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The Physics Of Remodeling The Transmitting Loop Antenna Using The Schrodinger-Maxwell Equation

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Abstract

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Using the Schrödinger equation and the Legendre polynomial, vital parameters were worked out for a theoretically reconfiguring of the transmitting loop antenna. The directional/angular position of the loop antenna was resolved for greater efficiency. The complex feed capacitors were resolved with respect to the length of the conductor used for the construction of the transmitting loop antenna. The efficiency, bandwidth and size of the transmitting loop were accounted for.

Introduction

Much has been discussed of the design methodology of the antennas. Recently, the coplanar folded –slot antennas (CFSA) have been the area of interest due to their typically large bandwidth and their broadside radiation pattern.⁽¹⁾ On the other hand, loop antennas (especially the small loop antenna (SCA)) have poor efficiency and are mainly used as transmitting and receiving antennas at low frequency. Unlike the CFSA's whose circumference is one guided wavelength, most loop antennas have a circumference of one tenth of a wavelength or less. The loop antenna is circular shaped object which can be distorted into different closed shape. Such closed shape still maintains the loop characteristics. The current in the small loop antenna develops a standing wave if the frequency or the size is increased.

One of the advantages of the transmitting loop antennas is its ability to function perfectly when the space of a full sized antenna is limited. Before now, it is believed that small transmitting loop antennas sacrifice bandwidth for small size and efficiency. The more efficient they are, the more narrow the frequency range in which they can operate. In other words, the greatest achievement has been to attain two options out of the three options i.e small size (in terms of wavelength), efficiency and broadband. Small loop antennas are often referred to as magnetic antennas because they mostly respond to the magnetic component of an electromagnetic wave and transmit a large magnetic component in the extreme near field (<1/10 wavelength distance). In the far field (>1 wavelength distance) the RF from a small

loop is the same as that from any other antenna being composed of both electric and magnetic fields. At distances between about 1/10 and 1 wavelength it responds more to the electric field than the magnetic field.

The self resonant loop antenna has some interesting features which would be discussed in the later part of the paper. Its resonant frequency is determined by the circumference of the loop while small loop antennas are determined by the area of the area enclosed by the loop. In this paper, the standing wave function using the Schrödinger equation was discussed analytically. Vital parameters which determine the efficiency of transmitting loop antennas were worked for further industrial research. The resonance frequencies with respect to our theoretized model (Legendre polynomial) were worked out.

Theoretical Modelling Of The Transmitting Loop Antenna

Electromagnetic waves carry energy as they propagate through space and this energy can be transferred to objects placed in their path.^(2,3) An antenna radiates and receives electromagnetic wave. The fundamental of electromagnetic wave phenomena are the Maxwell equation which has been further remodeled with respect to different antennas according to their efficiency. The Schrödinger equation has been used in various publications to analyze the electromagnetic field.^(4,5) We hope to build upon past research work with the aim of employing its findings to reconfigure the transmitting loop antenna. The time-independent Schrödinger equation is given

$$\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x)\psi(x) = i\hbar \frac{\partial}{\partial t} \psi(x) \quad (i)$$

In the context of our study, we assume a loop antenna made up of a long cylindrical conductor whose potential is calculated

$$\phi(r, \theta) = V_o + E_o \left(\frac{a^2}{r} - r \right) \quad (ii)$$

Where V_o is a constant on the surface of the conducting cylinder, E_o is the field, a is the radius of the cylinder.

Equation (i) can therefore be rearranged as

$$i\hbar \frac{\partial}{\partial t} \psi - \frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = 0 \quad (iii)$$

The langrangian density related to eqn(iii) is given as

$$\mathcal{L}_1 = \frac{1}{2} \left[\left| \frac{\partial \psi}{\partial t} \right|^2 - \frac{\hbar^2}{2m} |\nabla \psi|^2 - V|\psi|^2 \right] \quad (iv)$$

Applying the minimum coupling rule to describe the interaction of ψ with the electromagnetic field i.e.

$$\frac{\partial}{\partial t} \mapsto \frac{\partial}{\partial t} + ie\phi,$$

$$\nabla \mapsto \nabla - ieA \quad \text{where } \phi = \phi(r, \theta)$$

Eqn(iv) transforms into

$$\mathcal{L}_1 = \frac{1}{2} \left[\left| \frac{\partial \psi}{\partial t} + ie\psi\phi \right|^2 - \frac{\hbar^2}{2m} |\nabla \psi - ieA\psi|^2 - V|\psi|^2 \right] \quad (v)$$

The cylindrical conductor is included into equation (v) to account for the solenoidal nature of the conductor used for the construction. Where $r = x$

$$\mathcal{L}_1 = \frac{1}{2} \left[\left| \frac{\partial \psi}{\partial t} + ie\psi V_o + iE_o e\psi \left(\frac{a^2}{x} - x \right) \cos \omega t \right|^2 - \frac{\hbar^2}{2m} |\nabla \psi - ieA\psi|^2 - V|\psi|^2 \right] \quad (vi)$$

Applying the solution of the standing wave $\psi(x, t) = e^{iS(x,t)} T(x, t)$ in equation(6)

Where $T, S: \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}$, the lagrangian density takes the form

$$\mathcal{L}_1 = \frac{1}{2} \left\{ T_t^2 - |\nabla T|^2 - \left[\frac{\hbar^2}{2m} |\nabla S - eA|^2 + |S_t + V_o e|^2 - \left(|S_t - E_o e \left(\frac{a^2}{x} - x \right) \right|^2 - |S_t| \right) + 2E_o V_o e^2 + V \right] T^2 \right\} \quad (vii)$$

Considering the lagrangian density of the particle electromagnetic field E-H field of an electrically small transmitting loop antenna

$$\mathcal{L}_o = \frac{1}{8\pi} (|E|^2 - |H_1|^2 - |H_2|^2)$$

Where the values of electric and magnetic was adapted from Glenn(1972)

$$E = \frac{\xi \beta m}{4\pi r} \left(1 - \frac{j}{\beta r} \right) e^{-j\beta r} \sin \theta \quad (viii)$$

$$H_1 = -\frac{\mu_o \beta^2 m}{4\pi r} \left(1 - \frac{j}{\beta r} - \frac{1}{\beta^2 r^2} \right)$$

$$e^{-j\beta r} \sin \theta \quad (ix)$$

$$H_2 = \frac{\mu_o \beta^2 m}{2\pi r} \left(\frac{j}{\beta r} + \frac{1}{\beta^2 r^2} \right) e^{-j\beta r} \cos \theta \quad (x)$$

Therefore the total action of lagrangian density is given by $S = \iint \mathcal{L}_1 + \mathcal{L}_o$

Then the Euler-Lagrange equation associated to the function $S = S(T, S, \theta, A)$ gives rise to the following systems of equation

$$T_{tt} - \Delta T + \left[\frac{\hbar^2}{2m} |\nabla S - eA|^2 + |S_t + V_o e|^2 - \left(|S_t - E_o e \left(\frac{a^2}{x} - x \right) \right|^2 - |S_t| \right) + 2E_o V_o e^2 + V \right] T = 0 \quad (xi)$$

$$\frac{\partial}{\partial t} [(S_t + V_o e)T^2] - \frac{\partial}{\partial t} \left[\left(S_t + E_o e \left(\frac{a^2}{x} - x \right) \right) T^2 \right] - \frac{1}{2} \frac{\partial S_t}{\partial t} + \text{div}[(\nabla S - eA)T^2] = 0 \quad (\text{xii})$$

$$E_o e \left(\frac{a^2}{x} - x \right) \sin\theta \left(S_t - E_o e \left(\frac{a^2}{x} - x \right) \cos\theta \right) T^2 = \frac{1}{8\pi} \left[-\frac{\xi\beta^2 m}{4\pi r} \left(1 - \frac{j}{\beta r} \right) e^{-j\beta r} \cos\theta - \frac{\mu_o \beta^2 m}{4\pi r} \left(1 - \frac{j}{\beta r} - \frac{1}{\beta^2 r^2} \right) e^{-j\beta r} \cos\theta - \frac{\mu_o \beta^2 m}{2\pi r} \left(\frac{j}{\beta r} + \frac{1}{\beta^2 r^2} \right) e^{-j\beta r} \sin\theta \right] \quad (\text{xiii})$$

$$\frac{\hbar^2}{2m} (\nabla S - eA)e = 0 \quad (\text{xiv})$$

The importance of the system of equations(xi-xiv) is as follows;

(i) Find the standing waves of equations (xi-xiv) whose solution is of the form

$$T = T(x), \quad S = \omega t, \quad A = 0, \quad e = 1, \quad \emptyset = \emptyset(r, \theta), \quad \omega \in \mathbb{R}$$

(ii) To workout vital parameters required for the reconfiguration of the transmitting loop antenna.

Equations (xii) and (xiv) are identically satisfied.

Mathematical analysis on equation (xi) enables the following results

$$\nabla T + (\omega + V_o)^2 T - \left[\omega - E_o e \left(\frac{a^2}{x} - x \right) \cos\theta \right]^2 T - \omega T + 2E_o V_o T - VT = 0 \quad (\text{xv})$$

Equation (xv) is referred to as the Euler-Lagrange equation of the functional $F: H^1 X D^{1,2} \rightarrow \mathbb{R}$

$$F(T, \theta) = \frac{1}{2} \int_{\mathbb{R}^3} |\nabla U|^2 dx - \frac{(\omega + V_o)}{2} \int_{\mathbb{R}^3} T dx - \frac{\left[\omega - E_o e \left(\frac{a^2}{x} - x \right) \right]}{2} \int_{\mathbb{R}^3} |\cos\theta|^2 dx$$

Therefore $(T, \theta) \in H^1 X D^{1,2}$ is a critical point of F. The graphical outlook of the first critical point of F i.e. (θ) is shown in figure (1, 2, 3&4) below. Equation(xiii) generates three system of equations i.e equation (xvi), (xix) and (xxi) (when e=1) as shown below

$$E_o \omega \left(\frac{a^2}{x} - x \right) T^2 - E_o^2 \left(\frac{a^2}{x} - x \right)^2 T^2 \cos\theta = \frac{\mu_o \beta m j}{16\pi^2 r^2} + \frac{\mu_o m}{16\pi^2 r^3} \quad (\text{xvi})$$

The solution of equation (xvi) is given as

$$E_o^2 \left(\frac{a^2}{x} - x \right)^2 + \frac{\omega^2}{4\cos^2\theta e^{ij\beta r}} = \frac{\mu_o \beta r m j + \mu_o m}{16\pi r^3 T^2} \quad (\text{xvii})$$

$$E_o^2 \left(\frac{a^2}{x} - x \right)^2 = \frac{\mu_o \beta r m j + \mu_o m}{16\pi r^3 T^2} \quad (\text{xviii})$$

$$\frac{32\pi^2 r E_o^2 \left(\frac{a^2}{x} - x \right) T^2 e^{ij\beta r} + \beta^2 m \xi - \frac{\beta m \xi j}{r}}{\frac{\beta}{r} + \frac{m}{r^2} - \beta^2 m} = \mu_o \quad (\text{xix})$$

When $m \rightarrow \infty$

The wave impedance of free space is therefor

$$\frac{32\pi^2 r E_o^2 \left(\frac{a^2}{x} - x \right) T^2 e^{ij\beta r}}{\frac{\beta}{r}} = \mu_o \quad (\text{xx})$$

The last equation from (xiii)

$$r = \left(\frac{\mu_0 m}{2\pi(T^2 \omega E_0 \sin\theta (a^2-1)e^{j\beta} - T^2 e E_0^2 \sin\theta \cos\theta (a^2-1)e^{j\beta} - \frac{\mu_0 \beta m}{4\pi} (\cos\theta - 2\sin\theta))} \right) \quad (xxi)$$

When $e = 1, j \rightarrow \infty$

The radius of the transmitting loop antenna is therefore given as

$$r = \frac{\mu_0 m}{2\pi(T^2 \omega E_0 \sin\theta (a^2-1) - T^2 e E_0^2 \sin\theta \cos\theta (a^2-1)e^{j\beta} - \frac{\mu_0 \beta m}{4\pi} (\cos\theta - 2\sin\theta))} \quad (xxii)$$

The relationship between the radius of the transmitting loop antenna and its propagation in free space is shown in Figure(6)

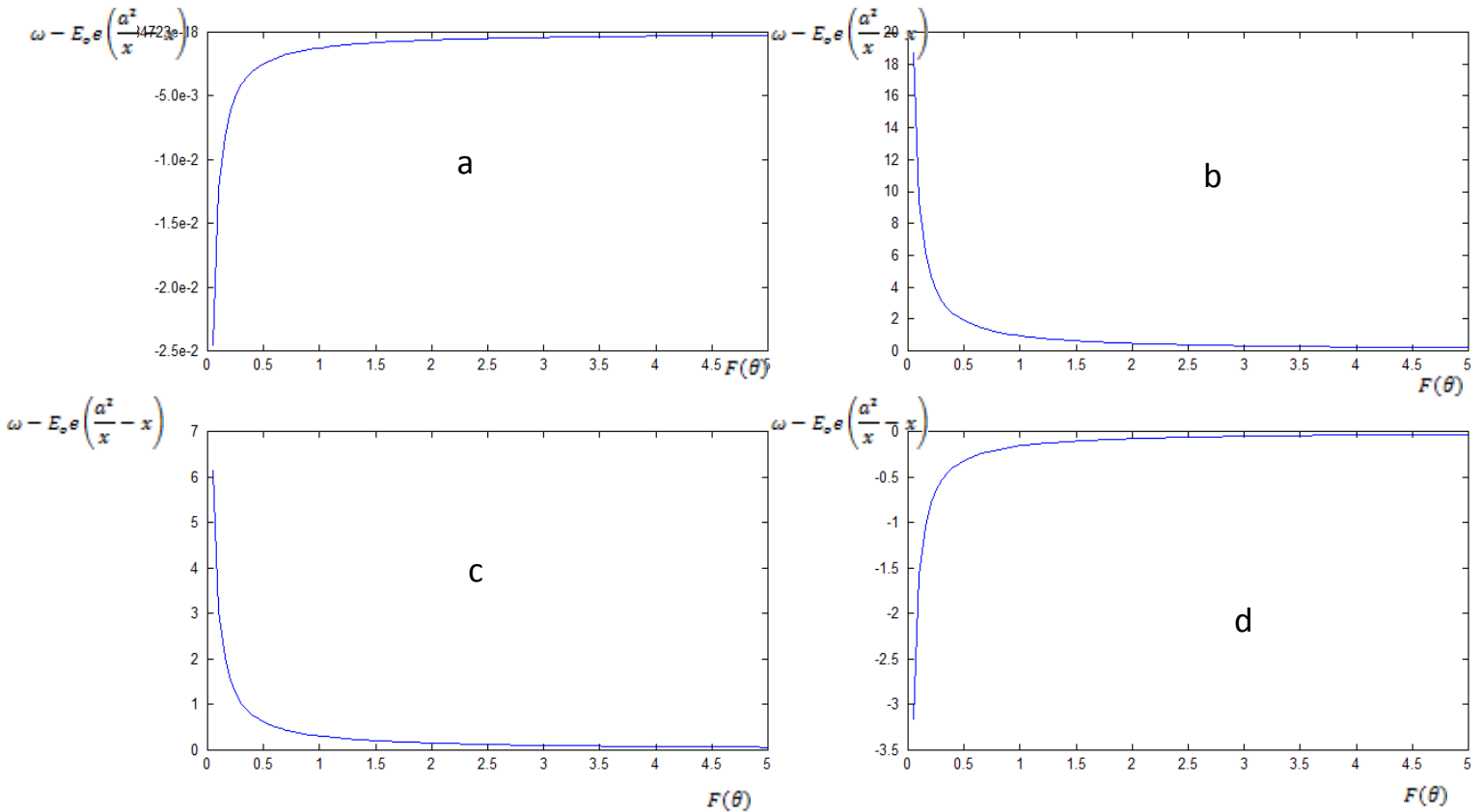


Figure (1a): The standing waves when the phase angle is 30° and functional phase ratio ranges from 0 to 5. (1b): when the phase angle is 60° and functional phase ratio ranges from 0 to 5. (1c): when the phase angle is 120° and functional phase ratio ranges from 0 to 5. (1d): when the phase angle is 150° and functional phase ratio ranges from 0 to 5. $E_0 e \left(\frac{a^2}{x} - x \right) \ll \omega$

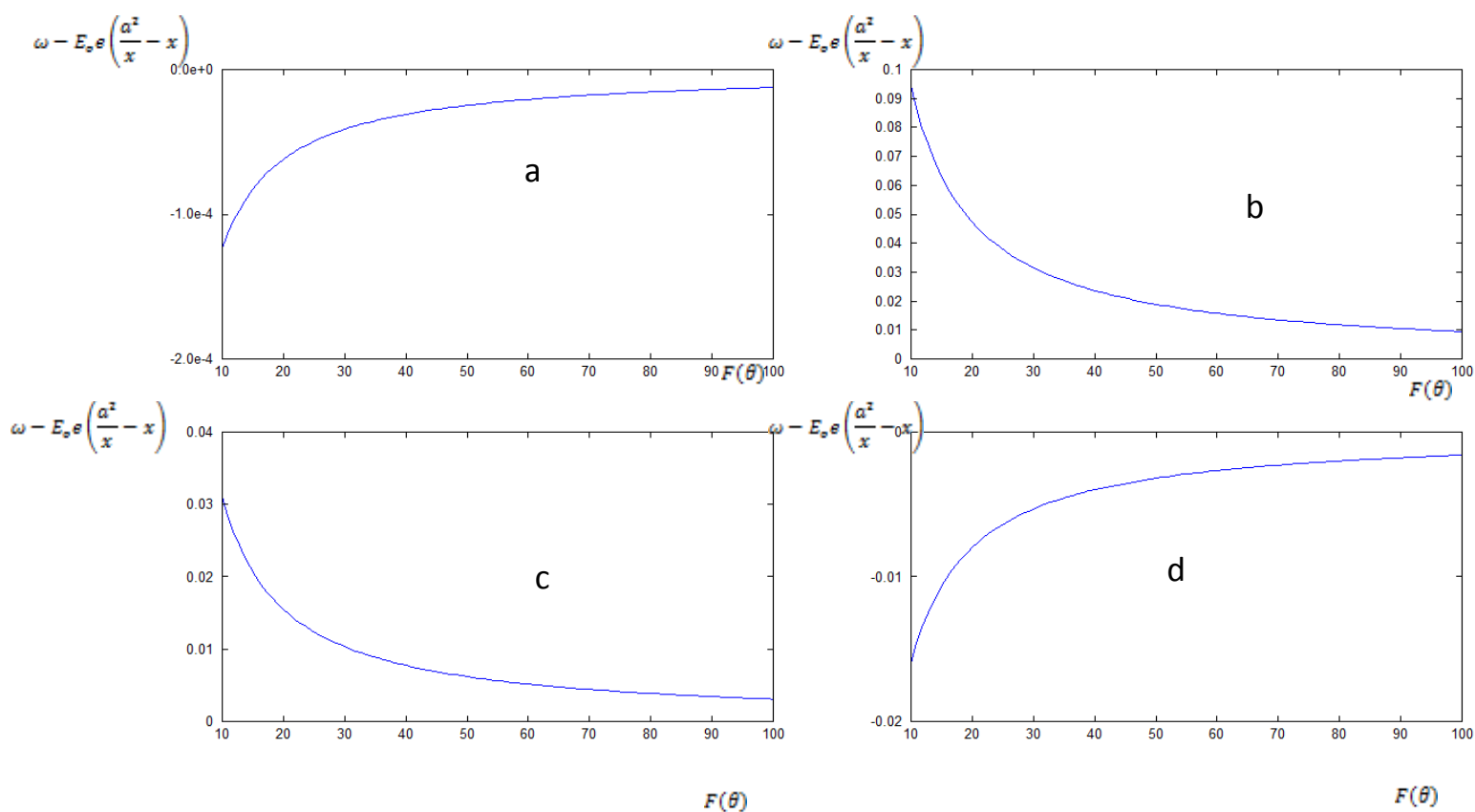


Figure (2a): The standing waves when the phase angle is 30° and functional phase ratio ranges from 10 to 100. (2b): when the phase angle is 60° and functional phase ratio ranges from 10 to 100. (2c): when the phase angle is 120° and functional phase ratio ranges from 10 to 100. (2d): when the phase angle is 150° and functional phase ratio ranges from 10 to 100. $E_o e^{\left(\frac{a^2}{x} - x\right)} \ll \omega$

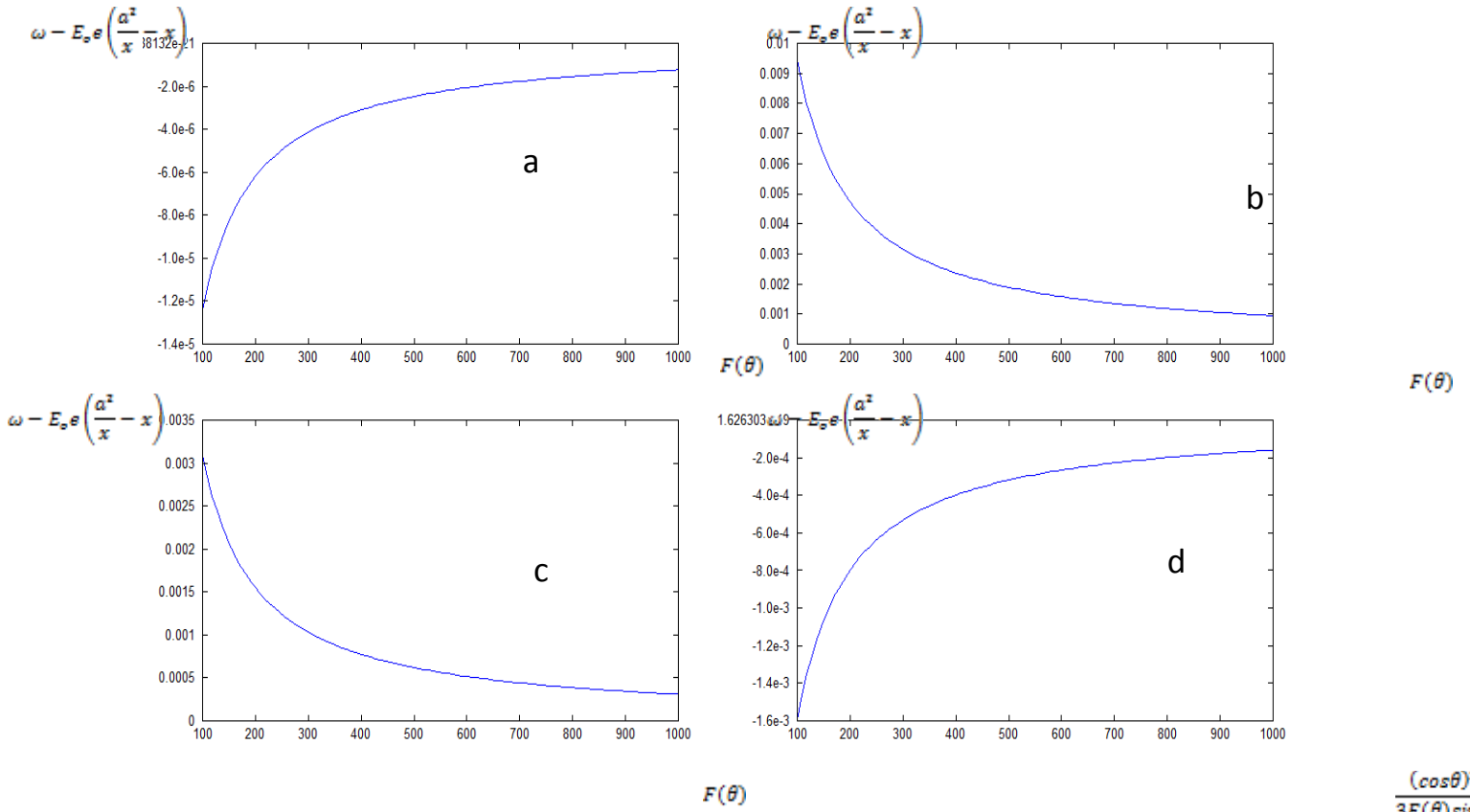


Figure (3a): The standing waves when the phase angle is 30° and functional phase ratio ranges from 100 to 1000. (3b): when the phase angle is 60° and functional phase ratio ranges from 100 to 1000. (3c): when the phase angle is 120° and functional phase ratio ranges from 100 to 1000. (3d): when the phase angle is 150° and functional phase ratio ranges from 100 to 1000. $E_\sigma e^{\left(\frac{\alpha^2}{x} - x\right)} \ll \omega$

$$\frac{(\cos\theta)^2}{3F(\theta)\sin\theta}$$

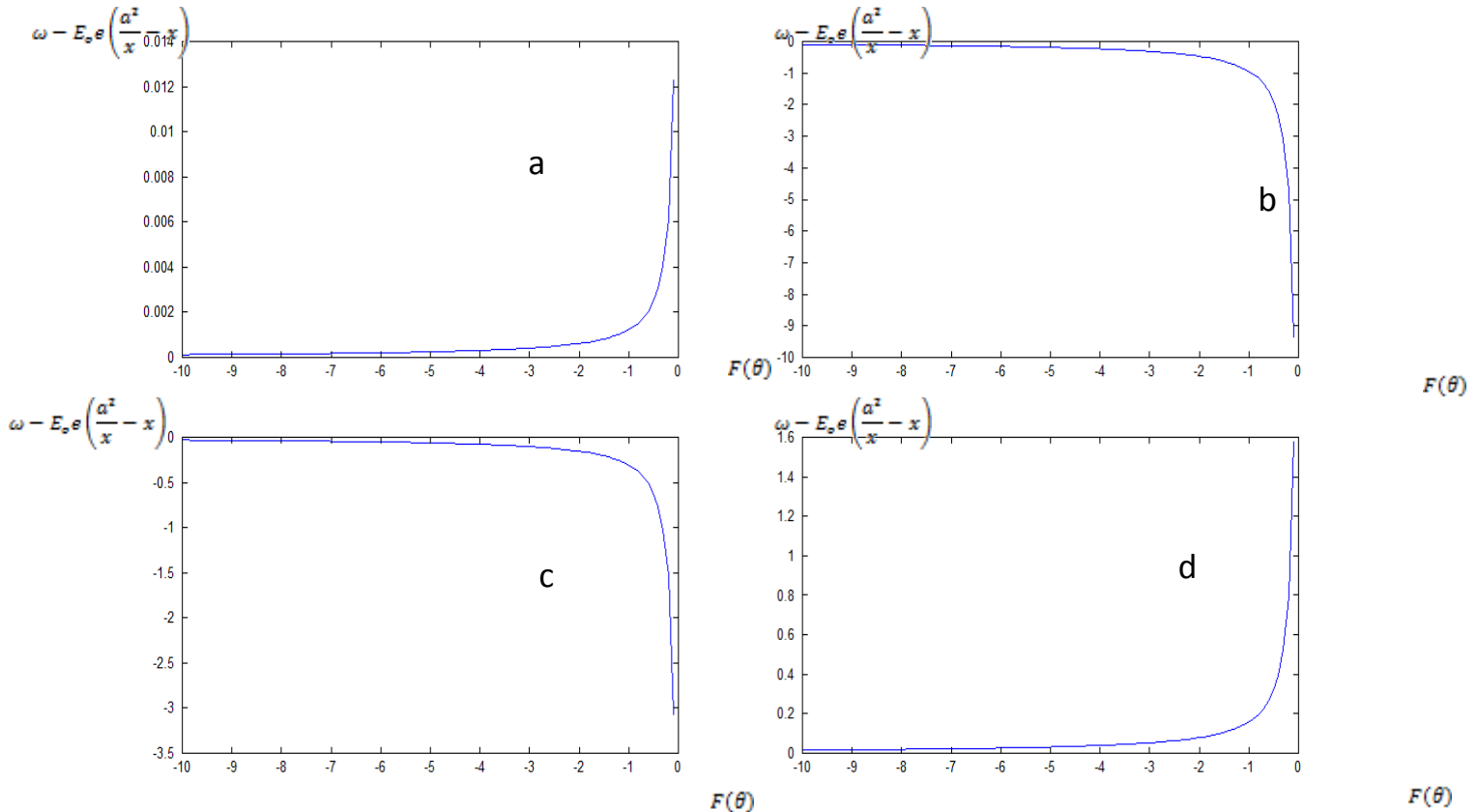


Figure (4a): The standing waves when the phase angle is 30° and functional phase ratio ranges from -10 to 0. (4b): when the phase angle is 60° and functional phase ratio ranges from -10 to 0. (4c): when the phase angle is 120° and functional phase ratio ranges from -10 to 0. (4d): when the phase angle is 150° and functional phase ratio ranges from -10 to 0. $E_o e^{\left(\frac{a^2}{x} - x\right)} \ll \omega$

The graphical outlook of the second critical point of F i.e. (T) is shown in figure (5)

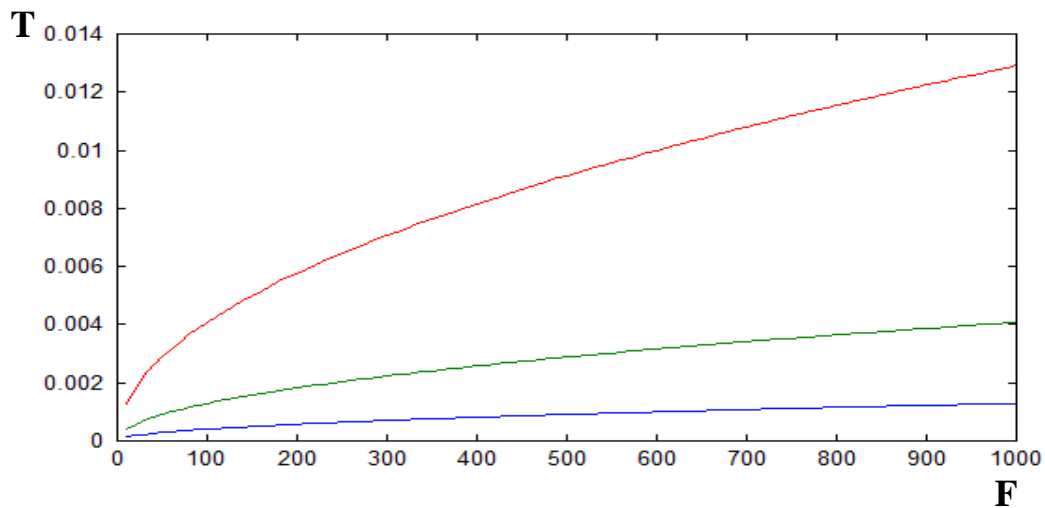


Figure 5: A graph of T against F when the $f=2.4\text{GHz}$, 240MHz and 24MHz

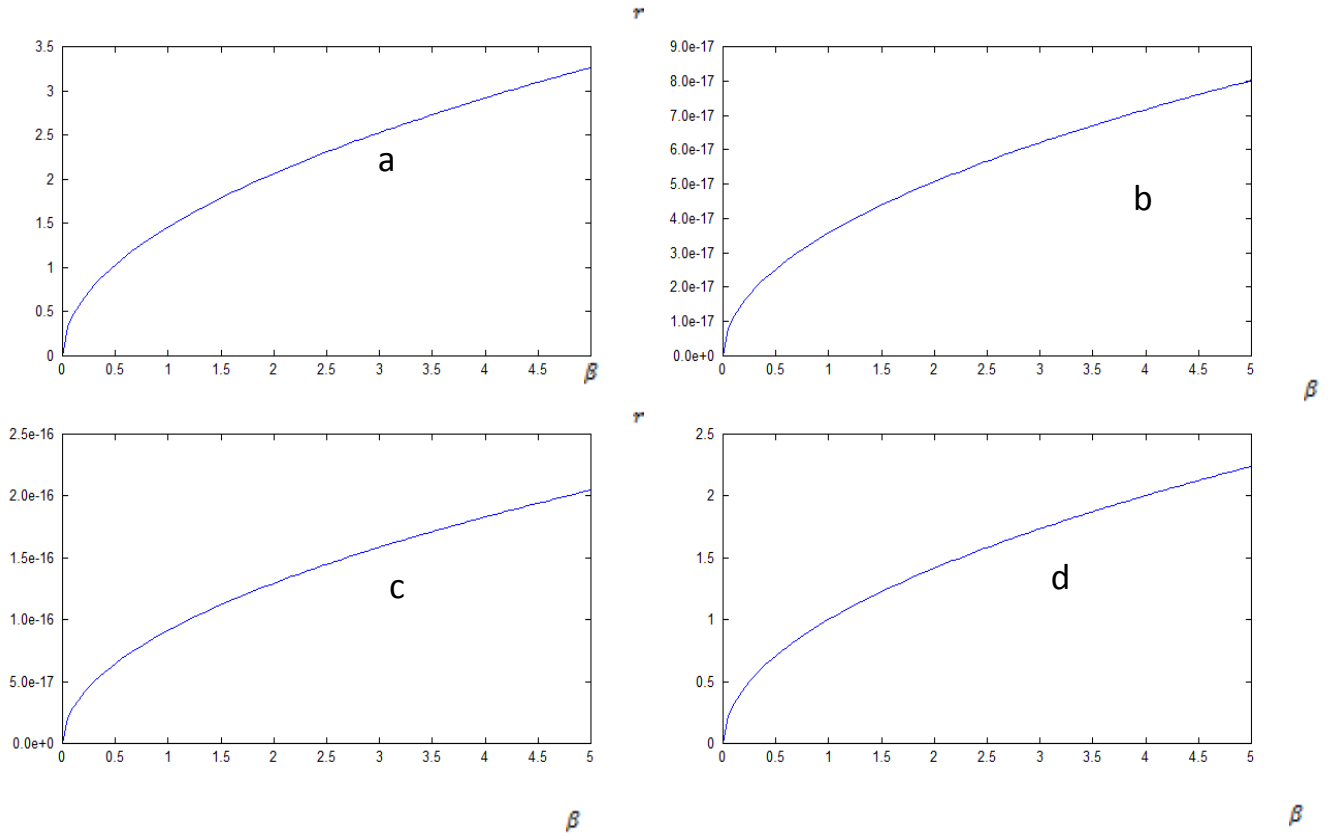


Figure6: proportionality between the radius of the loop and its propagation in free space.Fig6a: proportionality at 30° , Fig6b: proportionality at 60° , Fig6c: proportionality at 90° Fig6d: proportionality at 180°

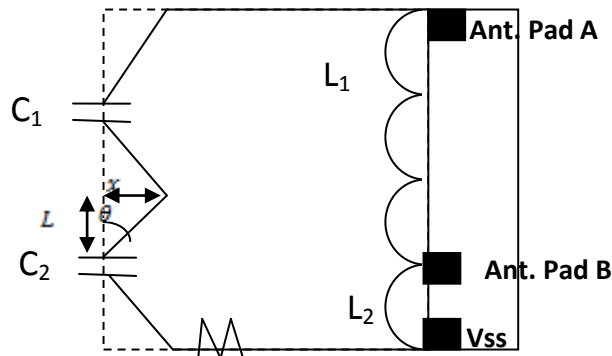
Equation (xxii) explains the size of the transmitting loop antenna

Applying equation(xvii) when $j \rightarrow \infty$, the bandwidth of the transmitting antenna is given as

$$B = \frac{\cos\theta}{\pi Q} \left(\frac{\mu_o m}{16\pi r^3 T^2} - E_o^2 \left(\frac{a^2}{x} - x \right) \right)$$

Where Q is the quality factor.

The circuitry of a small loop is such that a lumped capacitance C is sometimes placed in parallel with impedance Z to account for the distributed capacitance. Ironically, a loop with uniform current distribution would have no capacitance because no charge exists within the loop. Our objective in this section is analyzing the possible resonance frequencies in a multi-capacitor small loop circuit when there is uniform current distribution. Our theoretical loop design is shown in Figure (7)



Therefore, the total capacitance of n-number of capacitance is given as

$$C_T(x) = \frac{\text{product } C_n(x)}{\sum_{i=0}^n C_n(x)} \quad (\text{xxiii})$$

In the theoretical design of the loop antenna in figure(5), the length of the conductor (x) which is written in equation (xxiv) is transformed by applying the Legendre polynomial to calculate the influence of the number of the capacitor when the current distribution in the circuit is uniform. (See Table 2). The advantage of this design is to minimize resistive losses close to 90%.

$$x = 4\pi r \tan\left(\frac{\theta}{2}\right) \quad (\text{xxiv})$$

Result And Discussion

From equation (xxii), it has been established that the increase in the diameter of the loop, it begins to diverge from being a "magnetic dipole" and the pattern and polarization can change. In other words, larger the loop diameter, the greater the efficiency as shown in Figure (5a & b). Theoretically, it has been proved that equation (xxii), is true only if the phase angle is 30° and 180°. The angle of elevation was resolved –observing figure (1-4), the best position of the transmitting loop antenna was found to be 30° when the function of the standing wave ranges from 10 – 1000. At this position, the loop antenna transmits at a good level of megahertz. When the standing wave function is negative (as shown in figure (4)), there is a reversal which result to losses which transforms to heat in conductor e.g. braided

conductor. Also, the heat loss is related to the angle of elevation of the loop antenna. The losses in the transmitting loop antenna were analyzed under a theoretical construction whose tuning capacitor is dependent on the length and nature of conductor. Figure (6) explains the relationship between radius of transmitting loop antenna and the propagation in free space. At 30° and 180°, (the tilting angle of the antenna) there exist a realistic relation which supports figure (3a). Figure (5), reveals the efficiency of the transmitting loop antenna at three different frequencies i.e. 2.4GHz, 240MHz and 24MHz. The function of the standing wave when it ranges from 0 – 1000 favors the reconfiguration of the transmitting loop antenna to operate at desired frequency.

Conclusion

In conclusion, a small transmitting loop antenna will perform well when mounted vertically at an elevation angle of 30° when the standing wave function is between 100-1000. The transmitting loop at these position exhibit useful sharp nulls in the azimuth pattern. The loops were theoretically built to reduce radiation losses. It was observed that the higher the radiation resistance, the higher the efficiency of the transmitting loop antenna. The angle elevation can account for the loss which transforms to heat in the conductor.

Figure(7) Two inductor and two inductor loop circuit

The equivalent capacitance for n-number of capacitors is shown in Table(1)

No of capacitors	Equivalent capacitance(F)
1	C_1
2	$\frac{C_1 C_2}{(C_1 + C_2)}$
3	$\frac{C_1 C_2 C_3}{C_1 + C_2 + C_3}$
n	$\frac{C_1 C_2 C_3 \dots \dots C_n}{C_1 + C_2 + C_2 + \dots \dots C_3}$

Table(1) Equivalent capacitance of n capacitors

Number of capacitors	Total capacitance(F)	Corresponding resonant frequencies
1	$\frac{1}{\sqrt{2-2x}}$	$\left(\frac{\sqrt{2-2x}}{2\pi L}\right)^{0.5}$
2	$\frac{x}{\sqrt{2-2x}}$	$\left(\frac{\sqrt{2-2x}}{2\pi x L}\right)^{0.5}$
3	$\frac{3x^2-1}{2\sqrt{2-2x}}$	$\left(\frac{2\sqrt{2-2x}}{2\pi L(3x^2-1)}\right)^{0.5}$
4	$\frac{5x^3-3x}{2\sqrt{2-2x}}$	$\left(\frac{2\sqrt{2-2x}}{2\pi L(5x^3-3x)}\right)^{0.5}$

Table2: Influence of the number of the capacitor when the current distribution in the circuit is uniform when $x < 1$.

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