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THEORETICAL ANALYSIS OF REACTIVE FLOW OF MASS AND HEAT TRANSFER OVER A SEMI-INFINITE VERTICAL PLATE

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ABSTRACT

A general analysis has been developed to study unsteady reactive flow of mass and heat transfer over a semi-infinite vertical plate. The governing equations which are nonlinear partial differential equations were reduced to the coupled nonlinear ordinary differential equations using the similarity transformations. The resulting equations, nonlinear ordinary differential equations, are solved numerically by using Runge-Kutta the shooting method. The effects of the chemical reaction Parameters, the Prandtl number, the thermal Grashof number, concentration Grashof number and the Schmidt number are examined on the velocity, temperature and concentration profiles.

Keywords: Reactive flow, Chemical reaction, Unsteadiness, Mass and heat transfer, Vertical plate.

Contribution/ Originality

This study contributes in the existing literatures on free convective flow of mass and heat transfer over plate. This study investigated numerically using similarity transform together with the Runge-kutta shooting techniques the unsteady unidimensional problem of a reactive flow of mass and heat transfer over a semi-infinite vertical plate.

1. INTRODUCTION

Natural convection flows under the control of gravitational force have been studied most widely because they occur regularly in nature and in science and engineering applications. When a heated surface gets in touch with the fluid, the result of temperature difference brings about buoyancy force, which creates the natural convection. In recent times heat flux applications are extensively used in industries, engineering and science fields. The problem of boundary layer flow over a vertical plate under different conditions was studied by many researchers.

Theoretical solution of simultaneous heat and mass transfer, by free convection about a vertical flat plate, was investigated by Bottemanne [1] and also by Muthucumaraswamy and Ravi Shankar [2] and Aiyesimi, et al. [3]. Mohamed [4], Palani and Srikanth [5], Samad and Mohebujjaman [6], and Jyothi Bala and Vijaya Kumar Varma [7] analysed and investigated, the influence chemical reaction on hydromagnetic free convection heat and mass transfer for a viscous fluid past a semi-infinite vertical plates.

Soundalgekar and Ganesan [8] studied the finite difference of transient free convection with mass transfer of an isothermal vertical flat plate. Mahanti and Gaur [9] showed the effects of varying viscosity and thermal conductivity on steady free convective flow and heat transfer along an isothermal vertical plate in the presence of heat sink. Elbashbeshy and Ibrahim [10] investigated the steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate. Rani and Chang [11] analyses the natural convection flow over an isothermal semi infinite vertical cylinder with effects of variable viscosity and thermal conductivity. Takhar, et al. [12] investigated transient free convection past a semi-infinite vertical plate with variable surface temperature. Soundalgekar and Ganesan [13]analysed transient free convective flow past a semi-infinite vertical plate with mass transfer.

Prakash and Ogulu [14], Chaudhary and Kumar [15], Fasogbon [16], and Mohamed, et al. [17] employed the perturbation technique and investigated the combined mass and heat flow over vertical plate with chemical and magnetic effects. Ishak [18]) and Seddeek and Almushigeh [19]investigated the effect of a chemical reaction on the flow over a linearly stretching vertical sheet.

The objective of this study is to investigate numerically using similarity transform together with the Runge-kutta shooting techniques the unsteady reactive flow of mass and heat transfer over a semi-infinite vertical plate.

2. FORMULATION OF THE PROBLEM

This study considers an unsteady unidimensional problem of a reactive flow of mass and heat transfer over a semi-infinite vertical plate. This research is based on the following assumptions: The fluid physical properties are assumed to be isotropic and constant, which varies linearly with the fluid temperature and the density variation in the body force term in the momentum equation which are approximated by the Boussinesq relations. Also that, the effect of viscous dissipation is negligible in the energy equation.

The x-axis is taken to be along the plate in the vertically upward direction and the y-axis is chosen to be perpendicular to the plate.

The governing equations, continuity equation, momentum equation, heat transfer equation and the mass transfer equation for the problem takes the following form:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} + \lambda\left(C - C_\infty\right) \tag{4}$$

Subject to the following boundary conditions,

$$u = U, T = T_w, C = C_w, \text{ at } y = 0 \tag{5}$$

$$u \to 0, T \to T_\infty, C \to C_\infty, \text{ as } y \to \infty \text{ for } t > 0 \tag{6}$$

Nomenclature

$(u, v) =$ Velocity, $(x, y) =$ Coordinates, $t =$ Time, $T =$ Temperature of the flow,

$T_w =$ Plate surface temperature, $T_\infty =$ Temperature outside the flow, $\upsilon =$ Kinematic viscosity,

$g =$ Gravity, $\beta_T =$ Heat transfer coefficient, $\beta_C =$ Mass transfer coefficient

$C =$ Concentration of fluid, $C_w =$ Plate surface concentration, $C_\infty =$ Concentration outside the flow, $\alpha =$ Thermal diffusion, $D =$ Coefficient of mass diffusion and $\lambda =$ Rate of Chemical Reaction.

Solving for v, from equation (1), the suction velocity becomes,

$$v = V_o = \text{constant} \tag{7}$$

To transform the equations to dimensionless form, we introduced the following dimensionless quantities:

$$\frac{u}{u_\infty} = f(\eta), \text{ where } \eta = \frac{y}{2\sqrt{\upsilon t}},$$

$$\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},$$

$$G_C=\frac{4tg}{u}\beta_C(C_w-C_\infty),\quad G_T=\frac{4tg}{u}\beta_T(T_w-T_\infty),\quad \Pr=\frac{\upsilon}{\alpha},\quad Sc=\frac{\upsilon}{D}$$

Substituting the above dimensionless quantities into the governing Equations (2) – (4) and the boundary conditions, we obtain the following dimensionless form:

$$\frac{d^2f(\eta)}{d\eta^2}+2(\eta+c)\frac{df(\eta)}{d\eta}+G_T\theta(\eta)+G_C\phi(\eta)=0 \tag{8}$$

$$\frac{d^2\theta(\eta)}{d\eta^2}+2\Pr(\eta+c)\frac{d\theta(\eta)}{d\eta}=0 \tag{9}$$

$$\frac{d^2\phi(\eta)}{d\eta^2}+2Sc(\eta+c)\frac{d\phi(\eta)}{d\eta}+Sc\,\tau\phi(\eta)=0 \tag{10}$$

Subject to the transformed boundary conditions:

$$f=\theta=\phi=1 \text{ at } \eta=0 \text{ and } f=\theta=\phi=0 \text{ as } \eta\to 0 \tag{11}$$

Where,

c = Unsteadiness parameter, G_T = Thermal Grashof number

G_C = Concentration Grashof number $\Pr=\frac{\upsilon}{\alpha}$ = The Prandlt number,

$Sc=\frac{\upsilon}{D}$ = The Schmindt number and τ = The Chemical Reaction Parameter.

3. NUMERICAL SOLUTION METHOD

The system of non-linear and locally similar ordinary differential equations (8) – (10) together with the boundary conditions (11) have been solved numerically by using Nachtsheim-Swigert shooting iteration technique (1995) along with fourth order Runge-Kutta integration scheme.

The numerical computations were carried out using Maple Release 15. From the course of numerical computation, the skin-friction coefficient, the local Nusselt number and the local Sherwood number, are obtained and their numerical values are given in a tabular form as shown in Table 1 below.

Table-1. Numerical computations showing the Skin friction $F'(0)$, the Nusselt number $\theta'(0)$ and the Sherwood number $\phi'(0)$ for the various embedded flow parameters.

G_T	**G_C**	**C**	**Pr**	**Sc**	τ	$-F'(0)$	$-\theta'(0)$	$-\phi'(0)$
0.1	0.1	0.1	0.72	0.24	1	1.1017225079	0.3456276073	0.366483153
0.5	0.1	0.1	0.72	0.24	1	0.7603393511	0.3456276073	0.366483153
1.0	0.1	0.1	0.72	0.24	1	0.3336104051	0.3456276073	0.366483153
0.1	0.5	0.1	0.72	0.24	1	0.8151565808	0.3456276073	0.366483153
0.1	1.0	0.1	0.72	0.24	1	0.4569491718	0.3456276073	0.366483153
0.1	0.1	0.2	0.72	0.24	1	1.2357556660	0.3570428659	0.402490401
0.1	0.1	0.3	0.72	0.24	1	1.3754144122	0.3686115810	0.439142011
0.1	0.1	0.1	3.0	0.24	1	1.1039978234	0.3634395174	0.366483153
0.1	0.1	0.1	7.1	0.24	1	1.1044346872	0.3669912648	0.366483153
0.1	0.1	0.1	0.72	0.24	1	1.1212745534	0.5465161312	0.366483153
0.1	0.1	0.1	0.72	0.24	1	1.1299943107	0.6877591023	0.366483153
0.1	0.1	0.1	0.72	0.62	1	1.1213656634	0.3456276073	0.624057431
0.1	0.1	0.1	0.72	0.78	1	1.1256056135	0.3456276073	0.712703275
0.1	0.1	0.1	0.72	0.24	2	1.0869399111	0.3456276073	0.048000000
0.1	0.1	0.1	0.72	0.24	3	1.0497691195	0.3456276073	0.606873211

The representation of the numerical results is depicted graphically in Figures 1 to 8 to illustrate the influence of the various studied physical parameters on the flow.

Figure-1. Velocity profiles for various values of G_T when Gc = 0.1, c = 0.5, Pr = 0.72, Sc = 0.24, $\tau = 0.1$.

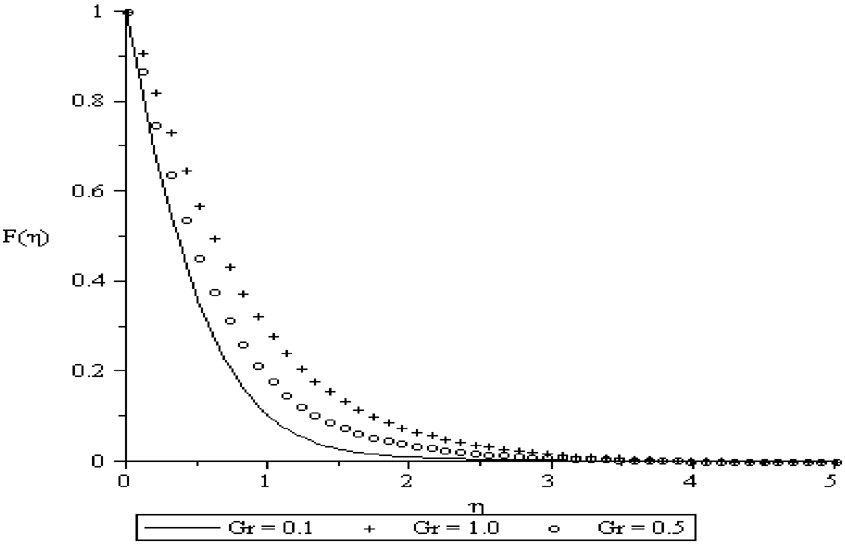


Figure-2. Velocity profiles for various values of Gc when G_T = 0.1, c = 0.5, Pr = 0.72, Sc = 0.24, $\tau = 0.1$.

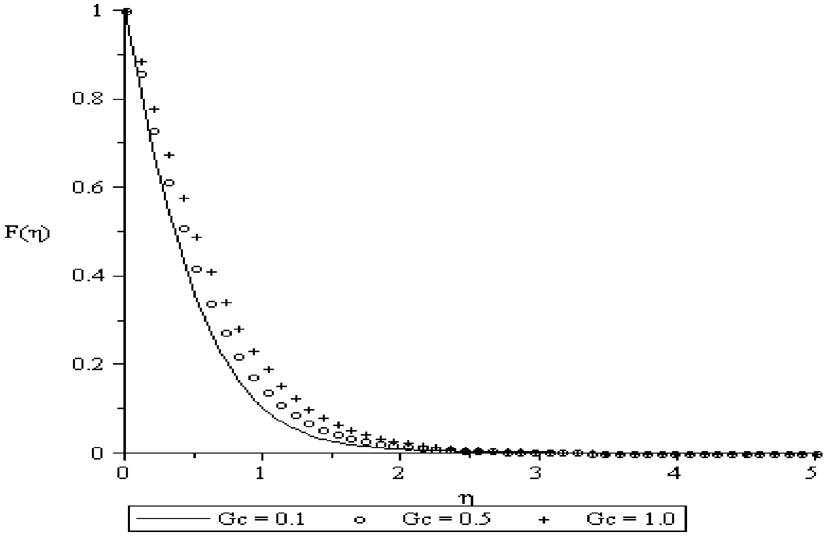


Figure-3. Velocity profiles for various value of c when G_T = 0.1, Gc = 0.1,Pr = 0.72, Sc = 0.24, $\tau = 0.1$.

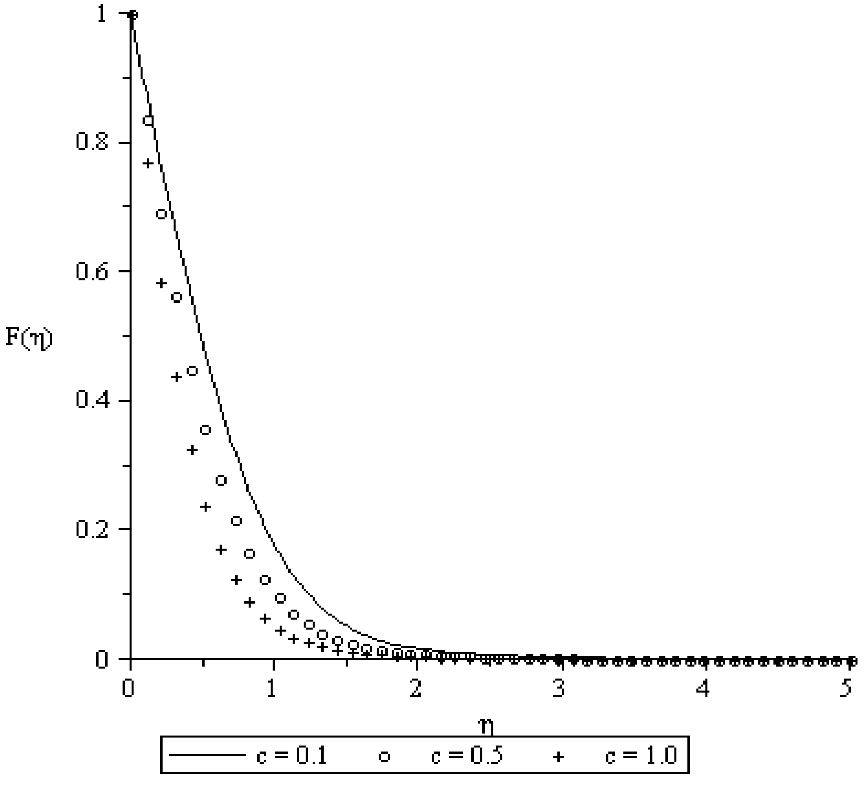


Figure-4. Concentration profiles for various value of c when $G_T = 0.1$, Gc = 0.1, Pr = 0.72, Sc = 0.24, $\tau = 0.1$.

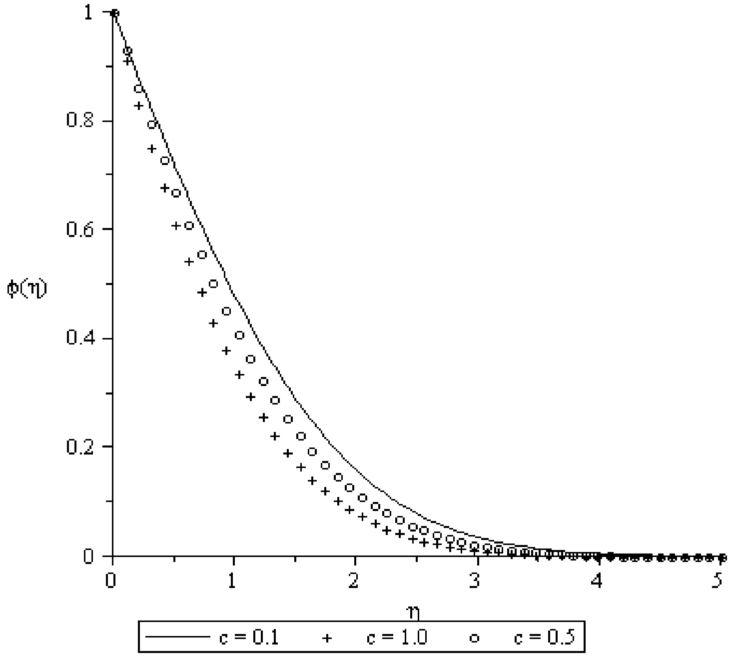


Figure-5. Temperature profiles for various value of c when $G_T = 0.1$, Gc = 0.1, Pr = 0.72, Sc = 0.24, $\tau = 0.1$.

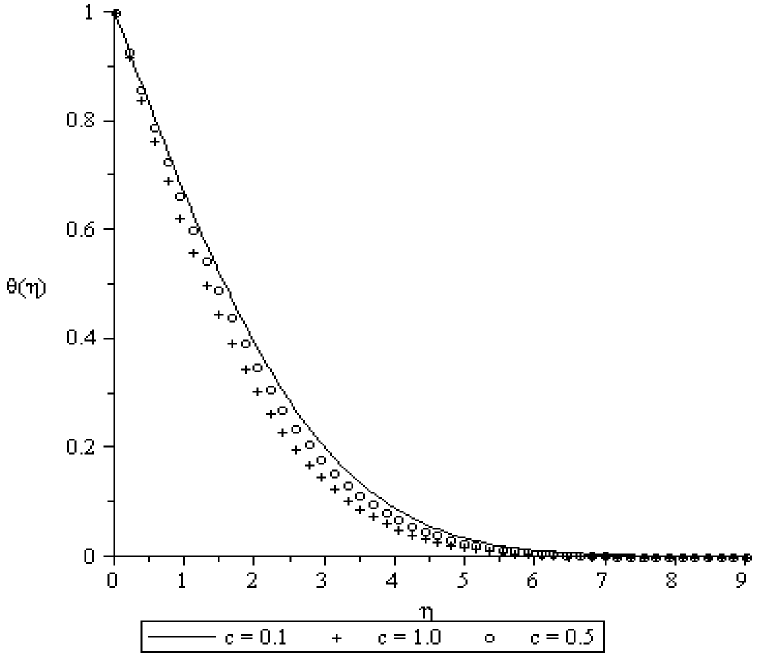


Figure-6. Temperature profiles for various values of Prandtl number P when $G_T = 0.1$, Gc = 0.1, c = 0.5, Sc = 0.24, $\tau = 0.1$.

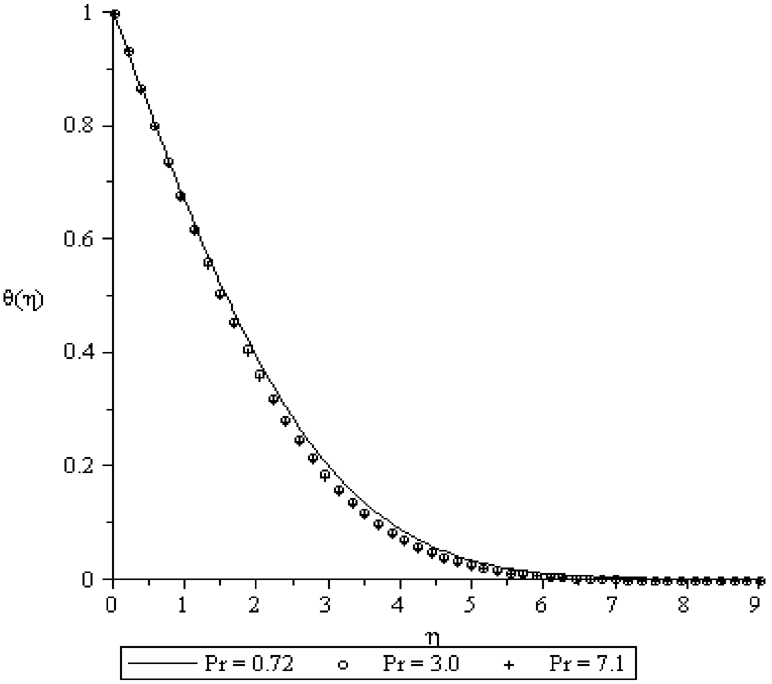


Figure-7. Concentration profiles for various values of Schmidt number Sc when $G_T = 0.1$, Gc = 0.1, c = 0.5, Pr = 0.72, $\tau = 0.1$.

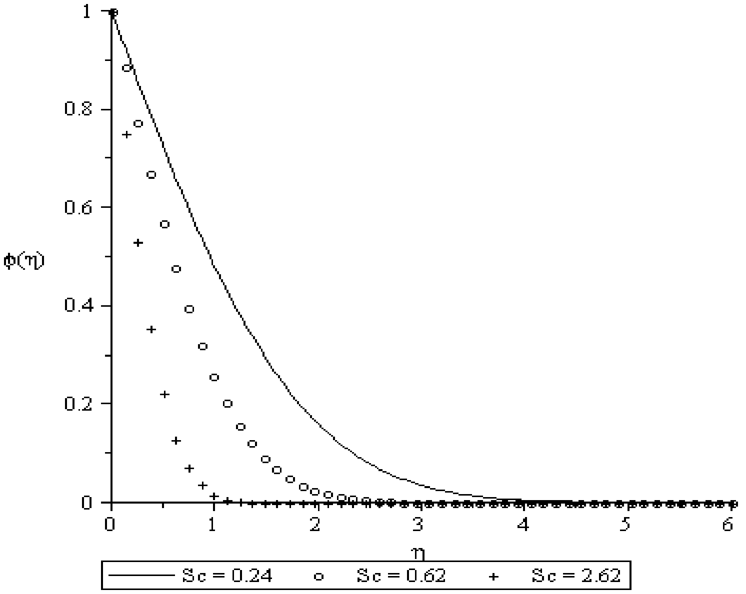
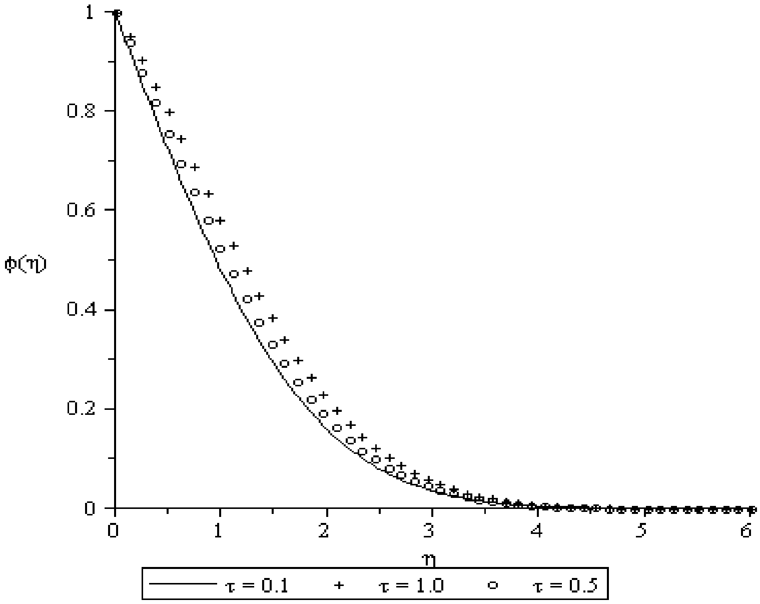


Figure-8. Concentration profiles for various values of chemical reaction parameter τ when G_T = 0.1, Gc = 0.1, c = 0.5, Pr = 0.72, Sc = 0.24.



4. RESULTS AND DISCUSSION

Numerical calculations have been carried out for different values of the physical parameters on the solution, showing the Skin friction, the Nusselt number and the Sherwood number for the various flows embedded by these parameters.

The velocity profiles for different values of thermal Grashof number G_T are described in Figure 1. It is observed that an increase in the thermal Grashof number G_T leads to a rise in the values of velocity. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the plate as the thermal Grashof number increases.

Figure 2 shows the velocity profiles for different values of the concentration Grashof number G_C. The results show that the effect of increasing values of the concentration Grashof number results in increase in velocity.

The effects on velocity for different values of the unsteadiness parameter c are shown in Figure3. It is observed that the increase in the unsteadiness parameter leads to a fall in the velocity.

The influence of the unsteadiness parameter c, on the concentration is being illustrated in Figure 4. It can be seen that concentration profiles decrease very rapidly and the concentration boundary layer thickness reduce with the increase of the unsteadiness parameter. Hence, as the unsteadiness parameter increases the concentration decreases.

The temperature profiles for different values of the unsteadiness parameter c, was depicted in Figure 5. It is clearly seen that the temperature decreases when there is an increase in the values of the unsteadiness parameter.

In Figure 6 we analyze the effect of Prandtl number (*P*r) on the temperature of the flow field. The figure is a plot of temperature against the non-dimensional distance for different values of

Prandtl number. A comparison of the curves of the said figure shows that a growing Prandtl number decreases the temperature of the flow field at all points.

The concentration profiles for different values of the Schmidt number (Sc) are presented in Figure 7. The effect of the Schmidt number is foremost in concentration field. We found out that the wall concentration decreases with the increasing values of the Schmidt number.

Figure 8 represents the concentration profiles for different values of the Chemical reaction parameter. It is observed that the concentration increases when increasing values of the Chemical reaction parameter.

5. CONCLUSIONS

We studied theoretical, the characteristics of unsteady reactive flow of mass and heat transfer over a semi-infinite vertical plate. The governing nonlinear partial differential equations have been reduced to the coupled nonlinear ordinary differential equations by the similarity transformations and solved numerically.

Findings in this research work show that:

1. When the values of the thermal and concentration Grashof numbers increases it leads to the increase of the velocity of the fluid.
2. The fluid velocity, temperature and concentration within the boundary layer decreases with increasing the unsteadiness parameter.Increase in the Prandlt number leads to the decrease of the temperature of the fluid.
3. The influence of the Schmidt number is more pronounce on the concentration profile. The concentration profile tends to decrease with increase in the value of the Schmidt number.
4. The effect of chemical reaction on the velocity and temperature profiles is very minimal, it is more dominant on the concentration of the fluid. The more the value chemical reaction parameter the less the concentration profile of the flow.

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