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MATHEMATICAL MODELING OF DOMESTIC VIOLENCE AND ITS TRENDS, CASE STUDY TAMALE METROPOLIS, GHANA

D. Otoo

Department of Mathematics, University of Energy and Natural Resources, Sunyani, Ghana

C. Sebil

Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana

S. K. Amponsah

Department of Mathematics, Kwame Nkrumah University of Science and Technology, Kumasi, Ghana

ABSTRACT

Domestic violence has become one of the possible areas which can reduce the work force of a country. This paper presents a numerical modelling of domestic violence by using a continuous model for its spread. A modelling technique of abusive, susceptible and violence victims, similar to the susceptible, infectious and recovered model in epidemics, is used for the formulation of the spread of domestic violence as a system of differential equations. Data used were collected from Domestic Violence and Victims Support Unit (DOVVSU) in Tamale and was analysed by using MATLAB software. The study revealed that the population of Domestic Violence Victims is limited.

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Keywords: Domestic violence, Abusive, Susceptible and violence victims, Infectious and recovered.

Contribution/ Originality

The study contributes to the existing literature in the area of mathematical modeling; it used the existing data from DOVSU to formulate a model for the prevailing domestic violence in the area. The study has provided DOVSU with a growth model and a limiting value to the growth of Domestic violence in the long run.

1. INTRODUCTION

Tamale, the capital of the northern region, one of the ten regional capitals of Ghana is a bustling Regional Capital of the Northern Region about 400 miles north of the Atlantic Coast in West Africa. The geographical area of Tamale is about 750 kilometre square. The economy of the area is predominantly Agriculture with most of them with elementary or no basic education. The capital takes about 38% of the region's population of Two million four hundred and sixty-eight

thousand, five hundred and fifty seven (2,468,557). With this population of the capital, 53% are females and the rest males.Tamale Metropolis is located in the central part of the Northern Region and shares boundaries with five other districts namely the Savelugu- Nanton to the North, Yendi Municipal Assembly to the East, Tolon-Kumbungu to the West, Central Gonja to the South West and East Ganja to the South.

Most people in the capital are Muslims and they practice the polygamous system of marriage with large family size. The dominant tribe is the Dagombas but there are other tribes like Gonja, Mamprusi, Konkomba, Hausa, Dagarti, and Grusi. 90 percent of all reported cases of Domestic Violence have the abusers being men and the victims women and children. It is also estimated that one out of four women will experience intimate partner violence at some time in their life [1]. Domestic Violence is a form of mental torture that affect people physically, psychologically, sexually and economically. It is one of the most common human right abuses on mostly women and children, especially in developing countries where family issues are settled by elders of the community instead o law enforcing agencies for proper punishment and redress. This canker is affecting economic and social lifes of people in the third world countries especially. Even though most societies proscribe violence against women, the reality is that violations against one's human rights are often sanctioned under the garb of cultural practices and norms, or through misinterpretation of religious tenets [2].

2. RELATED WORKS

Many research works have been conducted in the area of population growth using logistic model, some research works have also been performed on domestic violence and its effect. Gelles

and Straus [3]defined violence as an act carried out with the intention of perceived intention of physically hurting another person or as the exercise of power in order to impose one's will on a person or to have ones will with a thing.

According to Seager [4], Domestic Violence is a leading cause of injury and death to women and children worldwide; it is often regarded as private matter. The author further explored domestic violence as a tool of power used to sustain particular sexual, family and household structures and to keep women subordinate to them.

Montroll [5] made the connection between physical and population trajectories clear by proposing laws of social dynamics based on Newton's laws of mechanics. Simple exponential growth models is probably the most popular general model for describing positive quantities that grow in time by many orders of magnitude, as both population and world product have. The use of the logistic growth model is widely established in many fields of modeling and forecasting Banks.

Competing results have also been obtained from other researchers in forecasting population growth Wali, et al. [6].

Olson [7]used a general conditional logistic model to detect linkage between marker loci and common disease with samples of affected sib pairs.

Mahapatra and Kant [8] used a multinomial logistic model to deal with estimation problems and shown that the results of multinomial logistic are more informative and robust compared to the results of binary logistic model.

Manjunath and Manjunath [9] developed an integrated logistic model using supply chain management system which clearly shows a greater acceptability of logistic model in industry.

## 3. SCOPE OF THE STUDY AREA

The study area, Tamale is one of the fastest growing cities in West Africa. It is the capital of the northern region of Ghana and the third largest city of Ghana after Accra and Kumasi. Livelihood in the city is largely dependent on agriculture (crop, livestock/poultry, and agro-processing) despite its Metropolitan status. It is estimated that 6 out of 10 people are poor in the northern region and Tamale happens to be the capital of the region with people around the region migrating into the city in search of jobs. Majority of people in the metropolis are Muslims, a religion that allows polygamous marriage. Literacy level in the area is very low

4. MATHEMATICAL FORMULATION OF THE LOGISTISTIC GROWTH MODEL

We shall divide the population into two groups:
1. Susceptible/Abusive individuals at time (t), $S(t)$
2. Domestic Violence Victim's at time (t), $V(t)$ with total population size, N

**Figure-4.1.** A Schematic diagram of growth model
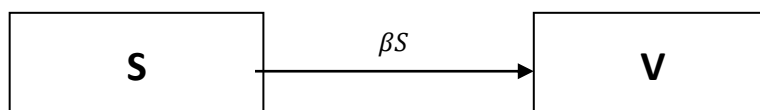
Model for the problem is formulated based on the following assumptions

Population size is large and constants, $S(t) + V(t) = N$ (4.1)

There is no birth, death, immigration or emigration, no recovery and the Violence spread rate is proportional to the number of Domestic Violence Victims, i.e. βV

Figure-4.2. A schematic diagram of spread of domestic violence

$$\beta VS$$

| S | → | V |
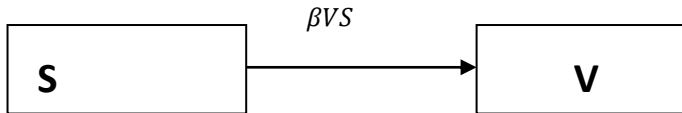
Ordinary differential equations (4.2) and (4.3) are used to describe the model:

$$\frac{dS}{dt} = -\beta V(t)S(t) \text{ (4.2)}$$

$$\frac{dV}{dt} = \beta V(t)S(t) \tag{4.3}$$

But $N = S(t) + V(t)$, is equivalent to $S(t) = N - V(t)$ and substituting into (4.3) gives

$$\frac{dV}{dt} = \beta V(t)[N - V(t)]$$

The differential equation is known as the **Logistic Growth Model.**
We will have a nonlinear ODE,

$$\frac{dV}{dt} = \beta V(t)[N - V(t)] \tag{4.4}$$

$$V(t)' = \beta V(t)\left[1 - \frac{V(t)}{N}\right] \tag{4.5}$$

Equating the derivative of (4.5) to zero gives $V(t) = N$. If the initial population is zero, there is no growth and the population stays at zero. If the population is in the range $0 < V(t) < N$, then $\frac{dV}{dt} > 0$ and hence $V(t)$ is increasing. At $V(t) = N$, the population stays at this level. Similarly, if we start with $V > N$, then $\frac{dV}{dt} < 0$ and hence $V(t)$ is decreasing. Using the analysis we construct the following phase line diagram shown in Figure 4.3.
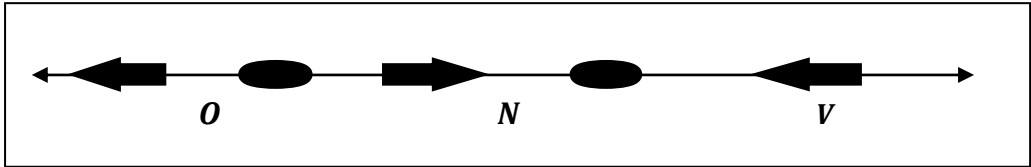
From the phase diagram, we see that solutions tend toward the equilibrium at N and hence the solution $V(t) = N$ is stable while the equilibrium at 0 is unstable. According to this model, if the population of victims of Domestic Violence is above 0, it will go to the carrying capacity N eventually.

From (4.5) we have $\dfrac{1}{V(t)[N-V(t)]}\dfrac{dV}{dt} = \beta$

$$\int \frac{1}{V(t)[N-V(t)]} dV = \int \beta dt$$

$$\int \frac{1}{u[N-u]} du = \int \beta dt$$

$$\frac{u}{N-u} = Ce^{\beta t}$$

$$\frac{V(t)}{N-V(t)} = Ce^{\beta t}$$

$$V(t) = \frac{NC^{\beta t}}{1+Ce^{\beta t}} \tag{4.6}$$

$$V(0) = \frac{NC}{1+C}$$

$$C = \frac{V(0)}{N-V(0)}$$

$$V(t) = \frac{N\left(\frac{V(0)}{N-V(0)}\right)e^{\beta t}}{1+\left(\frac{V(0)}{N-V(0)}\right)e^{\beta t}}$$

$$V(t) = \frac{NV(0)}{(N-V(0))e^{-\beta t} + V(0)} \tag{4.7}$$

Analyzing the solution above, we see that as time increases, the size of the population of victims reaches a finite limit. Mathematically, as $t \to +\infty$, $V \to N$ and almost all women and children becomes victims of Domestic Violence.
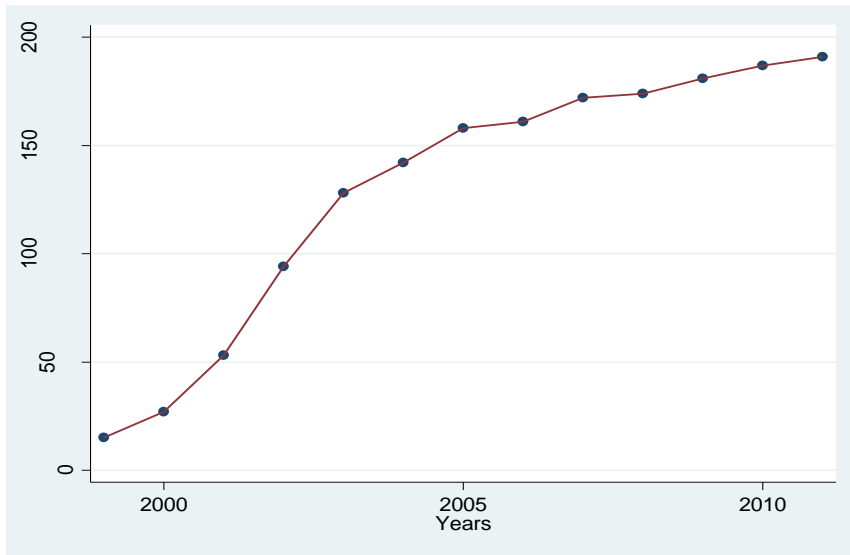
4.1 Analysis of Domestic Violemce

Population sizes for domestic Violence victims in Tamale for the years between 1999 and 2011

Table-4.1. Reported cases versus years

Years	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011
Cases	15	27	53	94	128	142	158	161	172	174	181	187	191

Figure-4.4. Graph of reported cases of Domestic Violence against time

4.1.2 Fitting Model for the Data

Consider the model of the form:

$$V(t)' = \beta V(t)\left(1 - \frac{V(t)}{N}\right)$$

(4.8)

In order to show that model (4.1) is logistic, we need to focus on the following questions:

(i) How to tell whether a given set of data is reasonably logistic?

(ii) What parameter β and N will be good fit?

Using difference equation, equation (4.8) can be expressed as

$$V(t+1) - V(t) = \beta V(t)\left(1 - \frac{V(t)}{N}\right)$$

(4.9)

$$\frac{\Delta V}{V} = \beta\left(1 - \frac{V}{N}\right)$$

(4.10)

The equation (4.10) says that the ratio of ΔV and V is a linear function of V.

Consider the left hand side (LHS) of equation (4.10). We calculate the difference of the populations for two consecutive years, and then use those differences against the corresponding function values. If a plot of ratios against functional values gives approximately linear graph, then

the model equation (4.10) is appropriate. That is to say, the model has the form (4.10) and it is Logistic.

Calculating the ratios on the left hand side of (4.10) yields:

Table-4.2. Ratio of consecutive population difference

a	$V(t)$
0.8000	15
0.9629	27
0.7736	53
0.3617	94
0.1094	128
0.1127	142
0.0189	158
0.0683	161
0.0116	172
0.0402	174
0.0331	181
0.0214	187

Plotting the Least Square approximation graph by using Table (4. 2)

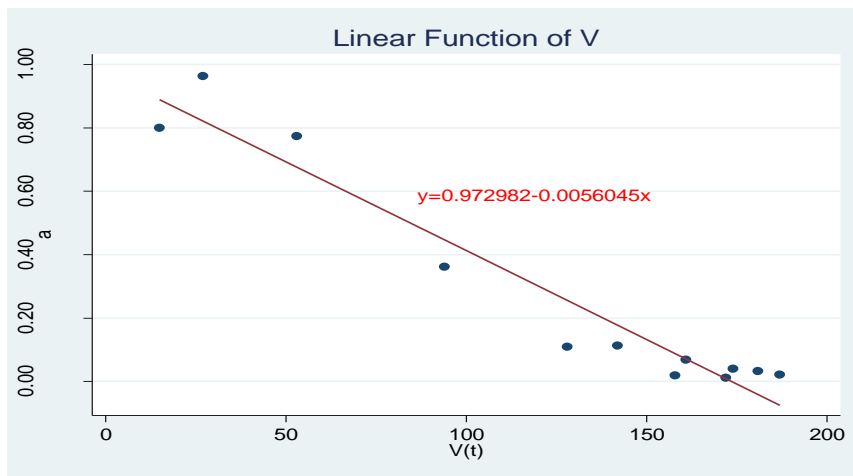**Figure-4.5.** Correlation of between Domestic violence versus ratio of population

Figure 4.3 depicts various cases of Domestic Violence Victim plotted levels $V(t)$ at time t, we can calculate corresponding ratios a . Based on these points we plot Least Square Approximation graph.

Figure 4.5 shows a reasonable correlation; therefore, our assumption for the equation (4.8) is reasonable and confirms that model (4.8) shows that the data is logistic.

4.2. Determining the Values of β and N

Least square line from Figure (1.3) is

$$y = 0.972982 - 0.0056045x \tag{4.11}$$

$$0.888915 = \beta \left(1 - \frac{15}{N}\right) \tag{4.12}$$

$$0.821661 = \beta \left(1 - \frac{27}{N}\right) \tag{4.13}$$

Suppose that $\beta, N \neq 0$, and dividing equation (4.12) by equation (4.13), we have:

$$\frac{0.888915}{0.821661} = \frac{\beta\left(1 - \frac{15}{N}\right)}{\beta\left(1 - \frac{27}{N}\right)}$$

$$N = 173.6103$$

$$N \approx 174$$

From equation (4.12), we obtain the value of β as 0.972775 and the model becomes

$$V(t)' = 0.972775V\left(1 - \frac{V}{174}\right) \tag{4.14}$$

This means that the size is bounded by 174 victims of Domestic Violence. In other words, the limiting number for this population model is 174 Victims.

4.3. The Logistic Model

$$\frac{dV}{dt} = 0.972775V\left(1 - \frac{V}{174}\right) \tag{4.15}$$

$$= 0.972775V - 0.005591V^2$$

By separation of variables, equation (4.15) gives

$$\int \frac{dV}{V(0.972775 - 0.005591V)} = t + C$$

$$\frac{1}{0.972775} \int \left(\frac{1}{V} + \frac{0.005591}{0.972775 - 0.005591V}\right) = t + C \tag{4.16}$$

Using the condition $V_0 = 15$ at $t = 0$, we obtain $C = 2.90489$

Thus equation (4.16) becomes

$$\frac{1}{0.972775}(\ln V - \ln(0.972775 - 0.005591V)) = t + 2.90489$$

$$V = \frac{173.9905}{1 + 10.5708e^{-0.972775t}} \tag{4.17}$$

If we take the limit of solution (4.17) as $t \to \infty$, gives, $V(t) \to 174$, this shows that there is a limit to the growth of V. From (4.17), we can get the predicted cases for each year. Table 4.6 shows time (t), actual cases (V) and predicted cases $(V(t))$ for Domestic Violence against women and children.

Table-4.6. Comparison of actual against predicted cases with time

Time (t)	Actual Cases [9]	Predicted Cases V(t)
0	15	15.0379
1	27	34.8271
2	53	69.3045
3	94	110.5040
4	128	143.1060
5	142	160.8710
6	158	168.7920
7	161	171.9940
8	172	173.2360
9	174	173.7100
10	181	173.8900
11	187	173.9590
12	191	173.9840
13		173.9940
14		173.9980
15		173.9990
16		174
17		174
18		174

Assuming a carrying capacity $N = 1000$, then from (4.12), we have

$$0.888915 = \beta\left(1 - \frac{15}{1000}\right) \text{giving } \beta = 0.902452$$

$$V(t)' = 0.902452V\left(1 - \frac{V}{1000}\right) \quad (4.18)$$

$$V(t) = \frac{1000.4900}{1 + 65.6987e^{-0.902454t}} \quad (4.19)$$

From (4.19), we obtain the predicted cases of Domestic Violence in the Table below;

Table-4.7. Predicted cases against time

Time (t)	0	1	2	3	4	5	6
Predicted cases V(t)	14.992	36.172	84.697	185.772	360.021	581.073	773.754
7	8	9	10	11	12	13	14	15	16
893.983	954.110	980.866	992.151	996.802	998.700	999.472	999.786	999.913	999.965
16	17	18	19	20	21	22
999.965	999.986	999.994	999.998	999.999	1000.000	1000.000

As $t \to \infty$, $V(t) \to 1000$, finally, assuming the carrying capacity is equal to the population

of females in the Northern which is 1257855 according to the provisional results of the Population and Housing Census of Ghana 2010.

we have $0.888915 = \beta\left(1 - \frac{15}{1257855}\right)$ which gives $\beta = 0.888926$

$$V(t)' = 0.888926V \left(1 - \frac{V}{1257855}\right) \tag{4.20}$$

$$V(t) = \frac{1250008.3333}{1 + 83333.3333e^{-0.888926t}} \tag{4.21}$$

From equation (4.21), we obtain the predicted cases of Domestic Violence in Table 4.8;

Table-4.8. Predicted cases with time

Time (*t*)	0	1	2	3	4	5
Predicted cases V(*t*)	14.9999	36.4860	88.7512	215.8669	524.9698	1276.2295

6	7	8	9	10	11	12
3099.9146	7513.8979	18121.6303	43184.3213	100092.9939	218423.1079	424947.3826

13	14	15	16	17	18
695155.9456	941182.6164	1101434.4142	1184333.0296	1222147.3674	1238402.4356

19	20	21	22	23	24
1245210.9520	1248031.6770	1249194.9782	1249673.8372	1249870.8013	1249951.7906

25	26	27	28	29	30
1249995.0881	1249998.7771	1250004.4047	1250006.7183	1250007.6693	1250008.0603

31	32	33	34	35	36
1250008.2211	1250008.2871	1250008.3143	1250008.3255	1250008.3301	1250008.3319

37	38	39	40	29	30
1250008.3327	1250008.3330	1250008.3332	1250008.333	1250008.6693	1250008.0603

The results show that as $t \to \infty$, $V(t) \to 1250008$.

Three cases as shown above indicate that there is a limit to the growth Domestic Violence Cases in Tamale Metropolis.

5. SUMMARY OF RESULTS

A model for the population of victims of Domestic Violence against Women and Children in the Tamale Metropolis from 1999 to 2011 was developed. That is, the differential equation that approximately models this population is $V' = 0.972775V \left(1 - \frac{V}{174}\right)$ (4.22)

The present model has shown to be Logistic. From the model, we can predict the population limitation of victims of Domestic Violence in Tamale.

Figure-4.6. Predicted Domestic violence against time
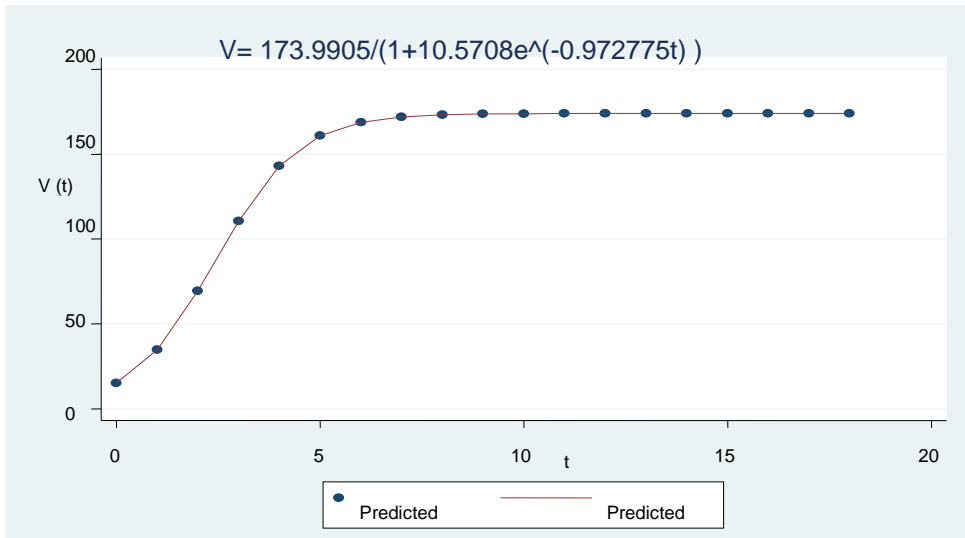
We then compared actual values with the predicted values and match the given data as shown in Figure 4.6. It shows that the mathematical model (4.14) has potential as a possible logistic growth model.

Figure-4.7. Combined graph of Actual and predicted cases against time
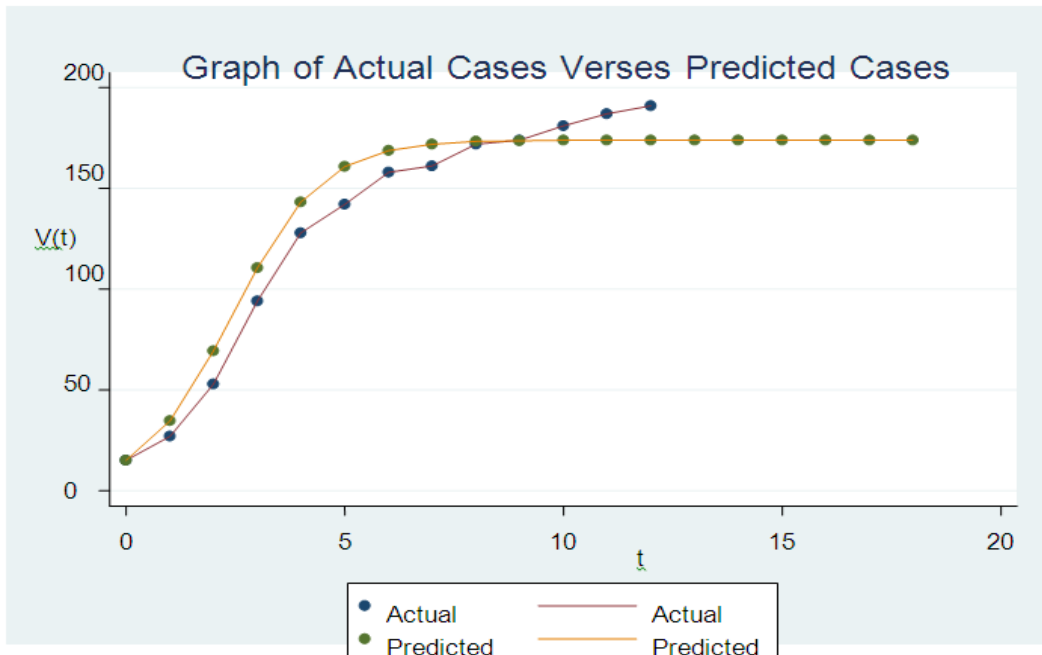
6. CONCLUSION

The Logistic Growth Model; $V(t)' = 0.972775V\left(1 - \frac{V}{174}\right)$ has the equation

$V = \frac{173.9905}{1+10.5708e^{-0.972775t}}$. The equation tested on the data from DOVVSU, Tamale shows that there

is a limit to the growth of Domestic Violence Victims V as time $t \to \infty$. The limiting value is 174 Victims. Assumptions of different carrying capacity of Tamale Metropolis were considered and its Logistic Growth Model was deduced.

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