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ESTIMATING THE SPECTRAL POWER DENSITY FUNCTION OF NON-GAUSSIAN SECOND ORDER AUTOREGRESSIVE MODEL

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ABSTRACT

This research focuses on estimation of the spectrum and the spectral power density function of non-Gaussian second order autoregressive model AR(2) through performing simulation's experiments to calculate the values of power spectral density function (PSD) for different sample sizes and various frequencies.

The random errors for the used model follow Non-Gaussian distribution (discrete, and continuous), and also the comparison between simulation results is made by using the criteria of Mean Square Error (MSE) and Mean Absolute Percentage Error (MAPE).

The main conclusions of the research are: There is no essential differences appeared for the spectrum values, or spectral power density function, and mean square errors and mean absolute percentage errors between discrete and continuous distributions for all frequencies.

In case of Cauchy distribution, whenever the sample size increases the value of p(w) and f(w) goes up, while the value of MSE, MAPE decreases.

Keywords: Spectrum, Non-gaussian, Autoregressive model, Density function, Simulation, Yule-walker, Distributions.

Contribution/ Originality

This study contributes in finding the best estimators of spectrum function by contrasting the condition of normal distribution .This has been done by studying the estimators when the random errors have other parametric distributions (discrete or continuous) because the length of period in time series leads to increase the possibility of facing Non Gaussian conditions that cause it to not follow its normal path and be away from its study conditions that hypothesized a normal distribution for errors . Variance is considered as a main indicator to study the behavior of time

series since there is a possibility to change it to spectrum function (studying time series in frequency field). So it is so important to study the behavior of spectrum function to know the behavior of time series, so this study is a remarkable addition for this field

1. INTRODUCTION

There are many natural phenomenons that lead to develop special concepts for spectral estimation such as the shapes of the moon, and the movement of celestial bodies.

The mathematical base for spectral estimation [1] belong to the 17^{th} century, especially for the work of Isaac Newton.

He noticed that the sunlight that is passing through prism is analyzing to different bundles of colors. Also, he discovered that each color represents a specific wavelength of light and the white light of sun contains all the wavelengths. Newton in 1671 used the word “spectrum”, a word derived from specter which means picture or ghost, as a scientific term to describe the color of bundles of light.

The interest in time series started in 1807 when Joseph Fourier [1] claimed that each time series can simplify to a set which consists of sine and cosine from that time ,it is named Fourier Series. The year 1930 [2] has faced a change point in the “main spectral analysis”.

This research puts forcefully the spectral analysis within statistical methods by dealing with random operations beside specifying accurate statistical definitions for both autocorrelation coefficient and power density function.

The research aims to estimate the Spectrum and power Spectral Density Function (PSD) of Non-Gaussian second order autoregressive model AR(2)when the random error of model follows Non-Gaussian distribution (discrete and continuous)for different sample sizes and various frequencies.

2. SPECTRUM [2] [3]

Spectrum of stationary process is the Fourier’s transformation for absolutely sample auto covariance function of the process:

$$f(w) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \gamma_k \; e^{-i\omega k} \qquad \dots (2.1)$$

$$f(w) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty} \gamma_k \, cos\; wk \qquad \dots (2.2)$$

And for the series with real values, as follows:

$$f(w) = \frac{1}{2\pi}\gamma_0 + \frac{1}{\pi}\sum_{k=1}^{\infty} \gamma_k \cos wk \quad , -\pi \le w \le \pi \qquad \dots (2.3)$$

Where we have used the properties: $\gamma_k = \gamma_{(-k)}$, Sin 0 = 0 , Sin w(-k) = -Sin w(k) , Cos w(-k) = Cos w(k) , γ_k can be represented spectrally through Fourier-Stieltjes integration, as follows: $\gamma_k = \int_{-\pi}^{\pi} e^{-i\omega k}\; dF(w) \qquad \dots (2.4)$

f(w) is known as the Spectral Density Function (SDF) similar to any statistical distribution since it is a nondecreasing function. It is noticed that f(w) doesn’t have the properties of probability distribution function completely, for: $\int_{-\pi}^{\pi} dF(w) \; = \gamma_0 \ne 1 \qquad \dots (2.5)$

And γ_0 is not necessarily equal to one, so it can be defined as:

$$G(w) = \frac{F(w)}{\gamma_0} \qquad , \; G(w) \geq 0 \quad \ldots(2.6)$$

as in:

$$\int_{-\pi}^{\pi} dG(w) \; = 1 \quad \ldots(2.7)$$

Since : dF(w)=f(w)dw

So :

$$P(w)dw = dG(w) = \frac{f(w)}{\gamma_0}\, dw \quad \ldots(2.8)$$

From the formula (2.1) and (2.3) Fourier's transformation will be:

$$P(w) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} r_{(k)} \; e^{-i\omega k} \qquad \ldots(2.9)$$

$$P(w) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} r_{(k)} \quad \cos wk \quad \ldots(2.10)$$

$$P(w) = \frac{1}{2\pi} + \frac{1}{\pi} \sum_{k=1}^{\infty} r_{(k)} \cos wk \quad , -\pi \leq w \leq \pi \qquad \ldots(2.11)$$

And

$$r(k) = \int_{-\pi}^{\pi} P(w)\, e^{-i\omega k}\, dw \quad , \; k = 0, \pm 1, \pm 2 \quad \ldots(2.12)$$

Since: $w_k = \frac{2\pi k}{n} \quad , k = 0,1,2, \ldots, \frac{n}{2}$

The P(w) function is named as Power Spectral Density (PSD) and it is known as parameter for distribution power as the frequency function since frequency represents the number of circles per second.

The properties of this function are:

1. $\int_{-\pi}^{\pi} P(w) \; dw = 1$
2. $P(w) \geq 0 \quad , \; \forall \, w$
3. $P(-w) = P(w)$ For operations with real values.

The spectrum that represents the average of square spectral density function for independent variable that is considered as variant at specific point and fix at another is known as power spectrum.

3. SECOND ORDER AUTOREGRESSIVE MODEL

It is possible to write the second order autoregressive model as the following formula [4]:

$$X_t = \emptyset_1 X_{t-1} + \emptyset_2 X_{t-2} + a_t \qquad \ldots(3.1)$$

Since : X_{t-i} $, i = 0,1,2$ Observation values of time series.

$\emptyset_i$ $, i = 1,2$ Autoregressive parameters.

a_t Random error.

3.1. Properties of the Model

3.1.1. Stationarity [5]

To achieve stationarity, it is conditioned for the roots equation:

$\emptyset(\mathrm{B}) = 1 - \emptyset_1 \mathrm{B}^1 - \emptyset_2 \mathrm{B}^2 = 0$ to be outside the unite circle that has a radius equal one.

For the model to be stationary it has to satisfy the following conditions:

$$\left.\begin{array}{c} -1 < \emptyset_2 < 1 \\ -2 < \emptyset_1 < 2 \\ \emptyset_1 + \emptyset_2 < 1 \\ \emptyset_2 - \emptyset_1 < 1 \end{array}\right\} \quad \dots (3.1.1)$$

3.1.2. Autocovariance [4]

We can write the auto covariance formula of second order autoregressive model as follows:

$$\gamma_{(k)} = \begin{cases} \emptyset_1 \gamma_{(k-1)} + \emptyset_2 \gamma_{(k-2)} + \sigma_a^2 & , k = 0 \\ \emptyset_1 \gamma_{(0)} + \emptyset_2 \gamma_{(k)} & , k = 1 \\ \emptyset_1 \gamma_{(k-1)} + \emptyset_2 \gamma_{(0)} & , k = 2 \\ 0 & , o.w \end{cases} \quad \dots (3.1.2)$$

Since : $\gamma_{(-k)} = \gamma_{(k)}$

3.1.3. Autocorrelation [6] [7]

We can write the autocorrelation's formula for AR(2) model as follows:

$$\rho_k = \begin{cases} 1 & , k = 0 \\ \emptyset_1/(1 - \emptyset_2) & , k = 1 \\ (\emptyset_1^2 + \emptyset_2 - \emptyset_2^2)/(1 - \emptyset_2) & , k = 2 \\ 0 & , o.w \end{cases} \quad \dots (3.1.3)$$

3.1.4. Partial Autocorrelation [6] [7]

We can write the partial autocorrelation's formula for AR(2) model as follows :

$$\emptyset_{kk} = \begin{cases} \frac{\emptyset_1}{1-\emptyset_2} & , k = 1 \\ \emptyset_2 & , k = 2 \\ 0 & , o.w \end{cases} \quad \dots (3.1.4)$$

3.2. Estimating of Parameter's Model [5] [8]

The parameters of second order autoregressive model AR(2) are estimated through Yule-Walker as follows:

$$R = \begin{bmatrix} 1 & r_1 \\ r_1 & 1 \end{bmatrix} \quad , \underline{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix}$$

$$\widehat{\emptyset}_1 = \frac{r_1(1-r_2)}{(1-r_1^2)} \quad , \widehat{\emptyset}_2 = \frac{r_2 - r_1^2}{(1-r_1^2)} \quad \dots (3.2.1)$$

Where:

$$C_k = \frac{1}{n} \sum_{t=1}^{n-k} X_t X_{t-k} \quad , \; r_k = \frac{C_k}{C_0} = \frac{\sum_{t=1}^{n-k} x_t x_{t-k}}{\sum_{t=1}^{n} x_t^2} \quad , k = 0,1,2 \quad \dots (3.2.2)$$

We can estimate the variance of random error $\hat{\sigma}_a^2$ for AR(2) model as follows :

$$\hat{\sigma}_a^2 = C_0 \left(1 - \acute{\underline{r}} \, \widehat{\underline{\emptyset}}\right) \quad \dots (3.2.3)$$

$$\hat{\sigma}_a^2 = C_0 \left(1 - \acute{\underline{r}} \, R^{-1} \underline{r}\right) = C_0 (1 - \acute{\widehat{\emptyset}} R \widehat{\emptyset}) \quad \dots (3.2.4)$$

$$\hat{\gamma}_0 = \frac{1}{n}\sum_{t=1}^{n}(X_i - \bar{X})^2 \qquad \text{....(3.2.5)}$$

4. THE SPECTRUM OF AR (2) MODEL [9, 10]

The spectrum of Non-Gaussian second order autoregressive model will be as follows :

$$f(w) = \frac{\sigma_a^2}{2\pi} \frac{1}{\left|1-\emptyset_1 e^{-i\omega k} - \emptyset_2 e^{-i\omega k}\right|^2} \qquad , \quad k = 0 \pm 1, \pm 2, \ldots, \pm \frac{n}{2} \qquad \text{... (4.1)}$$

$$f(w) = \frac{\sigma_a^2}{2\pi} \frac{1}{|1-\emptyset_1(\cos w - i\sin w) - \emptyset_2(\cos 2w - i \sin 2w)|^2}$$

$$f(w) = \frac{\sigma_a^2}{2\pi} \frac{1}{|1-\emptyset_1 \cos w - \emptyset_2 \cos 2w + i(\emptyset_1 \sin w + \emptyset_2 \sin 2w)|^2}$$

$$= \frac{\sigma_a^2}{2\pi} \frac{1}{1+\emptyset_1^2 cos^2 w + \emptyset_2^2 cos^2 2w - 2\emptyset_1 \cos w - 2\emptyset_1 \cos 2w + 2\emptyset_1\emptyset_2 \cos w\, cos 2w + \emptyset_1^2 sin^2 w + \emptyset_2^2 sin^2 2w + 2\emptyset_1\emptyset_2 \sin w \sin 2w}$$

$$= \frac{\sigma_a^2}{2\pi} \frac{1}{1+\emptyset_1^2 + \emptyset_2^2 - 2\emptyset_1 \cos w - 2\emptyset_2 \cos 2w + 2\emptyset_1\emptyset_2 \cos w\, cos 2w + 4\emptyset_1\emptyset_2 \sin w \sin 2w}$$

$$= \frac{\sigma_a^2}{2\pi} \frac{1}{1+\emptyset_1^2 + \emptyset_2^2 - 2\emptyset_1 \cos w - 2\emptyset_2 \cos 2w + 2\emptyset_1\emptyset_2 \cos w}$$

$$f(w) = \frac{\sigma_a^2}{2\pi} \frac{1}{1+\hat{\emptyset}_1^2 + \hat{\emptyset}_2^2 - 2\hat{\emptyset}_1\,(1-\hat{\emptyset}_2)\cos w - 2\hat{\emptyset}_2 \cos 2w} \qquad \text{.... (4.2)}$$

The (PSD) function will be as follows : $$P(w) = \frac{f(w)}{\gamma_0} \qquad \text{... (4.3)}$$

5. SIMULATION

We design (12) simulation experiments to calculate (estimate) the spectrum and (PSD) function of Non-Gaussian second order autoregressive model AR(2).

The experiment is repeated 5000, sample sizes are (25, 75, 150), and the parameters initial values are ($\emptyset_1 = 0.75$, $\emptyset_2 = -0.125$).We estimate the parameters of the model through sample auto covariance and the model random errors are distributed as Non-Gaussian distribution (discrete and continuous).

We using the following comparison tools : MSE(p(w)), MAPE(P(w)).

Table-1. General formula for generating variables follows Discrete and Continuous distributions [5]

Distribution	Formula	Distribution	Formula
Binomial (n,p)	$a_t = \begin{cases} 1 & if \quad 0 < u \le p \\ 0 & if \quad p < u \le 1 \end{cases}$	(α, β) Weibull	$a_t = \alpha - \beta[\ln(1-u)]^{\frac{1}{\alpha}}$
Poisson (λ)	$a_t = \begin{cases} 1,2,\ldots\ldots & if \quad -\ln\prod_{i=1}^{n} u_i < \lambda \le -\ln\prod_{i=1}^{n+1} u_i \\ 0 & if \quad -\ln u > \lambda \end{cases}$	Pareto (α, β)	$a_t = \left[\frac{\alpha^{\beta}}{1-u}\right] exp\left(\frac{1}{\beta}\right)$
Exponential (λ)	$a_t = -\lambda \ln(1-u)$	Geometric(P)	$a_t = \ln U / \ln(1-P)$
Beta (α, β)	$a_t = y_1/(y_1+y_2)$ $y_1 = u_1^{\frac{1}{\alpha}}$, $y_2 = u_2^{\frac{1}{\beta}}$, $y_1 + y_2 < 1$	Logistic (α, β)	$a_t = \alpha - \beta \ln(\frac{1}{u} - 1)$
Cauchy (α, β)	$a_t = \alpha + \beta \tan[\pi(u - 0.5)]$	Laplace (α, β)	$a_t = \alpha - \beta \ln[2(1-u)]$
Log-Normal(μ, σ^2)	$a_t = \exp[(-2\ln(u_1))^{\frac{1}{2}}.\cos(2\pi u_2)]$	Uniform (α, β)	$a_t = \alpha + (\alpha - \beta)u$

Table-2. Values of [f(w), p(w), MSE, MAPE] when random error of AR(2) are discrete (Binomial, Poisson, Geometric) distribution.

Distribution		Binomial				Poisson				Geometric			
N	W	f(w)	P(w)	MSE(p(w))	MAPE(p(w))	f(w)	P(w)	MSE(p(w))	MAPE(p(w))	f(w)	P(w)	MSE(p(w))	MAPE(p(w))
25	15	0.0190	0.0267	0.00075396	20.0797	581.1684	0.0315	0.0011	31.0078	0.1604	0.0261	7.1907e-04	19.2840
	30	0.0393	0.0549	0.0030	7.8783	1.1086e+03	0.0610	0.0039	14.5715	0.3231	0.0542	0.0030	6.8970
	45	0.0718	0.1519	0.0109	18.8788	2.0463e+03	0.1159	0.0151	29.6549	0.6021	0.1009	0.0106	16.6232
	60	0.0179	0.0252	0.00070545	27.0281	566.8342	0.0307	0.0011	42.5213	0.1460	0.0243	6.4985e-04	25.6281
	75	0.4382	0.6282	0.4590	29.8288	1.1161e+04	0.6496	0.5962	45.1440	3.6480	0.6246	6.4985e-04	28.2480
	90	0.0226	0.0316	0.0010	11.3816	665.2559	0.0360	0.0014	17.4012	0.1875	0.0311	9.8643e-04	11.1891
	120	0.1908	0.2726	0.0845	28.0991	5.3544e+03	0.3132	0.1328	46.3773	1.6034	0.2711	0.0820	26.4122
	135	0.0175	0.0246	0.00068090	28.4654	568.1270	0.0307	0.0012	46.4398	0.1433	0.0239	6.4054e-04	26.6587
	150	0.1173	0.1664	0.0302	24.2536	3.3025e+03	0.1903	0.0444	38.1947	0.9743	0.1649	0.0293	22.5952
	180	0.0207	0.0290	0.00086702	15.3307	615.3866	0.0332	0.0012	23.2814	0.1704	0.0283	8.2491e-04	14.6345
75	15	0.0163	0.0264	0.00071195	12.1842	3.0508e+04	0.0286	8.4996e-04	17.3652	0.1353	0.0262	7.0157e-04	12.2126
	30	0.0324	0.0524	0.0028	4.8805	5.9008e+04	0.0557	0.0031	7.9615	0.2691	0.0521	0.0027	4.6870
	45	0.0577	0.0933	0.0089	11.2500	1.0622e+05	0.1004	0.0105	16.8197	0.4779	0.0928	0.0088	10.7428
	60	0.0150	0.0243	0.00061296	15.9530	2.8415e+04	0.0268	7.7482e-04	23.2868	0.1255	0.0241	6.0561e-04	15.8944
	75	0.3409	0.5522	0.3196	17.0777	6.0202e+05	0.5707	0.3591	24.3373	2.8652	0.5556	0.3235	16.9267
	90	0.0194	0.0312	0.00098239	6.4079	3.5667e+04	0.0333	0.0011	9.1180	0.1601	0.0310	9.6740e-04	6.5062
	120	0.1471	0.2375	0.0586	15.8389	2.7181e+05	0.2577	0.0730	24.1179	1.2304	0.2376	0.0588	16.0317
	135	0.0148	0.0239	0.00059597	17.0964	2.7729e+04	0.0261	7.4247e-04	24.8053	0.1219	0.0236	5.8179e-04	16.6883
	150	0.0916	0.1481	0.0226	14.1811	1.6724e+05	0.1588	0.0270	21.5803	0.7602	0.1472	0.0223	14.0566
	180	0.0177	0.0286	0.00082781	9.0441	3.2405e+04	0.0306	9.5471e-04	12.8705	0.1464	0.0284	8.1773e-04	9.1901
150	15	0.156	0.0262	0.00069698	8.7712	4.2542e+05	0.0276	7.7960e-04	12.2695	0.1307	0.0262	6.9694e-04	8.6499
	30	0.0308	0.0518	0.0027	3.4195	8.4043e+05	0.0546	0.0030	5.5338	0.2562	0.0517	0.0027	3.3607
	45	0.0543	0.0913	0.0084	7.8560	1.4931e+06	0.0970	0.0096	11.6641	0.4509	0.0909	0.0083	7.7904
	60	0.143	0.0241	0.00059325	11.6934	3.9189e+05	0.0255	6.7442e-04	15.9675	0.1197	0.0241	5.9053e-04	11.4838
	75	0.3180	0.5350	0.2929	12.0603	8.5612e+06	0.5555	0.3227	16.8626	2.6422	0.5352	0.2931	11.8952
	90	0.185	0.0311	0.00097312	4.6289	5.0337e+05	0.0327	0.0011	6.4190	0.1549	0.0311	9.7134e-04	4.7525
	120	0.1373	0.2306	0.0543	11.2356	3.7590e+06	0.2440	0.0621	16.4001	1.1488	0.2305	0.0542	11.1802
	135	0.0141	0.0237	0.00057519	12.1405	3.8869e+05	0.0252	6.6433e-04	17.2553	0.1166	0.0237	5.7351e-04	12.3480
	150	0.0853	0.1435	0.0209	9.8906	2.3461e+06	0.1527	0.0241	14.6396	0.7111	0.1426	0.0207	9.8279
	180	0.0170	0.0286	0.00082091	6.5112	4.6089e+05	0.0299	9.0671e-04	8.8830	0.1412	0.0285	8.1651e-04	6.5135

Table-3. Values of [f(w), p(w), MSE, MAPE] when random error of AR(2) are continuous (Cauchy, Laplace, Exponential) distribution.

Distribution		Cauchy				Laplace				Exponential			
N	W	f(w)	P(w)	MSE(p(w))	MAPE(p(w))	f(w)	P(w)	MSE(p(w))	MAPE(p(w))	f(w)	P(w)	MSE(p(w))	MAPE(p(w))
25	15	2.2222e+06	0.0386	0.0018	23.7140	0.2849	0.0275	0.00080306	20.5613	0.0191	0.0261	0.00071888	19.2604
	30	1.4994e+04	0.1010	0.0121	24.6103	0.5872	0.0568	0.0033	8.5793	0.0387	0.0542	0.0030	6.8069
	45	1.0483e+06	0.1885	0.0420	29.6359	1.0838	0.1061	0.0119	19.1448	0.0726	0.1011	0.0107	17.3113
	60	1.6458e+04	0.0340	0.0015	27.6374	0.2598	0.0255	0.00072327	26.8773	0.0178	0.0244	0.00066164	26.5855
	75	7.0861e+04	0.4714	0.2474	25.9759	6.3134	0.6269	0.4595	29.9059	0.4392	0.6310	0.4586	28.6548
	90	1.8384e+05	0.0504	0.0029	20.9743	0.3401	0.0327	0.0011	11.5106	0.0225	0.0312	0.00098780	10.9356
	120	4.7647e+06	0.3428	0.1320	23.7867	2.8155	0.2802	0.0894	28.9595	0.1932	0.2698	0.0811	26.5089
	135	9.7076e+03	0.0340	0.0020	30.2852	0.2664	0.0256	0.00073996	29.2933	0.0174	0.0238	0.00063324	27.1011
	150	5.7683e+06	0.2656	0.0810	28.8062	1.7573	0.1713	0.0322	25.2662	0.1167	0.1653	0.0294	22.8036
	180	7.9756e+03	0.0443	0.0023	22.3490	0.3025	0.0299	0.00092089	15.3775	0.0205	0.0285	0.00083912	14.8498
75	15	2.7083e+05	0.0352	0.0013	10.5671	0.2523	0.0270	0.00074664	12.3426	0.0164	0.0263	0.00070498	12.1810
	30	1.5517e+05	0.0940	0.0093	11.8543	0.5139	0.0544	0.0030	5.4273	0.0322	0.0521	0.0027	4.5970
	45	2.2194e+05	0.1739	0,0319	15.0321	0.9051	0.0973	0.0097	11.6814	0.0577	0.0927	0.0087	10.6127
	60	6.9098e+05	0.0304	0.0010	12.0877	0.2321	0.0250	0.00064894	16.3570	0.0149	0.0242	0.00061029	16.1999
	75	4.0161e+05	0.4871	0.2459	13.0383	5.3726	0.5678	0.3393	17.5653	0.3418	0.5528	0.3203	16.8527
	90	6.6569e+06	0.0464	0.0023	9.5509	0.3026	0.0323	0.0011	6.7284	0.0192	0.0310	0.00096979	6.4680
	120	7.9253e+05	0.3434	0.1225	12.5314	2.3061	0.2480	0.0643	16.7139	0.1474	0.2367	0.0582	15.7530
	135	7.7709e+05	0.0297	0.00099078	12.7651	0.2276	0.0244	0.00062250	16.9904	0.0148	0.0238	0.00059245	16.8271
	150	1.0840e+10	0.2538	0.0674	15.2157	1.4468	0.1544	0.0247	14.9321	0.0907	0.1465	0.0221	13.6020
	180	1.1388e+04	0.0404	0.0017	10.0070	0.2786	0.0295	0.00088312	9.2158	0.0176	0.0285	0.00082094	8.9708
150	15	7.9867e+04	0.0346	0.0012	6.9751	0.2452	0.0269	0.00073479	8.7348	0.0156	0.0262	0.00069359	8.7523
	30	2.2419e+06	0.0916	0.0085	7.1177	0.4896	0.0540	0.0029	3.8922	0.0310	0.0517	0.0027	3.3742
	45	4.4344e+06	0.1717	0.0304	10.0866	0.8749	0.0957	0.00090093	8.5335	0.0543	0.0912	0.0084	7.8665
	60	4.0435e+04	0.0299	0.00096451	8.1814	0.2237	0.0246	0…61859	11.6870	0.0144	0.0241	0.00059342	11.7355
	75	8.0110e+05	0.4918	0.2462	7.8837	5.0115	0.5535	0.3145	12.7287	0.3183	0.5346	0.2927	12.1618
	90	2.6176e+06	0.0457	0.0021	5.9250	0.2920	0.0321	0.0010	4.6775	0.0186	0.0311	0.00097125	4.5905
	120	2.7607e+08	0.3423	0.1193	8.4723	2.1932	0.2412	0.0595	11.7434	0.1369	0.2295	0.0537	11.0488
	135	2.3664e+06	0.0288	0.00086333	8.0337	0.2198	0.0242	0.00059945	12.2875	0.0141	0.0236	0.00056815	11.9785
	150	3.1046e+05	0.2529	0.0656	10.1191	1.3744	0.1505	0.0230	10.6987	0.0851	0.1426	0.0207	9.7931
	180	5.0532e+04	0.0395	0.0016	5.9372	0.2667	0.0243	0.00086391	6.7259	0.0170	0.0285	0.00081936	6.5369

Table-4. Values of [f(w), p(w), MSE, MAPE] when random error of AR(2) are continuous (Beta, Log-Normal, Uniform) distribution.

Distribution		Beta				Log-Normal				Uniform			
n	W	f(w)	P(w)	MSE(p(w))	MAPE(p(w))	f(w)	P(w)	MSE(p(w))	MAPE(p(w))	f(w)	P(w)	MSE(p(w))	MAPE(p(w))
25	15	0.0050	0.0244	0.00062498	18.1320	0.3298	0.0269	7.6443e-04	18.8401	0.0086	0.0234	5.7008e-04	16.9598
	30	0.0105	0.0521	0.0027	5.0063	0.6818	0.0557	0.0031	8.1360	0.0186	0.0514	0.0027	3.9130
	45	0.0195	0.0976	0.0098	14.3407	1.3314	0.1044	0.0114	17.1701	0.0347	0.0975	0.0097	12.7545
	60	0.0046	0.0223	0.00054072	24.2067	0.2986	0.0247	6.8225e-04	24.7172	0.0077	0.0211	4.7473e-04	21.8704
	75	0.1255	0.6327	0.4474	27.0662	6.8078	0.6221	0.4366	26.1444	0.2276	0.6447	0.4604	25.1958
	90	0.0060	0.0295	0.00088353	10.8858	0.3868	0.0320	0.0010	11.2643	0.0103	0.0284	8.1992e-04	10.6113
	120	0.0518	0.2634	0.0753	23.6323	3.3923	0.2756	0.0845	25.7999	0.0947	0.2663	0.0761	21.5157
	135	0.0045	0.0221	0.00053674	25.2637	0.3039	0.0244	6.7371e-04	25.9489	0.0075	0.0206	4.5551e-04	22.7294
	150	0.0319	0.1601	0.0271	19.9533	2.0442	0.1685	0.0306	22.3731	0.0573	0.1606	0.0271	18.0138
	180	0.0055	0.0269	0.00074463	14.5527	0.3662	0.0292	8.8392e-04	14.9472	0.0094	0.0258	6.8027e-04	13.6825
75	15	0.0039	0.0251	0.00064364	11.8632	0.2938	0.0269	7.3914e-04	11.7403	0.0062	0.0244	6.0472e-04	11.3890
	30	0.0077	0.0498	0.0025	3.7449	0.5946	0.0543	0.0030	6.0628	0.0125	0.0492	0.0024	3.1459
	45	0.0137	0.0888	0.0080	9.6097	1.1113	0.0973	0.0097	11.8799	0.0224	0.0879	0.0078	8.5416
	60	0.0036	0.0230	0.00054971	15.2955	0.2661	0.0247	6.3648e-04	15.4054	0.0056	0.0222	5.0834e-04	14.7031
	75	0.0837	0.5412	0.3049	16.1552	6.0505	0.5685	0.3403	16.6343	0.1371	0.5414	0.3041	15.4775
	90	0.0046	0.0297	0.00088957	6.4412	0.3474	0.0321	0.0010	6.8271	0.0074	0.0291	8.5161e-04	6.2691
	120	0.0352	0.2285	0.0539	14.3601	2.8506	0.2476	0.0642	16.2881	0.0578	0.2276	0.0533	13.3060
	135	0.0035	0.0226	0.00053103	16.4407	0.2604	0.0242	6.1556e-04	16.1137	0.0056	0.0219	4.9601e-04	15.4065
	150	0.0217	0.1401	0.0201	12.3578	1.7391	0.1535	0.0244	14.5938	0.0353	0.1396	0.0199	11.6417
	180	0.0042	0.0272	0.00074761	8.9598	0.3145	0.0293	8.7055e-04	9.1158	0.0068	0.0265	7.1047e-04	8.7110
150	15	0.0036	0.0253	0.00064927	8.7844	0.2821	0.0269	7.3249e-04	8.7437	0.0056	0.0247	6.1769e-04	8.1973
	30	0.0070	0.0494	0.0024	2.8023	0.5935	0.0543	0.0030	4.8529	0.0110	0.0486	0.0024	2.5159
	45	0.0123	0.0865	0.0075	6.9565	1.0328	0.0962	0.0094	9.0532	0.0193	0.0854	0.0073	6.5823
	60	0.0033	0.0233	0.00055552	11.2025	0.2588	0.0245	6.1422e-04	11.2959	0.0051	0.0227	5.2582e-04	10.7788
	75	0.0734	0.5155	0.2712	11.4205	5.7043	0.5505	0.3108	11.6949	0.1152	0.5095	0.2644	10.7991
	90	0.0043	0.0299	0.00089585	4.6100	0.3481	0.0322	0.0010	5.1896	0.0066	0.0293	8.6261e-04	4.6505
	120	0.0312	0.2185	0.0485	10.1963	2.6524	0.2413	0.0597	11.8755	0.0488	0.2156	0.0472	9.6066
	135	0.0033	0.0228	0.00053358	11.9957	0.2513	0.0241	5.9164e-04	11.5915	0.0051	0.0224	5.1078e-04	11.6047
	150	0.0194	0.1356	0.0186	8.9405	1.6309	0.1512	0.0233	10.8152	0.0302	0.1339	0.0181	8.3950
	180	0.0039	0.0275	0.00076001	6.5051	0.3056	0.0294	8.6801e-04	6.5975	0.0061	0.0269	7.2579e-04	6.4496

Table-5. Values of [f(w), p(w), MSE, MAPE] when random error of AR(2) are continuous (Pareto, Logistic, Weibull) distribution.

Distribution		Pareto				Logistic				Weibull			
n	W	f(w)	P(w)	MSE(p(w))	MAPE(p(w))	f(w)	P(w)	MSE(p(w))	MAPE(p(w))	f(w)	P(w)	MSE(p(w))	MAPE(p(w))
25	15	1.6938e+04	0.0298	9.8607e-04	18.5335	0.8011	0.0367	0.0015	27.8529	0.3728	0.0238	5.9268e-04	17.3731
	30	2.3045e+05	0.0662	0.0046	17.1945	1.9556	0.0913	0.0101	29.7596	0.7850	0.0518	0.0027	4.2764
	45	2.2385e+05	0.1214	0.0159	22.2347	3.7184	0.1753	0.0392	41.0948	1.4740	0.0977	0.0098	13.0040
	60	2.3974e+04	0.0273	8.8957e-04	21.3038	0.7135	0.0329	0.0013	33.6553	0.3426	0.0216	5.0679e-04	22.0042
	75	6.6333e+05	0.5969	0.4049	23.2765	11.3688	0.5062	0.2969	32.2102	9.5355	0.6441	0.4595	25.4380
	90	1.2430e+04	0.0365	0.0014	14.4969	0.9927	0.0464	0.0025	22.7995	0.4453	0.0289	8.4798e-04	10.2302
	120	1.3492e+06	0.2973	0.0978	23.7016	7.5305	0.3343	0.1326	33.5491	4.0029	0.2655	0.0755	21.9866
	135	6.8712e+04	0.0265	8.4051e-04	21.6180	0.6981	0.0321	0.0012	34.7785	0.3373	0.0212	4.9290e-04	22.9338
	150	1.0196e+07	0.1906	0.0397	24.9935	5.5455	0.2564	0.0816	40.1942	2.4061	0.1602	0.0269	18.1927
	180	2.5591e+05	0.0329	0.0012	16.1450	0.8851	0.0408	0.0018	24.0656	0.4087	0.0261	6.9934e-04	13.4756
75	15	7.0764e+04	0.0306	9.9414e-04	11.0646	0.7629	0.0328	0.0011	14.6740	0.2856	0.0248	6.2688e-04	11.3617
	30	5.5092e+05	0.0701	0.0051	15.0119	1.8366	0.0791	0.0065	15.3753	0.5673	0.0496	0.0025	3.4295
	45	1.7633e+05	0.1290	0.0175	18.8841	3.4710	0.1496	0.0241	21.9133	1.0122	0.0885	0.0079	9.2425
	60	6.1419e+05	0.0272	7.8965e-04	11.6599	0.6792	0.0291	8.8994e-04	17.8230	0.2618	0.0226	5.3048e-04	15.0683
	75	6.8049e+05	0.5597	0.3287	12.2158	12.5016	0.5342	0.2983	16.7392	6.1193	0.5394	0.3022	15.2282
	90	4.4800e+04	0.0381	0.0015	10.7167	0.9713	0.0416	0.0018	11.4583	0.3373	0.0295	8.7281e-04	6.5114
	120	9.2846e+07	0.2995	0.0947	16.8852	7.6081	0.3264	0.1133	20.4469	2.6091	0.2286	0.0539	13.7511
	135	4.4829e+05	0.0264	7.4999e-04	11.9656	0.6589	0.0282	8.3518e-04	18.2284	0.2558	0.0223	5.1926e-04	15.6730
	150	5.8657e+06	0.2003	0.0424	19.4839	5.2286	0.2255	0.0550	22.8786	1.5992	0.1403	0.0201	12.0138
	180	1.0202e+05	0.0342	0.0012	10.0856	0.8578	0.0368	0.0014	12.6228	0.3062	0.0269	7.3028e-04	8.6965
150	15	4.4448e+04	0.0312	9.9597e-04	7.6415	0.7530	0.0320	0.0010	10.2676	0.2643	0.0215	6.3947e-04	8.7011
	30	1.6770e+05	0.0739	0.0056	13.0233	1.8170	0.0768	0.0060	10.1799	0.5104	0.0490	0.0024	2.7496
	45	1.9679e+07	0.1363	0.0192	16.0881	3.3638	0.1424	0.0210	14.9864	0.8992	0.0862	0.0075	6.9732
	60	3.4058e+05	0.0275	7.9128e-04	7.7677	0.6623	0.0280	8.0171e-04	12.2646	0.2396	0.0230	5.4217e-04	11.2974
	75	4.6551e+07	0.5510	0.3104	8.5914	12.9781	0.5475	0.3060	11.5420	5.3518	0.5131	0.2687	11.1379
	90	6.2450e+05	0.0396	0.0016	8.8998	0.9600	0.0406	0.0017	7.7879	0.3086	0.0297	8.8447e-04	4.5879
	120	1.6570e+06	0.3095	0.0989	12.1901	7.5647	0.3201	0.1058	14.3870	2.2622	0.2179	0.0482	9.9367
	135	5.5258e+04	0.0268	7.6382e-04	8.2406	0.6464	0.0273	7.6299e-04	12.9318	0.2370	0.0227	5.2641e-04	11.9637
	150	4.2632e+06	0.2087	0.0451	15.0947	5.1575	0.2190	0.0499	16.2153	1.4139	0.1349	0.0184	8.8415
	180	2.7737e+05	0.0350	0.0013	8.0559	0.8464	0.0359	0.0013	8.9428	0.2848	0.0273	7.4806e-04	6.4418

6. CONCLUSIONS

1- The value of MSE, MAPE and P(w) is decreased, whenever the value size of the sample is increased for all the distributions and frequencies.

2- In case of Cauchy and Poisson distribution, whenever the sample size increases the value of p(w) and f(w) goes up, while the value of MSE, MAPE decreases.

3- For all distributions and samples sizes, the highest value for p(w), f(w), MSE and MAPE is when W=75.

4- For all distributions and samples sizes, the lowest value of MSE is when W=135.

5- For most distributions, the highest value of MAPE is shown when W=135 and n=75.

6- There is no essential differences appeared for the spectrum values, or spectral power density function, and mean square errors and mean absolute percentage errors between discrete and continuous distributions for all frequencies.

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تقدير دالة قدرة كثافة الطيف لأنموذج الانحدار الذاتي غير الطبيعي من الدرجة الثانية

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المستخلص

تساهم هذه الدراسة في التوصل الى افضل المقدرات لدالة الطيف في ظل مخالفة شرط التوزيع الطبيعي من خلال دراسة تلك التقديرات في حالة كون الأخطاء العشوائية تتوزع توزيعات معلمية اخرى سواء كانت متقطعة او مستمرة لان طول الفترة في السلسلة الزمنية تؤدي الى زيادة احتمالية تعرضها لظروف غير طبيعية تجعلها تنحرف عن مسارها الطبيعي وبالتالي الأخلال بشروط دراستها التي تفترض توزيعا طبيعيا للأخطاء. يعتبر التباين أحد أهم المؤشرات لدراسة سلوك السلسلة الزمنية وبوجود إمكانية تحويله الى دالة الطيف (دراسة السلسلة الزمنية في حقل التكرار) لذا من المناسب جدا دراسة سلوك دالة الطيف لغرض التعرف على سلوك السلسلة الزمنية وتعتبر هذه الدراسة اضافة جيدة لأنها من الدراسات القليلة في هذا المجال.

يهدف هدا البحث الى احتساب الطيف وقيمة دالة قدرة كثافة الطيف لأنموذج الانحدار الذاتي من الدرجة الثانية بافتراض ان الاخطاء العشوائية للأنمودج تتوزع توزيعا غير طبيعيا ، ولاحجام عينات مختلفة ولمختلف الترددات ومن خلال 12 تجربة محاكاة، إذ تمت المقارنة بين نتائج المحاكاة باستخدام متوسط مربعات الخطأ ومتوسط الخطأ النسبي المطلق كمعيارين للمقارنة.