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VOLATILITY FORECASTING PERFORMANCE OF SMOOTH TRANSITION EXPONENTIAL SMOOTHING METHOD: EVIDENCE FROM MUTUAL FUND INDICES IN MALAYSIA

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ABSTRACT

This paper aims to empirically compare the performance of the smooth transition exponential smoothing (STES) method against the well-known generalized autoregressive conditional heteroskedasticity (GARCH) model in one-step-ahead volatility forecasting. While the GARCH model captured most of the stylized facts of the financial time series, threats of outliers in the leptokurtic distributed series remain unresolved. The study compared volatility forecasting performance of a total of 22 models and methods comprising STES, GARCH, and some ad-hoc forecasting. The daily returns of seven mutual fund indices (derived from 57 individual equity mutual funds) under two different economic conditions (sub-periods) were applied across all competing models. Findings revealed that the STES method with error and absolute error as transition variables emerged as the best post-sample volatility forecasting model in both sub-periods with and without financial crisis impact, as verified by model confidence set (MCS) procedure. The implications based on the results are: (1) both the sign and size of yesterday's news shock have an impact on today's volatility; (2) the STES method is resilient to outliers, and hence superior to GARCH and other volatility forecasting approaches examined. This study contributes an empirical approach in forecasting the risk of mutual funds investment for investors and fund managers, as well as extending the scope of volatility forecasting literature into the less explored mutual funds.

Contribution/Originality: This study contributes to the existing literature as the first to employ the robust STES method in forecasting the volatility of mutual funds. STES method outperformed GARCH with robustness of results verified by the MCS procedure. The seven newly created fund indices from 57 individual funds have enabled macro-analysis of fund risk.

1. INTRODUCTION

In the context of financial investment, volatility is the quantified measurement of risk arising from an uncertain situation impacting investment return. Although volatility is not the same as risk, its interpretation from the perspective of uncertainty becomes a crucial element in investment decisions and financial risk management. As such, accurate forecasting of an asset's return volatility is the prerequisite for assessing investment risk (Poon & Granger, 2003). Undeniably, measuring volatility is critical to portfolio risk management, securities pricing, and policies management. The emergence of large amount of literature on volatility forecasting over the past decades focusing predominantly on modeling and forecasting volatility signifies its importance.

Before the emergence of volatility studies, forecasting the volatility of financial assets has never been a topic of importance. The Random Walk Theory advocated that successive prices of an asset are unrelated or random. Historic prices are not relevant to the prediction of future prices (Figlewski, 1997). In short, the Random Walk Theory asserts that the best forecast for tomorrow's pricing is today's price. However, Mandelbrot (1963), in his seminal work, argued that financial time series are not normally and independently distributed. His findings provide insight into the existence of the "volatility clustering" phenomenon in financial time series, therefore invalidating the Random Walk Theory. The characteristics of financial time series often consist of high-frequency observations, which can intensify the influence of non-systematic factors such as the impact of news, triggering a reaction from investors which eventually forms the basis for volatility persistency over time. The variance of the random error term of financial time series is time-varying (non-constant) or heteroskedastic in nature. Poon & Granger (2003) reviewed 93 published works and working papers concerning the performance of various volatility forecasting models emphasizing the risk attributed to the unique stylized facts of financial time series. There are several salient features about financial time series and financial market volatility that is now well documented. These include fat tail distributions of risky asset returns, volatility clustering, asymmetry and mean reversion, and co-movements of volatilities across assets and financial markets. More recent research found that the correlation among volatilities is stronger than that among returns, and both tend to increase during bear markets and financial crises (Poon & Granger, 2003).

Although volatility is unobservable and latent, it persists over time (Bollerslev, Chou, & Kroner, 1992; Chou, 1988; Choudhry, 1995). This fact has enabled volatility to be modeled and forecast statistically. The GARCH (generalized autoregressive conditional heteroskedasticity) model by Bollerslev (1986) is an improvised version of the original ARCH (autoregressive conditional heteroskedasticity) model first introduced by Engle (1982) and has become an essential tool in examining heteroskedasticity in financial time series data. Extension of the GARCH model into the IGARCH (integrated GARCH) model by Engle & Bollerslev (1986) coupled with other asymmetric GARCH models, particularly EGARCH (exponential GARCH) by Nelson (1991) and GJR-GARCH by Glosten, Jagannathan, & Runkle (1993) has prompted the emergence of large quantity of literature on volatility studies, stocks in particular. However, volatility studies on mutual funds are scarce. A study of more than 300 different GARCH model specifications by Hansen & Lunde (2005) confirmed the supremacy of the GARCH model in post-sample volatility forecasting.

However, Taylor (2004a); Taylor (2004b) opened up new possibilities for volatility forecasting studies where his less explored STES (Smooth Transition Exponential Smoothing) method outperformed the well-known GARCH, as documented in his study of weekly volatility forecasting of time series for eight stock indices across both developed and developing markets using realized volatility generated from daily returns. To the best knowledge of the authors, the STES method has not been employed in forecasting the volatility of mutual funds thus far, either internationally or domestically within Malaysia. Furthermore, the application of the GARCH models in mutual funds is possibly non-existent in the Malaysian context with few mutual fund volatility studies from overseas (Busse, 1999; Tang, Wang, & Xu, 2012; Xie & Huang, 2013). Past studies on mutual funds found inconsistencies between funds' objectives and their risk-return relationship (Annuar, Shamsher, & Ngu, 1997;

DiBartolomeo & Witkowski, 1997; Jin & Yang, 2004; Kim, Shukla, & Tomas, 2000; Mohamad & Nassir, 1995). The inability of fund managers in "market timing" and "selectivity" (Chen, Adams, & Taffler, 2013; Haroon, Sadaqat, Jebran, & Ali Memon, 2018; Vieira, Neto, & Da Mota, 2017) further pose a greater risk in mutual funds investment. These factors certainly raise concerns about the risk involved in mutual funds investment.

This paper aims to empirically verify the robustness and supremacy of the STES method against the well-known GARCH family models and other historical volatility approaches through modeling volatility (risk) of seven equity-based private mutual funds indices (of different investment objectives) in Malaysia for two sub-periods with and without the impact of the financial crisis. The remainder of this paper is organized as follows: Section 2 provides a review of related past literature on GARCH under different error distribution assumptions and STES volatility forecasting studies; Section 3 explains the methodology and models employed; Section 4 presents the empirical results and discussion on the findings; Section 5 concludes the study with suggestions for future research.

2. LITERATURE REVIEW

2.1. Mutual Funds' Risk

The mutual fund industry commenced in Malaysia in 1959 and has now contributed a net asset value of 28.2% to the total market capitalization of Bursa Malaysia of the end of 2019. This implies its importance and growing acceptance amongst Malaysian as an alternative investment option. While mutual funds claim to offer the salient feature of risk diversification, many past studies on mutual funds revealed inconsistencies between funds' investment objectives and their risk-return relationship. DiBartolomeo & Witkowski (1997) found 40% of mutual funds examined have misclassified objectives. Kim et al. (2000) found that 33% of funds have deviated severely from their original objectives. Annuar et al. (1997) found riskier growth funds have only 53% risk diversification, while lower risk balanced funds have 60% risk diversification.

The inability of fund managers to time the market and select appropriate stocks for mutual funds' portfolios to diversify risk and optimize returns (Chen et al., 2013; Haroon et al., 2018; Vieira et al., 2017) further compounded the risk factor. Fund managers tend to disregard fund objectives in their daily trading operation due to "peer pressure" and "accountability to investment return", and therefore traded off risk diversification for riskier returns (Kim et al., 2000). This warrants a need to model and forecast risk in mutual funds investment.

2.2. Limitations of GARCH Models and Error Term Distribution Assumption

The generalized autoregressive conditional heteroskedasticity (GARCH) framework revolves around an autoregressive process where today's volatility (conditional variance) is conditioned on yesterday's squared error (due to news impact) and yesterday's conditional variance (Bollerslev, 1986). The GARCH model, regarded as the main workhorse of empirical volatility studies, offers flexible lag structures capable of accommodating longer memory effects attributed to the persistency of volatility over time. The integrated GARCH (IGARCH) is tasked to address the issue of volatility persistency (Bollerslev & Engle, 1993; Engle & Bollerslev, 1986). Meanwhile, both the symmetric GARCH and IGARCH models address the issue of excess kurtosis; however, the issue of skewness remains unresolved. The asymmetrical effect arises due to positive or negative news shock of similar magnitude (or size) and produces different impacts on conditional variance (volatility) of financial assets. The logarithmic form of the exponential GARCH (EGARCH) model eliminates the restriction of the non-negativity of parameter estimates imposed in the standard GARCH model by capturing asymmetric effects (Nelson, 1991). Inspired by the seminal work of American economist Fisher Black in 1976, Glosten et al. (1993) introduced the Glosten-Jagananthan-Runkle GARCH (GJR-GARCH) to capture the "leverage effect" where a negative new shock exerts a greater impact on the volatility of financial assets returns than positive news of a similar magnitude. Nevertheless, the GJR-GARCH is superior to the EGARCH in capturing the leverage effect, as the latter tends to produce conditional variance, which is even higher than the squared residual (a proxy for actual volatility) and should not be the case if the model is correctly specified (Engle & Ng, 1993).

Many prior works of literature have predominantly skewed towards a race to establish the best volatility model based on forecasting accuracy but lack discussion on the impact of innovations of the error term. Although the strength of asymmetric GARCH models lies in their capability to capture the asymmetrical leverage effect, the error term distribution assumption does influence the accuracy of volatility forecasting of GARCH models. Three commonly examined error term distribution assumptions are Gaussian (normal), Student-t, and General Error Distribution (GED). The asymmetric EGARCH and GJR-GARCH models under Student-t error term distribution assumption were commendable in accounting for an asymmetric leverage effect documented in past strands of literature on stock volatility, both internationally (Alberg, Shalit, & Yosef, 2008; Dritsaki, 2017; Hamilton & Susmel, 1994; Kuhe, 2018; Wilhelmsson, 2006) and domestically (Malaysian cases) (Angabini & Wasiuzzaman, 2011; Chong, Ahmad, & Abdullah, 1999; Lim & Sek, 2013; Shamiri & Isa, 2009). Meanwhile, the stock market of different volatility characteristics supported the asymmetric GJR-GARCH with the Student-t error distribution assumption (Musa, Adamu, & Dauran, 2020; Peters, 2001) and the symmetric GARCH under Student-t distribution (Luo, Pairote, & Chatpatanasiri, 2017; Rana, 2020) as the best post-sample volatility forecasting models.

Although the asymmetric GARCH models are efficient in capturing volatility clustering, volatility persistency and the asymmetric GARCH capture the asymmetric leverage effect, but the assumption of a Gaussian distribution of the error term does not entirely address the issue of leptokurtosis (Peters, 2001), typically in high-frequency financial time series where the issue of outliers prevails.

2.3. Outliers and Structural Changes Issues

The issue of outliers or extreme values of assets returns is one of the stylized facts of financial time series, where even a standard GARCH model with a Student-t innovation of the error term is not sufficient to address the outlier issue entirely (Poon & Granger, 2003). The presence of outliers in time series tends to cause overestimation of in-sample GARCH parameters under the maximum likelihood estimator, resulting in bias (overstated) post-sample volatility forecasting (Carnero, Peña, & Ruiz, 2012). The impact of outliers is even more prominent in a small to moderate sample size with a moderate to large magnitude of outliers (Grané & Veiga, 2014). Removal of additive outliers is one suggested remedy which can improve the accuracy of the GARCH model in volatility forecasting. A study conducted by Franses & Ghijsels (1999) using the Additive Outliers Corrected GARCH model found that the volatility forecasts of four out of five stock market indices had improved. The issue of addressing outliers depends on the appropriateness of the model applied in different economic conditions to ensure correct interpretation of volatility (Hossain, Akter, & Ismail, 2021).

2.4. Adaptive Exponential Smoothing

The adaptive exponential smoothing method was introduced (Trigg & Leach, 1967; Williams & Miller, 1999) to address the shortcomings of prior exponential smoothing methods. Leveraging on the adaptive exponential smoothing methods, Taylor (2004a) introduced the Smooth Transition Exponential Smoothing (STES) method of volatility forecasting where the adaptive smoothing parameter comes in the form of a logistic function of user-specified transition variables, which is highly resilient to the issues of outliers and structural changes in time series. Taylor (2004b) applied the STES technique across eight stock indices of developed and developing economies to forecast weekly volatility and compared it against the GARCH, IGARCH, GJRGARCH, Logistic Smooth-Transition GARCH, and Exponential Smooth-Transition GARCH as a comparison against five STES methods with five different transition variables, respectively. The STES method with the error and absolute error transition variables emerged as the best one-week-ahead volatility forecasting model.

Liu, Taylor, & Choo (2020) applied the STES method to stocks and empirically examined the resilience of STES to outliers using the different simulated magnitude of outliers. A comparison between the standard model set (comprised of four GARCH models) and the robust model set (comprising three exponential smoothing methods) revealed STES methods are the best for volatility forecasting indicated by the highest number of remaining models in the "Superior Set Models" of the Model Confidence Set procedure (Hansen, Lunde, & Nason, 2011). The application of the STES method in volatility forecasting studies has been very limited, apart from studies by Taylor (2004a); Taylor (2004b) and Liu et al. (2020).

Past literature on the STES method has focused mainly on stock volatility studies. The findings of this study will further enrich current literature and fill the volatility forecasting gap as STES methods are applied to model volatility of seven equity mutual fund indices in Malaysia under two different sub-periods. To the best knowledge of the authors, private equity mutual fund indices time series in Malaysia have never been applied in any mutual funds' volatility studies. We raised the following hypotheses:

H_{1A}: The STES method outperforms GARCH models and other methods in one-day-ahead volatility forecasting of mutual fund indices return in sub-periods during a financial crisis.

H_{1B}: The STES method outperforms GARCH models and other methods in one-day-ahead volatility forecasting of mutual fund indices return in sub-periods outside of a financial crisis.

3. METHODOLOGY

3.1. Volatility Forecasting Models and Methods

The analysis approach is designed to make comparison between models and methods to identify the best performing model in one-day-ahead volatility forecasting for seven equity private mutual fund indices by making comparisons between 22 models. These models are random walk; naïve forecasting; moving average; exponential weighted moving average (EWMA); EWMA optimized; GARCH models (comprising GARCH, EGARCH, IGARCH, GJR-GARCH, each under three different error term distribution assumptions of Gaussian, Student-t and GED); STES methods under five different "transition variables" of past error (E), past squared error (SE), absolute error (AE), past error and absolute error (E & AE), past error and past squared error (E & SE).

a) Random Walk (RW)

RW is a historical price model where today's variance σ_t^2 is a function of yesterday's variance σ_{t-1}^2, which is proxied by yesterday's residual squared ε_{t-1}^2 (Poon & Granger, 2003), specified as Equation 1:

$$\sigma_t^2 = \sigma_{t-1}^2 = \varepsilon_{t-1}^2 \qquad (1)$$

b) Naïve Variance Forecasting (NF)

NF defines today's variance σ_t^2 as an average of past squared residuals (proxy for past variances), specified as Equation 2:

$$\sigma_t^2 = \frac{1}{t-1}\left[\sum_{j=1}^{t-1} \varepsilon_j^2\right] \qquad (2)$$

c) Moving Average-30 days (MA-30)

MA is an extension of NF where the average of past squared residuals comes in the form of a rolling *n* number of days. An MA-30 implies that a smooth 30-day rolling contains the latest information with older information discarded, specified as Equation 3:

$$\sigma_t^2 = \frac{1}{30}\left[\sum_{j=t-30}^{t-1} \varepsilon_j^2\right] \qquad (3)$$

d) Exponential Weighted Moving Averages (EWMA)

EWMA addresses the weaknesses of MA by assigning higher weightage to more recent information and eliminating the issue of lag length determination in the form of exponential smoothing, and offering a pragmatic approach to measuring volatility, specified as Equation 4:

$$\sigma_t^2 = \beta\sigma_{t-1}^2 + (1-\beta)\frac{1}{L}\sum_{j=1}^{L} \sigma_{t-j}^2 \qquad (4)$$

where L is the length of moving average and β is the decaying factor. The JP Morgan RiskMetrics model suggested β values of 0.94 and 0.97 for daily and weekly, respectively (Chuang, Lu, & Lee, 2007; Mabrouk, 2017).

e) Standard GARCH (p,q)

The standard GARCH (p,q) model is specified in Equation 5 and Equation 6:

$$r_t = m_t + \varepsilon_t \text{ where } \varepsilon_t = v_t\sqrt{h_t} \text{ where } v_t \sim N(0,1) \qquad (5)$$

$$h_t = \,= \omega_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad (6)$$

where is Equation 5 is the mean equation and Equation 6 is the variance equation. r_t is the expected return of a financial asset, m_t is the conditional mean, ε_t is the residual series, h_t is the conditional variance of the residual, and v_t is the identical and independent sequence. ω_0, α and β are parameters to be estimated using the maximum likelihood estimation. The α value represents the impact of news shock on the volatility of the return of an asset, while the β value represents volatility clustering (impact of past volatility on current volatility). A higher value of $\alpha + \beta$ implies higher volatility persistency attributed to news impact. To ensure h_t is stationary and positive, a constraint $\alpha + \beta < 1$, hence $\omega_0 > 0, \alpha \geq 0$ and $\beta \geq 0$ and $i = 1, \ldots.q$, $j = 1, \ldots., p$. $\alpha + \beta$, measures volatility persistency. The GARCH model was developed under the assumption of normality in the error term distribution where $\varphi_t = \varepsilon_t / \sqrt{h_t} \sim$ N (0,1), and the conditional density of the error term ε_t in the likelihood function is given as follows:

$$f(\varepsilon_t | \gamma, \theta_{t-i}) = \frac{1}{\sqrt{2\pi h_t}} e^{-(\varepsilon_t^2 / 2h_t)}$$

where $\theta_{t-i} = \{\varepsilon_{t-1}, \varepsilon_{t-2}, \dots . \varepsilon_1\}$ and $\gamma = (\mu, \omega, \alpha, \beta)$

Based on the likelihood function above, the corresponding log-likelihood function (LLF) is re-written as:

$L(\theta,\varepsilon) = \sum_{t=2}^{T} \ln f(\varepsilon_t | \gamma, \theta_{t-i})$ where $\varepsilon = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_T)'$

and the maximum likelihood (ML) estimator $\hat{\theta}$ is obtained by maximizing the LLF above. The ML estimator estimates the distribution or density of the error terms as a function to the parameter estimates of the conditional variance based on the likelihood function.

f) *Integrated GARCH (IGARCH)*

The IGARCH model was developed by Engle & Bollerslev (1986) to specifically examine volatility persistency, specified as Equation 7:

$$h_t = \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \quad (7)$$

where $\alpha + \beta = 1$ implies an infinite variance of the unconditional distribution of the error term ε_t. The impact of current news shock on volatility will be infinitely persistent on future variance (Choudhry, 1995) could result in bias forecasting when a structural shift exists in the unconditional variance.

g) Exponential GARCH (EGARCH)

The non-negative restriction on parameter estimates in standard GARCH (p,q) has led to the introduction of the EGARCH model by Nelson (1991), specified as Equation 8:

$$\log h_t = \omega + \sum_{i=1}^{q} \alpha_i \, g(Z_{t-i}) + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) \quad (8)$$

where $g(Z_t) = \theta Z_t + \gamma\,(|Z_t| - E|Z_t|)$, $Z_t = \varepsilon_t | \sqrt{h_t}$ and $E|Z_t| = \sqrt{\frac{2}{\pi}}$ if $Z_t \sim$ N (0,1). The specification of $g(Z_t) \equiv \theta Z_t + \gamma\,(|Z_t| - E|Z_t|)$ enables the capturing of asymmetric effects. Component $\gamma\,(|Z_t| - E|Z_t|)$ measures the "magnitude effect" (size of the impact of news shock on volatility), while θZ_t denotes the "sign effect" (asymmetrical effect) of Z_{t-i}. The value $\gamma = 0$ signifies that no asymmetric or leverage effect exists, while $\gamma < 0$ indicates the presence of a "leverage effect" implying "that negative news" exerts a greater magnitude of shock on the conditional variance (volatility). Conversely, $\gamma > 0$ indicates the absence of a leverage effect but implies that a positive shock affects volatility more than negative shocks. The parameter θ measures the asymmetry effect where $\theta < 0$, and a negative news shock exerts greater volatility on asset returns than a positive news shock of a similar magnitude and vice versa.

h) Glosten-Jagannanthan-Runkle GARCH (GJR-GARCH)

Glosten et al. (1993) developed this model, which is capable of capturing the leverage effect, specified as Equation 9:

$$h_t = \omega + \sum_{i=1}^{q}(\alpha_i \varepsilon_{t-i}^2 + \gamma_i d_{t-i} \varepsilon_{t-i}^2) + \sum_{j=1}^{p} \beta_j h_{t-j} \quad (9)$$

where d_{t-i} is a functional index (dummy variable),; parameter γ_i measures the leverage effect,; $d_{t-i} = 1$ if a negative error occurs, indicated by $\varepsilon_{t-i} < 0$; and $d_{t-i} = 0$ if a positive error occurs, indicated by $\varepsilon_{t-i} > 0$. The conditional variance (volatility) is positive when , $\omega > 0$, $\alpha > 0$, $\gamma \geq 0, \alpha + \gamma \geq 0$ and $\beta \geq 0$. The GJR-GARCH process is deemed stationary when the following constraint $\alpha + \beta + \frac{\gamma}{2} < 1$ is fulfilled. $\gamma_i > 0$ signifies the existence of an asymmetric leverage effect. When $\gamma_i > 0$ and $\varepsilon_{t-i} < 0$ (negative news shock where the dummy $d_{t-i} = 1$), the total impact on volatility will be larger, as indicated by $(\alpha + \gamma_i)\, \varepsilon_{t-i}^2$. On the other hand, when a positive error occurs, indicated by $\varepsilon_{t-i} > 0$ (positive news shock where dummy $d_{t-i} = 0$), the total impact on the conditional variance (volatility) will be smaller, as indicated by $\alpha \varepsilon_{t-i}^2$.

i) Smooth Transition Exponential Smoothing (STES)

Taylor (2004a) extended the exponential smoothing method (Trigg & Leach, 1967; Williams & Miller, 1999) by incorporating an adaptive exponential smoothing parameter in the form of a logistic function of user-specified transition variables, which offers flexibility in the assignment of weight according to size ε_{t-1} and magnitude $|\varepsilon_{t-1}|$ of news impact on volatility. This addresses the issue of changing characteristics of time series attributed to seasonality, trends, or structural breaks where the issue of outliers often exists. The STES method is specified as Equation 10:

$$\widehat{\sigma_t^2} = (\alpha_{t-1})\varepsilon_{t-1}^2 + (1 - \alpha_{t-1})\, \widehat{\sigma_{t-1}^2} \quad (10)$$

where $\alpha_{t-1} = \frac{1}{1+\exp(\beta + \gamma V_{t-1})}$

is the "adaptive or smooth transition" parameter (modelled as a logistic function), and $0 < \alpha_{t-1} < 1$, depending on the user-specified "transition variable" V_{t-1} employed. The conditional variance in Equation 10 is a function of the calibration of the adaptive smoothing parameter α_{t-1} that depends on the choice of the transition variable

V_{t-1} and the optimization (using a solver) of parameters β and γ (coefficients of the transition variables concerned). The five different "transition variables" adopted in this study are, ε_{t-1}^2 (past squared error) denoted as STES-SE, ε_{t-1} (past error) denoted as STES-E model, $|\varepsilon_{t-1}|$ (absolute past error) denoted as STES-AE model, the combination of both past error and absolute past error denoted as STES-E-AE model, and the combination of past error ε_{t-1} and squared past squared error ε_{t-1}^2 denoted as STES-E-SE. If $\gamma < 0$, then α_{t-1} will be a monotonically increasing function of the chosen transition variable V_{t-1}, thus increasing the impact of the squared residual (past squared shocks) and reducing the impact of variance the day before on today's conditional variance and vice versa, as indicated by Equation 6. Since exponential smoothing is not based on any statistical theory, optimization of the STES parameters β and γ will be by the minimization of the forecasting error defined as the subtraction of the forecast variance error ($\widehat{\sigma_i^2}$) from the squared residual ε_i^2 (a proxy for actual volatility): min $\sum(\varepsilon_i^2 - \widehat{\sigma_i^2})^2$.

3.2. Data

The dataset comprises seven mutual fund indices of different investment objectives and risk exposure comprising Growth, Growth & Income, Income, Balanced Growth, Balanced Growth & Income, Balanced Income, and Mixed Asset Growth categories, which were created from the 57 individual private equity mutual funds in Malaysia (see Appendix A) sampled from January 3, 2005, to December 31, 2019. The full sample is divided into two sub-periods, one with financial crisis impact from January 3, 2005, to December 31, 2011, while the other is without financial crisis impact and runs from Jan 1, 2012, to December 31, 2019. The purpose of the analysis of the two sub-periods is to ascertain if financial crisis exerts a different impact on the volatility of mutual fund returns. The data were sourced from DATASTREAM, and the seven fund indices were created using a similar approach as adopted in generating the Dow Jones Industrial Average (DJIA) Index (Corielli & Marcellino, 2006; Parasuraman & Ramudu, 2014):

(Mutual Fund Index under specific fund objective)$_t$ = (sum of all funds NAV)$_t$ / divisor

where *t* denotes the period (daily in the case of this study). The divisor value of the respective fund index is obtained by dividing the summation of the net asset value (NAV) of all funds clustered within the respective fund index at a specific base date chosen by 100. The daily fund index is then obtained by dividing the summation of the daily NAV of all funds within a cluster against the calculated divisor of the respective fund index. The daily return of each fund index is specified as:

$$R_t = \ln\left[\frac{Fund\ Index_t}{Fund\ Index_{t-1}}\right]$$

where R_t is the compounded return of funds index derived from the first difference in logarithm form of the daily index, *Fund Index$_t$* is the index on day *t*, and *Fund Index$_{t-1}$* is the index on the previous day *t-1*. Appendices B and C respectively show the daily trend and plot of the return series of the respective mutual fund indices.

The descriptive statistics of the dataset for both sub-periods are shown in Table 1 and Table 2. The mean value for all seven fund indices in both sub-periods is close to zero, indicating the mean-reverting process in the long term. The mean, maximum, and minimum in the volatile sub-period of 2005-2011 (with financial crisis impact) is consistently higher across all fund indices compared to the less volatile sub-period of 2012-2019 (without financial crisis impact). Negative skewness in all fund indices for both sub-periods indicates a higher probability of making a loss (downside risk) than making a profit. Leptokurtosis (kurtosis above 3) distribution was found in all fund indices across both sub-periods; but was more prominent in the sub-period with financial crisis impact, implying the existence of outliers. Significant Jarque-Bera statistics in all fund indices for both sub-periods confirmed the non-normality of the return distribution. The augmented Dickey-Fuller statistics (see Tables 3 and 4) for all fund indices in both sub-periods are significant, implying that the data is stationary, and is therefore good for volatility estimation and forecasting.

Table -1. Descriptive statistics for sub-period with financial crisis (2005-2011).

Name of Fund Index	Obs.	Mean	Max.	Min.	Std. Dev.	Skew	Kurtosis	Jarque-Bera	Prob.
Equity Growth Fund Index	1824	1.890	0.040	-0.082	0.007	-1.452	15.79	13067.3	0.00
Equity Growth & Income Fund Index	1824	2.490	0.033	-0.074	0.007	-1.341	15.21	11881.0	0.00
Equity Income Fund Index	1824	1.030	0.033	-0.070	0.007	-1.308	12.59	7504.9	0.00
Balanced Growth Fund Index	1824	0.904	0.031	-0.081	0.007	-2.371	24.71	37544.8	0.00
Balanced Growth & Income Fund Index	1824	0.292	0.025	-0.055	0.005	-1.427	14.41	10510.6	0.00
Balanced Income Fund Index	1824	2.740	0.028	-0.056	0.005	-1.195	15.45	12206.3	0.00
Mixed Asset Growth Fund Index	1824	2.190	0.036	-0.066	0.006	-1.821	19.36	21341.0	0.00

Note: Mean has been multiplied by 10^4.

Table -2. Descriptive statistics for sub-period without financial crisis (2012-2019).

Name of Fund Index	Obs.	Mean	Max.	Min.	Std. Dev.	Skew	Kurtosis	Jarque-Bera	Prob.
Equity Growth Fund Index	2086	0.649	0.027	-0.025	0.004	-0.756	7.229	1753.0	0.00
Equity Growth & Income Fund Index	2086	0.664	0.024	-0.023	0.004	-0.749	6.719	1397.3	0.00
Equity Income Fund Index	2086	0.322	0.020	-0.028	0.005	-1.191	7.806	2501.1	0.00
Balanced Growth Fund Index	2086	0.046	0.021	-0.028	0.004	-1.820	14.07	11794.6	0.00
Balanced Growth & Income Fund Index	2086	0.064	0.015	-0.017	0.003	-0.855	6.660	1418.4	0.00
Balanced Income Fund Index	2086	1.930	0.018	-0.035	0.004	-0.793	9.312	3682.2	0.00
Mixed Asset Growth Fund Index	2086	0.000	0.070	-0.032	0.006	-0.005	19.41	23408.4	0.00

Note: Mean has been multiplied by 10^4.

Table -3. Unit root test with augmented Dickey-Fuller test for sub-period 2005-2011.

Fund Index	ADF-stats	p-value	sig.
Growth	-36.04	0.000	***
Growth & Income	-35.91	0.000	***
Income	-36.63	0.000	***
Balanced Growth	-38.66	0.000	***
Balanced Growth & Income	-37.21	0.000	***
Balanced Income	-38.67	0.000	***
Mixed Asset Growth	-38.45	0.000	***

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively.

Table -4. Unit root test with augmented Dickey-Fuller test for sub-period 2012-2019.

Fund Index	ADF-stats	p-value	sig.
Growth	-40.39	0.000	***
Growth & Income	-41.62	0.000	***
Income	-42.02	0.000	***
Balanced Growth	-41.87	0.000	***
Balanced Growth & Income	-41.03	0.000	***
Balanced Income	-41.11	0.000	***
Mixed Asset Growth	-42.19	0.000	***

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels respectively.

Table -5. Residual diagnostics of fund indices for sub-period 2005-2011.

	Ljung-Box Q-statistics Test			ARCH LM Test		
	Null Hypothesis:			Null Hypothesis:		
	Data are independently			ARCH effect does not exist		
	distributed (no autocorrelation)					
Fund Index	**Q-stat 12)**	**p-value**	**sig.**	**Obs*R²**	**p-value**	**sig.**
Equity Growth	90.75	0.000	***	79.33	0.000	***
Equity Growth & Income	84.97	0.000	***	77.94	0.000	***
Equity Income	69.60	0.000	***	66.94	0.000	***
Balanced Growth	32.99	0.001	***	20.97	0.000	***
Balanced Growth & Income	69.61	0.000	***	47.94	0.000	***
Balanced Income	46.01	0.000	***	68.97	0.000	***
Mixed Asset Growth	48.04	0.000	***	51.96	0.000	***

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels respectively.

Table -6. Residual diagnostics of fund indices for sub-period 2012-2019.

	Ljung-Box Q-statistics Test			ARCH LM Test		
	Null Hypothesis:			Null Hypothesis:		
	Data are independently			ARCH effect does not exist		
	distributed (no autocorrelation)					
Fund Index	**Q-stat (12)**	**p-value**	**sig.**	**Obs*R²**	**p-value**	**sig.**
Equity Growth	40.29	0.000	***	59.98	0.000	***
Equity Growth & Income	28.82	0.004	***	58.25	0.000	***
Equity Income	25.53	0.012	**	17.07	0.000	***
Balanced Growth	32.98	0.001	***	10.08	0.002	***
Balanced Growth & Income	32.15	0.001	***	57.10	0.000	***
Balanced Income	46.02	0.000	***	137.72	0.000	***
Mixed Asset Growth	31.55	0.000	***	6.92	0.009	***

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels respectively.

The p-values of the Ljung-Box Q-Statistics are significant up to lag 12 (see Tables 5 and 6) for all seven fund indices across both sub-periods indicating the existence of autocorrelation in the return series. The Lagrange multiplier (LM) test produces observed R^2 values that are all significant at a 1% significance level for all seven fund indices in both sub-periods, implying the existence of the ARCH effect. The combined results of autocorrelation up to lag 12 and the ARCH effect signifies that the GARCH model is appropriate to measure heteroskedasticity.

4. EMPIRICAL RESULTS AND DISCUSSIONS

4.1. In-Sample Estimation of GARCH Models

Since this study focuses on forecasting volatility, it is impractical to repeatedly specify the lag order of GARCH models for each of the fund indices. As such, following Taylor (2004b), the study opted for the GARCH (1,1) model specification and applied it across the standard GARCH, EGARCH, IGARCH, and GJRGARCH models. From the full sample period, there was an approximate 80:20 split between the in-sample and post-sample data running from Jan 3rd, 2005, to October 4th, 2010 (1500 observations) and Oct 5th, 2010, to Dec 30th, 2011 (324 observations), respectively. Regarding the sub-period without financial crisis, there was an approximate 70:30 split between in-sample and post-sample data covering Jan 3rd, 2012, to Oct 2nd, 2017 (1500 observations) and Oct 3rd, 2017, to Dec 31st, 2019 (586 observations), respectively. Parameters of the four GARCH family models were estimated under the Gaussian, Student-t, and Generalized Error Distribution (GED) innovations using the maximum likelihood estimation (MLE) method introduced by Bollerslev & Wooldridge (1992) with the EViews legacy optimization option.

Appendices D1 to D6 provide detailed results of the parameter estimates of the 12 GARCH models for the respective fund indexes in both sub-periods. The β coefficient values in all 12 GARCH models were consistently higher in the sub-period with financial crisis impact compared to the less volatile sub-period across all fund indices, implying a greater volatility clustering effect during a volatile period. The summation of $\alpha + \beta < 1$ consistently across all fund indices implies that volatility persists over time across all indices in both sub-periods. Table 7 shows the best-fitting GARCH model. Both the EGARCH and GJR-GARCH models under non-normal error term distribution (Stud-t and GED) are the best-fitting models in both sub-periods, implying the existence of both the asymmetry effect and the leverage effect, which are well captured by both models, and concur with past studies (Alberg et al., 2008; Angabini & Wasiuzzaman, 2011; Chuan, Mahdi, & Kenneth, 2021; Dritsaki, 2017; Hamilton & Susmel, 1994; Kuhe, 2018; Lim & Sek, 2013; Musa et al., 2020; Peters, 2001; Shamiri & Isa, 2009; Wilhelmsson, 2006).

Table -7. Best-fitting GARCH models by fund index and sub-period.

	Growth	Growth Income	Income	Bal. Growth	Bal. Growth Income	Bal. Income	Mixed Asset Growth
Sub-period (2005-2011)	GJR-GARCH GED	GJR-GARCH GED	EGARCH GED	EGARCH GED	EGARCH GED	EGARCH GED	EGARCH GED
Sub-period (2012-2019)	EGARCH GED	EGARCH-Stud T	EGARCH GED	EGARCH GED	EGARCH GED	EGARCH GED	EGARCH-Stud T

The residual diagnostic test (details available upon request) revealed insignificant p-values of the F-statistics and chi-squares in the ARCH LM test, as well as insignificant p-values of the Ljung-Box Q^2 statistics, implying that all information related to volatility has been well captured by the estimated models, and are therefore appropriate for volatility forecasting.

4.2. Post-Sample Forecasting Evaluation

Two criteria, MAE (mean absolute error) and RMSE (root mean square error), were employed to evaluate and determine the best post-sample one-day-ahead volatility forecasting model for the fund indices specified as:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{t=0}^{N}(v_t - \hat{v_t})^2}$$

$$\text{MAE} = \frac{1}{N}\sum_{t=1}^{N}|v_t - \hat{v_t}|$$

where the squared residual ε_t^2 is used as a proxy for actual volatility denoted by v_t, $\hat{v_t}$ is the forecast variance of the model concerned, and N denotes post-sample observations. The forecast variance of all 22 models and methods examined were compared against the squared residual to obtain forecast errors evaluated by both MAE and RMSE loss functions. Smaller MAE and RMSE denote a better model. Theil-U statistics were used for the relative performance of the models across each fund index, where the MAE and RMSE values of each model in a fund index were compared against the benchmark GJRGARCH-t model. The smallest value of the mean Theil-U is obtained by averaging the Theil-U values of each model across fund indices. The best performing model across the fund index is indicated by the lowest mean Theil-U value (ranked 1) to the poorest performing model with the highest mean Theil-U value (ranked 22) (see Appendices E1 to E4 for details). Table 8 summarizes the top five and bottom five ranked models/methods in terms of post-sample volatility forecasting performance.

Table -8. Post-sample volatility forecasting ranking of models.

		Sub-period 2005-2011		**Sub-period 2012-2019**	
		MAE	**RMSE**	**MAE**	**RMSE**
	Rank	**Model / method**	**Model / method**	**Model / method**	**Model / method**
Top 5 ranked	1	STES-E & AE	STES-E & AE	STES-E & AE	EGARCH-t
(in descending order)	2	STES-AE	IGARCH-N	STES-AE	GARCH-t
	3	EWMA-Riskmetric	STES-AE	Naïve Variance	STES-AE
	4	STES-SE	EWMA-Riskmetric	EGARCH-t	STES-E & AE
	5	IGARCH-N	STES-ESE	GARCH-N	GJRGARCH-t
Bottom 5 ranked	18	GARCH-t	GJRGARCH-GED	MA30	GJRGARCH-N
(in descending order)	19	EGARCH-t	GARCH-t	EGARCH-GED	EGARCH-GED
	20	GJRGARCH-t	GJRGARCH-t	GARCH-GED	GARCH-GED
	21	RW	Naïve Variance	GJRGARCH-GED	GJRGARCH-GED
	22	Naïve Variance	RW	RW	RW

Under MAE criteria, STES-E&AE emerged as the best model for post-sample volatility forecasting both sub-periods (see Table 8), followed by the STES-AE method. However, STES-AE is ranked third under the RMSE criteria in both sub-periods. The RW method was the worst-performing model and ranked last in both sub-periods, hence invalidating the Random Walk Theory, which claims that successive price movements are not correlated and random.

The model confidence set (MCS) of Hansen et al. (2011) was applied for a robustness check of the post-sample results. Unlike RMSE and MAE criteria, which aim to determine the "best post-sample forecasting model" based on a benchmark model (which is rather subjective), the MCS procedure, focuses on examining "equal predictive ability" among models that yields a set of remaining (surviving) models known as the model confidence set (MCS) through a sequential bootstrap elimination process. Table 9 summarizes the results of the MCS procedure (detailed MCS results are not presented here). Using the squared forecast error (SE), the MCS result is rather homogenous in the 2012-2019 sub-period where the EGARCH-GED model obtained a better mean ranking than the STES methods. However, when using the absolute forecast error (AE), the STES methods, especially the STES-E&AE method, were the least eliminated models with the best mean ranking across all fund indices in both sub-periods.

Table -9. MCS procedure results.

	Sub-period 2005-2011				Sub-period 2012-2019			
	Absolute Error		Squared Error		Absolute Error		Squared Error	
Model / Method	Count	Mean Rank	Count	Mean Rank	Count	Mean Rank	Count	Mean Rank
RW	1	15.0	1	22.0	0			
Naïve	2	20.0	5	20.6	7	10.7	7	19.3
EWMA-OP	7	6.1	7	7.3	6	13.2	7	12.3
EWMA-RM	7	4.9	7	10.0	6	15.3	6	15.5
MA30	7	8.9	7	16.9	7	17.9	5	20.4
GARCH-N	2	16.0	7	13.6	7	8.1	7	7.1
GJRGARCH-N	2	15.5	7	12.7	6	10.3	6	8.3
IGARCH-N	7	7.6	7	7.3	6	13.8	7	14.6
EGARCH-N	3	14.3	7	12.3	7	9.9	7	4.0
GARCH-t	2	15.5	6	16.3	7	11.3	7	10.9
GJRGARCH-t	3	18.0	6	16.3	7	13.0	7	10.6
IGARCH-t	6	6.5	6	8.8	6	14.5	7	15.3
EGARCH-t	2	13.0	6	13.2	7	10.0	7	5.0
GARCH-GED	3	15.0	6	11.0	7	5.4	7	7.7
GJRGARCH-GED	3	15.7	6	11.7	7	8.9	7	9.1
IGARCH-GED	6	7.7	6	7.8	6	13.2	7	15.3
EGARCH-GED	3	13.3	6	10.8	7	7.7	**7**	**3.3**
STES-SE	7	5.3	7	7.4	6	11.3	7	11.6
STES-E	7	7.3	7	5.9	6	13.0	7	11.9
STES-AE	7	3.0	7	8.1	6	3.2	7	9.0
STES-EAE	**7**	**1.0**	**7**	**3.0**	**7**	**1.9**	7	10.1
STES-ESE	7	8.1	7	4.7	6	10.5	7	9.6

Note: Mean rank is calculated from average of ranking scored by each fund index based on the p-value, significance at 15% level form MCS procedure. Count refers to remaining number of models/methods that remain uneliminated in the model confidence set after MCS procedure.

4.3. Discussion on Results

The STES method with the error (sign of previous period's shock) and absolute error (the size of previous period's shock) as transition variables emerged as the best one-day-ahead volatility forecasting method regardless of volatility condition. This implies that the STES-E&AE method has well-captured the volatility of the fund indices attributed to both sign (positive or negative) and size (magnitude) of the previous period's shock. The results support the superiority of the STES methods, as revealed by findings of Taylor (2004a), Taylor (2004b), and Liu et al. (2020). The strength and robustness of the STES methods are attributed to "their adaptive time-varying smoothing parameter" the form of a logistic function of user-specified transition variables, where the value of this parameter reduces to exert lower weight to the outlying observation (Liu et al., 2020).

It was shown that the five STES methods examined ranked higher than most GARCH models in both sub-periods (see Appendices E1 to E4), regardless of post-sample forecasting performance evaluation criteria. Although the EGARCH-t is the best post-sample model under RMSE criteria in the sub-period without financial crisis impact, the result is debatable. The reason being, that RMSE criteria tend to result in spurious inferences, particularly in the presence of outliers due to the nature of RMSE's quadratic function when a higher weight is assigned to a larger forecast error (Franses & Ghijsels, 1999). The results thus refuted the claim of GARCH's superiority in post-sample volatility forecasting by Hansen & Lunde (2005). The results have empirically proven the overall superiority of STES methods over the GARCH models in forecasting the volatility of mutual funds' returns. With this, the null hypotheses are rejected, or both alternative hypotheses, H_{1A} and H_{1B}, are supported.

5. CONCLUSION

This study examines the post-sample volatility forecasting performance of 22 models, aimed at verifying if the STES method can outperform the well-known GARCH family models. The results provide further evidence on the robustness and supremacy of the STES method in volatility forecasting, which was applied to seven mutual fund indices (created from 57 individual mutual funds' daily net asset values of different investment objectives and risk characteristics in Malaysia) for two different sub-periods, with financial crisis impact (2005-2011) and without financial crisis impact (2012-2019). The results revealed that the STES methods, particularly with error and absolute error as transition variables, provide the best one-day-ahead volatility forecasting across seven mutual fund indices in both sub-periods. The output from the applied MCS procedure reaffirmed the robustness of the results from both the MAE and RMSE evaluation criteria.

Although the asymmetric EGARCH and GJRGARCH under non-normal error term distribution assumptions have been effective in captured asymmetric and leverage effects, the post-sample forecasting performance was otherwise. It can be concluded that GARCH models with good in-sample estimation need not necessarily be a good model for post-sample volatility forecasting particularly during a volatile period where the issue of outliers is prominent. This study has provided strong empirical evidence of the supremacy of STES method, STES-E&AE particularly, over well-known GARCH models in one-day ahead volatility forecasting of private equity mutual funds' return in the Malaysian case, regardless of market volatility condition.

This study has expanded the literature on financial volatility through the application of STES methods to measure the volatility of mutual funds' returns, which to our best knowledge has never been examined in prior mutual fund studies. The STES method provides an empirical method for investing community (retail and institutional investors) to better manage the investment risk of mutual funds and can even be applied for the microanalysis of individual mutual funds' volatility. Though the study is confined to private equity mutual funds in Malaysia, the creation of the seven equity-based mutual fund indices is a notable contribution of this study, which, to the best of our knowledge, has never existed in Malaysia. It is hoped that the indices created in this study will inspire the generation of other sectorial or regional mutual fund indices to enable a more diversified macroanalysis. The inclusion of daily mutual fund flows as a transition variable in the STES method is suggested for future volatility studies.

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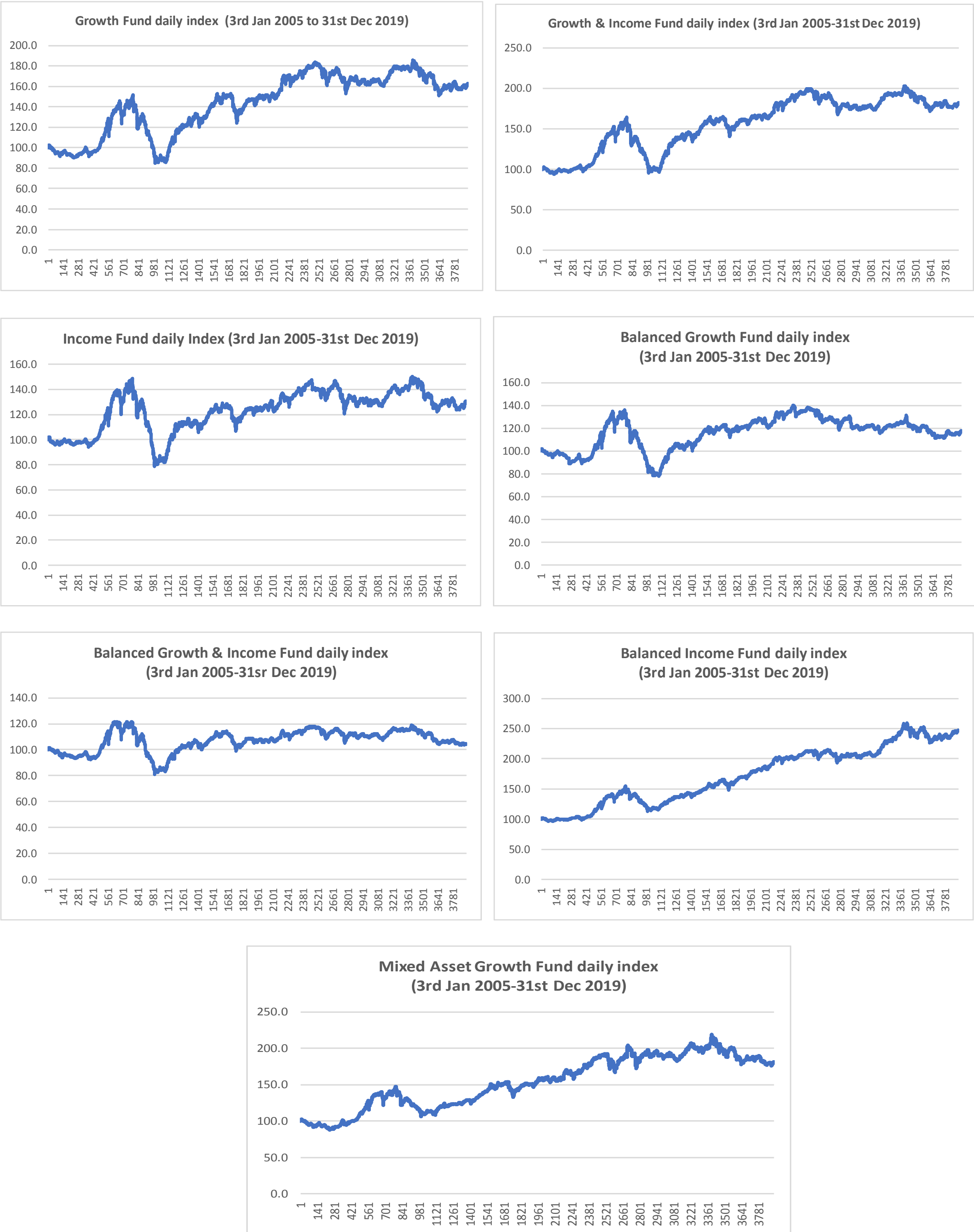
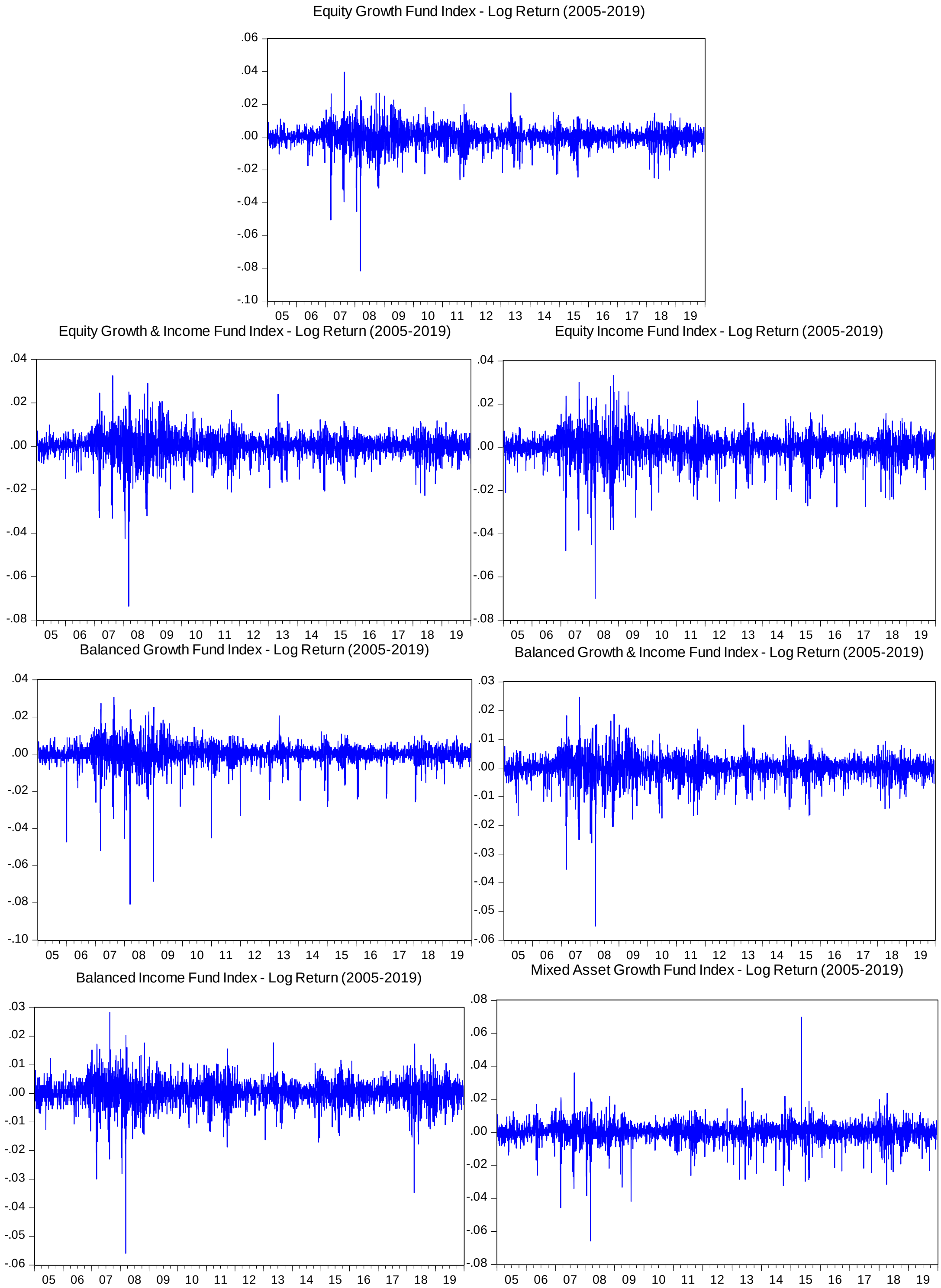
Appendix -A. List of 57 equity-based mutual funds.

Fund category	Name of Fund	Fund Management	Fund Inception
Growth	Public Aggressive Growth Fund	Public Mutual Bhd	25th Apr 1995
	Public Equity Fund	Public Mutual Bhd	15th Aug 2001
	Public Focus Select Fund	Public Mutual Bhd	25th Nov 2004
	Public Growth Fund	Public Mutual Bhd	11th Dec 1984
	Public Index Fund	Public Mutual Bhd	2nd Mar 1992
	Public Industry Growth Fund	Public Mutual Bhd	18th Nov 1993
	Public Islamic Equity Fund	Public Mutual Bhd	28th May 2003
	Public Ittikal Fund	Public Mutual Bhd	10th Apr 1997
	Public SmallCap Fund	Public Mutual Bhd	13th Jun 2000
	PB Growth	Public Bank Bhd	3rd Oct 2002
	Principal DALI Equity Fund	Principal	30th Apr 2003
	Principal DALI Equity Growth Fund	Principal	7th May 1998
	Principal Islamic Enhanced Opportunities Fund	Principal	15th June 1995
	Principal Islamic SmallCap Opportunities Fund	Principal	30th Apr 2003
	Principal Malaysia Enhanced Opportunities Fund	Principal	18th Aug 2004
	Principal Malaysia Opportunities Fund	Principal	12th Mar 1998
	Principal Malaysia Titans Plus Fund	Principal	28th Sep 1998
	Principal DALI Asia Pacific Equity Growth Fund	Principal	8th Oct 2004
	Principal KLCI-Linked Fund	Principal	8th Jun 2000
	AmIttikal Fund	AmInvest Bhd	12th Jan 1993
	Am Cumulative Growth Fund	AmInvest Bhd	24th Jul 1996
	Am Malaysia Equity Fund	AmInvest Bhd	15th Oct 2001
	Affin Hwang Aiiman Growth Fund	AffinHwang Asset	8th Oct 2002
	Affin Hwang Select Asia (ex-Japan) Quantum Fund	AffinHwang Asset	15th Apr 2004
	Affin Hwang Select Opportunities Fund	AffinHwang Asset	7th Sep 2001
	Manulife Investment Equity Index Fund	Manulife Investment	26th Jun 1997
	Manulife Investment Value Fund	Manulife Investment	28th Jul 1995
	Manulife Investment Regular Savings Fund	Manulife Investment	29th Sep 2004
	Manilife Dana Ekuiti Dinamik Fund	Manulife Investment	6th Oct 2003
	Manulife Equity Fund	Manulife Investment	10th Jul 2000
	Eastspring Investment Growth Fund	Eastspring Investment	29th May 2001
	Eastspring Investment SmallCap Fund	Eastspring Investment	29th May 2001
	Eastspring Investment Dana Al-Ilham Fund	Eastspring Investment	14th Aug 2002
Growth & Income	Public Regular Savings Fund	Public Mutual Bhd	25th Apr 1994
	Public Savings Fund	Public Mutual Bhd	29th Mar 1981
	Principal Titans Growth & Income Fund	Principal	15th May 1991
	AM Total Return Fund	AmInvest Bhd	10th Jan 1989
	Affin Hwang Equity Fund	AffinHwang Asset	29th Apr 1993
	Manulife Managed Fund	Manulife Investment	10th Jul 2000
Income	Principal Titans Income Plus Fund	Principal	1st Oct 2003
	Manulife Investment Syariah Index Fund	Manulife Investment	26th Jan 2002
	Eastspring Equity Income Fund	Eastspring Investment	18th Oct 2004
Balanced Growth	Principal Islamic Lifetime Balanced Growth Fund	Principal	26th May 2003
	Principal Lifetime Balanced Fund	Principal	12th Mar 1998
Balanced Growth & Income	Public Balanced Fund	Public Mutual Bhd	7th Jun 1995
	PB Balanced Fund	Public Bank Bhd	5th May 1998
	Principal Islamic Lifetime Balanced	Principal	8th Mar 2001
	Principal Lifetime Balanced Income Fund	Principal	10th Aug 1995
	Principal Dynamic Enhanced Malaysia Income Fund	Principal	12th Mar 1998
	Affin Hwang Aiiman Balanced Fund	AffinHwang Asset	11th Nov 2001
	Affin Hwang Select Balanced Fund	AffinHwang Asset	28th Jul 2003
	Manulife Investment Balanced Fund	Manulife Investment	2nd Jun 1991
	Eastspring Investment Balanced Fund	Eastspring Investment	29th May 2001
Balanced Income	Am Balanced Fund	AmInvest Bhd	16th Sep 2003
	Am Islamic Balanced Fund	AmInvest Bhd	10th Sep 2004
Mixed Asset Growth	Eastspring Investment Dana Dinamik Fund	Eastspring Investment	25th Feb 2004
	Eastspring Investment Dynamic Fund	Eastspring Investment	6th Nov 2003



Appendix -B. Daily index of seven equity private mutual fund indices.



Appendix -C. Plot of daily return series for all seven fund indices (2005–2019).

Appendix -D1. Parameter estimates under Gaussian distribution for sub-period with financial crisis (2005-2011).

Model	Parameters & Goodness of Fit Criteria	Growth Fund Index		Growth & Income Fund Index		Income Fund Index		Bal. Growth Fund Index		Bal. Growth & Income Fund Index		Bal. Income Fund Index		Mixed Asset Growth Fund Index	
GARCH-N	ω_0 (x 10^{-6})	0.437	***	0.434	***	0.392	***	1.640	***	0.269	***	0.472	***	1.200	***
	α_i	0.160	***	0.133	***	0.123	***	0.094	***	0.120	***	0.120	***	0.189	***
	β_j	0.850	***	0.869	***	0.882	***	0.884	***	0.882	***	0.866	***	0.799	***
	AIC	-7.407		-7.564		-7.380		-7.256		-8.017		-8.098		-7.764	
	BIC	-7.396		-7.553		-7.369		-7.246		-8.006		-8.087		-7.754	
	LogL	5562		5680		5538		5445		6016		6076		5826	
EGARCH-N	ω_0 (x 10^{-6})	-0.461	***	-0.408	***	-0.322	***	-0.214	***	-0.363	***	-0.451	***	-0.761	***
	α_i	0.281	***	0.232	***	0.226	***	0.079	***	0.204	***	0.212	***	0.320	***
	β_j	0.975	***	0.977	***	0.984	***	0.984	***	0.980	***	0.973	***	0.949	***
	γ_i	-0.046	***	-0.050	***	-0.035	***	-0.036	***	-0.046	***	-0.045	***	-0.030	***
	AIC	-7.404		-7.560		-7.391		-7.279		-8.033		-8.102		-7.765	
	BIC	-7.389		-7.546		-7.376		-7.265		-8.018		-8.087		-7.751	
	LogL	5560		5678		5547		5463		6028		6080		5828	
IGARCH-N	α_i	0.100	***	0.081	***	0.073	***	0.010	***	0.070	***	0.060	***	0.110	***
	β_j	0.900	***	0.919	***	0.927	***	0.990	***	0.930	***	0.940	***	0.890	***
	AIC	-7.379		-7.534		-7.352		-7.129		-7.994		-8.075		-7.692	
	BIC	-7.375		-7.530		-7.348		-7.125		-7.990		-8.071		-7.688	
	LogL	5539		5655		5515		5348		5996		6057		5770	
GJRGARCH-N	ω_0 (x 10^{-6})	0.503	***	0.548	***	0.415	***	1.660	***	0.305	***	0.572	***	1.250	***
	α_i	0.135	***	0.105	***	0.106	***	0.101	***	0.095	***	0.095	***	0.179	***
	$\gamma_i d_{t-i}$	0.054	***	0.063	***	0.030	**	-0.008		0.043	***	0.054	***	0.020	*
	β_j	0.846	***	0.862	***	0.882	***	0.882	***	0.881	***	0.858	***	0.796	***
	AIC	-7.409		-7.568		-7.380		-7.255		-8.019		-8.101		-7.763	
	BIC	-7.395		-7.554		-7.366		-7.241		-8.005		-8.086		-7.749	
	LogL	5564		5684		5539		5445		6018		6079		5826	

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively. AIC (Akaike Information Criteria), BIC (Schwarz Bayesian Information Criteria) and LL (Log Likelihood) are goodness of fit criteria for parameter estimates.

Appendix -D2. Parameter estimates under student-t distribution for sub-period with financial crisis (2005-2011).

Model	Parameters & Goodness of Fit Criteria	Growth Fund Index		Growth & Income Fund Index		Income Fund Index		Bal. Growth Fund Index		Bal. Growth & Income Fund Index		Bal. Income Fund Index		Mixed Asset Growth Fund Index	
GARCH-t	ω_0 (x 10^{-6})	0.418	***	0.444	***	0.289	**	0.761	***	0.271	**	0.330	***	1.310	***
	α_i	0.143	***	0.134	***	0.097	***	0.142	***	0.119	***	0.103	***	0.153	***
	β_j	0.866	***	0.872	***	0.909	***	0.865	***	0.890	***	0.891	***	0.823	***
	AIC	-7.473		-7.644		-7.493		-7.607		-8.139		-8.173		-7.938	
	BIC	-7.459		-7.630		-7.479		-7.593		-8.125		-8.159		-7.924	
	LogL	5612		5741		5624		5709		6108		6134		5958	
EGARCH-t	ω_0 (x 10^{-6})	-0.436	***	-0.386	***	-0.278	***	-0.267	***	-0.369	***	-0.386	***	-0.722	***
	α_i	0.269	***	0.239	***	0.201	***	0.156	***	0.214	***	0.203	***	0.260	***
	β_j	0.976	***	0.979	***	0.987	***	0.984	***	0.979	***	0.978	***	0.949	***
	γ_i	-0.047	***	-0.046	***	-0.036	**	-0.002		-0.042	***	-0.037	**	-0.062	***
	AIC	-7.474		-7.644		-7.501		-7.615		-8.143		-8.176		-7.944	
	BIC	-7.457		-7.626		-7.483		-7.598		-8.126		-8.159		-7.927	
	LogL	5614		5742		5631		5717		6113		6137		5963	
IGARCH-t	α_i	0.096	***	0.087	***	0.061	***	0.080	***	0.079	***	0.061	***	0.094	***
	β_j	0.904	***	0.913	***	0.939	***	0.920	***	0.921	***	0.939	***	0.906	***
	AIC	-7.460		-7.629		-7.485		-7.588		-8.126		-8.165		-7.906	
	BIC	-7.453		-7.622		-7.478		-7.580		-8.119		-8.157		-7.899	
	LogL	5601		5728		5616		5693		6096		6125		5932	
GJRGARCH-t	ω_0 (x 10^{-6})	0.500	***	0.536	***	0.3300	***	0.737	***	0.299	***	0.398	***	1.710	***
	α_i	0.124	***	0.114	***	0.089	***	0.145	***	0.103	***	0.087	***	0.106	***
	$\gamma_i d_{t-i}$	0.057	*	0.057	*	0.024		-0.008		0.031		0.043		0.120	***
	β_j	0.857	***	0.862	***	0.905	***	0.867	***	0.887	***	0.882	***	0.798	***
	AIC	-7.474		-7.645		-7.492		-7.605		-8.139		-8.174		-7.942	
	BIC	-7.456		-7.627		-7.475		-7.588		-8.121		-8.156		-7.924	
	LogL	5614		5742		5624		5709		6109		6135		5961	

Note: *, ** and *** denote significance at the10%, 5% and 1% levels, respectively. AIC (Akaike Information Criteria), BIC (Schwarz Bayesian Information Criteria) and LL (Log Likelihood) are goodness of fit criteria for parameter estimates.

Appendix -D3. Parameter estimates under GED distribution for sub-period with financial crisis (2005-2011).

Model	Parameters & Goodness of Fit Criteria	Growth Fund Index		Growth & Income Fund Index		Income Fund Index		Bal. Growth Fund Index		Bal. Growth & Income Fund Index		Bal. Income Fund Index		Mixed Asset Growth Fund Index	
GARCH-GED	ω_0 (x 10^{-6})	0.411	***	0.422	***	0.323	**	0.897	***	0.250	**	0.348	***	1.220	***
	α_i	0.148	***	0.128	***	0.103	***	0.138	***	0.114	***	0.104	***	0.157	***
	β_j	0.860	***	0.873	***	0.899	***	0.867	***	0.888	***	0.885	***	0.814	***
	AIC	-7.480		-7.653		-7.496		-7.610		-8.155		-8.177		-7.943	
	BIC	**-7.466**		**-7.639**		-7.481		-7.596		-8.141		**-8.163**		-7.929	
	LogL	5618		5748		5626		5711		6120		6137		5961	
EGARCH-GED	ω_0 (x 10^{-6})	-0.450	***	-0.388	***	-0.296	***	-0.244	***	-0.370	***	-0.405	***	-0.753	***
	α_i	0.274	***	0.232	***	0.205	***	0.140	***	0.209	***	0.202	***	0.274	***
	β_j	0.976	***	0.979	***	0.985	***	0.985	***	0.980	***	0.977	***	0.947	***
	γ_i	-0.047	***	-0.045	***	-0.035	**	-0.006		-0.043	**	-0.039	**	-0.052	***
	AIC	-7.481		-7.652		**-7.503**		**-7.617**		**-8.160**		**-8.180**		**-7.946**	
	BIC	-7.463		-7.635		**-7.485**		**-7.599**		**-8.142**		-8.162		**-7.929**	
	LogL	5619		5748		**5632**		**5718**		**6125**		**6140**		**5965**	
IGARCH-GED	α_i	0.096	***	0.084	***	0.064	***	0.016	***	0.075	***	0.060	***	0.098	***
	β_j	0.904	***	0.916	***	0.936	***	0.984	***	0.925	***	0.940	***	0.902	***
	AIC	-7.468		-7.640		-7.487		-7.590		-8.144		-8.169		-7.914	
	BIC	-7.461		-7.633		-7.480		-7.583		-8.137		-8.162		-7.907	
	LogL	5607		5736		5617		5695		6110		6129		5938	
GJRGARCH-GED	ω_0 (x 10^{-6})	0.491	***	0.515	***	0.359	***	0.835	***	0.280	**	0.421	***	1.490	***
	α_i	0.126	***	0.107	***	0.092	***	0.144	***	0.095	***	0.085	***	0.122	***
	$\gamma_i d_{t-i}$	0.057	*	0.055	*	0.025		-0.021		0.035		0.045	*	0.085	**
	β_j	0.853	***	0.864	***	0.896	***	0.871	***	0.886	***	0.878	***	0.797	***
	AIC	**-7.481**		**-7.654**		-7.495		-7.609		-8.155		-8.178		-7.944	
	BIC	-7.464		-7.637		-7.477		-7.591		-8.137		-8.160		-7.927	
	LogL	**5620**		**5750**		5626		5712		6121		6138		5963	

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively. AIC (Akaike Information Criteria), BIC (Schwarz Bayesian Information Criteria) and LL (Log Likelihood) are goodness of fit criteria for parameter estimates.

Appendix -D4. Parameter estimates under Gaussian distribution for sub-period without financial crisis (2012-2019).

Model	Parameters & Goodness of Fit Criteria	Growth Fund Index		Growth & Income Fund Index		Income Fund Index		Bal. Growth Fund Index		Bal. Growth & Income Fund Index		Bal. Income Fund Index		Mixed Asset Growth Fund Index	
GARCH-N	ω_0 (x 10^{-6})	1.020	***	0.903	***	1.490	***	0.441	***	0.661	***	0.817	***	8.620	***
	α_i	0.104	***	0.088	***	0.074	***	0.139	***	0.087	***	0.116	***	0.105	***
	β_j	0.842	***	0.852	***	0.863	***	0.848	***	0.832	***	0.806	***	0.624	***
	AIC	-8.195		-8.357		-7.920		-8.575		-8.953		-8.756		-7.587	
	BIC	-8.185		-8.347		-7.909		-8.564		-8.942		-8.746		-7.577	
	LogL	6150		6271		5943		6434		6718		6570		5694	
EGARCH-N	ω_0 (x 10^{-6})	-0.862	***	-1.172	***	-0.653	***	-1.136	***	-0.963	***	-1.089	***	-1.357	***
	α_i	0.187	***	0.167	***	0.107	***	0.294	***	0.133	***	0.176	***	0.071	***
	β_j	0.934	***	0.906	***	0.946	***	0.918	***	0.926	***	0.917	***	0.875	***
	γ_i	-0.108	***	-0.126	***	-0.087	***	-0.169	***	-0.100	***	-0.124	***	-0.143	***
	AIC	-8.215		-8.382		-7.936		-8.616		-8.971		-8.778		-7.626	
	BIC	-8.201		-8.367		-7.922		-8.602		-8.956		-8.763		-7.611	
	LogL	6166		6290		5956		6466		6732		6587		5723	
IGARCH-N	α_i	0.061	***	0.066	***	0.009	***	0.095	***	0.033	***	0.052	***	0.009	***
	β_j	0.939	***	0.934	***	0.991	***	0.905	***	0.967	***	0.948	***	0.991	***
	AIC	-8.153		-8.315		-7.832		-8.507		-8.888		-8.722		-7.572	
	BIC	-8.149		-8.311		-7.828		-8.602		-8.885		-8.719		-7.568	
	LogL	6116		6237		5875		6381		6667		6543		5680	
GJRGARCH-N	ω_0 (x 10^{-6})	1.520	***	1.900	***	1.500	***	1.150	***	0.714	***	1.240	***	8.770	***
	α_i	0.044	***	0.029	***	0.014		0.061	***	0.027	**	0.051	***	-0.005	
	$\gamma_i d_{t-i}$	0.153	***	0.168	***	0.083	***	0.346	***	0.108	***	0.191	***	0.176	***
	β_j	0.795	***	0.756	***	0.874	***	0.723	***	0.827	***	0.734	***	0.652	***
	AIC	-8.214		-8.373		-7.928		-8.600		-8.967		-8.773		-7.607	
	BIC	-8.199		-8.359		-7.914		-8.586		-8.953		-8.759		-7.593	
	LogL	6164		6284		5950		6454		6729		6584		5709	

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively. AIC (Akaike Information Criteria), BIC (Schwarz Bayesian Information Criteria) and LL (Log Likelihood) are goodness of fit criteria for parameter estimates.

Appendix -D5. Parameter estimates under student-t distribution for sub-period without financial crisis (2012-2019).

Model	Parameters & Goodness of Fit Criteria	Growth Fund Index		Growth & Income Fund Index		Income Fund Index		Balanced Growth Fund Index		Balanced Growth & Income Fund Index		Balanced Income Fund Index		Mixed Asset Growth Fund Index	
GARCH-t	ω_0 (x 10^{-6})	1.000	***	1.280	***	1.990	***	0.981	***	0.465	***	0.568	***	4.320	***
	α_i	0.114	***	0.126	***	0.112	***	0.146	***	0.092	***	0.107	***	0.139	***
	β_j	0.834	***	0.794	***	0.814	***	0.786	***	0.857	***	0.842	***	0.728	***
	AIC	-8.296		-8.458		-8.119		-8.810		-9.071		-8.836		-7.943	
	BIC	-8.282		-8.444		-8.105		-8.796		-9.057		-8.822		-7.929	
	LogL	6226		6348		6094		6612		6807		6631		5961	
EGARCH-t	ω_0 (x 10^{-6})	-0.778	***	-1.071	***	-0.869	***	-1.069	***	-0.759	***	-0.758	***	-1.035	***
	α_i	0.202	***	0.196	***	0.151	***	0.253	***	0.153	***	0.169	***	0.164	***
	β_j	0.943	***	0.917	***	0.929	***	0.922	***	0.945	***	0.945	***	0.912	***
	γ_i	-0.098	***	-0.112	***	-0.111	***	-0.072	***	-0.086	***	-0.096	***	-0.090	***
	AIC	-8.304		**-8.469**		-8.133		-8.815		-9.077		-8.844		**-7.954**	
	BIC	-8.287		**-8.451**		-8.115		-8.797		-9.059		-8.826		**-7.936**	
	LogL	6233		**6357**		6105		6616		6812		6638		**5970**	
IGARCH-t	α_i	0.079	***	0.079	***	0.020	***	0.011	***	0.058	***	0.061	***	0.016	***
	β_j	0.921	***	0.921	***	0.980	***	0.989	***	0.942	***	0.939	***	0.984	***
	AIC	-8.269		-8.426		-8.081		-8.747		-9.047		-8.816		-7.930	
	BIC	-8.262		-8.419		-8.074		-8.740		-9.040		-8.809		-7.923	
	LogL	6204		6322		6063		6563		6788		6614		5950	
GJRGARCH-t	ω_0 (x 10^{-6})	1.220	***	1.590	***	2.460	***	1.150	***	0.558	***	0.716	***	5.250	***
	α_i	0.051	**	0.045	*	0.036		0.116	***	0.040	*	0.050	**	0.032	
	$\gamma_i d_{t-i}$	0.130	***	0.153	***	0.136	***	0.073		0.091	***	0.117	***	0.191	***
	β_j	0.817	***	0.771	***	0.794	***	0.763	***	0.847	***	0.822	***	0.695	***
	AIC	-8.305		-8.467		-8.124		-8.810		-9.076		-8.842		-7.948	
	BIC	-8.287		-8.450		-8.107		-8.792		-9.058		-8.824		-7.931	
	LogL	6234		6356		6098		6612		6812		6637		5966	

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively. AIC (Akaike Information Criteria), BIC (Schwarz Bayesian Information Criteria) and LL (Log Likelihood) are goodness of fit criteria for parameter estimates.

Appendix -D6. Parameter estimates under GED distribution for sub-period without financial crisis (2012-2019).

Model	Parameters & Goodness of Fit Criteria	Growth Fund Index		Growth & Income Fund Index		Income Fund Index		Balanced Growth Fund Index		Balanced Growth & Income Fund Index		Balanced Income Fund Index		Mixed Asset Growth Fund Index	
GARCH-GED	ω_0 (x 10^{-6})	1.030	***	1.190	***	1.740	***	3.340	**	0.525	***	0.654	***	5.150	***
	α_i	0.113	***	0.113	***	0.096	***	0.573	**	0.088	***	0.106	***	0.128	***
	β_j	0.833	***	0.809	***	0.830	***	0.740	***	0.850	***	0.831	***	0.695	***
	AIC	-8.298		-8.457		-8.124		-8.871		-9.080		-8.842		-7.929	
	BIC	-8.283		-8.442		-8.110		-8.856		-9.066		-8.828		-7.915	
	LL	6227		6346		6097		6657		6814		6635		5951	
EGARCH-GED	ω_0 (x 10^{-6})	-0.811	***	-1.117	***	-0.806	***	-1.215	***	-0.844	***	-0.915	***	-1.275	***
	α_i	0.198	***	0.185	***	0.137	***	0.513	***	0.143	***	0.172	***	0.149	***
	β_j	0.940	***	0.913	***	0.934	***	0.903	***	0.937	***	0.932	***	0.889	***
	γ_i	-0.102	***	-0.116	***	-0.104	***	-0.150		-0.095	***	-0.106	***	-0.105	***
	AIC	**-8.307**		-8.468		**-8.135**		**-8.873**		**-9.087**		**-8.851**		-7.939	
	BIC	**-8.289**		-8.450		**-8.117**		-8.856		**-9.070**		**-8.833**		-7.921	
	LL	**6235**		6356		**6106**		**6660**		**6821**		**6643**		5959	
IGARCH-GED	α_i	0.072	***	0.073	***	0.017	***	0.008	***	0.048	***	0.056	***	0.014	***
	β_j	0.928	***	0.927	***	0.983	***	0.992	***	0.952	***	0.944	***	0.986	***
	AIC	-8.273		-8.430		-8.090		-8.824		-9.055		-8.823		-7.919	
	BIC	-8.266		-8.422		-8.083		-8.817		-9.048		-8.816		-7.912	
	LL	6207		6324		6069		6620		6794		6619		5941	
GJRGARCH-GED	ω_0 (x 10^{-6})	1.350	***	1.760	***	2.060	***	4.870	***	0.626	***	0.941	***	6.350	***
	α_i	0.049	***	0.038		0.028		0.436		0.031		0.049	*	-0.004	
	$\gamma_i d_{t-i}$	0.143	***	0.163	***	0.112	***	0.483		0.102	***	0.142	***	0.240	***
	β_j	0.804	***	0.758	***	0.821	***	0.668	***	0.838	***	0.787	***	0.658	***
	AIC	-8.307		-8.466		-8.128		-8.870		-9.086		-8.849		-7.935	
	BIC	-8.289		-8.448		-8.110		**-8.852**		-9.068		-8.831		-7.918	
	LL	6235		6354		6101		6658		6819		6642		5957	

Note: *, ** and *** denote significance at the 10%, 5% and 1% levels, respectively. AIC (Akaike Information Criteria), BIC (Schwarz Bayesian Information Criteria) and LL (Log Likelihood) are goodness of fit criteria for parameter estimates.

Appendix -E1. Post-sample volatility performance under MAE criteria (2005-2011).

Models	Growth Fund Index	Growth & Income Fund Index	Income Fund Index	Balanced Growth Fund Index	Balanced Growth & Income Fund Index	Balanced Income Fund Index	Mixed Asset Growth Fund Index	Mean Theil-U	Rank
RW	54.42	38.51	48.36	39.92	24.16	29.41	38.81	1.120	21
Naïve Variance	56.17	44.15	54.30	51.73	25.83	26.06	37.05	1.193	22
MA30	44.43	31.59	39.23	32.62	19.32	23.38	31.23	0.905	9
EWMA RiskMetric	43.95	31.28	39.00	32.41	19.20	23.17	30.70	0.897	3
EWMA Optimized	43.76	31.26	39.32	32.95	19.30	23.09	30.87	0.900	6
STES-SE	43.71	31.23	39.28	32.67	19.29	23.08	30.84	0.899	4
STES-E	43.81	31.29	39.35	33.16	19.31	23.10	30.90	0.902	8
STES-AbsE	42.76	30.62	38.53	**30.71**	19.02	22.75	30.16	0.877	2
STES E+AbsE	**42.24**	**30.37**	**37.89**	31.29	**18.72**	**22.16**	**29.47**	**0.866**	**1**
STES E+SE	43.82	31.31	39.33	33.06	19.31	23.09	30.88	0.901	7
GARCH-N	46.41	32.83	41.63	37.91	20.47	23.60	30.94	0.952	12
EGARCH-N	47.31	34.29	42.62	36.54	20.92	23.74	33.18	0.971	14
IGARCH-N	43.75	31.24	39.09	32.83	19.21	23.17	30.94	0.899	5
GJRGARCH-N	46.81	33.24	41.73	37.82	20.66	23.52	33.35	0.966	13
GARCH-t	46.48	41.43	41.85	36.92	21.07	23.84	32.78	0.991	18
EGARCH-t	47.79	42.56	43.14	34.28	21.63	24.05	32.92	0.999	19
IGARCH-t	43.75	37.74	39.00	32.69	19.23	23.16	30.83	0.920	11
GJRGARCH-t	47.08	42.63	42.03	36.73	21.22	23.80	33.28	1.000	20
GARCH-GED	46.09	40.87	41.04	37.15	20.45	23.51	32.12	0.977	16
EGARCH-GED	47.21	41.77	42.17	34.72	21.00	23.67	32.12	0.983	17
IGARCH-GED	43.75	37.74	39.02	32.26	19.22	23.17	30.85	0.918	10
GJRGARCH-GED	43.75	41.98	41.18	36.70	20.63	23.44	32.32	0.974	15

Note: MAE has been multiplied by 10^6.

Appendix -E2. Post-sample volatility performance under RMSE criteria (2005-2011).

Models	Growth Fund Index	Growth & Income Fund Index	Income Fund Index	Balanced Growth Fund Index	Balanced Growth & Income Fund Index	Balanced Income Fund Index	Mixed Asset Growth Fund Index	Mean Theil -U	Rank
RW	97.99	67.55	88.01	176.19	43.95	52.40	75.29	1.229	22
Naïve Variance	84.61	60.36	77.47	133.26	36.55	40.38	61.47	1.017	21
MA30	81.95	56.78	73.29	132.81	34.72	39.81	60.63	0.984	14
EWMA (0.6) RiskMetric	80.63	55.80	71.99	132.80	34.52	39.31	59.90	0.973	4
EWMA Optimized	80.42	55.70	71.78	133.92	34.62	39.27	60.01	0.973	8
STES-SE	80.41	55.69	71.77	133.72	34.62	39.27	59.99	0.973	6
STES-E	80.42	55.70	71.77	134.06	34.61	39.26	60.02	0.973	9
STES-AbsE	80.32	55.65	71.77	132.80	34.62	39.26	59.87	0.971	3
STES E+AbsE	**79.97**	**55.53**	**71.38**	133.41	**34.45**	39.01	59.79	0.969	**1**
STES E+SE	80.40	55.69	71.74	133.91	34.59	39.25	60.00	0.973	5
GARCH-N	81.30	55.93	72.16	133.30	34.80	39.15	60.07	0.976	10
EGARCH-N	80.97	56.11	72.46	131.74	34.75	**38.83**	60.03	0.973	7
IGARCH-N	80.42	55.70	71.86	**131.47**	34.53	39.31	60.07	0.971	2
GJRGARCH-N	81.58	56.05	71.93	133.23	34.77	38.92	60.72	0.977	11
GARCH-t	81.18	63.04	72.27	135.31	35.01	39.21	60.08	0.995	19
EGARCH-t	81.13	63.18	72.63	131.82	35.07	38.97	59.68	0.990	16
IGARCH-t	80.42	59.81	71.98	133.33	34.54	39.30	59.97	0.982	13
GJRGARCH-t	81.65	64.88	72.06	135.05	35.02	39.02	60.80	1.000	20
GARCH-GED	81.06	62.69	71.94	135.09	34.78	39.13	60.00	0.991	17
EGARCH-GED	80.90	62.56	72.23	131.77	34.80	38.86	**59.54**	0.986	15
IGARCH-GED	80.42	59.81	71.94	131.68	34.54	39.31	59.99	0.980	12
GJRGARCH-GED	81.44	64.35	71.69	134.52	34.76	38.91	60.32	0.995	18

Note: RMSE has been multiplied by 10^6.

Appendix -E3. Post-sample volatility performance under MAE criteria (2012-2019).

Models	Growth Fund Index	Growth & Income Fund Index	Income Fund Index	Balanced Growth Fund Index	Balanced Growth & Income Fund Index	Balanced Income Fund Index	Mixed Asset Growth Fund Index	Mean Theil-U	Rank
RW	31.13	24.46	39.35	19.95	12.16	29.36	43.62	1.238	22
Naïve Variance	24.96	19.60	29.55	16.54	10.02	**20.50**	37.25	0.981	3
MA30	26.34	20.29	31.57	16.77	10.08	24.59	37.17	1.033	18
EWMA RiskMetric	25.79	19.90	31.36	16.56	10.00	23.98	36.63	1.017	15
EWMA Optimized	25.78	19.98	31.26	16.40	9.95	24.14	36.8	1.016	14
STES-SE	25.72	19.94	31.16	16.36	9.94	24.05	36.58	1.013	10
STES-E	25.78	19.98	31.26	16.40	9.95	24.14	36.76	1.016	13
STES-AbsE	24.83	19.29	29.91	15.76	9.75	23.06	34.51	0.975	2
STES E+AbsE	**24.41**	**19.00**	**27.44**	**15.60**	**9.61**	22.51	**33.53**	**0.949**	**1**
STES E+SE	25.72	19.94	31.16	16.36	9.94	24.05	36.58	1.013	9
GARCH-N	25.02	19.28	30.18	16.83	9.92	21.97	36.96	0.992	5
EGARCH-N	24.85	19.16	30.32	18.33	10.02	21.31	37.73	1.003	8
IGARCH-N	25.79	19.90	31.17	16.83	9.99	24.29	37.06	1.022	16
GJRGARCH-N	25.31	19.38	30.23	19.71	9.97	22.34	38.43	1.027	17
GARCH-t	25.17	19.44	30.98	16.65	10.02	21.91	36.46	0.995	6
EGARCH-t	25.11	19.34	30.85	16.59	10.16	21.25	36.41	0.990	4
IGARCH-t	25.82	19.92	31.26	16.18	10.00	23.97	36.95	1.015	12
GJRGARCH-t	25.43	19.59	31.22	16.86	10.02	21.76	36.66	1.000	7
GARCH-GED	25.09	19.27	30.28	37.92	9.91	21.61	35.50	1.163	20
EGARCH-GED	24.98	19.19	30.29	36.27	10.05	21.01	35.68	1.147	19
IGARCH-GED	25.80	19.91	31.26	16.13	9.97	24.02	36.94	1.014	11
GJRGARCH-GED	25.38	19.42	30.39	39.91	9.92	21.51	36.40	1.186	21

Note: MAE has been multiplied by 10^6.

Appendix -E4. Post-sample volatility performance under RMSE criteria (2012-2019).

Models	Growth Fund Index	Growth & Income Fund Index	Income Fund Index	Balanced Growth Fund Index	Balanced Growth & Income Fund Index	Balanced Income Fund Index	Mixed Asset Growth Fund Index	Mean Theil-U	Rank
RW	68.55	53.51	84.71	60.32	24.32	77.25	99.03	1.319	22
Naïve Variance	53.92	41.92	62.84	43.40	18.77	62.53	76.89	1.015	17
MA30	54.02	41.95	62.83	43.93	18.77	61.60	76.71	1.014	16
EWMA RiskMetric	53.27	41.46	62.41	43.37	18.59	60.71	75.89	1.003	11
EWMA Optimized	53.20	41.37	62.18	43.33	18.55	60.76	75.76	1.001	8
STES-SE	53.19	41.36	62.17	43.31	18.55	60.73	75.73	1.001	7
STES-E	53.20	41.37	62.18	43.33	18.55	60.76	75.76	1.001	9
STES-AbsE	53.13	41.33	62.14	**43.19**	18.54	60.57	75.61	0.999	3
STES E+AbsE	53.09	41.34	62.22	43.30	18.56	60.38	75.68	1.000	4
STES E+SE	53.19	41.36	62.17	43.31	18.55	60.73	75.73	1.001	6
GARCH-N	52.97	41.24	62.21	43.50	18.68	61.23	76.12	1.003	14
EGARCH-N	**52.27**	**40.72**	**61.56**	45.41	18.53	60.77	75.98	1.002	10
IGARCH-N	53.28	41.49	62.46	43.50	18.62	61.26	76.47	1.006	15
GJRGARCH-N	52.91	41.07	61.98	48.24	18.69	62.08	76.52	1.020	18
GARCH-t	53.02	41.27	62.45	43.49	18.53	60.27	75.36	0.999	2
EGARCH-t	52.39	40.79	61.71	43.37	18.41	**59.72**	**75.13**	**0.991**	**1**
IGARCH-t	53.38	41.57	62.17	43.44	18.59	60.72	76.00	1.003	13
GJRGARCH-t	52.95	41.20	62.58	43.94	18.48	59.90	75.74	1.000	5
GARCH-GED	53.01	41.24	62.34	63.89	18.51	60.24	75.28	1.065	20
EGARCH-GED	52.35	40.76	61.64	61.33	**18.38**	59.76	75.22	1.049	19
IGARCH-GED	53.34	41.53	62.20	43.48	18.57	60.71	76.08	1.003	12
GJRGARCH-GED	52.96	41.14	62.25	74.68	18.46	59.90	75.99	1.099	21

Note: RMSE has been multiplied by 10^6.