THE DYNAMIC VOLATILITIES AND CORRELATIONS BETWEEN GERMAN STOCK MARKET INDICES AND COMMODITIES: EVIDENCE FROM WAVELET AND MGARCH-DCC APPROACHES

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ABSTRACT

This research investigates the possibilities of diversifying Islamic and conventional stock indices in the German stock market with other commodities, such as crude oil, Bitcoin, and gold. The EUR/USD exchange rate was selected as the robustness test. Our assessment, which was established based on the vector error correction model, reveals that crude oil and German conventional stock index returns have a greater impact on the other variables under consideration. Based on the MGARCH-DCC method, our results indicate that Bitcoin has the least correlation with most variables under consideration, which benefits portfolio diversification. Bitcoin has been exceptionally volatile over the last eight years; therefore, investors would benefit more from including EUR/USD or gold in their portfolio than Bitcoin. According to continuous wavelet transforms (CWT) analysis, the holding period between German stock indices and gold should not be shorter than 128 days because the correlation is exceptionally high within that period.

Contribution/Originality: This study contributes to the academic literature by examining the time-varying and time-dependent volatilities. Our results emphasize the necessity of employing modern techniques to uncover diversification possibilities for investors with varying stock holding periods.

1. INTRODUCTION

The commodity futures markets are a popular investment option for individuals and financial institutions because they provide potential diversification benefits that partly drive the trend of financialization (Algieri & Leccadito, 2017). As a result, vast sums of money have poured into the commodities market, thereby facilitating the development of future commodities as a desirable asset class made up of different types of commodities, such as metals, agricultural products, and energy (Tang & Xiong, 2012).

Financialization strengthened the relationship between energy and non-energy commodities, further complicating portfolio allocation choices. Numerous studies have been conducted on the relationships among precious metals, oil, and agricultural commodities, e.g., Cabrera and Schulz (2016), who discovered that several critical non-energy commodities are responsive to changes in energy commodities (Ji, Bouri, Roubaud, & Kristoufek, 2019). According to Kumar (2017), gold prices are highly susceptible to rising oil costs. Yahya, Ogled, and
and Dahl (2019) discovered growing connections between oil and agricultural commodities, particularly after 2006, and that the pace of these connections becomes increasingly significant in the long term.

Historically, commodity markets are susceptible to stock market shocks due to financialization and portfolio diversification by global investors. Stock and commodities markets move in the same direction with a high degree of correlation because of their increasing interconnectedness, thereby increasing the difficulty for investors to benefit from cross-market diversification. Additionally, fund managers and investors are concerned with different investment horizons throughout the investing cycle, wherein market returns vary in length and depend on periods associated with particular investment vistas ( Gençay, Selçuk, & Whitcher, 2001). Nonetheless, commodities markets continue to provide substantial benefits for portfolio diversification.

Despite the abundance of research on the relationship of the stock market with commodities, research on cryptocurrencies such as Bitcoin is limited. Thus, Bitcoin is included as one of the variables in our research. Academics, investors, regulators, and the general public are increasingly interested in cryptocurrencies, particularly Bitcoin. This interest stems from Bitcoin's appealing characteristics, such as its ability to function autonomously (decentralization) and encryption methods to authenticate transactions and control supply and demand. The total market capitalization of the cryptocurrency market surpassed US$1 trillion on January 9, 2021 (Forbes, 2021). New regulatory reforms in the cryptocurrency market and increasing institutional demand for cryptocurrencies, resulted in a strong demand for Bitcoin in the final quarter of 2020, with the price increasing from US$10,800 in October to US$40,000 in January 2021 (Coin Market Cap, 2021). A high return will result in high volatility in Bitcoin investing. Despite its extreme volatility, Bitcoin is often used as a valuable financial tool in a diversified portfolio (Eisl, Gasser, & Weinmayer, 2015).

The reaction of Islamic stocks to crises and uncertainty differs from that of conventional stock markets (Al-Yahyaee, Mensi, Rehman, Vo, & Kang, 2020). During the international economic meltdown, Islamic stock markets performed better than comparable markets (Jawadi, Jawadi, & Louhichi, 2014). Consequently, after the 2008 global economic crisis, the Islamic stock market drew investors seeking to diversify their portfolios, allowing the market to grow steadily (Hassan, Aliyu, Saiti, & Abdul Halim, 2021). In 2016, global investment in Islamic assets reached US$2.2 trillion, accounting for about 7% of worldwide investments (Alahouel & Loukil, 2020).

Shariah-compliant screening removes those companies with high outstanding debt from Islamic stocks and those involved with unlawful activities, gambling, and interest-based businesses. Previous research on German stock indices, such as Tilfani, Ferreira, Dionisio, and Youssef El Boukfaoui (2020) and Celebi and Hönig (2019), focused mostly on conventional stock indices and ignored Islamic stock indices. Therefore, we aim to add to the academic literature by examining the German Islamic stock index and comparing it with the conventional index and other commodities, such as gold, crude oil, and Bitcoin. German stock indices were chosen as the variable because the German stock exchange, also known as the Frankfurt Stock Exchange, is the third-biggest stock exchange in Europe, with a market capitalization of US$2 trillion. With a GDP that is the fourth-biggest in the world, Germany is an economic powerhouse and is regarded as "Europe's economic superstar" (Dustmann, Fitzenberger, Schönberg, & Spitz-Oener, 2014). Considerable research has been conducted on German stock indices, but the concentration of time-varying volatilities and correlations remains limited. For the robustness test, we included additional control variables, such as the nominal exchange rate of the Euro to the US dollar (EUR/USD).

The following research objectives were developed for this study: (i) estimate whether German indices influence other commodities; (ii) determine the relative endogeneity/exogeneity among the variables under study; (iii) identify variables that can potentially offer portfolio diversification benefits; and (iv) compare how portfolio diversification benefits differ depending on investment horizon strategies.

Our main contribution is an empirical study of the effect of time scales on time-varying volatilities and correlations between sample variables. By adding scale dependence, the present research may identify distinct
portfolio diversification potentials for various market participants with varied investment intervals. The present research expects that the findings will help investors seeking to diversify their portfolio allocations across various investment horizons. Our research attempts to obtain comprehensive information for strategic portfolio allocations to investors who aim to minimize risk in their portfolios through diversification in German stock indices, Bitcoin, gold, crude oil, and the EUR/USD exchange rate using the most recent data and techniques.

The present research is designed as follows: Section 2 looks at the foundations of essential theory for this study and summarizes past studies on gold, Bitcoin, portfolio diversification, and stock market indices; Section 3 provides an in-depth discussion of the methods utilized to accomplish the study’s objectives; Section 4 discusses the data analysis and outcomes; and Section 5 summarizes and offers reasonable explanations of the findings.

2. THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1. Modern Portfolio Theory

The theory used in the present research was established by Markowitz (1959) and is known as the portfolio diversification theory. The modern portfolio theory (MPT) created by Markowitz claims that the volatility of a portfolio should be lower than the volatility of the weighted average of stocks if such a portfolio is constructed from assets with low correlated returns (Markowitz, 1959). The early version of the MPT assumed that portfolio variances were normal. By contrast, Markowitz believes that normally distributed variance is insufficient to measure risk. Therefore, efforts were made to improve the model and reflect real-world trends to account for asymmetric and fat-tailed distributions. The MGARCH-DCC technique was employed in the present research because the Student-t distribution of variances is highly flexible in dealing with fat-tailed distributions of index returns (Pesaran & Pesaran, 2010). Moreover, the wavelet transform method does not need distributional assumptions, which results in highly realistic outcomes (In & Kim, 2013). Section 3 describes in full the techniques utilized to address the study's objectives.

2.2. Literature Review

Commodity markets have seen a significant inflow of money in the past two decades, mainly from index funds (Just & Łuczak, 2020). This phenomenon may be related to the stock market crash in the early 2000s, caused by overpriced stocks. This collapse facilitated investors’ awareness of the negative correlation between commodity returns and stock returns, which resulted in massive capital inflows into commodity markets because commodities are seen as a strategy for reducing risk in a stock portfolio (Tang & Xiong, 2012).

Fund companies, especially index funds, play a significant role in commodity asset allocation, which is often referred to as commodity financialization (Irwin & Sanders, 2011). The commodities markets have drawn enormous money due to the vast wave of commodity financialization (Huynh, Burggraf, & Nasir, 2020). Commodity financialization has increased the comovement of several commodities (Hu, Li, & Liu, 2020). Therefore, we aim to investigate the comovement of commodity volatility by investigating the dynamic volatility and correlation between the commodity and stock markets.

Tilfani et al. (2020) investigated the relationships among three major stock markets, namely Germany, the United Kingdom, and the United States, as well as the European Union nations, which were separated into two factions: Eurozone and non-Eurozone. They found that, in general, Germany and other Eurozone countries have substantial levels of comovement, but Brexit reduced these connections. Market comovement also emerged as a result of the subprime mortgage crisis. Meanwhile, Celebi and Höning (2019) examined the effect of macroeconomic variables, German government bond rates, mood, and other prominent indicators on the DAX30, Germany’s major stock index, from 1991 to 2018. They found that most data, including the composite leading indicator (CLI) and the German three-year government bond rates, had a delayed impact on stock returns. Several bodies of research on the
German stock market fall short of addressing time-varying volatility and correlations among the sample variables. We aim to fill this gap by researching this area.

The search for new risk-reducing instruments does not end with traditional commodities, such as oil and gold, but continues with cryptocurrency. According to one school of thought, Bitcoin is digital gold because it resembles a safe haven asset, such as gold (Bouri, Jain, Biswal, & Roubaud, 2017). Other studies view Bitcoin as an unsubstantiated instrument and a Ponzi scheme due to its lack of intrinsic value (Baur, Hong, & Lee, 2018); they attribute Bitcoin's extreme volatility to speculation (Brandvold, Molnár, Vagstad, & Valstad, 2015). Thus, adopting Bitcoin as a currency would be a financial disaster. Even though Bitcoin appears to be very volatile, its presence in a diversified portfolio may be quite helpful in portfolio diversification because of its uncorrelated or weakly correlated relationship with major equities, oil, and currencies (Uddin, Ali, & Masih, 2020).

In summary, limited research was conducted on the connection between stock indices and commodities, such as Bitcoin, gold, and crude oil, and their effect on portfolio diversification possibilities, particularly on the Islamic stock market. Thus, further research is required on this topic.

3. METHODOLOGY

3.1. Data

The present research examines daily Bitcoin, crude oil, and gold prices, the Euro exchange rate to the US dollar (EUR/USD), and Islamic and conventional stock indices in the German stock market from September 1, 2011, to June 28, 2019. DataStream was used as the data source.

3.2. Time Series Techniques

This research uses error correction methods to examine the connections between German stock indices and other commodities. Following the discovery of the cointegrating connection, several existing studies examined the lead–lag relationship using either the vector error correction model (VECM) or the variance decomposition (VD) technique (Granger causality). The following methods are utilized to examine the Granger causality relationship. The unit root test must be run first, followed by the Johansen cointegration test, to determine the VAR order. The cointegration test cannot be used to establish whether variables are leading or trailing, so VECM is used to determine Granger causality in the short and long runs (Masih, Al-Elg, & Madani, 2009). However, the VECM cannot express the relative exogeneity or endogeneity of variables. Generally, the VD is considered the best approach for evaluating the exogeneity and endogeneity of variables. However, the statistical programme employed (Microfit 5.0) is limited because it can only regress data for a maximum of 150 observations. The data collected for this study has 2,042 daily observations to date. Only five months of data were obtained when Microfit 5.0 was applied to the data, which is insufficient to reach a solid conclusion. Due to the constraints of the time series statistical tools employed in the research, the lead–lag connection was analyzed using a wavelet, which accounts for the time scale impact.

3.3. Maximum Overlap Discrete Wavelet Transformation (MODWT)

Discrete wavelet transform (DWT) and MODWT can decompose the sample variance of a time series on a scale-by-scale basis using their squared wavelet coefficients. However, the MODWT-based estimate outperforms the DWT-based estimator (Gallegati, 2008). Thus, the MODWT is used in this research.

Whitcher and Guttorp (2000) employed the MODWT to compute the estimates and approximate confidence intervals of wavelet covariance and wavelet correlation between variables. Wavelet covariance measures the strength of relationships between variables X and Y across various periods. Gallegati (2008) clarified that the wavelet covariance at scale j corresponds to the covariance between the wavelet coefficients of X and Y at that
particular j scale, that is, $\gamma_{XYj} = \text{Cov}(\tilde{\omega}_{j1}^X, \tilde{\omega}_{j1}^Y)$. The below equation offers an unbiased estimator of the wavelet covariance using the MODWT, where all boundary conditions are fulfilled.

$$\hat{\gamma}_{XYj} = \frac{1}{N_j} \sum_{t=L_j-1}^{N-1} \tilde{\omega}_{jt}^X \tilde{\omega}_{jt}^Y$$

Following this equation, the MODWT cross-correlation coefficients, $\hat{\rho}_{t,XYj}$ for scale j and lag τ, may be determined by dividing the wavelet cross-correlation, $\hat{\gamma}_{t,XYj}$, by the square root of the wavelet variances $X(\sigma_{Xj})$ and $Y(\sigma_{Yj})$ as follows:

$$\hat{\rho}_{t,XYj} = \frac{\hat{\gamma}_{t,XYj}}{\sigma_{Xj}\sigma_{Yj}}$$

Wavelet cross-correlation coefficients $\hat{\rho}_{t,XYj}$ have the same properties as conventional unconditional cross-correlation coefficients and should be between 0 and 1 over different time scales.

The asymptotic variance $V_j$ of the MODWT-based wavelet variance (covariance) estimator can be calculated, starting with the spectrum $S_{\omega X,j}$ of scale j wavelet coefficients. A random interval is created that generates a $100(1-2p)$ percent confidence interval. Gallegati (2008) offered methods for an estimated $100(1-2p)$ percent confidence intervals of the MODWT estimator resistant to the non-Gaussianity for $\tilde{\omega}_{X,j}^2$. The empirical evidence for the wavelet variance indicates that $N_j = 128$ is a reasonable number of wavelet coefficients under the large sample theory (Gallegati, 2008).

3.4. Multivariate GARCH Dynamic Conditional Correlation (MGARCH-DCC)

The MGARCH-DCC model from Pesaran and Pesaran (2010) is utilized to address the third study topic. We compared normal and t distributions to decide on the best model. Unconditional correlation coefficients may offer sufficient empirical data to address the third research issue. However, the calculation of conditional cross-asset correlations is needed to meet the third goal comprehensively through the MGARCH-DCC quantification as below:

$$\tilde{\rho}_{ij,t-1}(\phi) = \frac{q_{ij,t-1}}{\sqrt{q_{ii,t-1}q_{jj,t-1}}}$$

where $q_{ij,t-1}$ are given by:

$$q_{ij,t-1} = \tilde{\rho}_{ij}(1 - \phi_1 - \phi_2) + \phi_1 q_{ij,t-2} + \phi_2 \tilde{\rho}_{ij,t-1} \tilde{\rho}_{ij,t-1}.$$

In the above equation, $\tilde{\rho}_{ij}$ denotes the (ij)th unconditional correlation, and $\phi_1$ and $\phi_2$ are parameters, such that $\phi_1 + \phi_2 < 1$, and $\tilde{\rho}_{ij,t-1}$ denote the standardised asset returns. We also estimate $(1 - \lambda_1 - \lambda_2)$ to determine whether the calculated volatility is mean-reverting. Several validation tests are conducted to ensure the resilience of our models.

3.5. Continuous Wavelet Transform (CWT)

Our fourth research goal is addressed via the use of CWT. Several researchers, including Abdullah, Saiti, & Masih (2016), have begun to utilize CWT in financial economics studies. The original time series includes a time function with a single variable. CWT enables us to map a time function with a single variable into time-frequency variables. Among the essential benefits of CWT is its ability to automatically define the number of time scales based on the length of the data. Additionally, CWT enables mapping of the series correlation in a two-dimensional space, improving the capacity to highlight hidden information correctly. Compared to the discrete technique, the CWT approach has increased visibility owing to the redundancy of its characteristics, which facilitates ease of understanding. Thus, the analysis is highly visible and interpretable.
For the MODWT and CWT, the least asymmetric wavelet filter of Daubechies (1992) is used, with a length of $L = 8$, $\text{LA}\ (8)$. According to existing research, a length filter of $L = 8$ is adequate to handle high-frequency data (In & Kim, 2013). The $\text{LA}\ (8)$ filter also produces smoother wavelet coefficients than other filters, such as the Haar filter. Refer to In and Kim (2013) for additional information.

4. EMPIRICAL FINDINGS AND INTERPRETATIONS

4.1. Description of Data

Figure 1 shows the raw time series data of all chosen variables. The graph illustrates that the price of Bitcoin is highly volatile given the dramatic rise and fall of Bitcoin prices for a short period in 2017. We also observed a decline in oil and gold prices, which indicates a lack of demand for these commodities. Meanwhile, the German stock market's Islamic and conventional stock indices are rising, indicating an expanding economy backed by a steady EUR/USD exchange rate.
Table 1. Descriptive statistics.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Min.</th>
<th>Max.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Number of Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>BITCOIN</td>
<td>0.0016</td>
<td>0.0256</td>
<td>-0.2883</td>
<td>0.2105</td>
<td>-0.9530</td>
<td>20.2262</td>
<td>2,042</td>
</tr>
<tr>
<td>OIL</td>
<td>-0.0001</td>
<td>0.0091</td>
<td>-0.0466</td>
<td>0.0505</td>
<td>0.0270</td>
<td>3.2838</td>
<td>2,042</td>
</tr>
<tr>
<td>GOLD</td>
<td>-0.0001</td>
<td>0.0044</td>
<td>-0.0441</td>
<td>0.0236</td>
<td>-0.7989</td>
<td>9.5063</td>
<td>2,042</td>
</tr>
<tr>
<td>GERMIS</td>
<td>0.0001</td>
<td>0.0050</td>
<td>-0.0258</td>
<td>0.0208</td>
<td>-0.1773</td>
<td>1.9812</td>
<td>2,042</td>
</tr>
<tr>
<td>GERMCON</td>
<td>0.0001</td>
<td>0.0050</td>
<td>-0.0289</td>
<td>0.0226</td>
<td>-0.1876</td>
<td>2.7430</td>
<td>2,042</td>
</tr>
<tr>
<td>USDEUR</td>
<td>0.0000</td>
<td>0.0023</td>
<td>-0.0113</td>
<td>0.0098</td>
<td>-0.0376</td>
<td>2.0766</td>
<td>2,042</td>
</tr>
</tbody>
</table>

Table 1 displays the descriptive statistics for the return series, written as $\text{rt} = \ln \left( \frac{P_t}{P_{t-1}} \right)$, where $\text{rt}$ is the series return computed using the natural log and $P_t$ is the price index at time $t$. The average return of Bitcoin outperforms that of German stock markets and other commodities. Bitcoin is also the most volatile asset compared to gold, crude oil, and German stock indices. Consequently, Bitcoin seems more lucrative than German stock indices and other commodities regarding risk–return.

4.2. Empirical Findings of Standard Time Series Techniques

The VECM method with one cointegrating vector is used to determine the exogeneity and endogeneity of the variables. As shown in Table 2, crude oil and the conventional stock index return (CSIR) in the German stock market are exogenous, whereas Islamic stock index returns (ISIR), the EUR/USD exchange rate, Bitcoin, and gold are endogenous. This finding suggests that the German ISIR, Bitcoin, gold, and the EUR/USD exchange rate would react to the price of crude oil and the German CSIR. The VECM helps us capture short- and long-term Granger-causalties. The error correction term stands for the long-term relations among the variables. The short-term effect of each variable is determined by the F-test of the combined significance or non-significance of the lags of each of the differentiated variables.

The percentage of variance decomposition of a variable clarified by its previous shocks may be used to assess its relative exogeneity/endogeneity. However, the programme used to evaluate the variance decomposition restricts our horizon to only 150 points, even though our entire horizon is 2,042. Thus, the MODWT was used to determine the lead–lag connection among the chosen exogenous variables.

Table 2. Error correction model of Bitcoin and other variables.

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>DBitcoin</th>
<th>DCrudeoil</th>
<th>DGold</th>
<th>DGerman</th>
<th>DGermis</th>
<th>DGermcon</th>
<th>DEurousd</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECM (-1)</td>
<td>-0.09 (0.03)</td>
<td>0.01 (0.01)*</td>
<td>-0.01 (0.00)</td>
<td>0.01 (0.01)</td>
<td>0.01 (0.01)</td>
<td>-0.01 (0.00)</td>
<td>1.13 (0.29)</td>
</tr>
<tr>
<td>Chi-square</td>
<td>3.40 (0.06)</td>
<td>15.00 (0.00)</td>
<td>0.64 (0.42)</td>
<td>0.01 (0.02)</td>
<td>1.02 (0.31)</td>
<td>-0.01 (0.00)</td>
<td>1.13 (0.29)</td>
</tr>
<tr>
<td>SC(1)</td>
<td>6.01 (0.01)</td>
<td>1.96 (0.16)</td>
<td>0.09 (0.76)</td>
<td>0.11 (0.75)</td>
<td>0.75 (0.38)</td>
<td>0.21 (0.05)</td>
<td>0.21 (0.05)</td>
</tr>
<tr>
<td>Chi-square</td>
<td>32902 (0.00)</td>
<td>912 (0.00)</td>
<td>7632 (0.00)</td>
<td>367 (0.00)</td>
<td>675 (0.00)</td>
<td>370 (0.00)</td>
<td>370 (0.00)</td>
</tr>
<tr>
<td>FF(1)</td>
<td>267 (0.01)</td>
<td>11.1 (0.94)</td>
<td>11.1 (0.94)</td>
<td>24.30 (0.00)</td>
<td>36.80 (0.00)</td>
<td>2.22 (0.07)</td>
<td>2.22 (0.07)</td>
</tr>
</tbody>
</table>

Notes: Standard errors are shown in parentheses. Chi-squared statistics for serial correlation (SC), functional form (FF), normality (N), and heteroskedasticity (Het) are used as diagnostics. The findings show that the equations are well specified.

* Indicates significance at the .5% level.

4.3. Empirical Findings of the MODWT

Figure 2 shows the MODWT-based wavelet cross-correlations between crude oil price return and the German CSIR over all periods. It also shows the corresponding approximate confidence intervals, time leads, and lags for all scales associated with each scale-specific period. Individual cross-correlation function wavelet scales $\lambda_1, \ldots, \lambda_8$ are described as variations over 1–2, 2–4, 4–8, 8–16, 16–32, 32–64, 64–128, and 128–256 days. The red lines limit the 95% confidence range of wavelet cross-correlation. The first variable leads if the curve on the left side of the graph is significant. The reverse is true if the curve is prominent on the right side of the graph. A significant positive
wavelet cross-correlation exists when the 95% confidence intervals are above the horizontal axis. A significant negative wavelet cross-correlation exists when the 95% confidence intervals are below the horizontal axis. Figure 2 shows the following:

i) No indication of a lead–lag relationship between these two variables at wavelet levels 1, 2, 3, 4, and 5.

ii) The graph tilted to the right at wavelet level 6 suggests that the German CSIR leads the crude oil.

iii) The graph tilted to the left at wavelet levels 7 and 8 suggests that crude oil leads the German CSIR.

Thus, it can be inferred that the German CSIR leads crude oil on lower levels, whereas crude oil leads the German CSIR on higher levels. A long-term diversification advantage exists between these two variables. Our findings contradict those of Cueppers and Smeets (2015), who found that general oil price fluctuations had no substantial effect on the stock returns of German DAX firms. However, their data, which span from February 1982 to July 2007, differ from our sample data, which cover September 2011 to June 2019. These differences in data may explain the disparity in results.

4.4. Empirical Findings of the MGARCH–DCC

In this section, the MGARCH–DCC was used to evaluate the diversification advantages of the chosen commodities and German stock indices. Table 3 shows the maximum likelihood estimates of $\lambda_1$ and $\lambda_2$ for commodity price returns and stock indices, as well as $\delta_1$ and $\delta_2$, when comparing the multivariate normal distribution to the multivariate student t-distribution. The highest log-likelihood value for the t-distribution (49891) is higher than the normality assumption (49420). As anticipated for a multivariate normal distribution, the approximate degree of freedom for the t-distribution is 7.25, which is less than 30. This result implies that the t-distribution is suitable for the fat-tailed nature of the price return distribution. This research is based on t-distribution estimations.

The estimated unconditional volatilities (diagonal elements) and unconditional correlations (off-diagonal elements) of the six variables are shown in Table 4. The numbers in parentheses indicate the order of unconditional volatility (from highest to lowest). The ranking reflects the volatility of the six variables. Based on the
unconditional volatility, Bitcoin and crude oil prices tend to get a significant proportion of speculative transactions. The EUR/USD exchange rate has the least volatility, which indicates that the German economy is a robust and balanced capital market in Europe.

The relationships between commodity prices and stock indices are important for the third objective of this study. The unconditional correlations in Table 5 reveal that Bitcoin has the lowest correlations with other variables. We ordered the unconditional correlations from most significant to lowest to provide a good sense of the relative relationships among the variables.

Table 5. Estimates of the six variables under review.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Multivariate Normal Distribution</th>
<th>Multivariate $t$ Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>T-Ratio</td>
</tr>
<tr>
<td>Lamda 1 ($\lambda_1$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bitcoin</td>
<td>0.87</td>
<td>55.3</td>
</tr>
<tr>
<td>Crudeoil</td>
<td>0.93</td>
<td>87.7</td>
</tr>
<tr>
<td>Gold</td>
<td>0.93</td>
<td>74.9</td>
</tr>
<tr>
<td>Germis</td>
<td>0.94</td>
<td>129</td>
</tr>
<tr>
<td>Germcon</td>
<td>0.95</td>
<td>135</td>
</tr>
<tr>
<td>Eurusd</td>
<td>0.97</td>
<td>177</td>
</tr>
<tr>
<td>Lamda 2 ($\lambda_2$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bitcoin</td>
<td>0.12</td>
<td>9.16</td>
</tr>
<tr>
<td>Crudeoil</td>
<td>0.06</td>
<td>7.44</td>
</tr>
<tr>
<td>Gold</td>
<td>0.06</td>
<td>6.25</td>
</tr>
<tr>
<td>Germis</td>
<td>0.04</td>
<td>8.41</td>
</tr>
<tr>
<td>Germcon</td>
<td>0.05</td>
<td>8.55</td>
</tr>
<tr>
<td>Eurusd</td>
<td>0.03</td>
<td>6.42</td>
</tr>
<tr>
<td>Delta 1 ($\delta_1$)</td>
<td>0.94</td>
<td>173</td>
</tr>
<tr>
<td>Delta 2 ($\delta_2$)</td>
<td>0.03</td>
<td>14.2</td>
</tr>
<tr>
<td>Maximized log-likelihood</td>
<td>49420</td>
<td>49891</td>
</tr>
<tr>
<td>Degree of freedom (df)</td>
<td>-</td>
<td>7.25</td>
</tr>
</tbody>
</table>

Note: $\lambda_1$ and $\lambda_2$ are decay factors for variance and covariance, respectively.
Table 4. Estimated unconditional volatility matrix for German stock indices returns and other variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Bitcoin</th>
<th>Ranking</th>
<th>Crudeoil</th>
<th>Gold</th>
<th>Ranking</th>
<th>Germis</th>
<th>Ranking</th>
<th>Germcon</th>
<th>Ranking</th>
<th>Eurusd</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bitcoin</td>
<td>0.0255</td>
<td>(1)</td>
<td>0.0132</td>
<td>0.0487</td>
<td>0.0555</td>
<td>0.0279</td>
<td></td>
<td></td>
<td>-0.0006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crudeoil</td>
<td>0.0132</td>
<td></td>
<td>0.0090</td>
<td>(2)</td>
<td>0.111</td>
<td>0.2288</td>
<td>0.2350</td>
<td></td>
<td>-0.1163</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td>0.0487</td>
<td></td>
<td>0.1110</td>
<td>0.0041</td>
<td>(3)</td>
<td>-0.039</td>
<td>0.98</td>
<td>0.0048</td>
<td></td>
<td>-0.4062</td>
<td></td>
</tr>
<tr>
<td>Germis</td>
<td>0.0355</td>
<td></td>
<td>0.2288</td>
<td>-0.039</td>
<td>0.0048</td>
<td>(4)</td>
<td>0.98</td>
<td>0.0048</td>
<td></td>
<td>0.0029</td>
<td></td>
</tr>
<tr>
<td>Germcon</td>
<td>0.0279</td>
<td></td>
<td>0.2350</td>
<td>-0.044</td>
<td>0.98</td>
<td>0.0048</td>
<td>(3)</td>
<td></td>
<td>-0.0163</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Eurusd</td>
<td>-6E-04</td>
<td></td>
<td>-0.116</td>
<td>-0.406</td>
<td>0.0029</td>
<td>-0.0163</td>
<td>0.0023</td>
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</table>
Table 5. Ranking of unconditional correlations among the German stock indices returns and other variables.

<table>
<thead>
<tr>
<th>Bitcoin</th>
<th>Crudeoil</th>
<th>Gold</th>
<th>German ISIR</th>
<th>German CSIR</th>
<th>EUR/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Bitcoin)</td>
<td>(Oil)</td>
<td>(Gold)</td>
<td>(German)</td>
<td>(German)</td>
<td>(Eurusd)</td>
</tr>
<tr>
<td>Gold</td>
<td>Germsc</td>
<td>Eurusd</td>
<td>Germcon</td>
<td>Germis</td>
<td>Gold</td>
</tr>
<tr>
<td>Germsc</td>
<td>Germcon</td>
<td>Oil</td>
<td>Oil</td>
<td>Oil</td>
<td>Oil</td>
</tr>
<tr>
<td>Germcon</td>
<td>Eurusd</td>
<td>Bitcoin</td>
<td>a</td>
<td>Gold</td>
<td>Germcon</td>
</tr>
<tr>
<td>Oil</td>
<td>Gold</td>
<td>Germcon</td>
<td>Bitcoin</td>
<td>a</td>
<td>Bitcoin</td>
</tr>
<tr>
<td>Eurusd</td>
<td>Bitcoin</td>
<td>Germsc</td>
<td>Eurusd</td>
<td>Eurusd</td>
<td>Bitcoin</td>
</tr>
</tbody>
</table>

A number of essential findings can be inferred from the above table. The price of Bitcoin has the lowest correlation with nearly all variables under consideration (see symbol ‘a’ in Table 5). This result means that investors need to incorporate Bitcoin to obtain the full benefits of portfolio diversity. However, as shown in Table 4, the price return of Bitcoin is highly volatile, which reduces its potential as a portfolio diversifier. Our findings are consistent with Baur and Dimpfl (2021), who indicated that the excess volatility of Bitcoin limits its potential role in portfolios. Therefore, investors might consider the EUR/USD exchange rate as an alternative. Table 4 shows that EUR/USD exchange rate has the lowest volatility and, at the same time, the lowest correlation with German ISIR, CSIR, and Bitcoin. This finding complements the work by Garcia & Rodrigues (2019), in which they examined the long-term relationship between the FSTE 100 and Euro STOXX 50 indices, as well as the USD/EUR and USD/GBP exchange rates; they discovered a cointegration relationship among all variables under review, which implied a long-term relationship among the variables.

We have conducted volatility and correlation research and drawn findings on an unconditional premise. The average volatility and correlation of the sample period has been evaluated on an unconditional basis. The assumption that volatility and correlation stay constant over eight years contradicts intuition as volatility and correlation are likely to be dynamic. The MGARCH-DCC model used in this article focuses on this standpoint.

We begin by looking at the time component of volatility. Figure 3 depicts the conditional volatilities for the six variables. During the eight-year study, we discovered that the price return of Bitcoin had the most significant volatility. The price return of Bitcoin is exceptionally volatile, particularly between June and September 2013, owing to the positive and negative publications about Bitcoin during that period. Moreover, Bitcoin is randomly volatile throughout the eight-year study period. The EUR/USD exchange rate return had the lowest volatility during that period. We may infer from the graph that investing in Bitcoin is exceptionally risky because it is highly volatile and unstable compared with the other variables. Additionally, Figure 4 demonstrates that the EUR/USD exchange rate is the most stable variable, followed by gold. The outcome is consistent with the findings in Table 4.
We examine the correlation between the price returns of Bitcoin with the Islamic and conventional stock indices returns in the German stock market using conditional correlations, as shown in Figure 5. The graph shows that the correlation between Bitcoin and the German stock indices was consistent from 2011 to 2019. However, Bitcoin has a more significant correlation with German ISIR than CSIR, thereby making ISIR a poor option for diversifying an investment portfolio. This observation is consistent with those in Table 5. As a result, investors exposed to Bitcoin should invest in German CSIR rather than German ISIR to obtain a significant diversification advantage.
Crude oil price returns are strongly correlated with the German CSIR and ISIR. Thus, it is ineffective as a portfolio diversifier for the indices (see Table 5). The correlation between crude oil price returns and German CSIR and ISIR declined from 2011 to 2015, as seen in the graph above. The decreasing trend suggests that future diversification benefits between the variables may be possible. However, the correlation rises marginally from 2015 to 2019, but, generally, the correlation remains lower than in 2011. Additionally, Figure 6 demonstrates that the correlation between crude oil and German CSIR is stronger than the correlation between crude oil and German ISIR. This result is consistent with the findings in Table 5.

The lowest correlation is between gold price returns and German ISIR, as indicated in Table 5. Investors with a gold exposure portfolio are more suitable for diversifying German ISIR than German CSIR because the volatility and correlations of German CSIR are higher than those of German ISIR. Figure 7 shows the declining correlation of gold with the German stock index returns from 2011 to 2017. This finding suggests that the potential for diversification between gold price returns and German stock indices returns has increased. The correlation is negative from 2015 to 2019, which indicates a strong potential for diversification gain. The above graph also reveals that the correlation between gold and the German ISIR is lower than the correlation between gold and the CSIR. This result is consistent with the findings in Table 5.

4.5. Empirical Findings of Continuous Wavelet Transforms (CWT)

Figures 8–13 show the projected CWT and phase differences for the variables under consideration at scales ranging from 1 (one day) to 8 (nearly two years or 512 trading days). The horizontal axis represents trading days, whereas the vertical axis represents the investment horizon. Table 6 presents the date of the horizontal axis for Figures 8–13. The curving line below depicts the 5% significance threshold, calculated using Monte Carlo simulations. As shown on the right, the image employs a color-coding scheme that ranges from blue (low correlations) to red (high correlations).

Investors interested in the German CSIR and ISIR and those diversifying their portfolios via investments in Bitcoin, crude oil, and gold must understand the correlations between the variables. According to Chart 8, any investor who wishes to invest in Bitcoin while also being exposed to German Islamic and conventional stock indices can hold their portfolio for as long as they aim to gain diversification benefits because Bitcoin and German indices do not correlate in the short and long terms.

To enjoy the advantages of diversity, investors who wish to hold a portfolio of crude oil and German stock indices should do so for no more than three months (between one day and 64 days) to maximize their returns. Investors will be exposed to a strong correlation if their investment lasts longer than three months or 64 days (see Figures 10 and 11). Figures 10 and 11 demonstrate that the connection of crude oil with the German ISIR and
CSIR is almost comparable because both show a similar sensitivity area. The German economy is more susceptible to crude oil price fluctuations than Bitcoin price fluctuations because Germany is a net importer of crude oil and relies primarily on crude oil for energy. Germany is one of the world's biggest oil consumers, with a daily use of 2,357,500 barrels, 98% of which is imported. Thus, Germany is one of the top oil importers in the European Union (Cueppers & Smeets, 2015); these facts may explain why German stock indices are more susceptible to fluctuations in crude oil prices than Bitcoin. However, our findings contradict those of Cueppers and Smeets (2015), who discovered that overall oil price fluctuations had no substantial impact on the stock returns of German DAX firms during their study period (February 1982 to July 2007).

Figures 12 and 13 show that the correlation between German stock indices and gold is low on the upper scale beyond 128 days of investment holding time. This result implies that an investor would profit from diversification throughout this holding period. The correlation between the variables under consideration is strong between 16 and 128 days of investment horizon, which limits diversification possibilities.

Table 6. Horizontal axis date.

<table>
<thead>
<tr>
<th>Horizontal Axis</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>June 2012</td>
</tr>
<tr>
<td>400</td>
<td>March 2013</td>
</tr>
<tr>
<td>600</td>
<td>December 2013</td>
</tr>
<tr>
<td>800</td>
<td>September 2014</td>
</tr>
<tr>
<td>1000</td>
<td>July 2015</td>
</tr>
<tr>
<td>1200</td>
<td>April 2016</td>
</tr>
<tr>
<td>1400</td>
<td>January 2017</td>
</tr>
<tr>
<td>1600</td>
<td>October 2017</td>
</tr>
<tr>
<td>1800</td>
<td>July 2018</td>
</tr>
<tr>
<td>2000</td>
<td>May 2019</td>
</tr>
</tbody>
</table>

![Figure 8. CWT – Bitcoin PR vs. German ISIR.](image)
Figure 9. CWT – Bitcoin PR vs. German CSIR.

Figure 10. CWT – Crude Oil PR vs. German ISIR.
Figure 11. CWT – Crude Oil PR vs. German CSIR.

Figure 12. CWT – Gold PR vs. German ISIR.
5. CONCLUSION AND POLICY IMPLICATION

5.1. Conclusion

This research has four objectives. The VECM and many newly developed econometric techniques, such as the MGARCH-DCC, MODWT, and CWT, were used to accomplish these objectives, and the key results are as follows.

First, the VECM findings indicate that the price returns of crude oil and German CSIR are exogenous, whereas the German ISIR, Bitcoin, gold, and the EUR/USD exchange rate returns are endogenous. This suggests that the German ISIR, gold, Bitcoin, and the EUR/USD exchange rate will react to the German CSIR and crude oil price returns.

Second, the MODWT findings infer that the crude oil price return at high levels leads the German CSIR. This indicates that the price of crude oil can affect the German CSIR on trading days ranging from 64 to 256 days. Germany is a net importer of crude oil and relies significantly on this fuel for energy, which explains why the German CSIR is sensitive to changes in crude oil prices. We may also infer other possibilities for diversification for both variables.

Third, the MGARCH-DCC findings reveal that the EUR/USD exchange rate return has the lowest volatility, which indicates that Germany is a stable economy with one of the biggest capital markets in Europe. Moreover, the EUR/USD showed the least correlation with most variables under consideration. Thus, investors exposed to Bitcoin, German CSIR, and ISIR might consider using EUR/USD as a portfolio diversifier to achieve diversification benefits.

Finally, the CWT results indicate that diversification advantages may be achieved in Bitcoin with German stock indices in low- and high-scale holding periods. The CWT results also corroborate the MGARCH-DCC findings, which show that Bitcoin is one of the least correlated with the German stock indices. Investors exposed to German indices may use gold as a diversification tool for a holding duration of 128 days onward. Holding time ranging from 32–128 days exposes investors to significant correlation. Meanwhile, investors interested in crude oil
and German indices should not hold the variables for more than three months (64 days), as indicated in Figures 10 and 11. The CWT contributes to the understanding of portfolio diversification possibilities of investors with various investment perspectives or those holding stocks for different durations.

5.2. Policy Implication

The present research findings have potentially significant consequences for investors, portfolio managers, and regulators. Our findings also offer clarity on Bitcoin, gold, EUR/USD, and crude oil price pressures on German stock market indices. Furthermore, the results shed light on the significance of Bitcoin, gold, EUR/USD, and crude oil as critical financial instruments for asset allocation and risk management using German stock market indices. In terms of risk management, the results can also potentially assist investors and financial market participants. First, investors may utilize our discovery for hedging and diversification purposes. For example, the correlations between crude oil and German stock market indices are typically modest for holding periods shorter than 64 trading days. Thus, crude oil may be used as a hedge (or diversification) strategy against German stock market indices in the short term. Long-term investors should use gold as a hedging (or diversification) strategy because the correlation between gold and German stock market indices is minimal after 128 trading days. Second, the study found a highly substantial and time-varying correlation between gold, Bitcoin, and crude oil with German stock market indices. Thus, long and short strategies may be devised and utilized efficiently. Investors may tailor the risk exposure of their portfolio to their preferences. Third, long-short hedging and diversification strategies for investors exposed to German stock market indices may be improved based on strong and weak correlations between variables under consideration. Finally, the results show varying degrees of positive correlations among German stock indices and Bitcoin, gold, and crude oil. This implies that the financialization of the commodities market resulted in an increased commodity–stock correlation. Thus, commodities have reduced their appeal as a hedge or diversifier for investors in the German stock market. The interaction of German stock market indices with the selected commodities suggests that policymakers may complement market monitoring methods and investment activity with alternative commodities market variables, such as gold price uncertainty, volatility, and gold–stock correlations.

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