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MODELLING ENGINEERING NETWORKS BY USING NODAL AND MESH INCIDENCE MATRICES

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ABSTRACT

Matrices have a lot of applications in all areas of human endeavour. In fact, the use of matrices in modelling engineering networks has made matrices an essential part of research in the fields of science and engineering. In this paper, we used nodal and mesh incidence matrices to construct electrical networks. This thereby results in the characterization of connections in electrical networks and modelling of net of roads connecting cities.

Keywords: Nodal incidence matrix, Mesh incidence matrix, Mathematical modelling, Electrical networks, Nodes, Meshes.

2010 Mathematics Subject Classification: 97M50

INTRODUCTION

A matrix is a rectangular array of real or complex numbers (or elements), Ajayi and Joseph (2009), Riley *et al.* (2002) and Stroud (1995). An incident matrix is a matrix that describes the topology of a network, Dass and Verma (2011) and Oke (2008). It shows the relationship between two classes of objects in the network. If the first class is *X* and the second is *Y*, the matrix has one row for each element of *X* and one column for each element of *Y*. The entry in row *i* and column j is 1or -1 if *i* and j are related or incident and 0 if they are not, Dada (2010), Kreyszig (1987) and Oke (2012).

Incidence matrices are mostly used in graph theory. In graph theory, an undirected graph has two kinds of incidence matrices. These are oriented (or directed) and unoriented (or undirected) incidence matrices, Gross and Yellen (2006) and Diestel (2005).

The incidence matrix of an oriented (or directed) graph is an m x n matrix A_{ij} where

$$A_{ij} = \begin{cases} +1 \; if \; edge \; j \; leaves \; vertex \; i \\ -1 \; if \; edge \; j \; enters \; vertex \; i \\ 0 \qquad\qquad\qquad otherwise \end{cases}$$

m and n are the number of vertices and edges respectively, Kreyszig (1987) and Binoy (2009). It is to be noted that opposite sign convention may also be used for this purpose depending on what we want to achieve in the design and construction, Dada (2010) and Kreyszig (1987).
The incidence matrix of an unoriented (or undirected) graph is an m x n matrix B_{ij} where

$$B_{ij} = \begin{cases} +1 \; if \; vertex \; i \; and \; edge \; j \\ \qquad are \; indented \\ 0 \qquad\qquad otherwise \end{cases}$$

m and n are the number of vertices and edges respectively, Kreyszig (1987) and Binoy (2009).

The column of an incidence matrix can only have at most two non-zero entries because the edges of ordinary graphs can only have two vertices (one at each end). Although, a hypergraph can have multiple vertices assigned to one edge; thus, the general case describes a hypergraph, Dada (2010), Kreyszig (1987) and Gupta (2009).

MATERIALS AND METHODS

In this paper, we would form nodal incidence matrices from electrical networks by defining our nodal incidence matrix as

$$N_{ij} = \begin{cases} +1 \qquad\qquad if \; branch \; i \; leaves \; node \; j \\ -1 \qquad\qquad if branch \; i \; enters \; node \; j \\ 0 \; if \; branch \; i \; does \; not \; touch \; node \; j \end{cases}$$

Opposite sign convention may also be used for this purpose depending on what we want to achieve in the design and construction, Dada (2010) and Kreyszig (1987).

We would also form mesh incidence matrices from electrical networks by defining our mesh incidence matrix as

$$m_{jk} = \begin{cases} +1 \; if \; branch \; k \; is \; in \; mesh \;\; j \;\; and \; has \; the \\ \qquad same \; orientation \\ -1 \; if \; branch \; k \; is \; in \; mesh \;\; j \;\; and \; has \\ \qquad opposite \; orientation \\ 0 \; if \; branch \; k \; is \; not \; in \; mesh \;\; j \end{cases}$$

We can also use opposite sign convention for this purpose depending on what we want to achieve in the design and construction.
Electrical networks would also be constructed from nodal and mesh incidence matrices by following a reverse operation for the formation of nodal and mesh incidence matrices from the networks.

RESULTS AND DISCUSSIONS

Example 1

Figure 1 represents an electrical network having four branches and three nodes. One of the nodes is a reference node. All the nodes are numbered except the reference node. We also number and direct the branches.

We can now define our nodal incidence matrix as

$$N_{ij} = \begin{cases} +1 & if\ branch\ i\ leaves\ node\ j \\ -1 & if branch\ i\ enters\ node\ j \\ 0\ if\ branch\ i\ does\ not\ touch\ node\ j & \end{cases}$$

To form the nodal incidence matrix, we will need a computation table to relate the nodes and branches. This is contained in Table 1.

The nodal incidence matrix, constructed from table 1, is now a 2x4 matrix given by

$$\mathrm{N_{ij}} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ -1 & 0 & -1 & -1 \end{pmatrix}$$

To construct the electrical network in figure 1 from the nodal incidence matrix above, we will follow the reverse operation of the steps above.

From the nodal incidence matrix, we will construct table 1 to show us clearly how the nodes and branches are related. From table 1, we can see that branch 1 enters node 2 from the reference node (which is marked as x). The sketch is as shown in figure 2(a). Branch 2 leaves node 1 and enters the reference node. The sketch is as shown in figure 2(b). Branch 3 leaves node 1 and enters node 2 and branch 4 also leaves node 1 and enters node 2. We have the sketches in figures 2(c) and 2(d) respectively.

Putting the sketches in figure 2 together, we have the electrical network in figure 1.

Example 2

Now, let us consider a net of one way street as shown in figure 3. The directions are as indicated by the arrows.

To determine the analog of the nodal incidence matrix, we will also need a computation table to relate the nodes and branches. This is shown in Table 2.

The nodal incidence matrix, constructed from table 2, is now a 3x5 matrix given by

$$N_{ij} = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{pmatrix}$$

To construct the electrical network in figure 3 from the nodal incidence matrix above, we will follow the reverse operation of the steps above. From the nodal incidence matrix, we will construct table 2 to show us the nodes and branches clearly. From table 2, we can see that branch 1 enters node 1 from the reference node (which is marked as x). The sketch is as shown in figure 4(a). Branch 2 leaves node 1 and enters the reference node. The sketch is as shown in figure 4(b). Branch 3 leaves node 2 and enters node 3, branch 4 leaves node 3 and enters node 2 and branch 5

enters node 2 from the reference node. We have the sketches in figures 4(c), 4(d) and 4(e) respectively.
Putting the sketches in figure 4 together, we have the electrical network representing net of one – way street in figure 3.

Example 3

Figure 5 represents an electrical network having six branches and four meshes. The meshes are numbered and directed in an arbitrary fashion. We also number and direct the branches.
Let us define our mesh incidence matrix as

$$m_{jk} = \begin{cases} +1 \; if \; branch \; k \; is \; in \; mesh \;\; j \;\; and \; has \; the \\ \qquad same \; orientation \\ -1 \; if \; branch \; k \; is \; in \; mesh \;\; j \;\; and \; has \\ \qquad opposite \; orientation \\ 0 \; if \; branch \; k \; is \; not \; in \; mesh \;\; j \end{cases}$$

To form the mesh incidence matrix, we will need a computation table to relate the meshes and branches. This is given by Table 3
The mesh incidence matrix is now a 4x6 matrix given by

$$M_{jk} = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

To construct the electrical network in figure 5 from the mesh incidence matrix above, we will follow the reverse operation of the steps above.

From the mesh incidence matrix, we will construct table 3 to show us the meshes and branches clearly. From table 3, we can see that branches 4, 5 and 6 are in mesh 1 and have the same orientation with it. The sketch therefore is as given in figure 6(a). Branch 1 is in mesh 2 with the same orientation, but branches 3 and 5 are in mesh 2 with opposite orientation. The sketch is as shown in figure 6(b). Branches 2 and 4 are in mesh 3 with the same orientation but branch 3 is in mesh 3 with opposite orientation. The sketch of this is as shown in figure 6(c). Branches 1 and 6 are in mesh 4 with the same orientation but branch 2 is in mesh 4 with opposite orientation. The sketch is in figure 6(d).
Putting the sketches in figure 6 together, we have the electrical network in figure 5.

Example 4

Let us consider another electrical network having six branches and four meshes. The meshes are numbered and directed in an arbitrary fashion. We also number and direct the branches as before. The electrical network is as shown in figure 7.
To form the mesh incidence matrix, we will need a computation table to relate the meshes and branches. This is given by Table 4
The mesh incidence matrix, as constructed from table 4, is now a 4x6 matrix given by

$$M_{jk}=\begin{pmatrix}0 & -1 & -1 & 0 & -1 & 0\\ 1 & -1 & 0 & 0 & 0 & -1\\ -1 & 0 & -1 & -1 & 0 & 0\\ 0 & 0 & 0 & -1 & 1 & -1\end{pmatrix}$$

The electrical network in figure 7 can also be constructed by using the same procedure we applied for the construction of the network in figure 6.

Looking at the results from problems 1 to 4, we can easily see how to constructed nodal and mesh incidence matrices from electrical networks. The results also show how to model engineering networks form nodal and mesh incidence matrices by following the reverse operations.

CONCLUSION

In this paper, we have considered how to derive nodal and mesh incidence matrices from electrical networks. The reverse operation of these steps now leads to modelling engineering networks using nodal and mesh incidence matrices. This reverse operation is very useful for engineers in modelling networks for designs and constructions.

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FIGURES

Figure-1. An Electrical Network

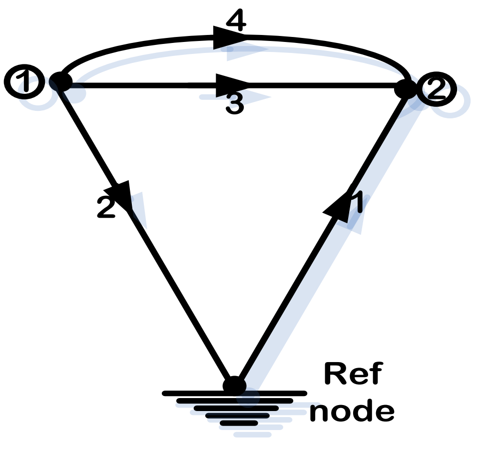


Figure-2. Sketches of Connections between Nodes and Branches for the Construction of an Electrical Network

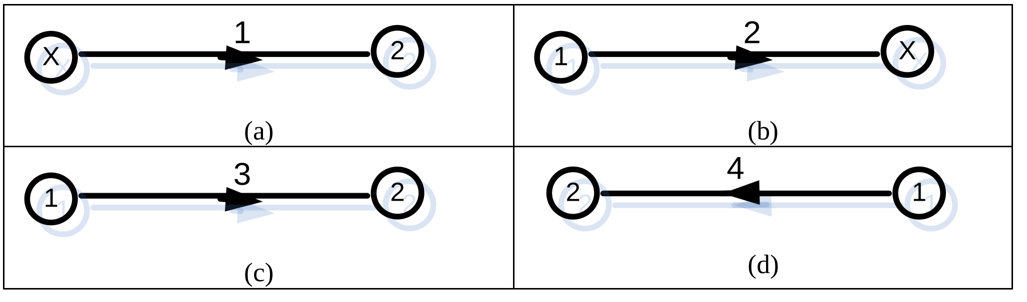


Figure-3. Net of one-way street

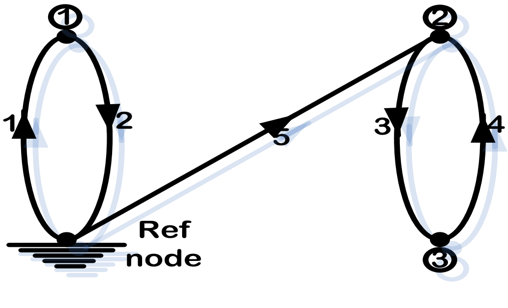
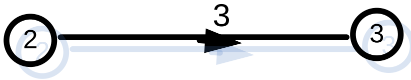
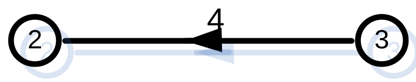


Figure-4. Sketches of Connections between Nodes and Branches for the Construction of a Network on Net of one-way street.

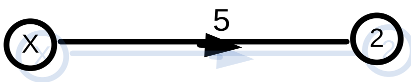
(a) (b)



(c)



(d)



(e)

Figure-5. An Electrical Network

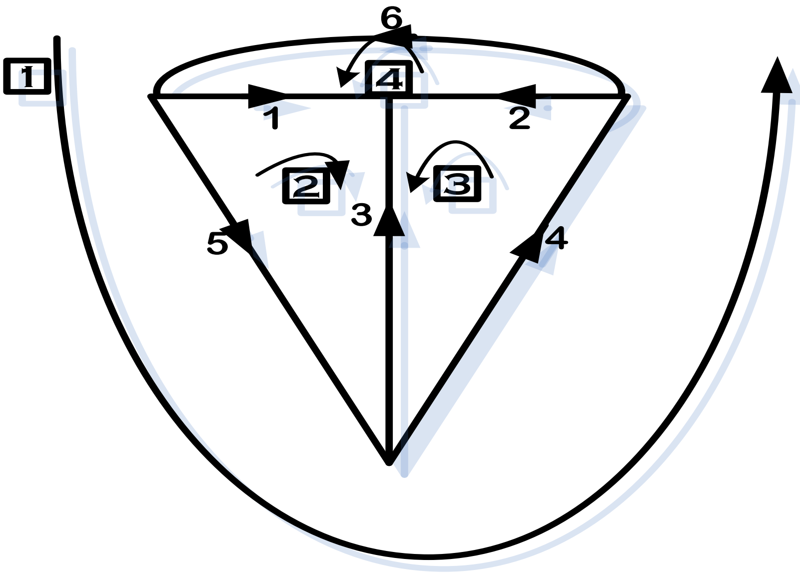
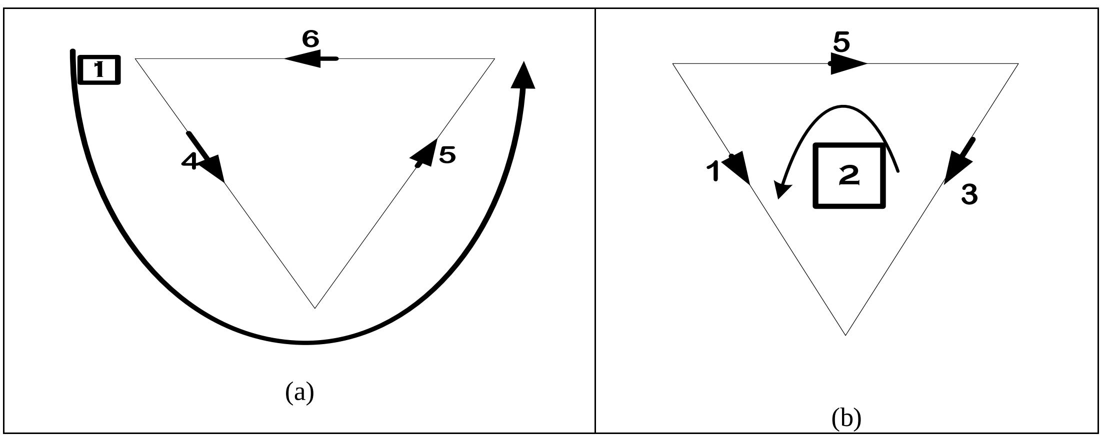


Figure-6. Sketches of Connections between Meshes and Branches for the Construction of an Electrical Network



(a)

(b)

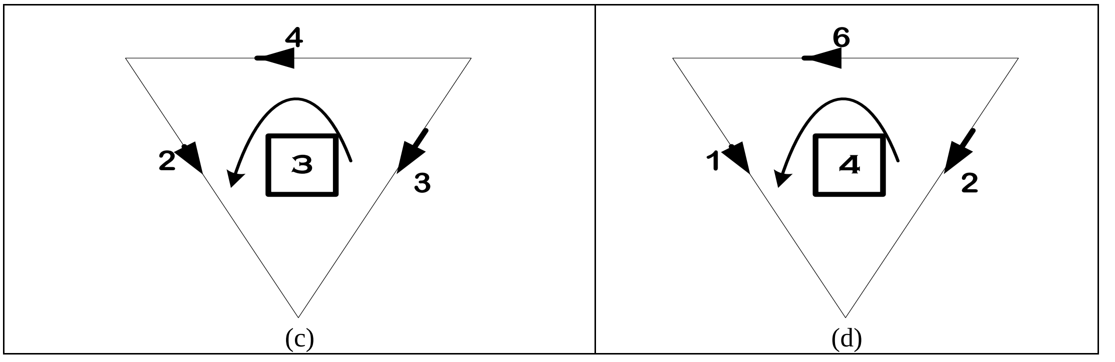
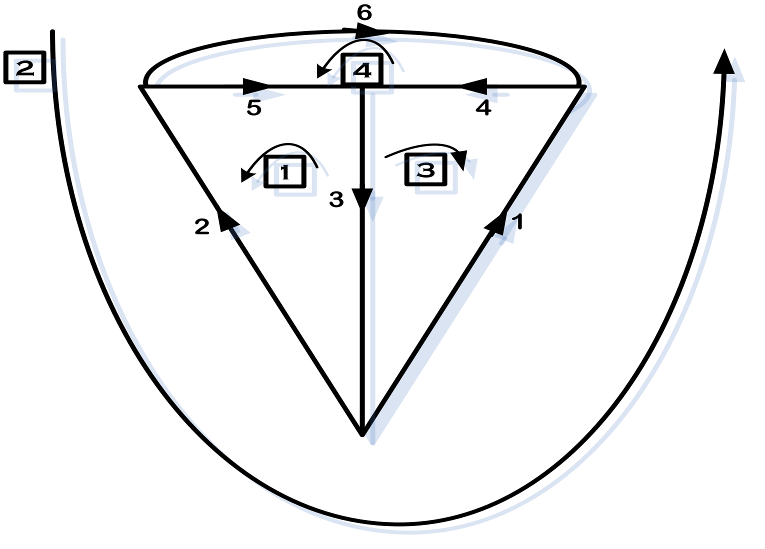


Figure-7. An Electrical Network



TABLES

Table-1. Computation Table for the Electrical Network in Figure 1

	Branch 1	Branch 2	Branch 3	Branch 4
Node 1	0	1	1	1
Node 2	-1	0	-1	-1

Table-2. Computation Table for the Net of one-way street in Figure 3

	Branch 1	Branch 2	Branch 3	Branch 4	Branch 5
Node 1	-1	1	0	0	0
Node 2	0	0	1	-1	-1
Node 3	0	0	-1	1	0

Table-3. Computation Table for the Electrical Network in Figure 5

	Branch 1	Branch 2	Branch 3	Branch 4	Branch 5	Branch 6
Mesh 1	0	0	0	1	1	1

Mesh 2	1	0	-1	0	-1	0
Mesh 3	0	1	-1	1	0	0
Mesh 4	1	-1	0	0	0	1

Table-4. Computation Table for the Electrical Network in Figure 7

	Branch 1	Branch 2	Branch 3	Branch 4	Branch 5	Branch 6
Mesh 1	0	-1	-1	0	-1	0
Mesh 2	1	-1	0	0	0	-1
Mesh 3	-1	0	-1	-1	0	0
Mesh 4	0	0	0	-1	1	-1