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COMBINED PARAMETERS ESTIMATION METHODS OF LINEAR REGRESSION MODEL WITH MULTICOLLINEARITY AND AUTOCORRELATION

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ABSTRACT

Multicollinearity and autocorrelation are two major problems often encounter in regression analysis. Estimators for their separate investigation have been developed even though they are not without challenges. However, the two problems occasionally do occur together. In this paper effort is made to provide some combined estimators based on Feasible Generalized Linear Estimator (CORC and ML) and Principal Components (PCs) Estimator that estimate the parameters of linear regression model when the two problems are in the model. A linear regression model with three explanatory variables distributed normally and uniformly as well as exhibiting multicollinearity and autocorrelation was considered through Monte Carlo experiments at four levels of sample size .The experiments were conducted and the performances of the various proposed combined estimators with their separate ones and the Ridge estimator were examined and compared using the Mean Square Error (MSE) criterion by ranking their performances at each level of multicollinearity, autocorrelation and parameter. The ranks were further summed over the number of parameters. Results show that the proposed estimator MLPC1 is generally best even though the CORCPC1 and PC1 often compete favorably with it. Moreover with increased sample size, the CORCPC12 and MLPC12 are often best.

Keywords: OLS estimator, FGLS estimators, Combined estimators, Mean square error, Linear regression model, Monte-Carlo experiments.

Contribution/ Originality

This study combines the Feasible Generalized Least Square Estimators (Cochrane and Maximum Likelihood Estimators) with Principal Components Extraction method to jointly handle the problem of Multicollinearity and Autocorrelation problem in Linear Regression Model.

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1. INTRODUCTION

Multicollinearity is a problem associated with strong intercorrelation among the explanatory variables of linear regression model which is often encountered in social sciences [1, 2]. Solving this problem has attracted and is still attracting the attention of many researchers because of the challenges associated with parameter estimation and hypothesis testing while the Ordinary Least Square (OLS) estimator is used. These challenges include imprecise estimates, large standard error, non-conformity of the sign of regression coefficient estimates with the prior and insignificance of the true regression coefficients [2-4]. Various estimation developed methods to overcome this problem include the Ridge Regression Estimator [5, 6] estimator based on Principal Component Analysis Regression [7-9] and estimator based on Partial Least Squares [10-12].

When the error terms of linear regression model are no longer independent as often encountered in time series data there is a problem of autocorrelation. Parameter estimation of linear model with autocorrelated error tem using the OLS estimator is known to produce inefficient but unbiased estimates and inefficient predicted values with underestimated sampling variance of the autocorrelated error terms [2, 3, 13, 14]. Adjusting for this lost of efficiency has lead to the development of several feasible generalized least squares (FGLS) estimators including Cochrane and Orcutt [15]; Paris and Winstein [16]; Hildreth and Lu [17]; Durbin [18]; Theil [19]; the maximum likelihood and the maximum likelihood grid Beach and Mackinnon [20] and Thornton [21]. However, these two problems occasionally occur together in practice.

Consequently, this paper provides estimators for parameter estimation of the linear regression model when these two problems are evident in a data setby combing the two feasible generalized linear squares estimators (CORC and ML) with the Principal Component Estimator (PCs). The performances of these combined estimators, their separate ones and the Ridge Estimator were examined through Monte Carlo Studies by examining the Mean Square Error (MSE) property of estimator.

2. MATERIALS AND METHODS

Consider the linear regression model of the form:

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + U_t \qquad (1)$$

Where $U_t = \rho U_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N\left(0, \sigma^2\right)$, $t = 1, 2, ..., n$

For Monte-Carlo simulation studies, the parameters of equation (1) were specified and fixed asβ_0 = 4, β_1 = 2.5, β_2 = 1.8 and β_3 = 0.6. The levels of multicollinearityamong the independent variables and autocorrelation were respectively specified as $\lambda = 0.9, 0.95, 0.99$ and $\rho = 0.7, 0.8, 0.9, 0.95, 0.99$. Furthermore, the experiment was replicated in 1000 times (R =1000) under four (4) levels of sample sizes (n =10, 20, 30, 50). The correlated uniform regressors were generated by using the equations provided by Ayinde [22] and Ayinde and Adegboye [23] to generate normally distributed random variables with specified intercorrelation. With P= 3, the equations give:

$$X_1 = \mu_1 + \sigma_1 Z_1$$

$$X_2 = \mu_2 + \rho_{12}\, \sigma_2 Z_1 + \sqrt{m_{22}}\, Z_2 \qquad (2)$$

$$X_3 = \mu_3 + \rho_{13}\, \sigma_3 Z_1 + \frac{m_{23}}{\sqrt{m_{22}}} Z_2 + \sqrt{n_{33}}\, Z_3$$

Where $m_{22} = \sigma_2^2\left(1-\rho_{12}^2\right)$, $m_{23} = \sigma_2\sigma_3\left(\rho_{23}-\rho_{12}\rho_{13}\right)$ and $n_{33} = m_{33} - \frac{m_{23}^2}{m_{22}}$;

and $Z_i \sim N(0, 1)$ i = 1, 2, 3. The study assumed $X_i \sim N(0, 1)$, i = 1, 2, 3 and further utilized the properties of random variables that cumulative distribution function of Normal distribution produces U (0, 1) without affecting the correlation among the variables [24] to generate $X_i \sim U(0,1), \mathrm{i}=1,2,3.$

The error terms were generated by assuming $e_t \sim \mathrm{N}(0,1), \mathrm{t}=1,2,3,4,\ldots\mathrm{n}$ and using one of the distributional properties of the autocorrelated error terms $U_t \sim N\left(0, \frac{\sigma_\varepsilon^2}{1-\rho^2}\right)$ and the AR(1) equation as follows:

$$U_1 = \frac{\varepsilon_1}{\sqrt{1-\rho^2}} \quad (3)$$

$$U_t = \rho U_{t-1} + \varepsilon_t \qquad t = 2,3,4,\ldots n \quad (4)$$

After the data have been simulated, we adopted the same technique of how the Principal Component does its estimation using OLS estimator by regressing the extracted components (PCs) on the standardized dependent variable for the combined estimators. The technique is such we regressed the extracted components (PCs) on the standardized dependent variable using the FGLS estimators, Cochrane and Orcutt [15] and the Maximum Likelihood (ML) estimators [20] instead of using the OLS estimator. Since the FGLS estimators require an iterative methodology for its estimation, the proposed combined estimators do not result back into the FGLS feasible estimators when all the possible PCs are used like the OLS estimator does. Consequently, the parameters of (1) are estimated by the following twelve (12) estimators: OLS, PC1, PC12, CORC,CORCPC1, CORCPC12, CORCPC123,ML, MLPC1, MLPC12, MLPC123 and Ridge as suggested by Sclove [25] and described in Amemiya [26]. This Ridge estimator is an empirical Bayesian estimator. The prior is that coefficients are zero with a variance estimated from the data as the sums of squared of the fitted values of the dependent variable divided by the trace of the design matrix. The Ridge parameter in this case is a consistent estimate of the residual variance divided by the variance of the coefficient prior.

Since some of these estimators have now been incorporated into the Time Series Processor (TSP 5.0, 2005) software, a computer program was written with TSP software and the Mean Square Errors of each parameter of the model for all twelve estimators are computed.

Mathematically for any estimator $\hat{\beta}_i$ of β_i i = 0, 1, 2, 3 of (1), the Mean Square Error (MSE) is defined as follows:

$$MSE(\hat{\beta}_i) = \frac{1}{R}\sum_{j=1}^{R}\left(\hat{\beta}_{ij} - \beta_i\right)^2 \quad (5)$$

This was used to evaluate and compare the estimators. The mean square errors of the estimators were ranked at each level of multicollinearity, autocorrelation and parameter. The ranks were further summed over the number of parameters.An estimator is best if it has minimum total ranks.

3. RESULTS AND DISCUSSION

The summary of the performances of the estimators in term of their total rank over the model parameters of three explanatory variables at various levels of multicollinearity and sample size is given for both normal and uniform regressors in Table 1 and 2 respectively. The effect of the constant term is same at each level of multicollinearity in each sample size. A sample of the Mean Square errors of the estimators that were ranked when autocorrelation level is 0.9 and sample size is 20 is provided in the appendix.

Table-1. Total rank of the Mean Square Error of the Estimators over the Parameters at different levels of multicollinearity, autocorrelation and sample size [Normally Distributed Regressors]

n	ρ	λ	Estimators											
			OLS	PC1	PC12	CORC	CORC+PC1	CORC+PC12	CORC+PC123	ML	ML+PC1	ML+PC12	ML+PC123	RIGDE
10	0.7	0.9	25	8	22	36	7	25	33	23	5	17	20	13
		0.95	22	7 ·	19	36	8.	21	33	29	3	15	26	15
		0.99	25	8	18	36	6	21	33	29	4	14	25	15
	0.8	0.9	27	9	29	36	7	23	32	19	5	19	16	12
		0.95	29	8	26	36	6	19	32	22	4	14	19	19
		0.99	27	8	23	36	6	16	33	27	4	15	24	15
	0.9	0.9	29	9	30	34	7	19	31	19	5	21	16	14
		0.95	31	8	28	34	6	17	31	23	4	15	20	17
		0.99	29	8	27	34	6	16	31	26	4	15	23	15
	0.95	0.9	29	9	30	32	7	19	27	21	5	22	14	19
		0.95	32	8	27	34	6	15	29	25	4	15	21	18
		0.99	30	8	27	34	6	16	29	27	4	15	23	15
	0.99	0.9	29	9	30	30	7	19	28	19	5	24	14	20
		0.95	32	8	27	31	6	13	30	24	4	19	19	21
		0.99	25	8	18	36	6	21	33	29	4	14	25	15
20	0.7	0.9	25	25	30	15	23	22	12	6	24	21	9	22
		0.95	35	21	31	18	20	18	15	7	19	13	10	27
		0.99	35	9	27	32	8	16	28	20	7	12	23	17
	0.8	0.9	26	25	29	14	23	22	13	6	24	21	9	22
		0.95	35	21	31	16	20	20	14	6	19	16	9	27
		0.99	35	9	30	30	8	14	28	19	7	11	22	21
	0.7	0.9	30	24	32	14	22	22	13	6	21	19	9	22
		0.95	35	21	31	18	20	18	15	7	19	13	10	27
		0.99	35	9	27	32	8	16	28	20	7	12	23	17
	0.8	0.9	26	25	29	14	23	22	13	6	24	21	9	22
		0.95	35	21	31	16	20	20	14	6	19	16	9	27
		0.99	35	9	30	30	8	14	28	19	7	11	22	21
	0.9	0.9	30	24	32	14	22	22	13	6	21	19	9	22
		0.95	35	21	31	15	20	21	14	6	19	16	9	27
		0.99	35	9	30	29	8	13	28	19	7	11	23	22
	0.95	0.9	32	23	31	15	22	21	12	6	21	20	9	22
		0.95	35	20	34	15	19	19	12	6	18	20	9	27
		0.99	34	9	35	27	6	14	20	14	3	21	21	30
	0.99	0.9	35	22	30	13	20	21	11	11	19	20	7	25
		0.95	35	19	32	13	18	20	10	12	17	19	7	32
		0.99	35	9	34	26	8	10	20	25	7	13	21	26
30	0.7	0.9	27	25	25	8	23	24	11	15	24	23	8	21
		0.95	29	25	22	13	24	11	16	22	23	10	13	26
		0.99	36	9	23	24	8	15	25	32	7	14	23	18
	0.8	0.9	29	25	27	8	23	20	12	11	24	21	11	23
		0.95	29	25	27	13	24	11	17	17	23	7	16	25
		0.99	36	9	25	22	8	12	26	26	7	16	25	22
	0.9	0.9	29	25	27	7	24	18	13	13	23	22	10	23
		0.95	35	22	30	10	21	10	19	18	20	8	16	25
		0.99	36	9	33	18	7	13	23	28	8	14	23	22
	0.95	0.9	32	23	30	9	22	19	13	9	21	20	11	25
		0.95	35	20	30	14	20	7	18	15	19	10	17	29
		0.99	36	9	33	21	8	12	25	21	7	16	24	22

	0.99	0.9	35	21	30	11	20	19	12	10	19	20	9	28
		0.95	34	19	34	17	18	7	18	16	17	8	15	31
		0.99	35	9	34	22	8	11	23	23	7	14	22	26
50	0.7	0.9	30	23	20	15	25	8	18	21	24	7	16	27
		0.95	30	27	20	15	26	5	18	21	25	4	16	27
		0.99	36	18	27	19	15	4	20	24	16	5	19	31
	0.8	0.9	30	23	20	13	25	6	16	20	24	9	21	27
		0.95	30	27	20	13	26	3	16	21	25	6	20	27
		0.99	36	18	27	17	15	5	18	24	16	4	23	31
	0.9	0.9	33	26	21	16	23	6	19	15	22	7	20	26
		0.95	33	24	24	16	23	3	19	15	22	6	20	29
		0.99	36	18	27	19	15	5	21	18	16	4	24	31
	0.95	0.9	33	24	24	19	21	7	20	16	20	6	15	29
		0.95	36	22	28	19	21	4	20	16	20	5	15	28
		0.99	36	21	28	19	19	5	20	16	20	4	15	31
	0.99	0.9	32	24	27	17	20	7	19	18	19	6	16	29
		0.95	35	22	28	17	19	3	19	18	20	6	16	31
		0.99	35	23	29	17	18	5	19	18	19	4	16	31

From Table 1, it can be observed that estimator is generally MLPC1. Specifically, it is best when the sample size is very small, n=10. At $20 \leq n \leq 30$, MLPC1 is best when multicollinearity tends to unity and either ML or MLPC123 or CORC is best when multicollinearity is severe. With increased sample size, the MLPC12 or CORCPC12 is often best. Moreover, the CORCPC1 and PC1 estimators often compete favorably with the PC1 estimator.

From Table 2, it can be seen that the best estimator is still generally MLPC1. However, at $20 \leq n \leq 30$ with high autocorrelation and severe multicollinearity, the best estimator is either ML, MLPC12, MLPC123, CORC, or CORCPC12. With increased sample size (n =50), the MLPC12 and CORCPC12 are generally best even though MLPC1 performs much better with multicollinearity level tending to unity.

Table-2. Total rank of the Mean Square Error of the Estimators over the Parameters at different levels of multicollinearity, autocorrelation and sample size [Uniformly Distributed Regressors]

n	ρ	λ	**Estimators**											
			OLS	PC1	PC12	CORC	CORC+PC1	CORC+PC12	CORC+PC123	ML	ML+PC1	ML+PC12	ML+PC123	RIGDE
10	0.7	0.9	25	8	22	36	7	25	33	23	5	17	20	13
		0.95	24	8	18	36	6	21	33	28	4	14	25	17
		0.99	25	8	18	36	6	21	33	29	4	14	25	15
	0.8	0.9	28	9	20	36	10	18	33	26	4	10	23	17
		0.95	29	10	18	36	10	19	32	26	4	11	23	16
		0.99	29	9	18	36	10	20	32	26	4	12	23	15
	0.9	0.9	32	9	23	35	10	16	32	23	4	10	20	20
		0.95	30	10	19	35	10	18	32	26	4	10	23	17
		0.99	30	9	19	35	10	19	32	26	4	12	23	15
	0.95	0.9	32	9	24	35	10	16	31	23	4	10	20	20
		0.95	30	10	22	35	10	18	31	25	4	10	22	17
		0.99	30	9	22	35	10	19	31	25	4	12	22	15
	0.99	0.9	32	9	25	34	10	16	31	23	4	10	20	20
		0.95	32	10	23	34	10	18	31	23	4	10	20	19
		0.99	31	9	23	34	10	19	30	25	4	12	22	15
	0.7	0.9	25	25	30	15	23	22	12	6	24	21	9	22
		0.95	35	21	31	18	20	18	15	7	19	13	10	27
		0.99	35	9	27	32	8	15	29	20	7	12	23	17
	0.8	0.9	35	9	28	29	7	16	25	19	6	12	20	28
		0.95	35	7	26	31	7	20	28	20	4	14	23	19
		0.99	36	7	29	31	7	19	27	21	4	14	25	14

20	0.9	0.9	35	9	34	25	7	15	23	18	6	11	21	30
		0.95	35	7	34	27	7	17	24	17	4	13	23	26
		0.99	36	7	29	31	7	14	28	22	4	17	25	14
	0.95	0.9	35	9	34	26	7	15	24	16	6	13	19	30
		0.95	35	7	34	26	7	16	25	17	4	14	23	26
		0.99	36	7	29	31	7	14	28	22	4	17	25	14
	0.99	0.9	35	9	33	25	7	15	22	17	6	12	22	31
		0.95	35	7	33	27	7	16	26	17	4	14	21	27
		0.99	35	9	34	26	8	10	20	25	7	13	21	26
30	0.7	0.9	27	25	25	8	23	24	11	15	24	23	8	21
		0.95	29	25	22	13	24	11	16	22	23	10	13	26
		0.99	36	9	23	24	8	15	25	32	7	14	23	18
	0.8	0.9	35	6	27	23	10	13	23	20	5	12	28	32
		0.95	35	7	28	27	10	12	28	24	4	15	27	17
		0.99	35	7	28	27	10	13	28	26	4	16	25	15
	0.9	0.9	35	6	28	20	10	11	22	27	5	14	25	31
		0.95	35	7	28	24	10	12	25	27	4	15	30	17
		0.99	35	7	28	23	10	13	25	29	4	16	29	15
	0.95	0.9	35	7	28	21	10	12	20	25	4	13	28	31
		0.95	35	7	28	22	10	12	23	30	4	15	27	21
		0.99	35	6	21	24	12	13	23	29	3	16	30	22
	0.99	0.9	35	21	30	11	20	19	12	10	19	20	9	28
		0.95	34	19	34	17	18	7	18	16	17	8	15	31
		0.99	35	9	34	22	8	11	23	23	7	14	22	26
50	0.7	0.9	30	23	20	15	25	8	18	21	24	7	16	27
		0.95	30	27	20	15	26	5	18	21	25	4	16	27
		0.99	36	18	27	19	15	4	20	24	16	5	19	31
	0.8	0.9	36	22	26	19	16	4	22	17	14	7	20	31
		0.95	36	11	27	22	16	9	25	20	6	8	23	31
		0.99	36	7	28	26	9	12	29	25	4	15	26	17
	0.9	0.9	36	20	27	14	24	5	15	19	17	4	22	31
		0.95	36	12	26	20	16	7	21	27	6	10	22	31
		0.99	36	7	28	26	9	15	25	28	4	12	27	17
	0.95	0.9	36	21	27	20	16	6	21	15	18	5	18	31
		0.95	36	18	26	22	16	8	23	22	6	7	19	31
		0.99	36	9	28	26	9	14	27	24	4	11	29	17
	0.99	0.9	32	24	27	17	20	7	19	18	19	6	16	29
		0.95	35	22	28	17	19	3	19	18	20	6	16	31
		0.99	35	23	29	17	18	5	19	18	19	4	16	31

4. CONCLUSION

In this study, efforts have been made to combine two feasible Generalized Estimators with the estimator based on the principal components regression and compared their performances with that of the existing ones. These combined estimators when all the principal components are not used generally performed better thanthe OLS estimator and very precisely, the recommended combined MLPC1 is generally best even though the CORCPC1 and PC1 often compete favorably with it. Moreover with increased sample size, the CORCPC12 and MLPC12 are often best.

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APPENDIX

Table-3. The Mean Square Error of the Estimators of the Parameters at different levels of multicollinearity when n = 20 and $\rho = 0.9$

REGRESSORS	MSE	λ	Estimators											
			OLS	PC1	PC12	CORC	CORC+PC1	CORC+PC12	CORC+PC123	ML	ML+PC1	ML+PC12	ML+PC123	RIGDE
NORMAL	B0	0.9	2.75	2.83	2.76	1.6E5	2.79	2.78	2.78	2.57	2.79	2.77	2.77	2.76
		0.95	2.75	2.83	2.76	1.6E5	2.79	2.78	2.78	2.57	2.79	2.77	2.77	2.76
		0.99	2.75	2.83	2.75	1.6E5	2.79	2.78	2.78	2.57	2.79	2.77	2.77	2.78
	B1	0.9	0.77	0.77	0.94	0.23	0.71	0.28	0.22	0.21	0.71	0.26	0.21	0.60
		0.95	1.91	0.76	2.14	0.48	0.71	0.50	0.47	0.45	0.71	0.47	0.45	1.23
		0.99	12.1	0.76	13.1	2.60	0.73	2.53	2.55	2.45	0.73	2.50	2.48	3.24
	B2	0.9	1.56	0.05	1.22	0.32	0.02	0.58	0.31	0.31	0.02	0.57	0.31	1.13
		0.95	3.06	0.05	1.91	0.62	0.02	0.71	0.62	0.61	0.02	0.70	0.61	1.72
		0.99	15.1	0.04	6.65	3.12	0.03	1.58	3.07	3.02	0.03	1.55	3.03	2.79
	B3	0.9	0.73	0.89	0.79	0.27	0.92	0.68	0.27	0.27	0.92	0.67	0.27	0.54
		0.95	1.44	0.93	0.90	0.53	0.95	0.66	0.53	0.53	0.95	0.66	0.53	0.87
		0.99	7.13	1.00	2.47	2.65	1.01	0.88	2.66	2.63	1.01	0.88	2.65	1.76
UNIFORM	B0	0.9	2.75	2.71	2.72	1.7E5	2.82	2.79	2.80	2.58	11.2	2.75	2.74	2.64
		0.95	2.73	2.70	2.70	1.8E5	2.82	2.79	2.80	2.58	11.6	2.76	2.76	2.62
		0.99	2.72	2.69	2.69	1.8E5	2.82	2.79	2.79	2.59	12.1	2.77	2.76	2.62
	B1	0.9	7.41	1.04	8.01	2.48	1.65	2.39	2.39	2.19	0.78	2.18	2.20	3.41
		0.95	17.0	1.00	17.7	4.93	1.64	4.71	4.73	4.37	0.77	4.40	4.41	5.26
		0.99	101.7	0.96	94.1	25.0	1.66	21.9	24.0	22.8	0.79	22.0	23.0	7.04
	B2	0.9	14.4	0.31	6.44	3.54	0.36	1.74	3.44	3.36	0.09	1.65	3.36	5.85
		0.95	28.9	0.27	8.69	7.06	0.36	2.19	6.83	6.59	0.08	2.10	6.63	7.14
		0.99	147.6	0.23	21.7	35.0	0.37	4.83	33.8	32.5	0.08	4.90	32.7	6.45
	B3	0.9	6.04	1.13	2.64	2.53	0.35	0.88	2.53	2.52	0.99	0.88	2.53	2.60
		0.95	12.2	1.15	5.76	4.76	0.36	1.45	4.76	4.74	1.02	1.44	4.76	3.38
		0.99	63.33	1.20	35.7	22.3	0.39	7.76	22.3	22.1	1.07	7.96	22.2	4.03