13)$$

We use $K_j(t)$ and $\bar{K}_j(t)$ to denote the capital stocks employed by and the value of wealth owned by country j . We have

$$K_j(t) = \sum_q K_{jq}(t), \quad \bar{K}_j(t) = \sum_q \bar{K}_{jq}(t). \quad (14)$$

The world capital stocks $K(t)$ employed by the production sectors is equal to the total wealth owned by the households of all the countries. That is

$$K(t) = \sum_{j=1}^J K_j(t) = \sum_{j=1}^J \bar{K}_j(t). \quad (15)$$

2.7. Demand and Supply for Services

A region's supply of services is consumed by the region

$$\hat{c}_{jq}(t)N_j(t) = \hat{F}_{js}(t). \quad (16)$$

2.8. Full Employment of the Regional Labor

The labor force is fully employed in each region

$$\tilde{N}_{jq}(t) + \hat{N}_{jq}(t) = N_j(t). \quad (17)$$

2.9. Full Employment of the National Labor Force

The national labor force is fully used

$$\sum_{q=1}^{Q_j} N_{jq}(t) = N_j(t). \quad (18)$$

2.10. The Regional Land Being Fully Used

Each region's land is fully occupied for residential use

$$l_{jq}(t)N_{jq}(t) = L_{jq}(t). \quad (19)$$

We thus built the model. The model is a special case of Zhang's model (Zhang, 2016) in the sense that it is the same as Zhang's model if all the parameters are constant. The model is general in the sense that it includes, for instance, the Solow growth model, the Uzawa two-sector growth model, and the Oniki-Uzawa trade.

3. SIMULATING THE MODEL

The world economy consists of any (finite) number of national economies. Although the dynamic system contains many time-dependent parameters, we provide a computational procedure for calculating all the variables at any point in time. In the appendix, we show that the dynamics of the global economy can be expressed as $1 + \sum_j Q_j$ differential equations (in which one is actually algebra equation as shown in the appendix). First, we introduce a variable $z_{11}(t)$

$$z_{11}(t) \equiv \frac{r(t) + \delta_{11}}{w_{11}(t)}.$$

Lemma

The motion of the global economy is given by the following $1 + \sum_j Q_j$ differential equations with $z_{11}(t)$, $(\bar{k}_j(t))$ and t as variables

$$\begin{aligned} \dot{\bar{k}}_{jq}(t) &= \Phi_{jq}(z_{11}(t), \bar{k}_{jq}(t), t), \quad j = 1, \dots, J, \quad q = 1, \dots, Q_j, \\ \dot{z}_{11}(t) &= \Phi_0(z_{11}(t), (\bar{k}_{jq}(t)), t), \end{aligned} \quad (20)$$

where Φ_{jq} and Φ_0 are functions of $z_{11}(t)$, $(\bar{k}_j(t))$ and t defined in the appendix. For any given positive values of $z_{11}(t)$, $(\bar{k}_j(t))$ and t at any point in time, the other variables are uniquely determined by the following procedure:

$r(t)$ by (A3) $\rightarrow w_{jq}(t)$ by (A4) $\rightarrow \hat{y}_{jq}(t)$ by (A9) $\rightarrow p_{jq}(t)$ by (A6) $\rightarrow N_{j1}(t)$ by (A12) $\rightarrow N_{jq}(t)$ by (A13) $\rightarrow R_{jq}(t)$ by (A13) $\rightarrow l_{jq}(t) = L_{jq} / N_{jq}(t) \rightarrow \hat{N}_{jq}(t)$ by (A14) $\rightarrow \tilde{N}_{jq}(t)$ by (A14) $\rightarrow \hat{K}_{jq}(t)$ and $\tilde{K}_{jq}(t)$ by

$$(A1) \rightarrow \tilde{F}_{jq}(t) \text{ by (1)} \rightarrow \hat{F}_{jq}(t) \text{ by (3)} \rightarrow c_{jq}(t), \tilde{c}_{jq}(t), \text{ and } s_{jq}(t) \text{ by (10)} \rightarrow K_{jq}(t) \text{ by (10)} \rightarrow \bar{K}_{jq}(t) \text{ by (13)} \rightarrow K_j(t) \text{ by (14)} \rightarrow \bar{K}_j(t) \text{ by (14)} \rightarrow K(t) \text{ by (15)} \rightarrow Y_{jq}(t) = \tilde{F}_{jq}(t) + p_{jq}(t)\hat{F}_{jq}(t) \rightarrow Y_j(t) = \sum_q Y_{jq}(t) \rightarrow Y(t) = \sum_j Y_j(t).$$

This result allows us to simulate the model with computer. As it is difficult to interpret the analytical results, to study properties of the system we simulate the model for a 3-country economy. The rest of this section follows Zhang (2016) in examining the motion of the system when all the parameters are constant. We specify parameter values as follows

$$N_1 = 20, N_2 = 15, N_3 = 40, d_j = -0.05, \delta_{1q} = 0.05, \delta_{2q} = 0.06, \delta_{3q} = 0.053,$$

$$\begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.85 \\ 0.75 \\ 0.65 \end{pmatrix}, \begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.1 \\ 0.12 \\ 0.14 \end{pmatrix}, \begin{pmatrix} \eta_{10} \\ \eta_{20} \\ \eta_{30} \end{pmatrix} = \begin{pmatrix} 0.07 \\ 0.08 \\ 0.08 \end{pmatrix}, \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \end{pmatrix} = \begin{pmatrix} 0.07 \\ 0.08 \\ 0.08 \end{pmatrix},$$

$$\begin{pmatrix} L_{11} \\ L_{12} \\ L_{21} \\ L_{22} \\ L_{31} \\ L_{32} \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 8 \\ 12 \\ 15 \\ 20 \end{pmatrix}, \begin{pmatrix} \tilde{A}_{11} \\ \tilde{A}_{12} \\ \tilde{A}_{21} \\ \tilde{A}_{22} \\ \tilde{A}_{31} \\ \tilde{A}_{32} \end{pmatrix} = \begin{pmatrix} 1.2 \\ 1.15 \\ 1 \\ 0.9 \\ 0.8 \\ 0.85 \end{pmatrix}, \begin{pmatrix} \hat{A}_{11} \\ \hat{A}_{12} \\ \hat{A}_{21} \\ \hat{A}_{22} \\ \hat{A}_{31} \\ \hat{A}_{32} \end{pmatrix} = \begin{pmatrix} 1.1 \\ 1.05 \\ 0.9 \\ 0.9 \\ 0.75 \\ 0.65 \end{pmatrix}, \begin{pmatrix} \tilde{\alpha}_{11} \\ \tilde{\alpha}_{12} \\ \tilde{\alpha}_{21} \\ \tilde{\alpha}_{22} \\ \tilde{\alpha}_{31} \\ \tilde{\alpha}_{32} \end{pmatrix} = \begin{pmatrix} 0.32 \\ 0.3 \\ 0.32 \\ 0.3 \\ 0.32 \\ 0.29 \end{pmatrix}, \begin{pmatrix} \tilde{\alpha}_{11} \\ \tilde{\alpha}_{12} \\ \tilde{\alpha}_{21} \\ \tilde{\alpha}_{22} \\ \tilde{\alpha}_{31} \\ \tilde{\alpha}_{32} \end{pmatrix} = \begin{pmatrix} 0.33 \\ 0.31 \\ 0.33 \\ 0.31 \\ 0.33 \\ 0.3 \end{pmatrix}, \begin{pmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{21} \\ \theta_{22} \\ \theta_{31} \\ \theta_{32} \end{pmatrix} = \begin{pmatrix} 3.8 \\ 3.5 \\ 4.1 \\ 4.5 \\ 3.5 \\ 3.8 \end{pmatrix}, \tag{21}$$

We specify the initial conditions as follows

$$z_1(0) = 0.085, \bar{k}_{11}(0) = 6.5, \bar{k}_{12}(0) = 6.1, \bar{k}_{21}(0) = 3.6, \bar{k}_{22}(0) = 2.5, \bar{k}_{31}(0) = 1.8, \bar{k}_{32}(0) = 2.3.$$

The motion of the variables is plotted in Figure 1.

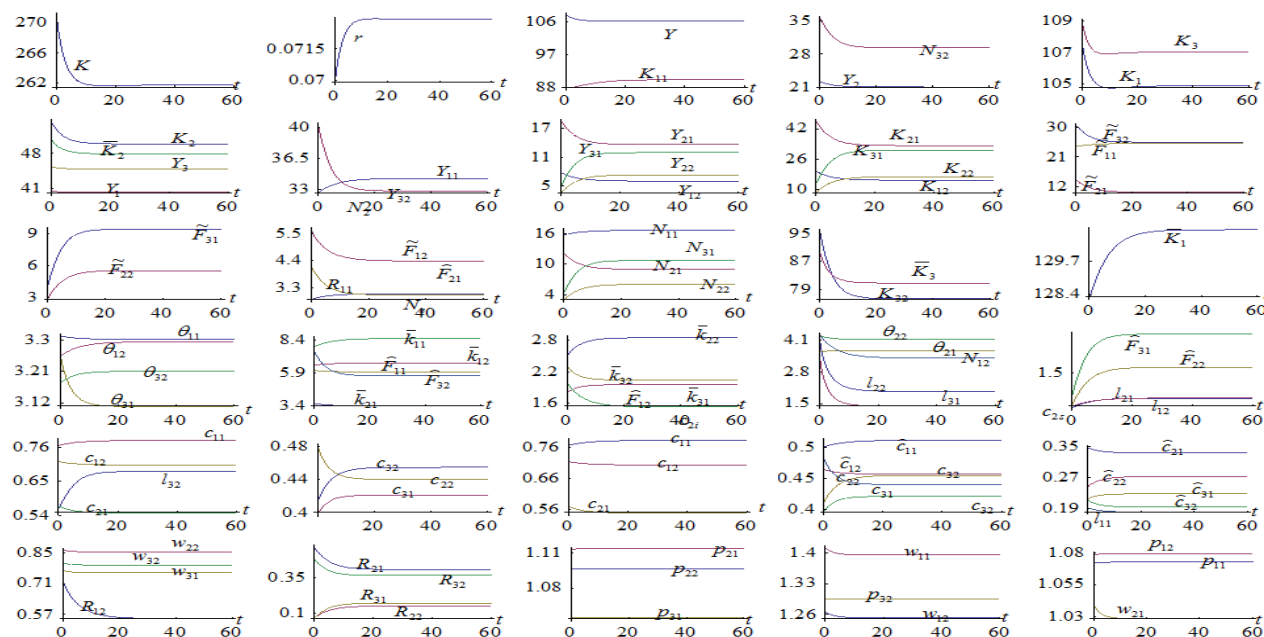


Figure-1. The Motion of the Economic System

The simulation confirms that the system has a unique equilibrium. We list the equilibrium values in (22)

$$Y = 105.9, \quad K = 259.6, \quad r = 0.074, \quad Y_1 = 40.2, \quad Y_2 = 20.95, \quad Y_3 = 44.76,$$

$$\begin{pmatrix} K_1 \\ K_2 \\ K_3 \\ \bar{K}_1 \\ \bar{K}_2 \\ \bar{K}_3 \end{pmatrix} = \begin{pmatrix} 104 \\ 49.5 \\ 106.2 \\ 131 \\ 47.8 \\ 80.7 \end{pmatrix}, \quad \begin{pmatrix} Y_{11} \\ Y_{12} \\ Y_{21} \\ Y_{22} \\ Y_{31} \\ Y_{32} \end{pmatrix} = \begin{pmatrix} 34.2 \\ 6.06 \\ 13.6 \\ 7.34 \\ 11.9 \\ 32.8 \end{pmatrix}, \quad \begin{pmatrix} N_{11} \\ N_{12} \\ N_{21} \\ N_{22} \\ N_{31} \\ N_{32} \end{pmatrix} = \begin{pmatrix} 16.64 \\ 3.36 \\ 8.99 \\ 6.01 \\ 10.68 \\ 29.32 \end{pmatrix}, \quad \begin{pmatrix} K_{11} \\ K_{12} \\ K_{21} \\ K_{22} \\ K_{31} \\ K_{32} \end{pmatrix} = \begin{pmatrix} 89.22 \\ 14.76 \\ 32.84 \\ 16.62 \\ 30.39 \\ 75.76 \end{pmatrix}, \quad \begin{pmatrix} \tilde{F}_{11} \\ \tilde{F}_{12} \\ \tilde{F}_{21} \\ \tilde{F}_{22} \\ \tilde{F}_{31} \\ \tilde{F}_{32} \end{pmatrix} = \begin{pmatrix} 25.04 \\ 4.37 \\ 10.33 \\ 5.52 \\ 9.37 \\ 25.45 \end{pmatrix},$$

$$\begin{pmatrix} \hat{F}_{11} \\ \hat{F}_{12} \\ \hat{F}_{21} \\ \hat{F}_{22} \\ \hat{F}_{31} \\ \hat{F}_{32} \end{pmatrix} = \begin{pmatrix} 8.51 \\ 1.54 \\ 2.99 \\ 1.64 \\ 2.43 \\ 5.69 \end{pmatrix}, \quad \begin{pmatrix} \tilde{N}_{11} \\ \tilde{N}_{12} \\ \tilde{N}_{21} \\ \tilde{N}_{22} \\ \tilde{N}_{31} \\ \tilde{N}_{32} \end{pmatrix} = \begin{pmatrix} 12.24 \\ 2.45 \\ 6.85 \\ 4.54 \\ 8.41 \\ 22.81 \end{pmatrix}, \quad \begin{pmatrix} \hat{N}_{11} \\ \hat{N}_{12} \\ \hat{N}_{21} \\ \hat{N}_{22} \\ \hat{N}_{31} \\ \hat{N}_{32} \end{pmatrix} = \begin{pmatrix} 4.4 \\ 0.92 \\ 2.14 \\ 1.48 \\ 2.27 \\ 6.51 \end{pmatrix}, \quad \begin{pmatrix} \tilde{K}_{11} \\ \tilde{K}_{12} \\ \tilde{K}_{21} \\ \tilde{K}_{22} \\ \tilde{K}_{31} \\ \tilde{K}_{32} \end{pmatrix} = \begin{pmatrix} 64.84 \\ 10.1 \\ 24.73 \\ 12.39 \\ 23.69 \\ 58.3 \end{pmatrix}, \quad \begin{pmatrix} \hat{K}_{11} \\ \hat{K}_{12} \\ \hat{K}_{21} \\ \hat{K}_{22} \\ \hat{K}_{31} \\ \hat{K}_{32} \end{pmatrix} = \begin{pmatrix} 24.38 \\ 4.16 \\ 8.1 \\ 4.23 \\ 6.69 \\ 17.46 \end{pmatrix},$$

$$\begin{pmatrix} p_{11} \\ p_{12} \\ p_{21} \\ p_{22} \\ p_{31} \\ p_{32} \end{pmatrix} = \begin{pmatrix} 1.07 \\ 1.08 \\ 1.1 \\ 1.11 \\ 1.06 \\ 1.3 \end{pmatrix}, \quad \begin{pmatrix} w_{11} \\ w_{12} \\ w_{21} \\ w_{22} \\ w_{31} \\ w_{32} \end{pmatrix} = \begin{pmatrix} 1.39 \\ 1.25 \\ 1.03 \\ 0.85 \\ 0.76 \\ 0.79 \end{pmatrix}, \quad \begin{pmatrix} R_{11} \\ R_{12} \\ R_{21} \\ R_{22} \\ R_{31} \\ R_{32} \end{pmatrix} = \begin{pmatrix} 3.04 \\ 0.55 \\ 0.41 \\ 0.15 \\ 0.17 \\ 0.37 \end{pmatrix}, \quad \begin{pmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{21} \\ \theta_{22} \\ \theta_{31} \\ \theta_{32} \end{pmatrix} = \begin{pmatrix} 3.3 \\ 3.29 \\ 3.67 \\ 4.11 \\ 3.11 \\ 3.21 \end{pmatrix}, \quad \begin{pmatrix} \bar{k}_{11} \\ \bar{k}_{12} \\ \bar{k}_{21} \\ \bar{k}_{22} \\ \bar{k}_{31} \\ \bar{k}_{32} \end{pmatrix} = \begin{pmatrix} 6.66 \\ 5.99 \\ 3.42 \\ 2.84 \\ 1.95 \\ 2.04 \end{pmatrix},$$

$$\begin{pmatrix} l_{11} \\ l_{12} \\ l_{21} \\ l_{22} \\ l_{31} \\ l_{32} \end{pmatrix} = \begin{pmatrix} 0.18 \\ 0.89 \\ 0.89 \\ 2 \\ 1.4 \\ 0.68 \end{pmatrix}, \quad \begin{pmatrix} c_{11} \\ c_{12} \\ c_{21} \\ c_{22} \\ c_{31} \\ c_{32} \end{pmatrix} = \begin{pmatrix} 0.78 \\ 0.7 \\ 0.55 \\ 0.46 \\ 0.42 \\ 0.44 \end{pmatrix}, \quad \begin{pmatrix} \hat{c}_{11} \\ \hat{c}_{12} \\ \hat{c}_{21} \\ \hat{c}_{22} \\ \hat{c}_{31} \\ \hat{c}_{32} \end{pmatrix} = \begin{pmatrix} 0.51 \\ 0.46 \\ 0.33 \\ 0.27 \\ 0.23 \\ 0.19 \end{pmatrix}. \tag{22}$$

It is straightforward to calculate the six eigenvalues of the differential equations evaluated at the equilibrium point as follow

$$\{-0.218, -0.216, -0.170, -0.163, -0.003, -1.04 \times 10^{-17}\}.$$

The six eigenvalues are real and negative. The equilibrium is locally stable. This result is important as it guarantees the validity of comparative dynamic analysis in the next section.

4. COMPARATIVE DYNAMIC ANALYSIS

We simulated the motion of the global economy when all the parameters are constant as in (21). This study is interested in how the national and global economies behave when the system experiences time-dependent shocks. As the

lemma in the previous section shows the computational procedure to calibrate the motion of all the variables with any type of exogenous shocks, it is straightforward to examine effects of change in any parameter. We consider the parameters in (21) as the long-term average values. We introduce different exogenous periodic perturbations around these long-term values. We use a variable $\bar{\Delta}x_j(t)$ to stand for the change rate of the variable, $x_j(t)$, in percentage due to changes in a parameter.

4.1. Perturbations in the Total Factor Productivity of Region (1, 1)'S Capital Good Sector

We first study the effects of the following perturbations in the total factor productivity of region (1, 1)'s capital good sector

$$\tilde{A}_{11}(t) = 1.2 + 0.05\sin(t).$$

The simulation result is plotted in Figure 2. All the variables experience periodic changes with different amplitudes. The global income oscillates with greater amplitude than the global wealth. The wages of different parts of the global economy vary similarly. Country 1's variables have great amplitudes in the perturbations.

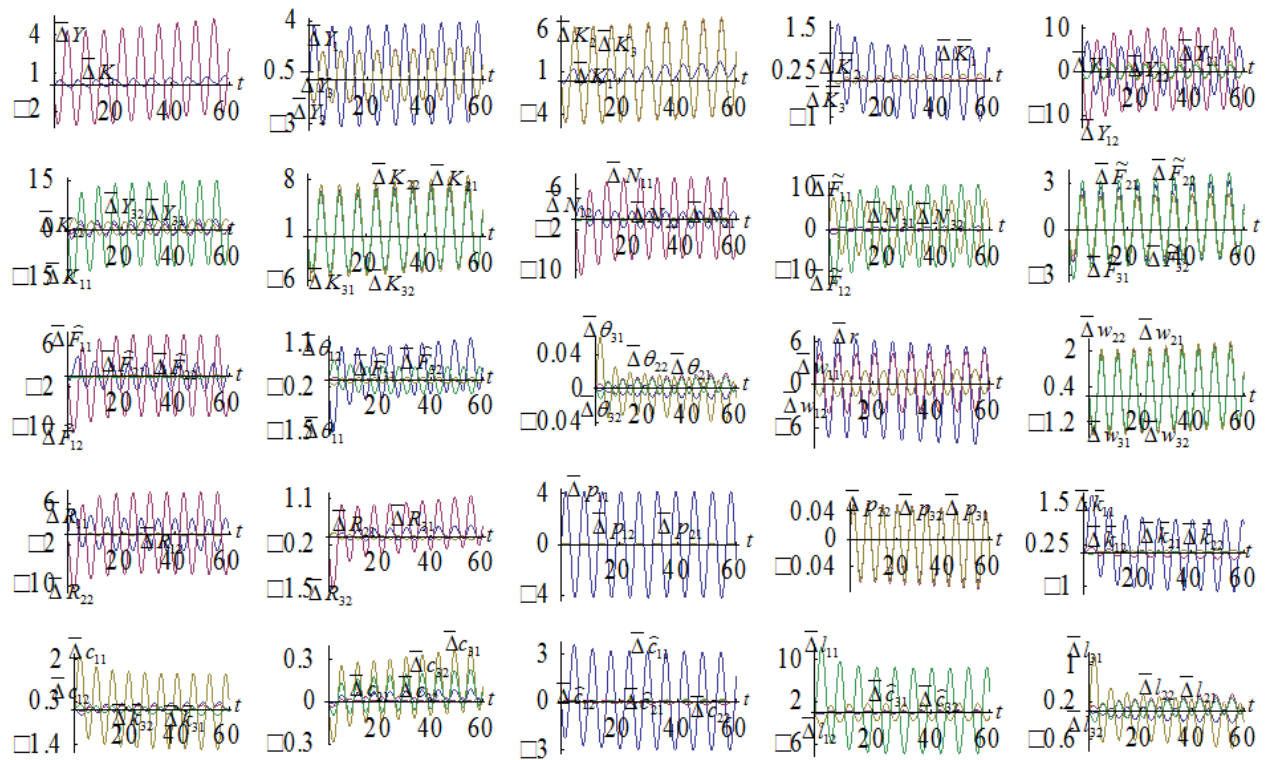


Figure-2. Perturbations in the Total Factor Productivity of a Capital Good Sector

4.2. Perturbations in the Total Factor Productivity of Region (2, 1)'S Service Sector

We now examine how the global and national economies react to the following periodic perturbations in the total factor productivity of region (2, 1)'s service sector

$$\hat{A}_{21}(t) = 0.9 + 0.05\sin(t).$$

The simulation result is plotted in Figure 3. All the variables experience periodic. The global income and wealth fluctuate slightly with the technological fluctuation in the service sector. This is different from the impact of fluctuations in the capital sector's total factor productivity. The wages of different parts of the global economy and rate

of interest are slightly affected. Region (2, 1)'s price and consumption levels of services fluctuate greatly, while the corresponding variables in the other regions change slightly. As services are locally consumed, the effects of technological changes tend to be reflected in the price and consumption of the local product.

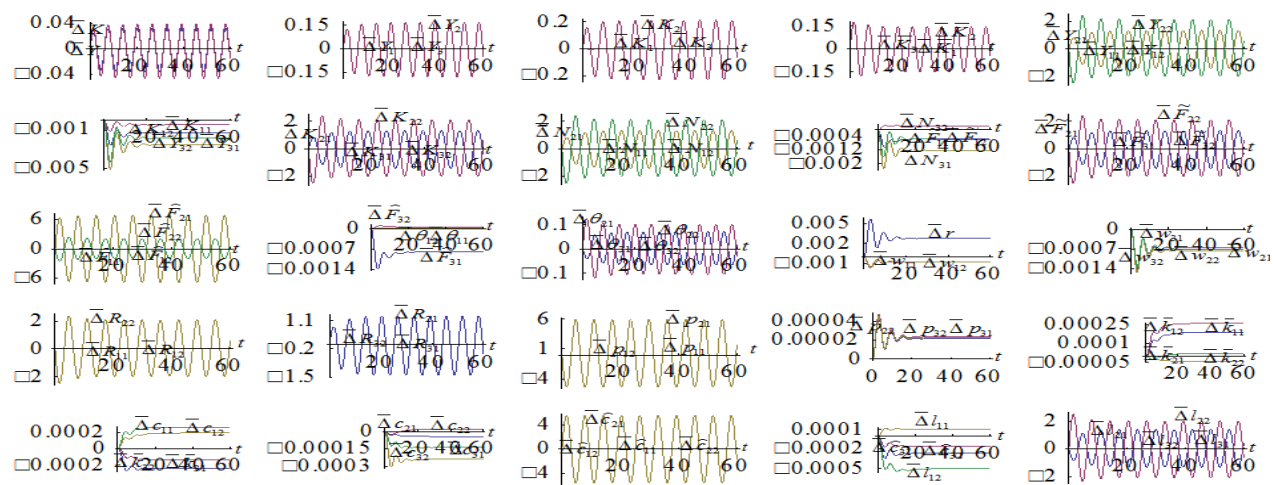


Figure-3. Perturbations in the Total Factor Productivity of a Service Sector

4.3. Exogenous Oscillations in Country 1's Propensity to Save

We now examine how the global and national economies react to the following periodic perturbations in country 1's propensity to save

$$\lambda_{10}(t) = 0.85 + 0.02\sin(t).$$

The simulation result is plotted in Figure 4. All the variables are oscillatory. The global income oscillates much less than the global wealth. The housing markets in country 1 change greatly.

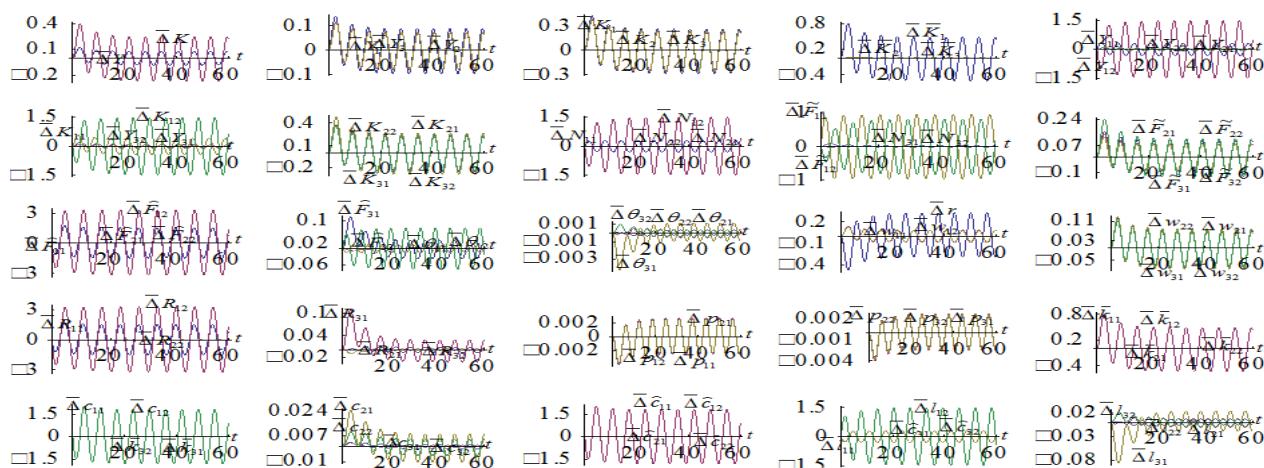


Figure-4. Exogenous Oscillations in Country 1's Propensity to Save

4.4. Exogenous Oscillations in Region (3, 2)'S Amenity Parameter

We now deal with how the global and national economies react to the following periodic perturbations in region (3, 2)'s amenity parameter

$$\bar{\theta}_{32}(t) = 3.85 + 0.1\sin(t).$$

The simulation result is plotted in Figure 5. The global and national economies experience business cycles due to exogenous periodic changes. The global income oscillates much less than the global wealth. The two regions in country

3 experience greater changes than the other two countries. Except the lot sizes, the microeconomic variables in the economic system are affected slightly.

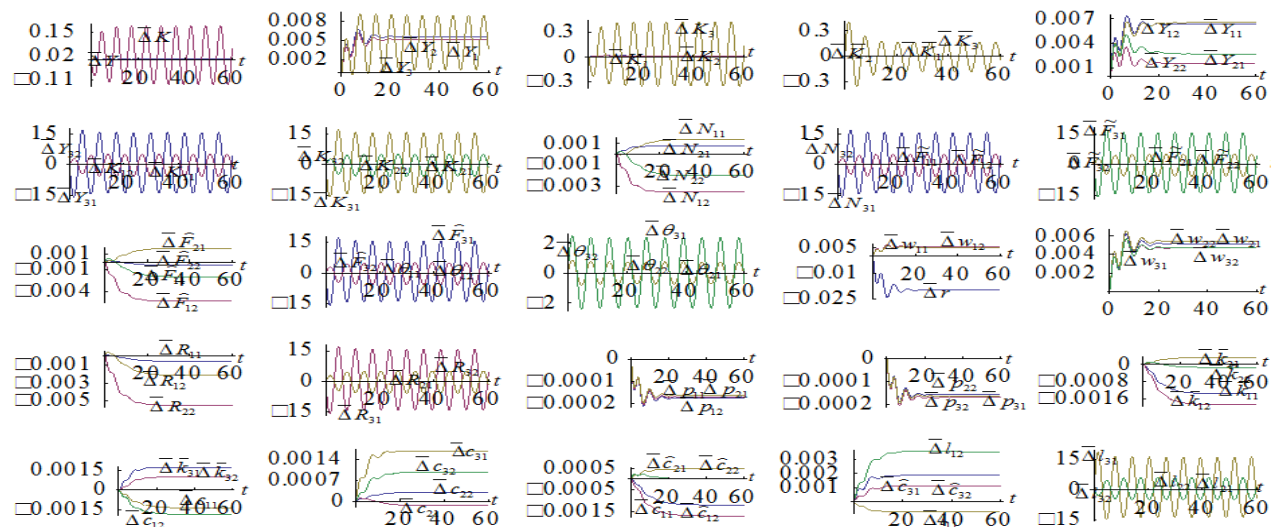


Figure-5. Exogenous Oscillations in a Region's Amenity Parameter

4.5. Perturbations in Country 3's Propensity to Consume Housing

We now study how the global and national economies react to the following periodic perturbations in country 3's propensity to consume housing

$$\eta_{30}(t) = 0.08 + 0.01\sin(t).$$

The simulation result is plotted in Figure 6. The global and national economies experience business cycles. The global income oscillates less than the global wealth.

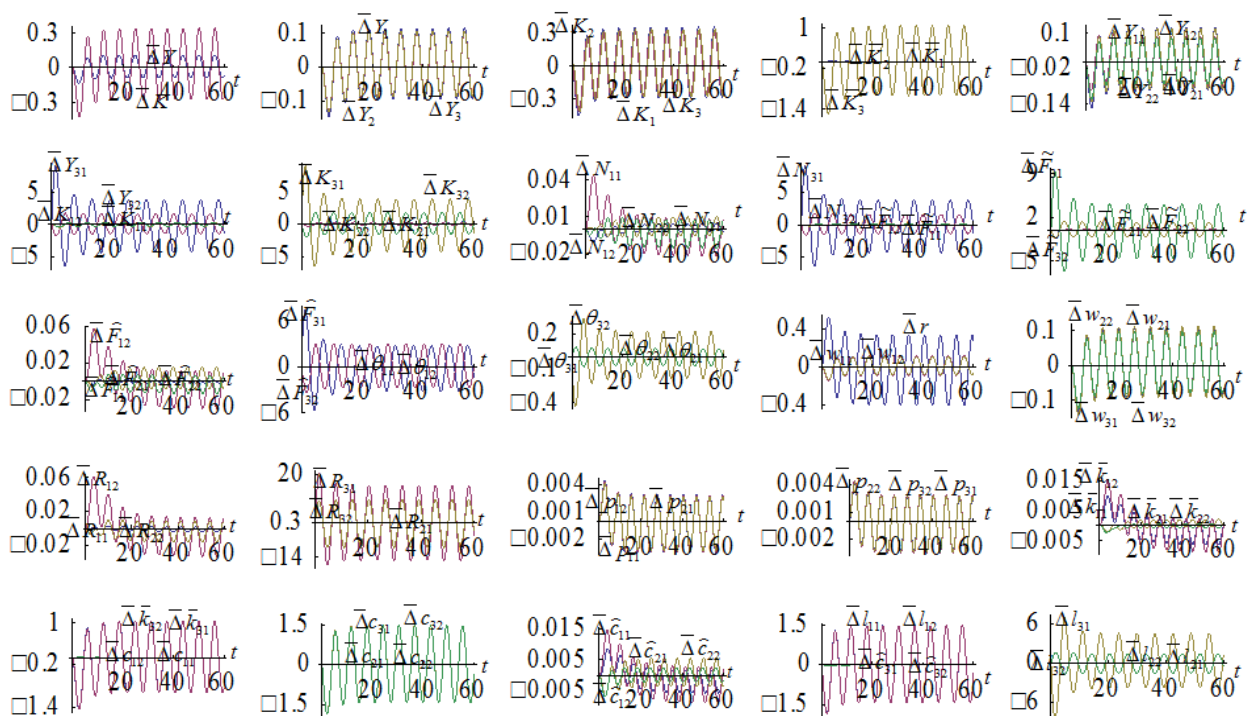


Figure-6. Perturbations in Country 3's Propensity to Consume Housing

5. CONCLUSIONS

This paper generalized the global economic growth model with any number of countries and each country with any number of regions recently proposed by Zhang (2016). Zhang's model extends Uzawa's two-sector growth model to a global economy for examining dynamic interactions between international trade, national and global growth, interregional migration, wealth accumulation, and regional amenities. This study generalized Zhang's model by allowing all the time-independent parameters to be time dependent. The generalization makes it possible to examine effects of any types of exogenous time-dependent shocks on the dynamic system cross regions over time. We simulated the model of the global economy with three countries and each country with two regions. We demonstrated the existence of equilibrium point and confirmed (local) stability of the equilibrium point. We also conducted comparative dynamic analysis with regard to exogenous periodic shocks in the total factor productivity of regions' capital good sectors, the total factor productivities of the service sectors, the propensities to save, the amenity parameters, and the propensities to consume housing. Our comparative analysis shows how business cycles are generated by periodic exogenous shocks. The study may be extended in different ways. For instance, we may further analyze behavior of the model with other forms of production or utility functions. Moreover, households in each country should be heterogeneous. Also issues related to tax competition between countries or regions have caused great attention in economic geography. The current model is deterministic. Random factors are important in making the model more relevant for explaining reality.

Appendix: Proving the lemma

We now prove the procedure to determine the dynamics of the economic system. First, equations (2) and (4) imply

$$z_{jq} \equiv \frac{r + \delta_{jq}}{w_{jq}} = \frac{a_{jq} \tilde{N}_{jq}}{\tilde{K}_{jq}} = \frac{b_{jq} \hat{N}_{jq}}{\hat{K}_{jq}}, \quad (\text{A1})$$

where

$$a_{jq} \equiv \frac{\tilde{\alpha}_{jq}}{\tilde{\beta}_{jq}}, \quad b_{jq} \equiv \frac{\hat{\alpha}_{js}}{\hat{\beta}_{js}}.$$

Insert $z_{jq} / a_{jq} \equiv \tilde{N}_{jq} / \tilde{K}_{jq}$ in $r + \delta_{jq} = \tilde{\alpha}_{jq} F_{jq} / K_{jq}$ from (2)

$$r(z_{jq}, t) = \frac{\tilde{\alpha}_{jq} \tilde{A}_{jq}}{a_{jq} \tilde{\beta}_{jq}} z_{jq}^{\tilde{\beta}_{jq}} - \delta_{jq}. \quad (\text{A2})$$

Especially we have

$$r(z_{11}, t) = \frac{\tilde{\alpha}_{11} \tilde{A}_{11}}{a_{11} \tilde{\beta}_{11}} z_{11}^{\tilde{\beta}_{11}} - \delta_{11}. \quad (\text{A3})$$

The rate of interest is determined as a unique function of z_{11} and t . From (A2) we have

$$z_{jq}(z_{11}, t) = a_{jq} \left(\frac{r + \delta_{jq}}{\tilde{\alpha}_{jq} \tilde{A}_{jq}} \right)^{1/\tilde{\beta}_{jq}}. \quad (\text{A4})$$

Hence we can treat all z_{jq} as a unique function of z_{11} and t . Equations (A1) and (A2) imply

$$w_{jq}(z_{11}, t) = \frac{r + \delta_{jq}}{z_{jq}}. \tag{A5}$$

From $z_{jq} = b_{jq} \widehat{N}_{jq} / \widehat{K}_{jq}$ and (1), we get

$$p_{jq}(z_{11}, t) = \frac{b_{jq}^{\beta_{jq}} (r + \delta_{jq})}{\widehat{\alpha}_{jq} \widehat{A}_{jq} z_{jq}^{\beta_{jq}}}. \tag{A6}$$

From (16) and $p_{jq} \widehat{c}_{jq} = \gamma_j \widehat{y}_{jq}$ we get

$$\gamma_j \widehat{y}_{jq} N_{jq} = p_{jq} \widehat{F}_{jq}. \tag{A7}$$

Inserting (4) in (A6) we have

$$\gamma_j \widehat{y}_{jq} N_{jq} = \frac{w_{jq} \widehat{N}_{jq}}{\widehat{\beta}_{jq}}. \tag{A8}$$

By (3) we have

$$\widehat{y}_{jq}(z_{11}, \bar{k}_{jq}) = (1 + r) \bar{k}_{jq} + w_{jq}. \tag{A9}$$

Substituting $l_{jq} = L_{jq} / N_{jq}$, $\theta_j = \bar{\theta}_j N_j^{d_j}$, and (7) into (6) yields

$$U_{jq} = \frac{\bar{\theta}_{jq} N_{jq}^{d_j - \eta_{j0}} L_{jq}^{\eta_{j0}}}{p_{jq}^{\gamma_{j0}}} \xi_j^{\xi_{j0}} \gamma_j^{\gamma_{j0}} \lambda_j^{\lambda_{j0}} \widehat{y}_{jq}^{\xi_{j0} + \gamma_{j0} + \lambda_{j0}}. \tag{A10}$$

Applying $U_{jq} = U_{jm}$ to (A10) implies

$$N_{jq} = \Lambda_{jq} N_{j1}, \quad q = 1, \dots, Q_j, \tag{A11}$$

where

$$\Lambda_{jq}(z_{11}, (\bar{k}_{jq})_j, t) \equiv \left(\frac{\bar{\theta}_{j1} L_{j1}^{\eta_{j0}} p_{jq}^{\gamma_{j0}}}{\bar{\theta}_{jq} L_{jq}^{\eta_{j0}} p_{j1}^{\gamma_{j0}}} \right)^{\frac{1}{(d_j - \eta_{j0})}} \left(\frac{\widehat{y}_{j1}}{\widehat{y}_{jq}} \right)^{\frac{\xi_{j0} + \gamma_{j0} + \lambda_{j0}}{d_j - \eta_{j0}}},$$

where $(\bar{k}_{jq})_j \equiv (\bar{k}_{j1}, \dots, \bar{k}_{jQ_j})$. Inserting (A11) in (18) we have

$$N_{j1}(z_{11}, (\bar{k}_{jq})_j, t) = \frac{N_j}{\sum_{q=1}^{Q_j} \Lambda_{jq}}, \quad \Lambda_{j1} = 1. \tag{A12}$$

With (A11) and (A12) we determine the population distribution within country j as functions of z_{11} , $(\bar{k}_{jq})_j$ and t .

By $l_{jq} R_{jq} = \eta \widehat{y}_{jq}$ and $l_{jq} N_{jq} = L_{jq}$, we get

$$R_{jq}(z_{11}, (\bar{k}_{jq})_j) = \frac{\eta_j \hat{y}_{jq} N_{jq}}{L_{jq}}. \quad (\text{A13})$$

Inserting (A8) in (A9) we have

$$\hat{N}_{jq}(z_{11}, (\bar{k}_{jq})_j, t) = \left(\frac{(1+r)\bar{k}_{jq}}{w_{jq}} + 1 \right) \gamma_j \hat{\beta}_{jq} N_{jq}. \quad (\text{A14})$$

Equations $\tilde{N}_{jq} + \hat{N}_{jq} = N_{jq}$ and (A11) imply

$$\tilde{N}_{jq}(z_{11}, (\bar{k}_{jq})_j, t) = N_{jq} - \hat{N}_{jq}. \quad (\text{A15})$$

Equations (13)-(15) imply

$$\sum_{j=1}^J \sum_{q=1}^{Q_j} (\tilde{K}_{jq} + \hat{K}_{jq}) = \sum_{j=1}^J \sum_{q=1}^{Q_j} \bar{k}_{jq} N_{jq}. \quad (\text{A16})$$

Inserting (A1) in (A16) we get

$$\sum_{j=1}^J \sum_{q=1}^{Q_j} \left(\frac{a_{jq} \tilde{N}_{jq} + b_{jq} \hat{N}_{jq}}{z_{jq}} \right) = \sum_{j=1}^J \sum_{q=1}^{Q_j} \bar{k}_{jq} N_{jq}. \quad (\text{A17})$$

Substitute (A15) into (A17)

$$\sum_{j=1}^J \sum_{q=1}^{Q_j} \left(\frac{(b_{jq} - a_{jq}) \hat{N}_{jq}}{z_{jq}} \right) = \sum_{j=1}^J \sum_{q=1}^{Q_j} \left(\bar{k}_{jq} - \frac{a_{jq}}{z_{jq}} \right) N_{jq}. \quad (\text{A18})$$

Inserting (A14) in (A18) we have

$$\sum_{j=1}^J \sum_{q=1}^{Q_j} \left(\frac{(1+r)\bar{k}_{jq}}{w_{jq}} + 1 \right) \frac{\bar{a}_{jq} N_{jq}}{z_{jq}} = \sum_{j=1}^J \sum_{q=1}^{Q_j} \left(\bar{k}_{jq} - \frac{a_{jq}}{z_{jq}} \right) N_{jq}, \quad (\text{A19})$$

where

$$\bar{a}_{jq} \equiv (b_{jq} - a_{jq}) \gamma_j \hat{\beta}_{jq}.$$

Inserting (A10) in (A19) we have

$$\Phi(z_{11}, (\bar{k}_{jq})_j, t) \equiv \sum_{j=1}^J N_{j1} \sum_{q=1}^{Q_j} \left(\left(\frac{(1+r)\bar{k}_{jq}}{w_{jq}} + 1 \right) \frac{\bar{a}_{jq}}{z_{jq}} - \left(\bar{k}_{jq} - \frac{a_{jq}}{z_{jq}} \right) \right) \Lambda_{jq} = 0. \quad (\text{A20})$$

Substituting $s_{jq} = \lambda_j \hat{y}_{jq}$ and $\hat{y}_{jq} = (1+r)\bar{k}_{jq} + w_{jq}$ into equations (9) yields

$$\dot{\bar{k}}_{jq} = \Phi_{jq}(z_{1q}, \bar{k}_{jq}, t) \equiv (1+r)\lambda_j \bar{k}_{jq} + \lambda_j w_{jq} - \bar{k}_{jq}. \quad (\text{A21})$$

Taking derivatives of equation (A20) with respect to t yields

$$\dot{z}_{11} = - \left(\frac{\partial \Phi}{\partial t} + \sum_{j=1}^J \sum_{q=1}^{Q_j} \frac{\partial \Phi}{\partial \bar{k}_{jq}} \dot{\bar{k}}_{jq} \right) \left(\frac{\partial \Phi}{\partial z_{11}} \right)^{-1}. \quad (\text{A22})$$

Inserting (A18) in (A19) we have

$$\dot{z}_{11} = \Phi_0(z_{1q}, \bar{k}_{jq}) \equiv - \left(\frac{\partial \Phi}{\partial t} + \sum_{j=1}^J \sum_{q=1}^{Q_j} \Phi_{jq} \frac{\partial \Phi}{\partial \bar{k}_{jq}} \right) \left(\frac{\partial \Phi}{\partial z_{11}} \right)^{-1}. \quad (\text{A23})$$

With the procedure in the lemma we follow the motion of the global economic system.

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