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SIMULATED ANNEALING APPROACH TO SOLUTION OF INTEGRATED CIRCUIT PROBLEMS

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ABSTRACT

This paper applies a simulated annealing (SA) algorithm to analyze and compute the power output of the interstage coupled tuned amplifier problems that are very hard to be optimized. The inverse of the power output of circuit problem is converted into multivariable unconstrained optimization problem to serve as the cost function. A variable cooling factor (VCF) incorporated into the SA algorithm to give a new algorithm, called Powell's-simulated annealing (PSA) algorithm, was used to find the global minimum of the cost function. The PSA algorithm has been compared with the conjugate gradient and Nelder and Mead Simplex methods using the same cost function. The PSA algorithm proved to be more reliable than the other algorithms as it was always able to find optimum at a point or very close to it in a very good execution time. The solution of integrated circuit problems can easily be found through the PSA algorithm. The PSA algorithm has also been programmed to run on android smartphone systems to facilitate the computations, design and analysis of the coupled tuned amplifier problem.

Keywords: Integrated circuit, Coupled tuned amplifier, Variable cooling factor, Simulated annealing, Powell-simulated annealing algorithm, Multivariable unconstrained nonlinear optimization problems, Global minimum, Android smart phone systems.

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Contribution/ Originality

The paper proposes the Powell's simulated annealing algorithm via an innovative cooling scheme (the variable cooling factor) to reduce the computation time of multivariable unconstrained nonlinear optimization problems and also to analyze circuit problems that can be ran on android smartphone systems, contributing to the existing knowledge in combinatorial optimization.

1. INTRODUCTION

In recent times, consumers demand wireless systems that are low-cost, power efficient, reliable and have a small form-factor. High levels of integration are desired to reduce cost and achieve compact form-factor for high volume applications. Hence, the long-term vision for wireless transceivers is to merge as many components as possible just to reduce cost of analysis and design. Many applications in telecommunications, such as in radio, radar, mobile phone, television receivers, etc., use coupled tuned amplifiers. In radio frequency (RF), communication system, control of frequency bands and impedance matching conditions between amplifier stages are problems that constantly face

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circuit designers. The tasks are so frequent that many analytical techniques and approximation methods suited for high-frequency circuits have evolved and strongly influenced the jargon of RF engineering. The single-tuned amplifier has a single resonant circuit and provides a good vehicle for introducing the analysis and design of any coupled tuned amplifier [1]. The voltage-gain function of the coupled tuned amplifiers have several circuit elements, such as resistors, capacitors and inductors [2] and their analysis requires massive algebra manipulations. The problem is how to convert the models of the power output of the interstage coupled tuned amplifiers into linear or nonlinear optimization problem so that the power output can be optimized through an iterative heuristic method. Simulated annealing (SA) algorithm has been an efficient optimization method for optimizing computer-aided design of very large system integrated (VLSI) circuit problems. Kirkpatrick, et al. [3], first introduced the SA algorithm, and used it not only as solution to the travelling salesman problem but also as a technique for computer design. Henderson, et al. [4] provides a comprehensive review of SA and practical guidelines for implementing cooling schedules of SA while other readings can be found in van Laarhoven and Aarts [5]; Aarts and Korst [6] and Lenstra, et al. [7] or in a study by Pirlot [8].

2. RELATED WORKS

Many methods and algorithms have been used for analyzing and designing integrated circuit problems while several studies have used optimization techniques to model circuit problems and other related problems in engineering science. Liu, et al. [9] developed a capacitive coupling circuit model for power transfer system. The authors derived expressions for the capacitors which were functions of physical alignment parameters that were variable in two dimensions and used them to optimize the power transfer from input end to the load. Hofsajer [10] and Wang and Lee [11] also proposed a capacitive coupling circuit model to deal with the inter-winding capacitance for minimization of the parasitic effect in a high frequency inductor where the self-capacitances were ignored. In attempting to derive a general RC capacitive coupling model, Jeruchim and Shanmugan [12] converted the output power of a nonlinear system with no memory into Taylor series to reasonably model the nonlinear behavior of circuit problems. Similarly, Schetzen [13] used Volterra series to model the variation in frequency caused by the impedances of circuit elements (capacitors and inductors), which introduced memory into the system. A detailed derivation of this approach has been provided by Bussgang, et al. [14] using the probing method. Similar results can also be obtained using the high-order convolutions and Fourier transforms Bussgang, et al. [14]. Pinsky [15] applied a very similar method for the IS-54 signals. This particular study used the cubic interpolation between adjacent data points to evaluate geometric and polynomial functions while Struble, et al. [16] applied the linear interpolation instead and performed a similar analysis for the IS-95 and personal handy phone signals (PHS). In 2010, Yang, et al. [17] presented a feedback analysis of inductive links power using tuned amplifier with a variable coupling factor. The results indicated that the inductive link performed at the optimum state with a constant output voltage in the case where the duty cycle and the frequency of the driving signal were adjusted.

All these methods that claim varying degrees of success in analyzing integrated circuit problems are very hard to be optimized. In fact, none of them attempts to optimize the power amplifier and other circuit elements using SA algorithm. They do not provide much insight for designers trying to optimize the power of the tuned amplifiers. In this paper we apply the Powell-simulated annealing (PSA) algorithm to compute the power of tuned amplifier problems. Also, the PSA algorithm will be programmed to run on android smartphone systems (ASS) to facilitate the computation of optimization problems.

3. CASE STUDY

The paper considers an interstage coupled tuned amplifier in which the two design variables are the variable capacitances C_a and C_b that can be adjusted to provide the maximum output power through the resistance R_b for a given carrier frequency. The inverse of the power output considered as cost function is optimized using the PSA algorithm which has also been programmed on android smartphone systems.

4. METHODOLOGY

The variable cooling factor (VCF) model is first proposed and then incorporated into the SA algorithm coupled with the Powell's cycle move techniques. The resultant algorithm, called PSA algorithm, is applied to find the global minimizer of a multivariable cost function.

4.1. VCF Model

The fixed cooling factor embedded in the SA cooling scheme is replaced by the VCF model (1) as detailed in a companion paper by:

$$\Phi_k = \left(1 + \frac{1}{\sqrt{k(\nu+1)+\nu}}\right)^{-1}. \tag{1}$$

The new cooling scheme in the SA process is then given by the temperature-update formula;

$$T_{k+1} = \Phi_k T_k \tag{2}$$

where $\Phi_k \in [0.60,\ 0.99]$, $k = 1, 2, ..., N$ is the number of cycle counts or transitions in the system and ν is the number of variables in the cost function.

4.2. Simulated Annealing (SA) Algorithm

This section presents a brief account of the basic SA algorithm for finding the global minima of functions of several variables without calculating their derivatives [18]. The algorithm is fully described in Kirkpatrick, et al. [3] and Venkataraman [19]:

- **Step I:** Choose the starting design x_0 and calculate $f_0 = f(x_0)$, (which needs stopping criteria).
- **Step II:** Choose a random point on the surface of a unit *n*-dimensional hyper-sphere (it is just a sphere in *n* dimensions) to establish a search direction $\mathbf{S}$.
- **Step III:** Using a step size γ, calculate $f_1 = f(x_0 + \gamma\mathbf{S})$; $\Delta f = f_1 - f_0$.
- **Step IV:** If $\Delta f \le 0$, then $p = 1$, else $p = \exp(-\beta\Delta f)$, where p is the Boltzmann probability function and $\beta \in [0,\ 1]$ is a fixed cooling factor.
- **Step V:** A random number $r : 0 \le r < 1$ is generated: If $r \le p$, the step is accepted and the design vector is updated; else a change is made to the design. Go to Step II.

4.3. PSA Algorithm

The PSA algorithm procedure is to replace the fixed cooling factor embedded in the SA cooling scheme by the VCF. The VCF is calculated by (1) at each transition by the PSA algorithm as outlined by the following procedure:

Given the problem, minimize the function $F = f(\xi_1, \xi_2, ..., \xi_n)$ for $a \le \xi \le b$.

- **Step 1**: **Initializations:**
  - (i) Choose the initial parameters: $x_0 = \xi_0$, N_k (cycle counts or transition states, k), $U_k = (e_1, e_2, ..., e_n)$ - columns of identity matrix as direction vectors, $k = 1, 2, ..., N$.
  - (ii) Compute $f_0 = f(\xi_0)$; if $f_0 \in \Re$, continue, else choose another ξ_0.
  - (iii) Set $k = 0$, select initial temperature, $T_k = T_0 = 0.6L$ by design, where $L = b - a$. Select the temperature update formula, $T_{k+1} = \Phi_k T_k$, where $\Phi_k \in [0.60,\ 0.99]$.
- **Step 2: 1-D Minimization problem stage:**

First, $F = f(\xi_1, \xi_2, ..., \xi_n)$ is reduced to $F(\gamma) = f(\xi_0 + \gamma_0 u_1)$ and is minimized as a single variable problem (for example, by the golden search method):

(i) Set $k = 1, 2, ..., N$ (transition states)

(ii) For each iteration, $i = 1: n$, calculate γ_i to minimize $f(\gamma_i) = f(\xi_{i-1} + \gamma_i u_i)$, using the Golden search method. Generate $\xi_i = \xi_{i-1} + \gamma_i u_i$

- **Step 3: Acceptance and rejection of solution stage:**

  (i) Find $f(\xi_i) = f(\xi_{i-1} + \gamma_i u_i)$ and hence calculate $\Delta f = f(\xi_i) - f(\xi_{i-1})$.

  If $\Delta f \leq 0,$, then $M_p = 1$. Select the next direction vector in the cycle in Step 2(i). Else $M_p = \exp(-\Phi \Delta f)$

  [or $M_p = \exp(-\Delta f (T_k)^{-1}$, the Boltzmann probability function].

  (ii) A random number ρ, $0 \leq \rho < 1$ is generated. If $\rho \leq M_p$, then the iteration step is accepted and the next direction vector in the cycle is selected. Else, no change is made to the vector. Go to Step 2(i).

- **Step 4: Average direction vector:**

  Compute the average direction vector, $U_d = \xi_N - \xi_0$ and find γ_i, $\xi_i = \xi_0 + \gamma_i u_d$ and $f(\xi_i) = f(\xi_i + \gamma_i u_d)$. End of iterations in the first state $k = 1$ and number of iterations at this state is $(\nu + 1)$.

- **Step 5: Direction vector and temperature updates:**

  For $k = 2$, update the direction vector; $U_k \rightarrow U_{k+1}$, where $U_k = [\hat{e}_1, \hat{e}_2, ..., \hat{e}_d]$. That is, we replace e_1 by $e_d = U_d / \|U_d\|$ so that $\|e_d\| = 1$, where $\|.\|$ denotes the Euclidean norm. Then put $\xi_0 = \xi_{i+1} + \gamma_i \hat{e}_2$ and update temperature using the formula: $T_{k+1} = \Phi_k T_k$. Go to Step 2 through step 5 until the global minimum is obtained or the stopping criterion, $\Delta f \leq 10^{-4}$, is achieved.

5. PROBLEM FORMULATION

An interstage coupled tuned amplifier problem is converted into continuous unconstrained optimization problem and used as a cost function. For simplicity, the design variables were limited to two variable capacitances C_a and C_b. The electric circuit in Figure-1 represents an interstage coupling of the tuned amplifier. The two design variables are the variable capacitances C_a and C_b that can be adjusted to provide the maximum output power through the resistance R_a and R_b for a given carrier frequency.

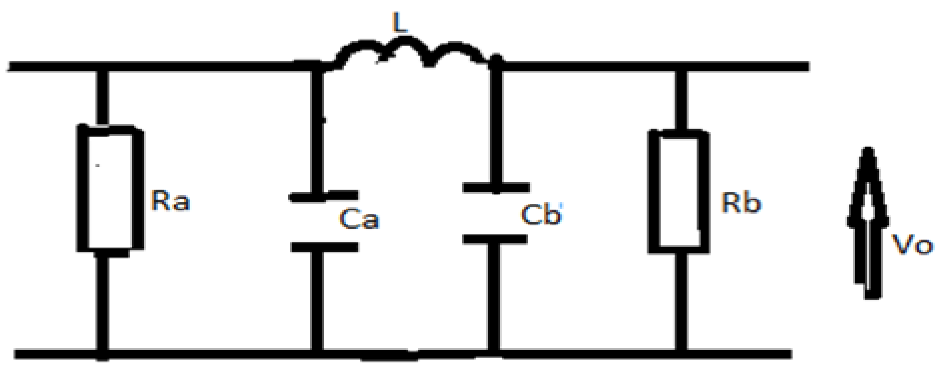

Figure-1. Electrical circuit of an interstage coupled turned amplifier problem for two variable capacitances, C_a and C_b with resistances, R_a and R_a, respectively.

The parameters in this circuit problem are the two resistances R_a and R_b , and the inductor L. The power output P_0 , by the circuit calculations, is given by (3):

$$P_0 = \frac{|V_0(j\omega)|^2}{R_b}\ , \tag{3}$$

where V_0 is the output voltage defined,

$$V_0(j\omega) = \frac{-R_a R_b I(j\omega)}{(j\omega)^3 L R_a C_a R_b C_b + (j\omega)^2 L(R_a C_a + R_b C_b) + (j\omega)(L + R_a R_b C_b +_a R_b C_a) + R_a + R_b} \tag{4}$$

$$|V_0(j\omega)| = \frac{IR_a}{\sqrt{\left(1 + R_a/R_b - [\omega^2 L R_a C_a]\big/R_b - [\omega^2 L R_b C_b]\big/R_b\right)^2 + \left(\omega L/R_b - [\omega R_a R_a C_b/R_b] + \omega R_a C_a + -[\omega^3 L R_a C_a R_b C_b]\big/R_b\right)^2}} \tag{5}$$

We define the new parameters and design variables to eliminate the dependence on the service frequency;

$$\begin{cases} \tau = \dfrac{R_a}{R_b} \quad ; \quad \rho = \dfrac{\omega L}{R_b}; \\ x_1 = \omega R_a C_a; \quad x_2 = \omega R_b C_b \end{cases} \tag{6}$$

where ω is the angular frequency.

To convert this to the standard form that defines a minimization problem, the cost function is set up to minimize the inverse of the power output and optimized for a chosen values of τ and ρ. Thus, the PSA algorithm is used to solve the multivariable unconstrained nonlinear optimization problem (7) to minimize the cost function, F :

$$\text{Minimize}\, F = f(x_1, x_2) = (1 + \tau - \rho x_1 - \rho x_2)^2 + (\rho + x_1 + \tau x_2 - \rho x_1 x_2)^2 \tag{7}$$

where τ and ρ are to be selected by the user. From (6), knowing the values of x_1 and x_2, ω can be found. The contour plot of the cost function in (7) for $\tau = 10$ and $\rho = 1$, showing the essential features is presented in **Figure-2**.

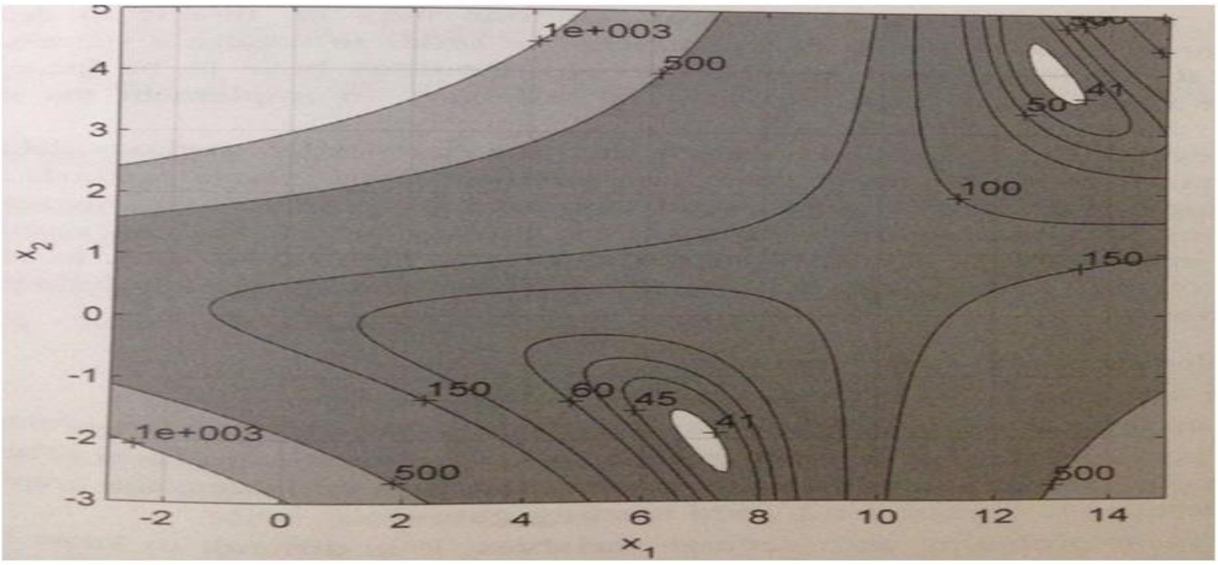

Figure-2. Contour plot of the cost function in (7) for $\tau = 10$ and $\rho = 1$.

6. IMPLEMENTATION PSA ALGORITHM

The PSA algorithm and two other selected algorithms, Nelder and Mead Simplex method (NMSM) and conjugate gradient method (CGM), were applied one after the other to solve the cost function (7). The NMSM was chosen because it has been reported in Nelder and Mead [20] and Brent [21] to be simple, reliable, and efficient global optimizer of multivariable unconstrained nonlinear functions. It is also observed, sometimes, able to follow the gross behaviour of test functions despite their many local minima. The CGM has been reported in Fletcher and Reeves [22] and Bendat [23] to converge quickly to the local minima of quadratic cost functions with the least number of iterations. The optimum minimum from Figure-2 served as a benchmark solution of the cost function. The PSA algorithm is expected to find the global optimizer of cost functions in the shortest computation time, and also to be ran on android smartphone system (ASS). Hence, the nature of optimum of the cost function can easily be identified to be local or global minimum by these methods.

6.1. Conjugate Gradient Method (CGM)

The CGM has the property that if $f(\mathbf{x})$ is quadratic, it will take exactly n iterations to converge, where n is the number of variables in the **x** vector. Although, it works especially well with the quadratic functions, it can also work with non-quadratic functions. Detailed description of the CGM can be found in Edgar and Lasdon [24]. However, a brief presentation of the method is given here:

Step 1: Choose a starting point $\mathbf{x}^0$ and calculate $f(\mathbf{x}^0)$. Let

$$\mathbf{d}^0 = -\nabla f(\mathbf{x}^0) \tag{8}$$

Step 2: Calculate $\mathbf{x}^1$ using (9):

$$\mathbf{x}^1 = \mathbf{x}^0 + \alpha^0 \mathbf{d}^0 \tag{9}$$

Find α^0 by performing a single-variable optimization on $f(\mathbf{x}^0 + \alpha^0 \mathbf{d}^0)$.

Step 3: Calculate $f(\mathbf{x}^1)$ and $\nabla f(\mathbf{x}^1)$. The new search direction is calculated using the equation (10):

$$\mathbf{d}^1 = -\nabla f(\mathbf{x}^1) + \mathbf{d}^0 \frac{\nabla^T f(\mathbf{x}^1)\cdot \nabla f(\mathbf{x}^1)}{\nabla^T f(\mathbf{x}^0)\cdot \nabla f(\mathbf{x}^0)}, \tag{10}$$

which can be generalized for the kth iteration:

$$\mathbf{d}^{k+1} = -\nabla f(\mathbf{x}^{k+1}) + \mathbf{d}^k \frac{\nabla^T f(\mathbf{x}^{k+1})\cdot \nabla f(\mathbf{x}^{k+1})}{\nabla^T f(\mathbf{x}^k)\cdot \nabla f(\mathbf{x}^k)} \tag{11}$$

Step 4: Determine the tolerance for stopping criteria:

$$\left| f(\mathbf{x}^{k+1}) - f(\mathbf{x}^k) \right| < \varepsilon_1 \tag{12}$$

For quadratic functions, this method will converge in n iterations ($k = n$). But for non-quadratic functions, after n iterations, the algorithm cycles again with $\mathbf{d}^{n+1}$ becoming $\mathbf{d}^0$.

6.2. PSA - Android Mobile Application

A recent survey revealed that the basic SA algorithm is inaccessible by most of the mobile phones due to incompatible operating systems. One achievement of the PSA algorithm is that it can be ran on ASS. There are two options to access the App, by mobile application or Web access:

- **Mobile Application:** Download and install the application (Scudd PSA), from Google Play store.
- **Web Access:** Follow the link: http://www.scudd-psa.byethost7.com./electronic.php

To run the PSA Pseudo code on ASS we solve the optimization problem (13):

Given the optimization problem in equation (13):

$$\text{Minimize}\, F = \sum_{i=1}^{N}\left(\sum_{j=1}^{N} k_i\left(A_i + C_j x_j^{m_j} + D_{j} x_{j-1} x_j\right)\right)^{n_i} \tag{13}$$

- Input the parameters: k_i, A_i, C_j, x_i, m_j and n_j.
- Press the run key for the results.

6.3. Test Results and Analysis

Different values for the two parameters, τ and ρ, were selected for testing for global optimum. The three algorithms, PSA, NMSM and CGM, used to minimize the cost function (7) were started from the same initial point $\mathbf{x}_0 = (10,\ 2)$, randomly chosen from the domain [-5, 15] of the cost function. Each test was ran twenty times, and the best set of solutions for each algorithm was reported in Table-1. For $\tau = 10$ and $\rho = 1$, the optimal power found by CGM and NMSM deviated slightly from the optimal power of 40.00 units (see Table-1). However, the best optimal power of 40 units with CPU time of 0.18 second was recorded by the PSA algorithm using 14 iterations. Figure-2 illustrates this solution graphically. Similarly, when $\tau = 100$ and $\rho = 10$, the optimal power recorded by PSA algorithm was 400.0101 units with CPU time of 0.024 second using 17 iterations. This is much closer to the optimal power value than the optimal values found by both CGM and NMSM. Similar least values were produced by the PSA algorithm when $\tau = 200$ and $\rho = 20$ with a closet optimum value of 900.0277 units and CPU time of 0.018 second.

Again, for the parameters, $\tau = 10$ and $\rho = 1$, the global minimizer vectors were found to be $[13.0070, 3.9987]^T$, $[13.0002, 4.0030]^T$ and $[13.0000, 4.0000]^T$ by CGM, NMSM and PSA, respectively. The best global minimizer vector and CPU time were obtained by the PSA algorithm which occurred at the point where $x_1 = 13$ and $x_2 = 4$. This means that, for any given values of τ and ρ, the solution of the cost function and magnification or quality factor of the amplifier in (14) and radio-frequencies can be optimized easily using the proposed model.

$$Q = \omega R C_T = \frac{\omega R_T}{\omega L} \tag{14}$$

Table-1. Comparison of test results of PSA algorithm, CGM and NMSM for minimizing the cost function

MEAN PARAMETERS	ALGORITHM CGM	NMSM	PSA
Parameters: $\tau = 10; \rho = 1$ (Opt = 40 Units)			
Number of Iterations	11	30	14
Global minimizer vector	$\begin{pmatrix}13.0070\\3.9987\end{pmatrix}$	$\begin{pmatrix}13.0002\\4.0030\end{pmatrix}$	$\begin{pmatrix}13.0000\\4.0000\end{pmatrix}$
Optimal functional value	40.0004	40.0001	40.000
Total running time (CPU)	1.2656	0.4063	0.1800
Distance from optimal value	0.0004	0.0001	0
Parameters: $\tau = 100; \rho = 10$ (Opt = 400 Units)			
Number of Iterations	12	43	17
Global minimizer vector	$\begin{pmatrix}10.1060\\0.2010\end{pmatrix}$	$\begin{pmatrix}10.0008\\0.1203\end{pmatrix}$	$\begin{pmatrix}10.0990\\0.1956\end{pmatrix}$
Optimal functional value	400.0140	400.0151	400.0101
Total running time (CPU)	1.2688	0.1463	0.0240
Distance from optimal value	0.0140	0.0151	0.0101
Parameters: $\tau = 200; \rho = 20$ (Opt = 900 Units)			
Number of Iterations	6	45	11
Global minimizer vector	$\begin{pmatrix}10.0017\\0.0670\end{pmatrix}$	$\begin{pmatrix}10.0013\\0.0665\end{pmatrix}$	$\begin{pmatrix}9.9998\\0.0582\end{pmatrix}$
Optimal functional value	900.1052	900.1010	900.0277
Total running time (CPU)	0.2813	0.2125	0.0180
Distance from optimal value	0.1052	0.1010	0.0277

6.4. Rate of Cooling in PSA Algorithm

The PSA algorithm was applied to solve the cost function (7) with the starting point (10, 2), randomly selected from the domain [-5, 15]: [-2, 48] for $\tau = 10$ and $\rho = 1$. The average CPU time and temperature computed at each transition stage K are presented in Table-2, among other parameters' values. The graph of temperature versus CPU time is also plotted in Figure-3, showing the cooling behaviour using the PSA algorithm.

The cooling curve declines faster during the first transition as seen in Figure-3, But unlike the cooling schedule proposed by Aarts and Korst [6] where the temperature reduction was faster at initial iterations, the temperature reduction rate in the VCF model was gentle and smooth right from the initial temperature to the final temperature and global minimizer. Starting at a suitable initial solution and using a low VCF value, it is found that the cooling curve increases its value within the order of 10^{-2}. This depicts typical application of the PSA algorithm to circuit problems such as the problem in (7), which would reduce the number of iterations and converge to the global optimum in a good time. These results are also in consistent with observations in other studies using a simulated annealing scheme, which doubles the execution time with the number of iterations and also show that larger cooling factor hastens the reliability process towards the global minimum [25].

Table-2. Average results of the PSA algorithm cooling scheme at various transition stage values. The results include the temperature (Temp), the number of iterations (Itr) at each transition stage K, the optimal value of objective function (Fval), the total execution time (CPU) and the step-size of the variable cooling factor (SSVCF).

	PARAMETER:					
K	**Itr**	**Fval**	**VCF**	**Temp**	**CPU**	**SSVCF**
0	0	75.220	-	12.000	0	-
1	3	49.512	0.691	8.298	0.012	-
2	6	41.123	0.739	6.126	0.018	0.048
3	9	41.013	0.768	4.707	0.024	0.029
4	12	40.999	0.789	3.714	0.030	0.021
5	15	40.889	0.805	2.989	0.036	0.016
6	18	40.658	0.817	2.443	0.042	0.012
7	21	40,012	0.828	2.022	0.048	0.011
8	24	40.003	0.836	1.690	0.054	0.008
9	27	40.002	0.843	1.425	0.060	0.007
10	30	40.001	0.849	1.211	0.132	0.006

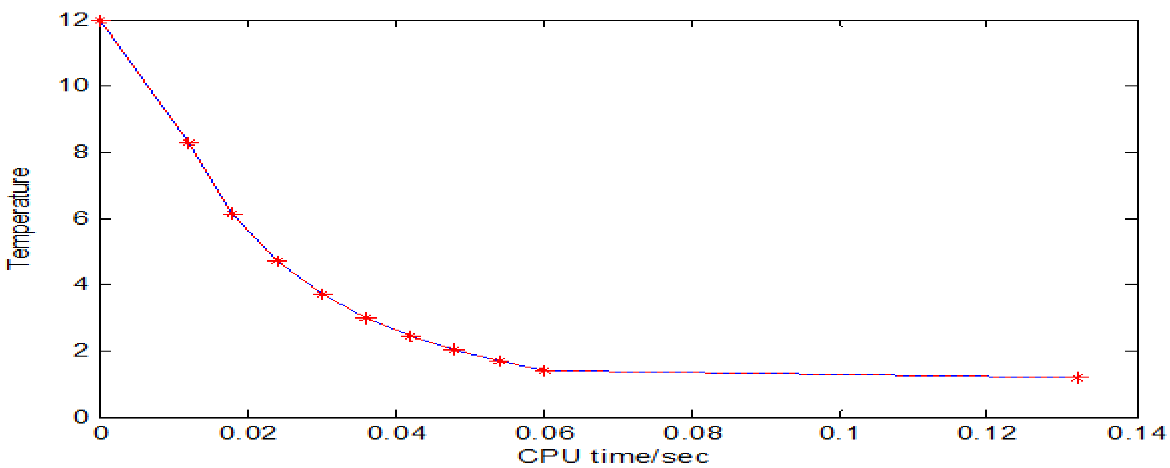

Figure-3. Temperature decline with CPU time in the PSA cooling scheme

7. DISCUSSION AND CONCLUSION

In this paper, we applied SA algorithm to analyze and compute the power output of the interstage coupled tuned amplifier problems that are very hard to be optimized. Specifically, the PSA algorithm, conjugate gradient and, Nelder and Mead Simplex methods were tested on the inverse of power cost function. The PSA algorithm proved to be more reliable than the CGM and NMSM. The PSA was always able to find the optimal minima, or at least a point very close to it in a very good execution time. Generally, it was observed that the local and global minima of the cost function were the same, one of the characteristics of quadratic functions exhibited by the CGM. The optimal minimum power was found by the PSA to be 40.00 units for the parameters' values, $\tau = 10$ and $\rho = 1$. This result compares favourably with the power gain at resonant frequency in literature when ratio is $R_a = 10R_b$. Similar results were obtained by the PSA algorithm when $\tau = 100$; $\rho = 10$ and $\tau = 200; \rho = 20$. The power, radio-frequency and magnification or quality factor of the amplifier can be computed using the PSA algorithm. The conversion of power of interstage coupled tuned amplifier problem into multivariable optimization problem was in line with the studies of Jeruchim and Shanmugan [12] and Schetzen [13] who used Taylors series and Volterra models, respectively to analyze integrated circuit problems. All these methods reduced the massive algebraic manipulations involve in the coupled tuned amplifiers problems but the PSA algorithm was found to perform better.

The solution of integrated circuit problems can easily be found through the PSA algorithm. The algorithm has been programmed to run on android smartphone systems and on the Web. This facilitates the computations, design and analysis of the coupled tuned amplifier problems faced by computer and electrical engineers.

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Contributors/Acknowledgement: All authors contributed equally to the conception and design of the study.

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