



## A NEW INDUCED BIDIRECTIONAL ASSOCIATIVE FUZZY COGNITIVE DYNAMICAL SYSTEM

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### ABSTRACT

*In this paper, we introduce a new Fuzzy bimodel called Induced Bidirectional Associative Fuzzy Cognitive Maps ( IBAFCM ). We also state the advantages of this system.*

**Keywords:** FCM, BAM, BAFCM, IBAFCM , Fixed point, Limit cycle , Hidden pattern.

### 1. INTRODUCTION

Neural networks and fuzzy systems estimate input-output functions. Both are trainable dynamical systems. Neural networks based fuzzy modeling is a well established area in Computational Intelligence. Fuzzy models are mathematical tools introduced by Zadeh [1] to study and analyze political decisions, neural networks, social problems etc., Using the concepts of neural networks and fuzzy logic Bart Kosko proposed many more models [2, 3]. These models provide a mathematical strength to capture the uncertainties associated with human cognitive processes, such as thinking and reasoning. These models are well suited to get a clear representation of the knowledge to support decision-making process and assist in the area of computational intelligence, which involves the application of soft computing methodologies even though the given inputs are vague, uncertain, ambiguous, imprecise, incomplete, inconsistent, redundant and even contradictory in nature [4].

Among the various fuzzy models, some of the important models which have been used to study these problems are Fuzzy Associative Memories ( FAM ), Fuzzy Cognitive Maps ( FCM ), Fuzzy Relational Maps ( FRM ), Fuzzy Cognitive Relational Maps (FCRM) and Bi Associative Memories ( BAM ). The BAM model was introduced by Bart Kosko [2, 3]. Mostly, Fuzzy models have been used to solve many real world problems in science and engineering which includes fuzzy expert system and fuzzy control.

In recent days, many engineering and mathematics researchers are trying to apply fuzzy models to non engineering fields such as social sciences and humanities. Many researchers have used these models to study and analyze the problems such as Rag Pickers problem, AIDS problem

and transportation problem, school dropout problem. Balasangu, et al. [5, 6] have studied the Agriculture labours problem. Yet already engineers have successfully applied fuzzy systems in many commercial areas. It is used to model several types of problems varying from gastric appetite behavior, popular political developments etc. It is also used to model in robotics like plant control. This fuzzy model works on the opinion of experts.

To give more importance to the respondent's feelings and expectations, and to give proper representation to the unsupervised data, in 2005, a new model called Induced Fuzzy Cognitive Maps(IFCM) has been introduced in the literature [7].

A new fuzzy bimodel called Bidirectional Associative Fuzzy Cognitive Maps (BAFCM) was introduced by Thirusangu, et al. [8]. This new bimodel can act simultaneously as a Bidirectional Associative Memories (BAM) as well as Fuzzy Cognitive Maps (FCMs). This model is best suited when the study under consideration is one with an unsupervised data. This model can give a bihidden pattern.

In this paper, we introduce a new Fuzzy bimodel called Induced Bidirectional Associative Fuzzy Cognitive Maps (IBAFCM) and state its advantages.

## 2. DESCRIPTION OF THE BAM MODEL

A group of neurons forms a field. Neural networks contain many fields of neurons. Suppose  $F_x$  denotes a neuron field which contains  $n$ -neurons and  $F_y$  denotes a neuron field which contains  $p$  – neurons. The neuronal dynamical system is described by a system of first order differential equations that govern the time evolution of the neuronal activation

$$\dot{x}_i = g_i(X, Y, \dots)$$

$$\dot{y}_j = h_j(X, Y, \dots)$$

where  $x_i$  and  $y_j$  denote, respectively, the activation time function of the  $i^{\text{th}}$  neuron in  $F_x$  and the  $j^{\text{th}}$  neuron in  $F_y$ . The over dot denotes time differentiation,  $g_i$  and  $h_j$  are some functions of  $X$  and  $Y$  etc., where

$$X(t) = (x_1(t), \dots, x_n(t))$$

$$Y(t) = (y_1(t), \dots, y_p(t))$$

define the state of the neuronal dynamical system at time  $t$ .

Let us suppose that the field  $F_x$  with  $n$ -neurons is synoptically connected to the field  $F_y$  with  $p$ -neurons. Let  $(m_{ij})$  be a synapse (junction) from the  $i^{\text{th}}$  neuron in  $F_x$  and the  $j^{\text{th}}$  neuron in  $F_y$ ,  $m_{ij}$  can be positive, negative or zero ( $1 \leq i \leq n$  and  $1 \leq j \leq p$ ). The synaptic matrix  $M$  is an  $n \times p$  matrix of real numbers whose entries are synaptic efficacies  $m_{ij}$ . The matrix  $M$  or the network describes the forward projection from neuronal field  $F_x$  to the neuronal field  $F_y$ . Similarly, a  $p \times n$  synaptic matrix  $N$  or the network describes the backward projection from  $F_y$  to  $F_x$ . The network is

said to be bi-directional network if  $M = N^T$  and  $N = M^T$ . When the activation dynamics of the neuronal fields  $F_x$  and  $F_y$  lead to the overall stable behaviour, the bi-directional networks are called as Bi-directional Associative Memories or BAM.

An additive model is defined by a system of  $n + p$  coupled first-order differential equations that interconnects the fields  $F_x$  and  $F_y$  through the constant synaptic matrices  $M$  and  $N$  described earlier.

$$\begin{aligned} \dot{x}_i &= -A_i x_i + \sum_{j=1}^p S_j(y_j) n_{ji} + I_i \\ \dot{y}_j &= -A_j y_j + \sum_{i=1}^n S_i(x_i) m_{ji} + J_j \end{aligned}$$

$S_i(x_i)$  and  $S_j(y_j)$  denote, respectively, the signal function of the  $i^{\text{th}}$  neuron in the field  $F_x$  and the signal function of the  $j^{\text{th}}$  neuron in the field  $F_y$ . Discrete additive activation models correspond to neuron with threshold signal functions. The neurons can assume only two values, ON and OFF, where ON represents the signal value +1 and OFF represents 0 or -1 (-1 when the representation is bipolar).

At each moment, each neuron can randomly decide whether to change state, or whether to emit a new signal given its current activation. The BAM is a non-adaptive additive bivalent neural network.

In real life problems the entries of the constant synaptic matrix  $M$  depend upon the experts opinion or the feelings of the investigator. The synaptic matrix is given a weightage according to the experts' opinion or their feelings.

If  $x \in F_x$  and  $y \in F_y$ , the forward projection from  $F_x$  to  $F_y$  is defined by the matrix  $M$  as

$$\{F(x_i, y_j)\} = (m_{ij}) = M, 1 \leq i \leq n, 1 \leq j \leq p.$$

The backward projections are defined by the matrix  $M^T$  as

$$\{F(y_j, x_i)\} = (m_{ji}) = M^T, 1 \leq i \leq n, 1 \leq j \leq p.$$

A BAM system ( $F_x, F_y, M$ ) is bi-directionally stable if all inputs converge to fixed point equilibria. The change of the state of the system leads to a fixed point or a limit cycle. The bi-directional stability is a dynamic equilibrium. The same signal information flows back and forth in a bi-directional fixed point.

Let us suppose that  $A$  denotes a binary  $n$ -vector and  $B$  denotes a binary  $p$ -vectors. Let  $A$  be the initial input to the BAM system. Then the BAM equilibrates to a bi-directional fixed point ( $A_f, B_f$ ) as

$$\begin{aligned}
 &A \rightarrow M \rightarrow B \\
 &A' \leftarrow M^T \leftarrow B \\
 &A' \rightarrow M \rightarrow B' \\
 &A'' \leftarrow M^T \leftarrow B' \\
 &\dots \\
 &A_f \rightarrow M \rightarrow B_f \\
 &A_f \leftarrow M^T \leftarrow B_f
 \end{aligned}$$

where  $A', A'', \dots$  and  $B', B'', \dots$  represent intermediate or transient signal state vectors between  $A$  and  $A_f$  and  $B$  and  $B_f$  respectively. The fixed point of a bi-directional system is time dependent. The fixed point for the initial input vectors can be attained at different times. Based on the synaptic matrix  $M$  which is developed by the expert's opinion or their feelings, the time at which bi-directional stability is attained also varies accordingly.

### 3. FUZZY COGNITIVE MAPS

Fuzzy Cognitive Maps can be described by a directed graph. The concepts are taken as nodes. If there is a relationship between the attributes, we draw an edge between them in the following way. The directed edge  $e_{ij}$  from casual concept  $C_i$  to concept  $C_j$  measures how much  $C_i$  causes  $C_j$ . The time varying concept function  $C_i(t)$  measures the non-negative occurrence of some fuzzy event, perhaps the strength of a political sentiment, historical trend or Military objective. If increase (or decrease) in one concept leads to increase (or decrease) in another, then we give the value 1. If there exists no relation between two concepts the value 0 is given. If increase (or decrease) in one concept decreases (or increases) another, then we give the value -1.

#### 3.1. Definition

When the nodes of the FCM are fuzzy sets then they are called as fuzzy nodes. The FCMs with edge weights or causalities from the set  $\{-1, 0, 1\}$ , are called simple FCMs.

#### 3.2. Definition

Consider the nodes/ concepts  $C_1, C_2, \dots, C_n$  of the FCM. Suppose the directed graph is drawn using edge weight  $e_{ij} \in \{0, 1, -1\}$ . The matrix  $M$  be defined by  $M = (e_{ij})$  where  $e_{ij}$  is the weight of the directed edge  $C_i C_j$ .  $M$  is called the adjacency matrix of the FCM, also known as the connection matrix of the FCM. It is important to note that all matrices associated with an FCM are always square matrices with diagonal entries as zero.

Let  $C_1, C_2, \dots, C_n$  be the nodes of an FCM.  $A = (a_1, a_2, \dots, a_n)$  where  $a_i \in \{0, 1\}$ .

The above vector  $A$  is called the instantaneous state vector and it denotes the ON-OFF position of the node at an instant.

$$a_i = \begin{cases} 0 & \text{if } a_i \text{ is OFF} \end{cases}$$

1 if  $a_i$  is ON , for  $i = 1, 2, \dots, n$ .

**3.3. Definition**

Let  $C_1, C_2, \dots, C_n$  be the nodes of an FCM. Let  $\overline{C_1C_2}, \overline{C_2C_3}, \overline{C_3C_4}, \dots, \overline{C_iC_j}$  be the edges of the FCM ( $i \neq j$ ). Then the edges form a directed cycle. An FCM is said to be cyclic if it possesses a directed cycle. An FCM is said to be acyclic if it does not possess any directed cycle. An FCM with cycles is said to have a feedback. When there is a feedback in an FCM, i.e., when the casual relations flow through a cycle in a revolutionary way, the FCM is called a dynamical system. Let  $C_1C_2, C_2C_3, \dots, C_{n-1}C_n$  be a cycle. When  $C_i$  is switched on and if the causality flows through the edges of a cycle and if it again causes  $C_i$ , we say that the dynamical system goes round and round. This is true for any node  $C_i$ , for  $i = 1, 2, \dots, n$ . The equilibrium state for this dynamical system is called the hidden pattern. If the equilibrium state of a dynamical system is a unique state vector, then it is called a fixed point. If the FCM settles down with a state vector repeating in the form  $A_1 \rightarrow A_2 \rightarrow A_3 \rightarrow A_4 \rightarrow \dots \rightarrow A_1$ , then this equilibrium is called a limit cycle.

**3.4. Example**

Consider an FCM with  $C_1, C_2, \dots, C_n$  as nodes. For example let us start the dynamical system by switching on  $C_1$ . Let us assume that the FCM settles down with  $C_1$  and  $C_n$  as ON. That is, the state vector remains as  $(1, 0, 0, \dots, 0, 1)$ . This state vector  $(1, 0, 0, \dots, 0, 1)$  is called the fixed point.

**4. NOTATION**

Suppose  $A = (a_1, a_2, \dots, a_n)$  is a vector which is passed into a dynamical system M. Then  $AM = (a'_1, a'_2, \dots, a'_n)$ . After thresholding and updating the vector suppose we get  $(b_1, b_2, \dots, b_n)$ , then we denote this vector by  $(a'_1, a'_2, \dots, a'_n) \rightsquigarrow (b_1, b_2, \dots, b_n)$

Thus the symbol,  $\rightsquigarrow$  means the resultant vector has been thresholded and updated.

The FCM model works as follows: We pass state vectors I repeatedly through the FCM connection matrix M, thresholding or non linearly transforming the result after each pass. Independent of the FCMs size, it quickly settles down to a temporal associative memory limit cycle or fixed point which is the hidden pattern of the system for that state vector I. The limit cycle or fixed-point inference summarizes the joint effects of all the interacting fuzzy knowledge.

**5. THE BAFCM BIMODEL**

Here we describe the BAFCM bimodel, the advantages of this model and its merits.

**5.1. Definition**

Let  $S = S_1 \cup S_2$ , where  $S_1$  and  $S_2$  are non-empty sets; with  $S_1 \not\subseteq S_2$  and  $S_2 \not\subseteq S_1$  then we call  $S$  as a biset.

**5.2. Example**

Let  $S = \{(2\ 5\ 1\ 0\ 1\ 4), (1\ 9\ 3\ 5\ 7\ 0), (8\ 1\ 0\ 2\ 2\ 1)\} \cup \{(2\ 2\ 1\ 3), (5\ 0\ 2\ 9), (0\ 0\ 0\ 0), (6\ 6\ 6\ 6)\} = S_1 \cup S_2$ , clearly  $S$  is a biset.

**5.3. Definition**

Let  $V_1 = (v_1, v_2, \dots, v_n)$  and  $V_2 = (v_1', v_2', v_3', \dots, v_m')$  be two vectors of length  $n$  and  $m$  respectively. Then  $V = V_1 \cup V_2$  is a bivector.

**5.4. Example**

Let  $V = V_1 \cup V_2 = (8\ 5\ 4\ 1\ 9\ 3) \cup (3\ 4\ 5\ 2)$ ,  $V$  is a bivector. If  $V = V_1 \cup V_2 = (000000) \cup (0000)$ , then  $V$  is a zero bivector. If  $V = V_1 \cup V_2 = (111111) \cup (1111)$  then  $V$  is a unit bivector.

**5.5. Definition**

A matrix  $E$  is said to be a bimatrix if  $E = E_1 \cup E_2$  where  $E_1$  and  $E_2$  are two different matrices.

**5.6. Example**

A matrix  $E$  of the form  $E = E_1 \cup E_2 = (3\ 7\ 5\ 0\ 0\ 1) \cup (1\ 9\ 0\ 1)$  is called a bimatrix or a row bimatrix.

**5.7. Example**

Let  $E = E_1 \cup E_2 =$

$$\begin{bmatrix} 6 & 5 & 0 & 0 & 2 \\ 1 & 1 & 4 & 0 & 1 \\ 7 & 0 & 1 & 2 & 6 \\ 5 & 2 & 0 & 8 & 3 \\ 2 & 4 & 1 & 7 & 0 \end{bmatrix} \cup \begin{bmatrix} 2 & 3 & 2 & 1 & 9 \\ 1 & 4 & 9 & 4 & 0 \\ 8 & 6 & 5 & 0 & 6 \\ 1 & 6 & 9 & 8 & 1 \\ 0 & 2 & 5 & 1 & 3 \end{bmatrix}$$

$E$  is a square bimatrix.

**5.8. Definition**

Let  $E = E_1 \cup E_2$  be a bimatrix where  $E_1$  is a  $m \times n$  matrix and  $E_2$  is a  $p \times s$  matrix. If  $V = V_1 \cup V_2$  is a bivector such that  $V_1$  has  $m$  components and  $V_2$  has  $p$  components then the product of  $V$  with  $E$  is defined as  $VE = (V_1 \cup V_2)(E_1 \cup E_2) = V_1 E_1 \cup V_2 E_2$  where  $V_1 E_1$  is a  $1 \times n$  matrix and  $V_2 E_2$  is a  $1 \times s$  matrix or more mathematically;  $V_1 E_1 \cup V_2 E_2 = A_1 \cup A_2$  is a bivector or a row bivector.

### 5.9. Definition

Let  $E = E_1 \cup E_2$  be a bimatrix. Then the bitranspose of the bimatrix  $E$  is defined as  $E^t = (E_1 \cup E_2)^t = E_1^t \cup E_2^t$ .

### 5.10. Definition

A Bidirectional Associative Fuzzy Cognitive Maps (BAFCM) is a directed special bigraph with concepts like policies, events, etc as nodes and causalities as edges. It represents causal relationship between concepts. In a BAFCM we call the pair of associated nodes as binodes. If the order of the bimatrix associated with BAFCM is a  $p \times m$  matrix and a  $n \times n$  square matrix then the binodes are bivectors of length  $(p,n)$  or length  $(m,n)$ .

Let  $V = V_1 \cup V_2 = (v_1^1, v_2^1, v_3^1, \dots, v_m^1) \cup (v_1^2, v_2^2, v_3^2, \dots, v_m^2)$  where  $v_i \in \{0,1\}$ ;  $V$  is called the instantaneous state bivector and it denotes the ON-OFF position of the binode at an instant.  $v_j^i = 0$  if  $v_j^i$  is OFF,  $v_j^i = 1$  if  $v_j^i$  is ON, for  $1 \leq i \leq 2$  and  $1 \leq j \leq m,n$ .

### 5.11. Definition

Consider the binodes biconcepts  $\{X_1, \dots, X_p\}$  and  $\{Y_1, \dots, Y_m\}$  of the BAM and  $\{C_1^1, C_2^1, \dots, C_n^1\}$  of the FCM of the BAFCM bimodel. Suppose the directed graph is drawn using the edge biweight  $e_{ij}^1 = \{0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5\}$ ;  $e_{ij}^2 = \{0, \pm 1\}$ . The bimatrix  $E = E_1 \cup E_2$  is defined by  $e_{ks}^1 \cup e_{ij}^2$  where  $e_{ks}^1$  is the directed edge of  $X_k Y_s$  and  $e_{ij}^2$  is the weight of the directed edge  $C_i C_j$ .  $E = E_1 \cup E_2$  is called adjacency bimatrix of the new BAFCM bimodel, also known as the connecting relational bimatrix of the new BAFCM bimodel.

### 5.12. Definition

The new BAFCMs with edge biweight  $\{1,0, -1\}$  are called simple BAFCMs. Let  $\{(X_1, \dots, X_m), (Y_1, \dots, Y_n)\} \cup \{C_1, \dots, C_p\}$  be the binodes of an BAFCM.  $V = V_1 \cup V_2 = (x_1, \dots, x_m)$  (or  $(y_1, \dots, y_n)$ )  $\cup (z_1, \dots, z_p)$  where  $x_i, y_j, z_k \in \{0,1\}$ ;  $1 \leq i \leq m$ ,  $1 \leq j \leq n$  and  $1 \leq k \leq p$ .  $V$  is called instantaneous state bivector and it denotes the ON-OFF position of the node at an instant.

$x_j = 0$  if  $x_j$  is OFF and  $x_j = 1$  if  $x_j$  is ON for  $1 \leq j \leq m$

$y_i = 0$  if  $y_i$  is OFF and  $y_i = 1$  if  $y_i$  is ON for  $1 \leq i \leq n$

$z_k = 0$  if  $z_k$  is OFF and  $z_k = 1$  if  $z_k$  is ON for  $1 \leq k \leq p$

$i = 1,2,3, \dots, m$

### 5.13. Definition

Let  $\{(X_1, \dots, X_m), (Y_1, \dots, Y_n)\} \cup \{C_1, \dots, C_p\}$  be the binodes of an BAFCM. Let  $X_k Y_s \cup C_i C_j$  be the bidedges of the BAFCMs;  $1 \leq k \leq m$ ,  $1 \leq s \leq n$ ,  $1 \leq i, j \leq p$ , ( $i \neq j$ ).

Then the bidedges form a directed bicycle. An BAFCM is said to be bicyclic if it possesses a directed bicycle. An BAFCM is said to be abicyclic if it does not possess any directed bicycle.

**5.14. Definition**

An BAFCM with bicycles is said to have a feedback. When there is a feed back in an BAFCM, i.e., when the casual relations flow through a cycle in a revolutionary way, the BAFCM is called a dynamical bisystem.

**5.15. Definition**

Let  $\{(X_i, Y_j) \text{ (or } Y_j, X_i) \mid 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{C_1, C_2, C_3, \dots, C_{n-1}, C_n\}$  be a bicycle. If  $Y_j$  (or  $X_i$ ) is switched ON and if the casuality flows through the edges of the bicycle and if it again causes  $Y_j$  (or  $X_i$ )  $\cup C_k$  we say that the dynamical bisystem goes round and round. This is true for the binodes  $Y_j$  (or  $X_i$ )  $\cup C_k$  for  $1 \leq j \leq n$  (or  $1 \leq i \leq m$ ),  $1 \leq k \leq n$ . The equilibrium bistrate for the dynamical bisystem is called the hidden bipattern. If the equilibrium bistrate of the dynamical bisystem is a unique bistrate bivector then it is called fixed bipoint.

**5.16. Example**

Consider the BAFCMs with  $(X_1, \dots, X_p) \text{ (or } Y_1, \dots, Y_m) (C_1, \dots, C_n)$  as binodes. For instant if we start the dynamical bisystem by switching  $Y_1$  (or  $X_1$ )  $\cup C_1$  ON. Let us assume the BAFCM settles down with  $((Y_1 \text{ and } Y_m) \text{ (or } X_1 \text{ and } X_p)) \cup C_1 \text{ and } C_n$ . That is the state bivector remains as  $(100\dots 1)$  in  $Y$  (or  $(100\dots 1)$  in  $X \cup (100\dots 01)$ ). This state bivector is called the fixed bipoint. It is to be noted that in the case of BAFCM we get a pair of fixed bipoint say  $A = A_1 \cup X$  and  $A = A_1 \cup Y$ ;  $X$  denote the state vector in the  $F_x$  field of the BAM component of the BAFCM and  $Y$  denotes the state vector of the  $F_y$  field of the BAM component of the BAFCM bimodel.

**5.17. Definition**

If the BAFCM settles down with a bistrate bivector repeating in the form  $B_1 \rightarrow B_2 \rightarrow \dots \rightarrow B_j \rightarrow B_1 \text{ (or } D_1 \rightarrow D_2 \rightarrow \dots \rightarrow D_k \rightarrow D_1) \cup A_1 \rightarrow A_2 \rightarrow \dots \rightarrow A_1$  then this equilibrium is called a limit bicycle.

**5.18. Definition**

The biedges  $e_{ij} = (e_{ks}^1) \cup (e_{ij}^2)$  take the values in fuzzy casual biinterval  $[-1, 1] \cup [-1, 1]$ .

- i)  $e_{ij} = 0$  indicates no causality between the binodes.
- ii)  $e_{ij} > 0$  implies that both  $e_{ij}^2 > 0$  and  $e_{ks}^1 > 0$ ; implies increase in the binodes  $C_i \cup X_k$  (or  $Y_s$ ); implies increase in the binodes  $C_j \cup Y_s$  (or  $X_s$ ).
- iii)  $e_{ij} < 0$  implies that both  $e_{ij}^2 < 0$  and  $e_{ks}^1 < 0$ ; similarly decrease in the binodes  $C_i \cup X_k$  (or  $Y_s$ ); implies decrease in the binodes  $C_j \cup Y_s$  (or  $X_k$ )

However unlike the FCM and BAM model we can have the following possibilities other than that of  $e_{ij} = 0, e_{ij} > 0$  and  $e_{ij} < 0$ .

- i)  $e_{ij} = (e_{ij}^2) \cup (e_{ks}^1)$  can be such that  $(e_{ij}^2) = 0$  and  $(e_{ks}^1) > 0$ . No relation in one binode and an increase in other node.
- ii)  $e_{ij} = (e_{ij}^2) \cup (e_{ks}^1)$  we can have  $(e_{ij}^2) = 0$  and  $(e_{ks}^1) < 0$ . No causality in the FCM node and decreasing relation in the BAM mode.
- iii)  $e_{ij} = (e_{ij}^2) \cup (e_{ks}^1)$  we can have  $(e_{ij}^2) = < 0$  and  $(e_{ks}^1) > 0$



- iv) In  $e_{ij} = (e_{ij}^2) \cup (e_{ks}^1)$  we can have  $(e_{ij}^2) < 0$  and  $(e_{ks}^1) = 0$
- v) In  $e_{ij} = (e_{ij}^2) \cup (e_{ks}^1)$  we can have  $(e_{ij}^2) > 0$  and  $(e_{ks}^1) = 0$
- vi) In  $e_{ij} = (e_{ij}^2) \cup (e_{ks}^1)$  we can have  $(e_{ij}^2) > 0$  and  $(e_{ks}^1) < 0$ .

Thus in the case of BAFCM we can have 9 possibilities where as in FCMs or BAMs we have only 3 possibilities. Thus the extra 6 possibilities can help in making the solution of the problem more sensitive or accurate.

## 6. INDUCED BAFCM BIMODEL

In this section we introduce a new bimodel called Induced BAFCM bimodel. It is found that in the analysis of BAFCM bimodel, the expert or the analyst chooses certain vectors on ON state with 1 and certain other vectors to OFF state with 0. If many vectors are chosen as ON state at the input level, naturally at the output level, the resultant vector will have too many 1s, that is, many attributes will be as ON state. There is no specific criteria or framework laid down to guide the expert or analyst while choosing the input vector. To avoid this state of affairs, in this paper we introduce a new guideline and methodology for the analysis. Hence first, each vector is given its due importance by keeping it on ON state and applying on the dynamical system; second, the impact of all the attributes are gathered into one induced graph for the final analysis. Third, in the process it is possible to detect the interrelationship between the attributes and how one induces or influences the other while reaching the state of equilibria. The fixed point obtained will include the impact of all the mentioned attributes and the interpretation of the results will be holistic rather than partial.

Let us take the connection bimatrix of BAFCM bimodel which has both BAM and FCM components and let  $A_1$  be the initial input bivector. In  $A_1$ , let a particular vector component, say  $b_1$  and  $c_1$ , be kept on ON state and all other components are on OFF state and pass the state vector  $A_1$  through the connection bimatrix  $E$ . To convert the resultant vector into a signal function, the values in the BAM component which are greater than or equal to one are made as ON state and all others as OFF state by giving values 1 and 0 respectively and the values in the FCM component which are greater than or equal to one are made as ON state and all others as OFF state by giving values 1 and 0 respectively and that component in FCM component which had been initially kept on ON state, will always be retained as ON state. Denote this process by the symbol  $\hookrightarrow$ . This new bivector is related with the connection bimatrix and that vector which triggers the highest number of attributes to ON state is chosen as  $A_{1NEW}$ . That is, for each positive entry we get a set of resultant vectors; among these vectors a vector which contains maximum number of 1s is chosen as  $A_2$ . If there are two or more vectors with equal number of 1s on ON state, choose the first occurring one in the set of vectors. Repeat the same procedure till a fixed point or a limit cycle is obtained. This process is done to give due importance to each vector separately as one vector induces another or many more vectors into ON state. Get the hidden pattern by the limit cycle or by getting a fixed point. Observe a pattern that leads one cause to another and may end up in one vector or a cycle. Next choose the vector with its second component in ON state and repeat the same to get another cycle. This process has been repeated for all the vectors separately. We

observe the hidden pattern of some vectors found in all or many cases. Inference from this hidden pattern summarizes or highlights the causes.

## 7. ALGORITHM

**Input:** The dynamical bisystem  $E = E_1 \cup E_2$  where,  $E_1$  is of order  $m \times p$  and  $E_2$  is of order  $n \times n$  and the initial bivector  $A_\omega = B_\omega \cup C_\omega$  of order  $(1 \times m)$  (or  $(1 \times p)$ )  $\cup$   $(1 \times n)$  as the input bivector.

### Step: 1

Let  $\omega = 1$

### Step 2:

Pass the bivector  $A_\omega = B_\omega \cup C_\omega$  into a dynamical bisystem  $E = E_1$  (or  $(E_1^T)$ )  $\cup$   $E_2$ .

Let the resultant be  $A_\omega E = B_\omega E_1$  (or  $(B_\omega E_1^T)$ )  $\cup$   $C_\omega E_2 = (y_1', y_2', \dots, y_p')$  (or  $(z_1', z_2', \dots, z_m')$ )  $\cup$   $(x_1', x_2', \dots, x_n')$ .

**Step 3 :** Threshold and update this  $A_\omega E$  using the signal function  $S$  as:

$$S(y_j') = 0 \text{ if } y_j' \leq 0; \quad 1 \leq j \leq p$$

$$S(y_j') = 1 \text{ if } y_j' > 0; \quad 1 \leq j \leq p$$

$$S(z_k') = 0 \text{ if } z_k' \leq 0; \quad 1 \leq k \leq m$$

$$S(z_k') = 1 \text{ if } z_k' > 0; \quad 1 \leq k \leq m$$

$$S(x_i') = 0 \text{ if } x_i' \leq 0; \quad 1 \leq i \leq n$$

$$S(x_i') = 1 \text{ if } x_i' > 0; \quad 1 \leq i \leq n$$

That is if  $(y_1', y_2', \dots, y_p')$  (or  $(z_1', z_2', \dots, z_m')$ )  $\cup$   $(x_1', x_2', \dots, x_n')$

$\hookrightarrow (y_1, y_2, \dots, y_p)$  (or  $(z_1, z_2, \dots, z_m)$ )  $\cup$   $(x_1, x_2, \dots, x_n)$ .

We use the symbol  $\hookrightarrow$  for thresholding.

Moreover, keep the state  $x_j$  as ON if it was in ON position in  $C_\omega$ .

Then, the values of  $x_i, y_j, z_k$  are either zero or 1, for  $1 \leq i \leq n; 1 \leq j \leq p; \text{ and } 1 \leq k \leq m$ .

**Step 4:** Assign:  $R_\omega = (D_\omega \cup F_\omega) \leftarrow ((y_1, y_2, \dots, y_p)$  (or  $(z_1, z_2, \dots, z_m)$ )  $\cup$   $(x_1, x_2, \dots, x_n)$ )

Assign:  $N \leftarrow (E_1^T$  (or  $(E_1)$ )  $\cup$   $E_2)$ .

Pass the bivector  $R_i$  into the system  $N$ .

Let the resultant be  $R_\omega N = D_\omega E_1^T$  (or  $(D_\omega E_1)$ )  $\cup$   $F_\omega E_2 = (y_1'', y_2'', \dots, y_m'')$  (or  $(z_1'', z_2'', \dots, z_p'')$ )  $\cup$   $(x_1'', x_2'', \dots, x_n'')$ .

**Step 5 :** Threshold and update this  $R_\omega N$  as:

$$S(y_j'') = 0 \text{ if } y_j'' \leq 0; \quad 1 \leq j \leq m$$

$$S(y_j'') = 1 \text{ if } y_j'' > 0; \quad 1 \leq j \leq m$$

$$S(z_k'') = 0 \text{ if } z_k'' \leq 0; \quad 1 \leq k \leq p$$

$$S(z_k'') = 1 \text{ if } z_k'' > 0; \quad 1 \leq k \leq p$$

$$S(x_i'') = 0 \text{ if } x_i'' \leq 0; \quad 1 \leq i \leq n$$

$$S(x_i'') = 1 \text{ if } x_i'' > 0; \quad 1 \leq i \leq n$$

That is if  $(y_1'', y_2'', \dots, y_p'')$  (or  $(z_1'', z_2'', \dots, z_m'')$ )  $\cup$   $(x_1'', x_2'', \dots, x_n'')$

$\hookrightarrow (y_1, y_2, \dots, y_m) \text{ (or } (z_1, z_2, \dots, z_p)) \cup (x_1, x_2, \dots, x_n)$  .

Moreover, keep the state  $x_j$  as ON if it was in ON position in  $F_\omega$ .

Then, the values of  $x_i, y_j, z_k$  are either zero or 1, for  $1 \leq i \leq n$ ;  $1 \leq j \leq m$ ; and  $1 \leq k \leq p$ .

Define:

$K_1 = \{ \alpha \mid y_\alpha = 1 \}$  where  $\alpha \in \{1, 2, \dots, m\}$  or  $\{ \gamma \mid z_\gamma = 1 \}$  where  $\gamma \in \{1, 2, \dots, p\}$

$K_2 = \{ \beta \mid x_\beta = 1 \}$  where  $\beta \in \{1, 2, \dots, n\}$

$\forall \alpha \in K_1$  ( or  $\gamma$  ) and  $\forall \beta \in K_2$ , define

$A_{\omega_{NEW}} = \{ (t_1, t_2, \dots, t_m) \text{ (or } r_1, r_2, \dots, r_p) \cup (q_1, q_2, \dots, q_n) \}$  such that  $t_\alpha = 1$  and  $t_\lambda = 0$  where  $\alpha \neq \lambda$   
 ( or  $r_\gamma = 1$  and  $r_\lambda = 0$  where  $\gamma \neq \lambda$  ) and  $q_\beta = 1$  and  $q_\lambda = 0$  where  $\beta \neq \lambda$  }

**Step: 6**

For every bivector in  $A_{\omega_{NEW}}$ , Do steps 2 through 5.

**Step: 7**

Let  $\mu = |K_1| + |K_2| - |(K_1 \cap K_2)|$ .

For all  $\theta \in \{1, 2, \dots, \mu\}$ , Calculate the row sums in each  $(B_i^\theta \cup C_i^\theta)$

**Step: 8**

Choose the maximum row sum among  $(B_i^\theta \cup C_i^\theta)$  where both  $B_i^\theta$  and  $C_i^\theta$  is a non zero vector to form a new input bivector  $A_{\omega+1}$

**Step: 9**

Repeat steps 2 through 8 till we get the limit point.

**Output:** The pair of Limit Points.

From the pair of limit points we can analyze the situation and derive a conclusion of any problem on which this proposed model is applied.

**8. CONCLUSION**

We proposed a new Induced Bidirectional Associative Fuzzy Cognitive Maps ( IBAFCM ) fuzzy bimodel. We also state the advantages of this system. We show that the system terminates in a finite number of steps as well.

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