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INTEGRATING ZAKAT AND PURIFICATION IN A MODIFIED BLACK–LITTERMAN MODEL FOR SHARIAH-COMPLIANT STOCK PORTFOLIOS

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ABSTRACT

Many articles demonstrate the advantages of applying Shariah principles in economic and financial activities, which motivates model development for Shariah investment. The development of the capital asset pricing model (CAPM) and mean-variance (MV) for Shariah portfolios shows various efforts within a Shariah framework. But the MV model has diversification weaknesses, and the Black–Litterman (BL) model solves this problem. According to Islamic principles, the CAPM's use in the BL model is problematic due to the risk-free rate. This study aims to introduce a modified BL model adapted to Islamic principles. We propose a modified BL model with a Shariah-compliant asset pricing model (SCAPM) that considers the *zakat* obligation and purification rate to develop the model with Islamic principles. The proposed model was implemented in the Indonesian stock market. A portfolio's Sharpe ratio using the BL-SCAPM outperforms BL-CAPM. This finding supports the idea that the modified BL model can be applied as a new alternative for a Sharia portfolio model.

Contribution/Originality: This study introduces a new procedure that modifies the Black–Litterman model for Shariah-compliant stock portfolio optimization. The empirical results show that considering the *zakat* calculation and applying the purification factor in the specific model can provide good diversification for investors and better performance. This research enriches portfolio strategy and management in Islamic economics and finance.

1. INTRODUCTION

In the late 80s and early 90s, oil refineries attracted many investors from the Islamic community (Abdalloh, 2020). Middle Eastern Muslims possessed abundant wealth, but the global market was overfunded. The Shariah capital market began to attract more attention, triggered by many excess funds from Middle Eastern countries. Then, some Shariah investments emerged as products that accommodated the surplus funds. Today, one can choose from many Shariah investment types. Hassan, Aliyu, Paltrinieri, & Khan (2019) provide an overview of their research on investments, such as mutual funds, sukuk, and equity. However, *zakat* and sadaqah must be encouraged in the stock market. *Zakat* is obligatory for Muslims (as written in the Holy Qur'an). Paying *zakat* is believed to be a way of receiving God's mercy, and the wealthy must assist the poor. The development of the literature studying Islamic principles in finance and management has increased in the last few decades.

One popular trading activity is stock investment, which is permitted if one adheres to the Shariah principles. Production and trade that generate profits also have a *zakat* obligation. But not all Muslims understand the procedures considered in stock investment (including the *zakat* calculation). Rahman (2015) provides an overview of the computation of the obligation to impose *zakat* on stock investments. *Zakat* is considered an annual Islamic tax with the underlying provisions associated with Shariah principles (Alam, Akbar, Shahriar, & Elahi, 2017). Hazny, Hasim, & Yusof (2012) introduced the study of *zakat* in the portfolio model, illustrating the development of the mean-variance (MV) method with *zakat* reduction and purification.

By adhering to the principle of not allowing short sales in the model, MV analysis remains widely used to determine the optimal allocation of stock portfolios. Sandwick & Collazzo (2021) reviewed MV for the Islamic asset portfolio and compared the portfolio growth results of Islamic and non-Islamic assets. Their results showed that the increase in Islamic asset portfolios' value exceeded that of other portfolios. Trichilli, Abbes, & Masmoudi (2020) examined the Markowitz mean-variance and Bayesian inferences for optimal portfolio formation. Their results stated the possibility that market sentiment affects Islamic and conventional portfolios. Market conditions (such as a bearish and bullish) also impact performance. So, the portfolio's efficiency frontier (using Bayesian analysis) outperforms the MV method through the market information.

But the MV model has weaknesses, such as diversification results that tend to be extreme. It is considered inferior in distributing assets in one basket. Black & Litterman (1992) solved this weakness by proposing a model known as the Black–Litterman (BL) model. The BL model's framework includes external factors from investor predictions and CAPM return starting points. The new expected returns from a combination of CAPM and investor predictions are included as a new portfolio target using the MV method. For more details on connecting both components, refer to Walters (2014). Several analysis methods have been proposed for the BL formula, such as Bayesian analysis (Kolm & Ritter, 2017; Satchell & Scowcroft, 2007), mixed estimation (Fabozzi, Focardi, & Kolm, 2006; He. & Litterman, 2002), and sampling theory (Mankert & Seiler, 2011).

Many studies also support the BL model's validity. Bessler, Opfer, & Wolff (2017) compared the outperformance of several models at once and showed that the BL model produced a higher Sharpe ratio than the MV model. Similar studies conducted by Allaj (2013) and Duqi, Franci, & Torluccio (2014) compared BL performance with several other models. Their efforts to propose BL as the optimal strategy were also executed for a multi-asset portfolio during a financial crisis by Bayram, Abdullah, & Meera (2018). The BL model's advantageousness encouraged many other authors to implement the model for Shariah stock portfolios. Dewandaru, Masih, Bacha, & Masih (2015) applied the augmented BL model with the application to the Islamic stock market. Widodo, Achsani, & Andati (2017) used the BL model for the Islamic stock market in Indonesia and substituted the risk-free rate in the CAPM with Sertifikat Bank Indonesia Syariah (SBIS). They used the ARIMA-ARCH model as an investor's prediction. The advantage of using the BL model for a portfolio is the main focus of their study. Their research indirectly showed that the use of the CAPM in the BL model must be adjusted for a portfolio with Shariah-compliant assets. The developing BL model (focused on a starting point) is one way to extend it (Subekti, Abdurakhman, & Rosadi, 2021a). Nocera (2016) modified the formula for BL returns with a new equilibrium return. Subekti, Abdurakhman, & Rosadi (2021b) also demonstrated the BL model's portfolio application using a different initial return. Subekti, Abdurakhman, & Rosadi (2022) proposed the modified BL method with a SCAPM that used inflation. They found that the Shariah version of the BL model performed better, which led to a study for developing the BL model with a new equilibrium return complying with Shariah principles.

Several SCAPMs (proposed by multiple authors) show various efforts to improve the conventional model to follow Shariah principles. CAPM adjustment with Shariah principles has been introduced and summarized by Subekti, Abdurakhman, & Rosadi (2020). There are five SCAPM types: SCAPM-1 (which uses CAPM without the interest rate), SCAPM-2 (which uses the *zakat* rate instead of the interest rate), SCAPM-3 (which uses national gross domestic product [NGDP]), SCAPM-4 (which uses the inflation rate), and SCAPM-5 (which uses the sukuk rate). Derbali, El Khaldi, & Jouini (2017) and Hazny, Hasim, & Yusof (2017) proposed a developed SCAPM that uses sukuk and

includes *zakat* reduction and purification. *Zakat* reduction indicates every Muslim's obligation to cleanse his property. Wealth cleansing through *zakat* is also sometimes referred to as purification. In Shariah-compliant stocks, a purification term purifies income that is not considered halal. CAPM points adjusted to Shariah principles should be used in applying Shariah-compliant stock portfolio formation.

This research fills the gap in the literature regarding applying the conventional BL model to Shariah portfolios by offering a new BL model with the SCAPM. The proposed BL model in this study used the SCAPM involving *zakat* and purification (abbreviated as BL-SZP).

Following this section, a literature review is presented in Section 2; Section 3 provides the methodology and the proposed Black–Litterman model under Islamic principles, such as the SCAPM, *zakat*, and purification; the empirical results are provided in Section 4, and the conclusion is summarized in Section 5.

2. LITERATURE REVIEW

2.1. Zakat and Purification in Shariah-Compliant Stock

Capital is essential to Islamic finance, but according to Islamic principles, capital cannot be separated from the obligation of *zakat*. Capital can be personal or company wealth. Islamic finance aims to reduce poverty and promote sustained growth and full employment (Krichene, 2013). Individual wealth differences can result in a social gap between the rich and poor. *Zakat* is an obligation for Muslims to sacrifice their wealth to others who are entitled to receive *zakat*. So, a principle of the *zakat* obligation in Islam is related to social justice.

*Zakat (*one of Islam's pillars) comprises *zakat* fitrah and maal. Many verses in the Holy Qur'an contain the *zakat* command. Some are noted in *At-Taubah: 103, Al-Baqarah: 267, An-Nur: 56, and Adz-Zaariyat: 19*. In these verses, Allah asserts that *zakat* is beneficial to cleanse and purify the community to foster peace of mind. In discussing the investment world, profit from transactions is one form of property where *zakat* must be issued. This *zakat* is a form of *zakat* maal.

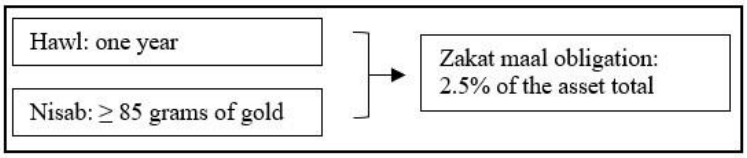

Figure 1. The requirements of Zakat Maal.

Zakat calculation must be considered by investors for short-term and long-term investments (Rahman, 2015). Consistent with the increasing number of Muslim investors, especially in Indonesia's Shariah-compliant stock market, *zakat* should be encouraged in stock market transactions. In Islam, Shariah principles exist for Muslim investors. Shariah investment products represent the Shariah capital market in the form of mutual funds to accommodate these investment funds. The Islamic stock market has attracted attention since the appearance of adjustments to the Dow Jones Islamic stock index (DJII) for stocks categorized as compatible with Shariah principles. Even the Islamic index trend shows aslightly above-average performance. An asset's selection process includes many rules in the Shariah category (Hassan et al., 2019). One characteristic of the Islamic economy and finance is the principle of justice. How does *zakat* become something that God has ordained for humanity? As one of Islam's fundamental pillars, *zakat* should be a concern, considering its vital role in the community. Several opinions exist about *zakat* provisions in stock investment. First, in Al-Qardawi's opinion, a *zakat* rate of 2.5% should be imposed on shares and profits. But if converted to the Gregorian calendar, the *zakat* rate is 2.58%. *Zakat* can be calculated from the total assets that have reached nisab (the minimum amount that a Muslim must have before being obliged to give zakat). The stock investment equals trading activities, where nisab *zakat* maal is worth 85 grams of gold, with a *zakat* rate of 2.5%. Besides the nisab rule, the *zakat* obligation applies when the portfolio has reached one year (hawl) (see Figure 1).

Rahman (2015) and Alam et al. (2017) reviewed the *zakat* calculation on stock investments and concluded that 2.5% is the calculation of *zakat* based on the Hijri calendar. Second, the share profits are subject to *zakat*, according to the scholars' agreement at the first International Muktamar about *zakat* in Kuwait on 29 Rajab 1404 (Islamic calendar) or April 30, 1984. *Zakat* on stock investments can be considered from the profit only. The *zakat* can be calculated as 5% or 10%. In this case, the profit from stock investments resembles that from a crop (Rahman, 2015).

The other essential term in Shariah-based investment activities is *purification* (distinguished from *zakat)*. This purification of business processes is a vital pillar (Albaity, 2011). Purification vocabulary originated from the idea that the income earned from trading on shares is based on calculating the profits. Purification is a prudent step in making a profit from trade or investment. Several explanations for the term purification exist. Based on the study by Hazny et al. (2017), this purification is defined separately from *zakat* because it is calculated as a follow-up step to avoid accepting non-halal actions.

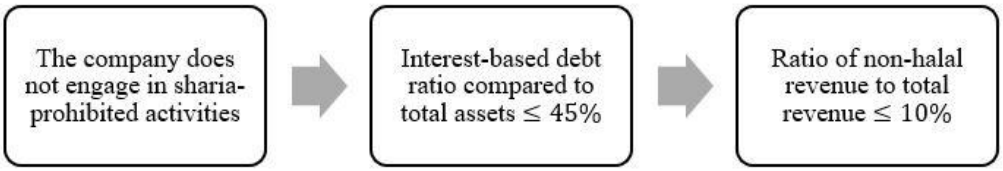

Figure 1. The selection process to become listed as a Shariah-compliant stock.

In Indonesia, the purification process is included in selecting Shariah-compliant stocks. The MUI National Sharia Board issued this rule concerning the categorization of Shariah-compliant stocks in Indeks Saham Syariah Indonesia (ISSI). As shown in Figure 2, the ratio of non-halal revenue to total revenue is equal to, or less than, 10%.

A Shariah investor must understand that capital market Shariah-compliant stock transactions are long-term investments. The long-term duration is influenced by substantial capital (Abdalloh, 2020). The greater the investment capital, the longer investors will hold the portfolio. If the portfolio generates a profit, the investment is a success. Additionally, investors should be aware that a *zakat* obligation exists.

2.2. Asset Pricing and Portfolio Modeling

The development of theoretical and applied models with Shariah principles in the economic and financial fields is increasing. Studies focusing on theory means more modeling. Most of the development in the application demonstrates that the traditional model is implemented on assets subject to Shariah principles. Studies on model development in Islamic investments have also been carried out by Hassan, Aliyu, Saiti, & Halim (2020) and Abdullahi (2018). This current study provides related works on developing models for asset valuation or portfolio construction with Shariah principles. The summary models are presented in Table 1.

The adjusted CAPMs on the risk-free rate components (SCAPM-1 to SCAPM-5) are summarized and applied with Indonesian stock market data (Subekti et al., 2020). Compared to the other four models, the model with the sukuk rate is better as a substitute for the risk-free rate in the CAPM under Islamic principles. Sukuk's replacement of the risk-free rate in the developed CAPM must be revisited because of *zakat* and purification in stock investment. Considering this reduction reminds investors of their *zakat* obligation and meticulously calculates a company's income underlying the assets.

Markowitz's MV model is the pioneer of modern portfolio theory in portfolio optimization. This MV model is modified by incorporating two Islamic principles: the obligation to issue *zakat*, and the concept of purification for Shariah shares (Hazny et al., 2012). Shariah-compliant stock portfolio modeling is done by adding the expected return on assets exceeding the *zakat* rate and using goal programming to accommodate several goals simultaneously (Masri, 2018). Consistent with developing MV by considering reducing *zakat* and purification, the SCAPM return was used

to construct a portfolio with Shariah-compliant stock in Malaysia (Hazny et al., 2017).

Table 1. Modeling related to the Shariah-compliant portfolio.

Models	Variables	Authors
SCAPM-1	Market returns only	Cyril & Karim (1987)
SCAPM-2	Market and *zakat*	Ashker (1987)
SCAPM-3	Market and NGDP	Shaikh (2009)
SCAPM-4	Market and inflation rate	Hanif (2011)
SCAPM-5	Market and sukuk rate	Hakim, Hamid, & Mydin Meera (2016)
SCAPM-6	Market, sukuk rate, *zakat*, and purification	Hazny et al. (2017); Derbali et al. (2017)
MV-modified 1	MV with *zakat* and purification	Hazny et al. (2012)
MV-modified 2	Multiple objective stochastic programs, additional constraint *zakat* selection	Masri (2018)
BL with a case study on the Islamic stock market	ARIMA-ARCH, BI Shariah certificate rate, Jakarta Islamic Index	Widodo et al. (2017)
The Augmented BL	Momentum, value, quality investing, Dow Jones Islamic Index	Dewandaru et al. (2015)
BL with SCAPM inflation	SCAPM, inflation rate as a substitute for the risk-free rate	Subekti et al. (2022)

However, the MV model has weaknesses due to using the mean of historical data as a parameter in portfolio calculations. The BL model, which uses the CAPM's expected return as the target return held by most investors, overcomes this problem. The BL model also combines the CAPM with subjective predictions from investors. Many studies confirm the advantages of the BL model by comparing it to other methods (Allaj, 2020; Bessler et al., 2017; Duqi et al., 2014). This model's advantages are used in asset allocation. Dewandaru et al. (2015) employed Islamic stocks (such as the Dow Jones Islamic Index) in stock portfolios using an augmented BL model from Cheung (2013). They used momentum, value, and quality investing for the portfolio. Widodo et al. (2017) implemented the BL model using predictions from the ARIMA-ARCH model on stocks listed on the Jakarta Islamic Index utilizing the BI Shariah certificate rate as a substitute for the interest rate in the conventional CAPM.

Trichilli et al. (2020) analyzed the MV and Bayesian models on portfolios with Islamic stocks, using investor sentiment as information for the Bayesian approach. The study results state the advantages of MV's diversification with conventional stocks over Islamic stocks when the market is depressed and bullish. Diversification is better in Islamic markets than traditional markets when using the Bayesian method. In a calm market, a portfolio with an Islamic index is more profitable when using MV. But the opposite is true when using Bayesian modeling. Hooi & Parsva (2012) suggested that the Shariah market would attract more Muslim investors. They examined the Islamic indices in Malaysia during and after the global financial crisis in 2008 to show that the risk was high but tolerable. Various approaches to obtaining BL returns exist, such as Theil's mixed estimation, Bayesian, and sampling theory (Walters, 2014). Black & Litterman (1992) used Theil's mixed estimation approach and provided Bayes' theorem. However, the explanation was not provided in detail, so the BL development emerged through the description of the method to obtain the formula for the expected BL return. Kolm & Ritter (2017) explained the description of the Bayesian method in the Black–Litterman model. The process toward the expected BL return starts from the return equilibrium (identical to the CAPM). But in Black and Litterman's original article, they did not state that the CAPM condition was equilibrium. Investor predictions are vital in preparing a portfolio related to assets. Combining the two returns results in a new target expectation. Several open problems have led to other studies developing the BL model, from its elaboration to its application (Subekti et al., 2021a). For example, Nocera (2016) proposed a modification of the BL model using the three-stage dividend discount model (DDM) instead of the CAPM. Krishnan & Mains (2005) developed the BL model using a multifactor CAPM. Subekti et al. (2022) used the SCAPM as a new starting point to adjust the BL strategy because it better suits portfolios containing Shariah-compliant stocks. The SCAPM uses the

inflation rate instead of the interest rate in the CAPM for the BL model. Their result is that the BL model with the SCAPM outperformed BL model with the CAPM.

So, these studies motivate a new development of the BL model by utilizing a unique SCAPM for the Shariah portfolio. We develop a modifying BL model with a new SCAPM for Shariah-compliant stocks regarding zakat and purification.

3. RESEARCH METHODOLOGIES AND DATA

3.1. Research Methodologies

The Black–Litterman model combines the CAPM return with investor predictions. We established the portfolio using two strategies to demonstrate the proposed model and offer a new Black–Litterman model with *zakat*, and purification reductions as a model for Shariah-compliant stock portfolios. This section briefly describes the stages in the BL-CAPM and the proposed BL-SZP model. The experimental results with Shariah-compliant stock data were taken from the Indonesian stock market, specifically the Jakarta Islamic Index (JII) 70.

3.2. Research Model

The development of the Black–Litterman model in the 90s provided critical solutions because the MV model is sensitive. The range of the historical means used in its calculation indicates the classical MV model's problem (Black & Litterman, 1992). Suppose a portfolio has *n* assets and assume that $r \sim N(\mu, \Sigma)$. The return on assets is normally distributed with a vector of expected return, μ, and a covariance matrix, Σ. The expected return and portfolio variances are calculated in Equations 1 and 2, respectively, as follows:

$$E(R_p) = \sum_{i=1}^{n} w_i \; E(R_i) \tag{1}$$

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j \neq i}^{n} w_i w_j \sigma_{ij} \tag{2}$$

Where *n* is the number of assets in a portfolio and w_i is the weight allocation for asset *i*.

The MV problem can be expressed in matrix form, such as Equation 3 that presents the objective function, and Equation 4 as the constraint.

$$max_w w^t \mu - \frac{\lambda}{2} w^t \Sigma w \tag{3}$$

$$\text{subject to } w^t 1 = 1 \tag{4}$$

Where $w^t = (w_1 \quad w_2 \quad \cdots \quad w_n)$, $\mu^t = (\mu_1 \quad \mu_2 \quad \cdots \quad \mu_n)$, Σ = a matrix of variance-covariance, and $1^t = (1 \quad 1 \quad \cdots \quad 1)$. The investor can set additional constraints, such as the absence of short sales, $w \geq 0$.

The new expected return from the Black–Litterman model derives from a combination of prior estimations from the CAPM (π), and the views vector (Q) is expressed in Equation 5 where the covariance values are $\tau\Sigma$ and Ω, respectively, and P is a pick matrix related to the Q (Subekti et al., 2022).

$$\hat{\mu}_{BL} = [(\tau\Sigma)^{-1} + P^t \Omega^{-1} P]^{-1} [(\tau\Sigma)^{-1} \pi + P^t \Omega^{-1} Q] \tag{5}$$

and

$$\hat{\Sigma}_{BL} = ((\tau\Sigma)^{-1} + P^t \Omega^{-1} P)^{-1} \tag{6}$$

Black–Litterman's portfolio weight can be obtained by maximizing the unrestricted utility function, U, as follows:

$$\max \; U(w) = w^t \bar{\mu} - \frac{\lambda}{2} w^t \bar{\Sigma} w \tag{7}$$

Where the new distribution of returns can be written as $r \sim N(\bar{\mu}, \bar{\Sigma})$, in which $\bar{\mu} = \hat{\mu}_{BL}$ and $\bar{\Sigma} = \Sigma + \hat{\Sigma}_{BL}$ (He & Litterman, 2002).

$$w_{BL} = (\lambda \, (\Sigma + \hat{\Sigma}_{BL}))^{-1} \hat{\mu}_{BL} \tag{8}$$

Therefore, Equation 8 presents the weight of asset in the Black–Litterman model.

3.3. Shariah-Compliant Asset Pricing Model with Zakat and Purification

Various research types dealing with investing according to Islamic principles have emerged. Some countries focus on Shariah financial products. For example, a sukuk appears as an alternative asset to replace the conventional bond and the Shariah capital market. Sukuk and bonds differ in the traditional context (Tiby, El, & Grais, 2015). The CAPM adjustment is continued by involving the sukuk's return rate rather than the interest rate (Hakim et al., 2016). This method is used because, in Islamic principles, no investment is profitable without risk. The CAPM's development in Shariah principles continued, including *zakat* reduction and purification in calculating investment returns (Derbali et al., 2017; Hazny et al., 2017). The proposed model uses the yield level of sukuk assets instead of the risk-free level. Therefore, in this Shariah adjustment model, one must consider the obligation of *zakat* and the existence of purification from the beginning of the expected return calculation.

Two investment reductions are *zakat* and purification. *Zakat* is a tax that becomes an obligation and applies to Muslims if the requirements have been met. *Zakat* purifies Muslims' wealth. But investment activities also require the consideration of the existence of prohibited activities. So, purification is a critical step in receiving profits. The amount of *zakat* to be paid is 2.5% in this study. Purification can be calculated as the percentage of total income that is non-halal. In Indonesia, the asset to be selected in the Indeks Saham Syariah Indonesia (ISSI) or the Islamic Composite Index must be included in the Indonesia Stock Exchange and follow the process illustrated in Figure 2. The term purification in this process is defined separately from *zakat*. The purification process can be seen in Figure 2. The ratio of non-halal revenue to total revenue is less than or equal to 10%. The MUI National Shariah Board issued this rule concerning the categorization of Shariah-compliant stocks. According to Hazny et al. (2012), this purification is denoted by δ_i (Equation 9):

$$\delta_i = \frac{\text{non} - \text{halal income}}{\text{total income}} \tag{9}$$

So, to model the determination of the CAPM under Shariah principles, *zakat* reduction and purification components are assumed for Islamic investors. Starting from the CAPM (with the sukuk as a substitute for the risk-free rate), it is developed further by considering *zakat* reduction and purification (Derbali et al., 2017; Hazny et al., 2017).

The following Lemma 3.1 re-formulate a new return that adjusted for the reduction of zakat and purification. To distinguish the notation of the conventional portfolio and Shariah, the new expected return of the Shariah portfolio is written as $E(R_{ps})$. Two reductions exist here, namely the reduction of *zakat* and the reduction of purification. Each expected return and variance of individual Shariah-compliant stock (i) is reformulated, with adjustments to *zakat* (z) and purification (δ).

Lemma 3.1

Let $\mu = (\mu_1, \cdots, \mu_n)^t$; the mean of $r = (r_1, \cdots, r_n)^t$; and $\sigma_{ij} \epsilon R^{n \times n}$ is the covariance matrix of $(r_1, \cdots, r_n)$. If the adjusted return, r^*, is defined as $r_i^* = (1-z)(1-\delta_i)r_i$, where z is a percentage of *zakat* and δ_i is the purification factor for asset i, then $\mu_s = \left((1-z)(1-\delta_i)\mu_1, \cdots, (1-z)(1-\delta_n)\mu_n\right)^t$ and

$$\Sigma_s = \begin{pmatrix} (1-z)^2(1-\delta_1)^2\sigma_{11} & \cdots & (1-z)^2(1-\delta_1)(1-\delta_n)\sigma_{1n} \\ \vdots & \ddots & \vdots \\ (1-z)^2(1-\delta_1)(1-\delta_n)\sigma_{1n} & \cdots & (1-z)^2(1-\delta_n)^2\sigma_{nn} \end{pmatrix}$$

Proof:

Let $a = (1-z)(1-\delta_i)$ and $r^* = ar$. Considering the property of mean and variance that $\mu_{ax} = a\mu_x$ and $Var(aX) = a^2 Var\,(X)$, the new mean and variance-covariance are:

$$\mu_i{}^* = E(r_i{}^*) = (1-z)(1-\delta_i)\ \mu_i \tag{10}$$

$$\sigma_{ii}{}^* = Var(r_i{}^*) = (1-z)^2(1-\delta_i)^2\sigma_i^2 \tag{11}$$

$$\sigma_{ij}{}^* = Cov(r_i{}^* r_j{}^*) = (1-z)^2(1-\delta_i)(1-\delta_j)\sigma_{ij} \tag{12}$$

$$\mu_s = \begin{pmatrix} (1-z)(1-\delta_1)\mu_1 \\ \vdots \\ (1-z)(1-\delta_n)\mu_n \end{pmatrix}$$

And the variance-covariance matrix is:

$$\Sigma_s = \begin{pmatrix} (1-z)^2(1-\delta_1)^2\sigma_{11} & \cdots & (1-z)^2(1-\delta_1)(1-\delta_n)\sigma_{1n} \\ \vdots & \ddots & \vdots \\ (1-z)^2(1-\delta_1)(1-\delta_n)\sigma_{1n} & \cdots & (1-z)^2(1-\delta_n)^2\sigma_{nn} \end{pmatrix} \blacksquare$$

Suppose that all investors consider the zakat (z) and purification factor (δ_i). A portfolio contains n Shariah-compliant assets with r^*, defined as $r^* = (1-z)(1-\delta_i)r$.

$$\mu_{ps} = \sum_{i=1}^{n} w_i(1-z)(1-\delta_i)\ \mu_i \tag{13}$$

Equation 13 is a new expected return of the portfolio and the portfolio variance is:

$$\sigma_{ps}^2 = \sum_{i=1}^{n} w_i^2(1-z)^2(1-\delta_i)^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i w_j(1-z)^2(1-\delta_i)\big(1-\delta_j\big)\sigma_{ij} \tag{14}$$

In this case, lending and borrowing is allowed at the sukuk rate (Hazny et al., 2017). Hence, a new sukuk profit should be reduced by the *zakat* obligation.

$$R_s = \sum_{i=1}^{n} w_i R_s$$

Therefore, the new goal is to derive the SCAPM formula from maximizing a new Sharpe ratio of a portfolio. The constraints are that short selling is prohibited, and the proportion is 100%.

$$\theta_{ps} = \frac{\mu_{ps} - (1-z)R_s}{\sigma_{ps}} \tag{15}$$

Subject to $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0$.

$$\theta_{ps} = \frac{\sum_{i=1}^{n} w_i\ (1-z)(1-\delta_i)\mu_i - (1-z)\sum_{i=1}^{n} w_i R_s}{\left(\sum_{i=1}^{n} w_i^2(1-z)^2(1-\delta_i)^2\sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i w_j(1-z)^2(1-\delta_i)\big(1-\delta_j\big)\sigma_{ij}\right)^{\frac{1}{2}}} \tag{16}$$

By setting the derivative of the Sharpe ratio in Equation 16 to the weight of zero, a new formula for the SCAPM is as follows in Equation 17 (see Derbali et al. (2017) and Hazny et al. (2017)).

$$\mu_i = \frac{1}{1-\delta_i}R_s + \beta_i\left(\mu_M - \frac{R_s}{1-\delta_M}\right) \tag{17}$$

Where δ_M is defined as market purification. The market purification can be obtained through a new definition of the Sharpe ratio for the market return.

$$\theta^* = \frac{(1-z)(1-\delta_M)\mu_M - (1-z)R_s}{\sigma_M} \tag{18}$$

This step generates:

$$\delta_M = 1 - \frac{\theta^*\sigma_M + (1-z)R_s}{(1-z)\mu_M} \tag{19}$$

Equation 18 is an ideal condition defined to derive the market purification (Equation 19). Therefore, it is assumed that θ^* is the observed market price of risk on the capital market line (Derbali et al., 2017; Hazny et al., 2017).

3.4. A New Black–Litterman Model with a SCAPM Incorporating Zakat and Purification

We focus, in this study, on incorporating *zakat* and purification in the Black–Litterman model for Shariah-compliant stock portfolios. The new procedure in the BL model, modified for Shariah-compliant stock portfolios, is as follows:

Step 1: The SCAPM return is utilized as a benchmark for asset returns in the market. According to Lai & Xing (2008) and Ruppert & Matteson (2015), the CAPM can be associated with a regression model. Thus, it is assumed that the SCAPM is viewed from the regression analysis. The SCAPM in regression form can be written as follows:

$$R_{it} - \frac{1}{1-\delta_i} R_{st} = \alpha_i + \beta_i \left(R_{Mt} - \frac{R_{st}}{1-\delta_M} \right) + \varepsilon_{it}. \quad (20)$$

We have a market series (R_{Mt}) and each asset return (R_{it}) in actual data. The sukuk rate, R_{st}, can be a constant profit rate; β_i is the beta of stock i; δ_M is the market purification rate; and ε_{it} is the corresponding error term. We designate a new equilibrium excess return for Shariah-compliant stock, π_s, and the assumption is $\varepsilon_\pi \sim N(0, \tau\Sigma^*)$ so that:

$$\mu_s \sim N(\pi_s, \tau\Sigma^*) \quad (21)$$

where

$$\pi_s = \beta_i \left(\mu_M - \frac{R_s}{1-\delta_M} \right) \quad (22)$$

In the analog of the CAPM, hypothesis H_0: $\alpha = 0$ indicates that the SCAPM is valid. The Black–Litterman model return with adjustments for Shariah-compliant assets is estimated similarly to the original Black–Litterman model.

Step 2: The Black–Litterman model is identical to blending the investor prediction, called a view return. The return prediction expressed quantitatively in this model is not the value of numbers taken arbitrarily. According to Islamic principles, buying and selling are allowed. Naturally, the experts predict the future through their experience in market transactions. In Islamic principles, *gharar* means uncertainty, obscurity, or ambiguity. Consider that choosing portfolio assets is a rational investment process by understanding fundamental analysis. We follow the rule that the expected return will be recalculated considering the *zakat* and purification reduction, like the step-by-step SCAPM process.

In a regular BL model, an investor's prediction of future return is expressed by Q. The Q is the return given by investors based on experience or calculation results using the statistical method. Then, the error of the view return is assumed to be normally distributed, $\varepsilon_Q \sim N(0, \Omega)$. In this model, the prediction of future returns is assumed to be reduced because of the *zakat* principle. So, the new return prediction, Q^*, is subject to reductions in *zakat* (z) and purification (δ). The error is assumed as $\varepsilon_{Q*} \sim N(0, \Omega_s)$, where $\Omega_s = \text{diag}(P\tau\Sigma^* P^t)$. By using Lemma 3.1, a new statement of views is defined by Q^* as follows:

$$Q^* = (1-z)(1-\delta)\mathrm{Q} = P\mu_s + \varepsilon_{Q*} \quad (23)$$

An additional assumption, such as short sales activity, is prohibited in Shariah modeling. So, we add a limitation to the BL model with Shariah principles. We suppose that BL-SZP is limited for absolute views prediction. In this case, relative views have a short sales effect on optimum allocation results.

The following step is the combination process of two sources. For that reason, the SCAPM used in this modified Black–Litterman model can be written as Equation 22. The investor's prediction must also consider the reduction of *zakat* and purification as per Equation 23.

Step 3: Theorem 3.2. (the BL model with the SCAPM-ZP for Shariah portfolios). If the prior is the SCAPM-ZP in Equation 22 and investor views in Equation 23, the posterior distribution of the expected excess return in a new Black–Litterman model with the SCAPM is:

$$\hat{\mu}_{BL-S} = \left((\tau\Sigma^*)^{-1} + P^t {\Omega_s}^{-1} P \right)^{-1} \left((\tau\Sigma^*)^{-1} \pi_S + P^t {\Omega_s}^{-1} (1-z)(1-\delta) Q \right), \quad (24)$$

and the posterior variance is: Σ_{BL}, is $\left((\tau\Sigma^*)^{-1} + P^t {\Omega_s}^{-1} P \right)^{-1}$ where $\Omega_s = \mathrm{P}^t \tau\Sigma^* P$.

Proof:

The combination of a new equilibrium return of the SCAPM as investors' prior and view information is as follows:

$$\begin{pmatrix} \pi_S \\ Q^* \end{pmatrix} = \begin{pmatrix} \mu_s \\ P\mu_s \end{pmatrix} + \begin{pmatrix} \varepsilon_{\pi S} \\ \varepsilon_{Q*} \end{pmatrix} \quad (25)$$

We can rewrite it as:

$$y = \begin{pmatrix} \mathrm{I} \\ P \end{pmatrix} \mu_s + \begin{pmatrix} \tau\Sigma^* & 0 \\ 0 & \Omega_s \end{pmatrix} \quad (26)$$

According to Rao, Toutenburg, Shalabh, & Heumann (2008) using the GLS, we can obtain that $\hat{\mu} = (X^t v^{-1} X)^{-1} X^t v^{-1} y$ where $y = X\mu + \varepsilon$ and

$X = \begin{pmatrix} I \\ P \end{pmatrix}; v = \begin{pmatrix} \tau\Sigma & 0 \\ 0 & \Omega_s \end{pmatrix}; y = \begin{pmatrix} \pi_S \\ Q^* \end{pmatrix} \varepsilon \sim N(0, v)$. Then, the estimator is $\hat{\mu}_{BL-SZP}$,

$$\hat{\mu}_{BL-SZP} = \left((I \quad P) \begin{pmatrix} \tau\Sigma^* & 0 \\ 0 & \Omega_s \end{pmatrix}^{-1} \begin{pmatrix} I \\ P \end{pmatrix} \right)^{-1} \left((I \quad P) \begin{pmatrix} \tau\Sigma^* & 0 \\ 0 & \Omega_s \end{pmatrix}^{-1} \begin{pmatrix} \pi_S \\ Q^* \end{pmatrix} \right)$$

$$= \left(I^t (\tau\Sigma^*)^{-1} I + P^t {\Omega_s}^{-1} P \right)^{-1} \left(I^t (\tau\Sigma^*)^{-1} \pi_S + P^t {\Omega_s}^{-1} Q^* \right)$$

$$\hat{\mu}_{BL-SZP} = \left((\tau\Sigma^*)^{-1} + P^t {\Omega_s}^{-1} P \right)^{-1} \left((\tau\Sigma^*)^{-1} \pi_S + P^t {\Omega_s}^{-1} Q^* \right) \quad (27)$$

and

$$\hat{\Sigma}_{BL-SZP} = (X^t v^{-1} X)^{-1} = \left((\tau\Sigma^*)^{-1} + P^t {\Omega_s}^{-1} P \right)^{-1} \quad (28)$$

Step 4: The same procedure is employed with the MV method to find the optimal weight of the Black–Litterman portfolio. We need to add a limitation in the optimization model since short selling is against Shariah principles. The portfolio variance is the sum of the posterior variance and prior (He & Litterman, 2002). Kolm & Ritter (2017) explained that $\Sigma_{BL} = \Sigma + [(\tau\Sigma)^{-1} + P^t \Omega^{-1} P]^{-1}$, considering that investors interested in wealth variance prefer the unconditional variance form. Therefore, a new objective function (see Equation 29) can be obtained by modifying Equation 7, in which μ_{BL-SZP} is $\left[(\tau\Sigma^*)^{-1} + P^t {\Omega_s}^{-1} P\right]^{-1} \left[(\tau\Sigma^*)^{-1} \pi_s + P^t {\Omega_s}^{-1} Q^*\right]$, and Σ_{BL-SZP} is a sum of Σ^* and $\left[(\tau\Sigma^*)^{-1} + P^t {\Omega_s}^{-1} P\right]^{-1}$ with the assumption that no short-sales have occured (Equation 32).

$$Min \ w^t \Sigma_{BL-SZP} w \quad (29)$$

$$subject \ to \ w^t \mu_{BL-SZP} = \mu_p \quad (30)$$

$$w^t 1 = 1 \quad (31)$$

$$w \geq 0 \quad (32)$$

3.5. Data

The portfolio sample is constructed based on the Jakarta Islamic Index (JII) 70 list issued in June 2019. We can choose Shariah-compliant stock from the ISSI that contains 423 stocks. We selected JII 70, which has 70 liquid Shariah-compliant stocks. We collected the historical monthly returns for ten years and chose complete data from June 2009 to 2019. Due to the availability, only 42 stocks from 70 have comprehensive data in the sample period. We found 12 stocks with normal distribution (see Table 2), but we limit the portfolio to only six assets. The six Shariah-compliant stocks are selected from different sectors in this portfolio, and the chosen assets are ADRO, ASII, CTRA, LSIP, SMGR, and UNTR (see Table 3 for details).

Table 2. Jarque–Bera (JB) and Augmented Dickey–Fuller (ADF) test.

No.	Stock	Company	Sector	JB test	ADF test
1.	AALI	Astra Agro Lestari Tbk	Agriculture	6.0918 (0.0475)	-5.3028 (0.0100)
2.	ADHI	Adhi Karya (Persero) Tbk	Property & Construction	3.2618 (0.1958)	-4.9976 (0.0100)
3.	ADRO	Adaro Energy Tbk	Mining	5.6487 (0.0593)	-4.5960 (0.0100)
4.	ASII	Astra International Tbk	Misc. Industries	3.5778 (0.1671)	-5.8971 (0.0100)
5.	CTRA	Ciputra Development Tbk	Property & Construction	5.2161 (0.0736)	-6.1205 (0.0100)
6.	INTP	Indocement Tunggal Prakarsa Tbk	Chemical Industry	0.5657 (0.7536)	-5.2411 (0.0100)
7.	LPKR	Lippo Karawaci Tbk	Property & Construction	2.8888 (0.2359)	-5.5120 (0.0100)
8.	LSIP	PP London Sumatra Indonesia Tbk	Agriculture	3.7944 (0.1500)	-5.1649 (0.0100)

No.	Stock	Company	Sector	JB test	ADF test
9.	PTBA	Tambang Batubara Bukit Asam Tbk	Mining	1.9128 (0.3843)	-4.0032 (0.0100)
10.	SMGR	Semen Indonesia (Persero) Tbk	Chemical Industry	1.3184 (0.5173)	-5.3073 (0.0100)
11.	UNTR	United Tractors Tbk	Trade, Service, & Investment	1.1167 (0.5721)	-5.5476 (0.0100)
12.	WIKA	Wijaya Karya (Persero) Tbk	Property & Construction	2.6504 (0.2657)	-4.7821 (0.0100)

Note: The p-values are in parentheses.

The Jarque–Bera test and stationarity test results can be seen in Table 2. The augmented Dickey–Fuller (ADF) value in the fifth column in Table 2 indicates that all assets are stationary.

Table 3. Descriptive statistics.

Stock	Sector	Min.	Median	Mean	Max.	Std. Dev
ADRO	Mining	-0.2620	0.0057	0.0082	0.4117	0.1213
ASII	Miscellaneous industries	-0.1492	0.0189	0.0146	0.2800	0.0798
LSIP	Agriculture	-0.3410	0.0000	0.0071	0.3508	0.1204
CTRA	Property and construction	-0.2631	0.0205	0.0190	0.4117	0.1285
SMGR	Chemical industry	-0.2140	0.0151	0.0115	0.2540	0.0835
UNTR	Trade, service, and investment	-0.2203	0.0173	0.0124	0.2663	0.0886

Table 3 reports the descriptive statistics for six assets. All assets have positive means and medians of around 1%, the standard deviations range from 7% to 12%, while ADRO, LSIP, and CTRA have greater volatility than the others. The market return is represented by the Jakarta Composite Index, which has a mean return of 1.15% and a standard deviation of 4.53%. The sukuk rate is calculated for monthly returns as 0.56%, and the risk-free rate is 0.51%, on average.

4. EMPIRICAL RESULTS AND ANALYSIS

4.1. Results

Step 1: CAPM and SCAPM Calculation

We present the calculation of both CAPM and SCAPM as a starting return point for BL. The results of beta CAPM and beta SCAPM can be seen in Tables 4 and 5. The details of the significance test for alpha and beta can be seen in Columns 4 and 6 in Table 4. Toward SCAPM, we calculate the delta purification for each asset based on the annual financial report. Worldwide, one condition for a share to be categorized into Shariah-compliant stock groups in Indonesia is that the purification level is less than 10%. Accounting experts assist in calculating individual purification of assets based on annual financial statements. The expected return of SCAPM in Table 5 resembles that of CAPM in Table 4. SCAPM's value is slightly below CAPM's expected return. This result is consistent with Hazny. et al. (2017) finding that *zakat* and purification reduce CAPM's expected return.

Table 4. Alpha, beta, and the excess return CAPM.

No.	Stock	Alpha	Pval(Alpha)	Beta	Pval(Beta)	CAPM
1	ADRO	-0.0024	0.8160	0.8594	3.38E-04	0.0053
2	ASII	0.0009	0.8487	1.3536	9.49E-25	0.0083
3	LSIP	-0.0020	0.8491	0.6261	9.38E-03	0.0038
4	CTRA	0.0044	0.6627	1.4976	5.32E-10	0.0092
5	SMGR	-0.0009	0.8813	1.1413	4.30E-14	0.0070
6	UNTR	0.0010	0.8776	0.9677	8.49E-09	0.0059

Step 2: The input of a return prediction reflects the investor's prediction based on the asset observation. One of the best ways to understand a company's condition is to use a fundamental analysis. The investment decision-making process is based on a fundamental analysis of macroeconomic conditions, the entire industry, and related companies' performance. Risks in stock investments can be anticipated under the "buy what you understand and understand what you buy" principle. Investors' return predictions can be determined from several considerations simultaneously, such as technical and fundamental analyses. The moving average (MA) method is often a simple consideration, but it can help to see the movement of stocks in the future. Based on financial statements, investors can also provide advice on which assets will perform well in the future. We chose to use MA 10 and discuss it with practitioners.

Table 5. Purification, p-values of alpha, beta, and SCAPM returns.

No.	Stock	δ_i	$\frac{R_s}{1-\delta_i}$	P-value (Alpha)	Beta SCAPM	P-value (Beta)	π_S
1	ADRO	0.0065	0.0056	0.8013	0.8606	3.25E-04	0.0052
2	ASII	0.0077	0.0056	0.8751	1.3558	7.22E-25	0.0082
3	LSIP	0.0154	0.0057	0.8306	0.6293	8.94E-03	0.0090
4	CTRA	0.0328	0.0058	0.6823	1.4968	5.14E-10	0.0038
5	SMGR	0.0059	0.0056	0.8614	1.1404	4.17E-14	0.0069
6	UNTR	0.0090	0.0056	0.9003	0.9693	7.72E-09	0.0058

These results were discussed with investors as practitioners of Shariah divisions in Indonesia. Each asset's purification was determined from stocks' financial statements in the portfolio. Then, three assets were selected, namely, ADRO, LSIP, and SMGR, for prediction with statistical tools.

ADRO's reported operating income increased 11% annually to US$846 million in the first quarter of 2019. This achievement was consistent with the annual production growth and a positive return for the next period. LSIP is a company engaged in agribusiness, such as palm oil plantations. The government policy on expanding palm oil land has been issued.

Palm oil is a raw material that is increasingly in demand. Analysts predict that it affects the increase of LSIP profits. SMGR's performance (PT Semen Indonesia (Persero) Tbk) until May 2019 was still not positive, but throughout the year, Semen Indonesia Group's sales volume increased. The consolidated sales volume of Semen Indonesia Group increased by 17.05% to 15.16 million tons from the previous year's volume of 12.95 million tons.

The predicted statistical methods and collaboration discussions were used with investment managers for academic purposes. The moving averages (MA) for ADRO, LSIP, and SMGR are 2.8%, 7.4%, and 2.1%, respectively. Formally, these predictions are stated in the Black–Litterman procedure as follows:

- View 1: Stock ADRO is predicted to yield an expected return of 2.8%.
- View 2: Stock LSIP is predicted to generate an expected return of 7.4%.
- View 3: Stock SMGR is predicted to yield an expected return of 2.1%.

Furthermore, matrices Q^* and Ω_s (we use $\tau = 0.1$) are as follows:

$$Q^* = \begin{bmatrix} 0.0271 \\ 0.0710 \\ 0.0203 \end{bmatrix}, \Omega_s = \begin{bmatrix} 0.0013 & 0.0000 & 0.0000 \\ 0.0000 & 0.0013 & 0.0000 \\ 0.0000 & 0.0000 & 0.0006 \end{bmatrix}$$

Step 3: The expected returns of the BL model with CAPM and SCAPM are calculated using Equation 5 and Equation 27, respectively. The results are presented in Table 6, and the results of both expectations appear almost the same. The return of BL-SZP expectations is only slightly below that of BL-CAPM. The *zakat* reduction and purification influence this difference.

Table 6. The expected returns of BL-CAPM and BL-SZP.

No.	Stock	$\hat{\mu}_{BL}$	$\hat{\mu}_{BL-SZP}$
1.	ADRO	0.0243	0.0236
2.	ASII	0.0173	0.0168
3.	LSIP	0.0402	0.0387
4.	CTRA	0.0128	0.0124
5.	SMGR	0.0162	0.0158
6.	UNTR	0.0157	0.0153

Step 4: Portfolio optimization using BL utilizes the mean-variance method by updating its target expectations. As mentioned, short selling is prohibited. We can consider using the quadprog package in the R program. The following weight is positive for each asset in the portfolio. Using BL-SZP, the weight allocation obtained from the BL-SZP in Figure 3 resembles that obtained from the BL-CAPM. But the BL-SZP process is modeled with Shariah principles. The result of asset allocation does not contain negative weighting, so the constraints have been met without short selling. The allocation weights for LSIP, SMGR, and ADRO are corrected due to the views.

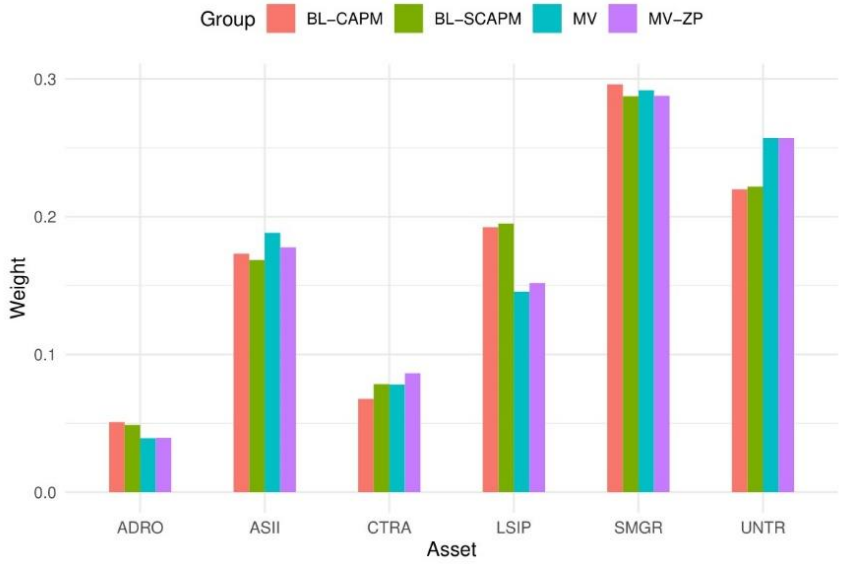

Figure 3. The weight allocations of MV, MV-ZP, BL, and BL-SZP.

Figure 3 shows the distribution of MV, BL-CAPM, and BL-SZP. Compared to the MV results without short selling, MV and BL weighting is dominated by SMGR. There is a correction for the proportion of UNTR and LSIP with the BL model. The difference in weight correction of BL and BL-SZP is quite small. However, this can cause differences in the allocation of capital to certain assets when the investment capital is large.

4.2. Portfolio Performance

We also demonstrate the portfolio results with the MV method as a consideration. Table 7 presents the comparison of the portfolio's expected return, risk, and Sharpe ratio.

Table 6. Portfolio performance results of MV, BL, and BL-SZP.

Portfolio	Expected Return	Risk	SR
MV	0.0121	0.0041	2.9683
MV-ZP	0.0117	0.0038	3.0746
BL-CAPM	0.0211	0.0043	3.7175
BL-SZP	0.0204	0.0040	3.8281

A portfolio using Black–Litterman analysis generates a higher Sharpe ratio than the MV model. This result is consistent with Bessler et al. (2017), who found that the BL model outperformed the MV model. The Shariah version of the BL model compared with the conventional model showed that the Sharpe ratio of the BL model with the integration of zakat and purification outperformed the BL-CAPM. Incorporating purification and zakat in the model reduces each asset's modified expected return and variance. Then the Sharpe ratio of the portfolio increases. This finding shows that despite changing the BL benchmark, it produces a better portfolio in diversifying assets and portfolio performance. The expected return and portfolio risks have changed from the traditional to the Shariah version with zakat reduction and purification. Thus, it embeds a new Shariah-compliant strategy.

5. CONCLUSIONS

Recently, confidence has increased in the Islamic economic and financial system nearly worldwide, consistent with the development of relevant research and the investment world of Shariah finance. But the literature regarding Shariah modeling for portfolios remains insufficient. The Shariah system's presence in the investment world is expected to accommodate Muslim investors. Some studies explain the advantages of the Black–Litterman model for portfolio optimization. But most portfolio construction for Shariah-compliant stock with the Black–Litterman model still uses the CAPM as an initial benchmark. This research is part of the development of Shariah compliance for investment. The idea of Shariah principles seems to be considered unfavorable. Many new terms are not well understood, and one such term is riba (usury). Usury is an exaggeration or addition prohibited in Islam, such as bank interest. The other related terms are *zakat* from income and purification in stock markets with Shariah principles.

This study proposes integrating the Black–Litterman model with Shariah principles by considering the *zakat* and purification concepts. The procedure and its application contribute to Islamic finance literature. *Zakat* can also be interpreted as a form of purification. The term purification also exists in business but is distinguished from the obligation of *zakat*. The term purification is identical to the cleansing process of returns from prohibited activities, such as interest rates. The SCAPM involving sukuk, *zakat*, and purification inspires portfolio construction with the BL process and Shariah framework. The new BL model is expected to produce results resembling the BL-CAPM but starts with a process that integrates Islamic principles.

The result shows a slight difference in allocation between the BL and BL-SZP models due to the reduction of *zakat* and purification. This research is expected to provide new discourse in the actualization of Islamic finance and business by adding illustrations of the *zakat* calculation and applying purification in the model. In future research, more sample experiments can supplement this model's development and are expected to strengthen the model offered to obtain an optimal portfolio and follow Shariah principles.

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REFERENCES

Abdalloh, I. (2020). *Islamic Capital market (A. Mamoedi (ed.)* (3rd ed.). Jakarta: Elex Media Komputindo.

Abdullahi, S. I. (2018). Contribution of mathematical models to Islamic economic theory: a survey. *International Journal of Ethics and Systems, 34*(2), 200–212.Available at: https://doi.org/10.1108/IJOES-09-2017-0129.

Alam, M. M., Akbar, C. S., Shahriar, S. M., & Elahi, M. M. (2017). The Islamic Shariah principles for investment in stock market. *Qualitative Research in Financial Markets, 9*(2), 132–146.Available at: https://doi.org/10.1108/QRFM-09-2016-0029.

Albaity, M. S. (2011). Impact of the monetary policy instruments on islamic stock market index return. *Economics Discussion Paper*, 2011-2026.

Allaj, E. (2013). The Black–Litterman model: A consistent estimation of the parameter tau. *Financial Markets and Portfolio Management*, *27*(2), 217-251.Available at: https://doi.org/10.1007/s11408-013-0205-x.

Allaj, E. (2020). The Black–Litterman model and views from a reverse optimization procedure: An out-of-sample performance evaluation. *Computational Management Science*, *17*(3), 465-492.Available at: https://doi.org/10.1007/s10287-020-00373-6.

Ashker, A. A.-F. E. l. (1987). *The Islamic business enterprise*. London: Croom Helm.

Bayram, K., Abdullah, A., & Meera, A. K. (2018). Identifying the optimal level of gold as a reserve asset using the Black–Litterman model: The case for Malaysia, Turkey, KSA and Pakistan. *International Journal of Islamic and Middle Eastern Finance and Management*, *11*(3), 334–356.Available at: https://doi.org/10.1108/IMEFM-06-2017-0142.

Bessler, W., Opfer, H., & Wolff, D. (2017). Multi-asset portfolio optimization and out-of-sample performance: an evaluation of Black–Litterman, mean-variance, and naïve diversification approaches. *The European Journal of Finance*, *23*(1), 1-30.Available at: https://doi.org/10.1080/1351847X.2014.953699.

Black, F., & Litterman, R. (1992). Global portfolio optimization. *Financial Analysts Journal*, *48*(5), 28-43.Available at: https://doi.org/10.2469/faj.v48.n5.28.

Cheung, W. (2013). The augmented Black–Litterman model: A ranking-free approach to factor-based portfolio construction and beyond. *Quantitative Finance*, *13*(2), 301-316.Available at: https://doi.org/10.1080/14697688.2012.714902.

Cyril, T., & Karim, R. A. A. (1987). The Shariah and its implications for Islamic financial analysis: An opportunity to study interactions among society, organization and accounting. *The American Journal of Islamic Social Sciences*, *4*(1), 101-115.

Derbali, A., El Khaldi, A., & Jouini, F. (2017). Shariah-compliant capital asset pricing model: New mathematical modeling. *Journal of Asset Management*, *18*(7), 527-537.Available at: https://doi.org/10.1057/s41260-017-0051-x.

Dewandaru, G., Masih, R., Bacha, O. I., & Masih, A. M. M. (2015). Combining momentum, value, and quality for the Islamic equity portfolio: Multi-style rotation strategies using augmented Black Litterman factor model. *Pacific-Basin Finance Journal*, *34*, 205-232.Available at: https://doi.org/10.1016/j.pacfin.2014.12.006.

Duqi, A., Franci, L., & Torluccio, G. (2014). The Black–Litterman model: The definition of views based on volatility forecasts. *Applied Financial Economics*, *24*(19), 1285-1296.Available at: https://doi.org/10.1080/09603107.2014.925056.

Fabozzi, F. J., Focardi, S. M., & Kolm, P. N. (2006). Incorporating trading strategies in the Black–Litterman framework. *The Journal of Trading*, *1*(2), 28-37.Available at: https://doi.org/10.3905/jot.2006.628192.

Hakim, S. A., Hamid, Z., & Mydin Meera, A. K. (2016). Capital asset pricing model and pricing of Islamic instruments. *Journal of King Abdulaziz University: Islamic Economics*, *29*(1), 21–39.Available at: https://doi.org/10.4197/Islec.29-1.2.

Hanif, M. (2011). Risk and Return under shari'a framework: an attempt to develop Shari'a compliant asset pricing (SCAPM). *Pakistan Journal of Commerce and Social Sciences*, *5*(2), 283-292.

Hassan, M. K., Aliyu, S., Paltrinieri, A., & Khan, A. (2019). A review of Islamic investment literature. *Economic Papers: A Journal of Applied Economics and Policy*, *38*(4), 345-380.

Hassan, M. K., Aliyu, S., Saiti, B., & Halim, Z. A. (2020). A review of Islamic stock market, growth and real-estate finance literature. *International Journal of Emerging Markets*.

Hazny, M. H., Hasim, H. M., & Yusof, A. Y. (2012). *Revisiting Markowitz's mean variance analysis: A review from shariah perspective*. Paper presented at the International Conference on Statistics in Science, Business and Engineering (ICSSBE), 1-6.

Hazny, M. H., Hasim, H. M., & Yusof, A. (2017). Mathematical modelling of a shariah-compliant capital asset pricing model. *Journal of Islamic Accounting and Business Research*.

He, G., & Litterman, R. (2002). The intuition behind Black–litterman model portfolios. *SSRN Electronic Journal 334304*.

Hooi, L. H., & Parsva, P. (2012). Performance of Islamic indices in Malaysia FTSE market: Empirical evidence from CAPM. *Journal of applied Sciences*, *12*(12), 1274-1281.Available at: https://doi.org/10.3923/jas.2012.1274.1281.

Kolm, P., & Ritter, G. (2017). On the bayesian interpretation of black–litterman. *European Journal of Operational Research*, *258*(2), 564-572.Available at: https://doi.org/10.1016/j.ejor.2016.10.027.

Krichene, N. (2013). Islamic capital markets. Singapore: John Wiley & Sons.

Krishnan, H. P., & Mains, N. E. (2005). Black–litterman: Generalizing to the two-factor case. *Risk-London-Risk Magazine*, *18*(7), 69-73.

Lai, T. L., & Xing, H. (2008). *Statistical models and methods for financial markets*. New York: Springer.

Mankert, C., & Seiler, M. (2011). Mathematical derivations and practical implications for the use of the Black–Litterman model. *Journal of Real Estate Portfolio Management*, *17*(2), 139-159.Available at: https://doi.org/10.1080/10835547.2011.12089901.

Masri, H. (2018). A Shariah-compliant portfolio selection model. *Journal of the Operational Research Society*, *69*(10), 1568-1575.Available at: https://doi.org/10.1057/s41274-017-0223-6.

Nocera, S. E. (2016). Portfolio construction and global asset allocation: A practitioner solution to a Black–litterman flaw. *Available at SSRN 2864954*, 1-29.Available at: https://doi.org/10.2139/ssrn.2864954.

Rahman, S. (2015). Ethical investment in stock screening and Zakat on stocks. *Journal of Islamic Finance*, *4*(1), 39-62.Available at: https://doi.org/10.12816/0024800.

Rao, C., Toutenburg, H., Shalabh, & Heumann, C. (2008). *Linear models and generalizations*. New York: Springer.

Ruppert, D., & Matteson, D. S. (2015). *Statistics and data analysis for financial engineering: With r examples* (2nd ed.). New York: Springer.

Sandwick, J. A., & Collazzo, P. (2021). Modern portfolio theory with sharia: A comparative analysis. *Journal of Asset Management*, *22*(1), 30-42.Available at: https://doi.org/10.1057/s41260-020-00187-w.

Satchell, S., & Scowcroft, A. (2007). A demystification of the Black–Litterman model: Managing quantitative and traditional portfolio construction. *Forecasting Expected Returns in the Financial Markets*, 39–53.Available at: https://doi.org/10.1016/B978-075068321-0.50004-2.

Shaikh, S. A. (2009). Corporate finance in an interest free economy: An alternate approach to practiced Islamic corporate finance. *Journal of Islamic Banking & Finance, International Association of Islamic Banks*.

Subekti, R., Abdurakhman, & Rosadi, D. (2020). Modified capital asset pricing model (CAPM) into sharia framework. *Journal of Physics: Conference Series*, *1581*(1), 012021.Available at: https://doi.org/10.1088/1742-6596/1581/1/012021.

Subekti, R., Abdurakhman, & Rosadi, D. (2021a). A short review over twenty years on the Black–litterman model in portfolio optimization. *Industrial Engineering & Management Systems*, *20*(4), 769-781.Available at: https://doi.org/10.7232/iems.2021.20.4.769.

Subekti, R., Abdurakhman, & Rosadi, D. (2021b). Reverse optimization and capital asset pricing model in the application of the Black–Litterman portfolio. *Journal of Physics: Conference Series*, *1918*(4), 042037.Available at: https://doi.org/10.1088/1742-6596/1918/4/042037.

Subekti, R., Abdurakhman, & Rosadi, D. (2022). Toward the Black–Litterman with Shariah-compliant asset pricing model: A case study on the Indonesian stock market during the COVID-19 pandemic. *International Journal of Islamic and Middle Eastern Finance and Management*. Available at: https://doi.org/10.1108/IMEFM-12-2020-0633

Tiby, A., El, M., & Grais, W. (2015). *Islamic finance and economic development*. New Jersey: Wiley.

Trichilli, Y., Abbes, M. B., & Masmoudi, A. (2020). Islamic and conventional portfolios optimization under investor sentiment states: Bayesian vs Markowitz portfolio analysis. *Research in International Business and Finance*, *51*, 101071.Available at: https://doi.org/10.1016/j.ribaf.2019.101071.

Walters, C. (2014). The Black–Litterman model in detail. *Available at SSRN 1314585*, *2*, 1–51.Available at: https://doi.org/10.2139/ssrn.1314585.

Widodo, B. W., Achsani, N. A., & Andati, T. (2017). An application of the Black–Litterman model with ARIMA-ARCH views for Islamic stock Portfolio in Indonesian stock exchange. *Asian Journal of Business and Management*, *5*(4), 133-140.