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A STUDY ON HOST MORTALITY RATE OF A THREE SPECIES MULTI ECOLOGY WITH UNLIMITED RESOURCES FOR THE FIRST SPECIES

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ABSTRACT

The purpose of this paper is to examine the stability analysis of a three species ecology with mortality rate for the host. The system comprises of a commensal (S1), two hosts S2 and S3 ie, S2 and S3 both benefit S1, without getting themselves affected either positively or adversely. Further the first species has unlimited resources. The model equations constitute a set of three first order non-linear coupled ordinary differential equations. Criteria for the asymptotic stability of all the four equilibrium states are established. Trajectories of the perturbations over the equilibrium states are illustrated and the global stability of the system is established with the aid of suitably constructed Liapunov's function and finally fourth order Runge-Kutta method is applied to obtain numerical solutions of the growth rate equations.

Contribution/ Originality: The paper contributes the first logical analysis in biological investigations with an iterative procedure of information collection. If such models are properly developed and used, they can provide insight into the relations between the physical variables and process influencing the system being studied. The resulting interplay between the experimental investigation and the theoretical model can be an essential factor in designing experiments and in the interpretation of data.

1. INTRODUCTION

Ecology is a branch of life sciences connected to the existence of diverse species in the same environment and habitat. It is natural that two or more species living in a common habitat interact in different ways. Significant research in the area of theoretical ecology has been thresholded by Lotka [1] and by Volterra [2]. Several mathematicians and ecologists contributed to the growth of this area of knowledge. Mathematical ecology can be broadly divided into two main sub-divisions, Aut-ecology and Multi-ecology, which are described by several authors. Multi-ecology is an ecosystem comprised of two or more distinct species. Species interact with each other in one way or other. The Ecological interactions can be classified as Ammensalism, Competition, Commensalism, Neutralism, Mutualism, Predation, Parasitism and so on.

Mathematical modeling has been playing an important role for the last half a century in explaining several phenomena concerned with individuals and groups of populations in nature. The general concept of modeling has been presented in the monographs of Meyer [3]; Kushing [4]; Paul [5]; Kapur [6]. Srinivas [7] studied competitive ecosystem of two species and three species with limited and unlimited resources. Lakshmi [8]; Lakshmi and Pattabhiramacharyulu [9] studied prey-predator ecological models with partial cover for the prey and alternate food for the predator. Stability analysis of competitive species was carried out by Archana, et al. [10] and by Bhaskara and Pattabhiramacharyulu [11] while Ravindra [12] investigated mutualism between two species. Further Phani [13] studied some mathematical models of ecological commensalism. The present author [14-20] discussed on the stability of a three and four species multi-ecosystems.

The present investigation is on an analytical study of a three species (S_1, S_2, S_3) multi ecology with mortality rate for the host and the first species has unlimited resources. The system comprises of a commensal (S_1), two hosts S_2 and S_3 ie, S_2 and S_3 both benefit S_1, without getting themselves affected either positively or adversely. Further S_2 is a commensal of S_3 and S_3 is a host of both S_1, S_2. **Commensalism** is a symbiotic interaction between two populations where one population (S_1) gets benefit from (S_2) while the other (S_2) is neither harmed nor benefited due to the interaction with (S_1). The benefited species (S_1) is called the commensal and the other (S_2) is called the host. Some real-life examples of commensalism are presented below.

i. A squirrel in an oak tree gets a place to live and food for its survival, while the tree remains neither benefited nor harmed.
ii. A flatworm attached to the horse crab and eating the crab's food, while the crab is not put to any disadvantage.
iii. Sucker fish (echeneis) gets attached to the under surface of sharks by its sucker. This provides easy transport for new feeding grounds and also food pieces falling from the sharks prey, to Echeneis.

2. METHODOLOGY

2.1. Notation

$N_i(t)$: The population strength of S_i at time t, $i=1,2,3$

t : Time instant

d_2 : Natural death rate of S_2

g_i : Natural growth rate of S_i, $i=1,3$

a_{ii} : Self inhibition coefficients of S_i, $i=2,3$

a_{12}, a_{13} : Interaction coefficients of S_1 due to S_2 and S_1 due to S_3

a_{23} : Interaction coefficient of S_2 due to S_3

$e_2 = \dfrac{d_2}{a_{22}}$: Extinction coefficient of S_2

$k_3 = \dfrac{g_3}{a_{33}}$: Carrying capacities of S_3

Further the variables N_1, N_2, N_3 are non-negative and the model parameters g_1, d_2, g_3, a_{12}, $a_{22}, a_{33}, a_{13}, a_{23}, e_2, k_3$ are assumed to be non-negative constants.

2.2. Theoretical Framework and Basic Equations

The model equations for the three species multi-ecosystem is given by the following system of first order non-linear ordinary differential equations.

(i) Equation for the first species (N_1):

$$\frac{dN_1}{dt} = g_1 N_1 + a_{12} N_1 N_2 + a_{13} N_1 N_3 \quad (1)$$

(ii) Equation for the second species (N_2):

$$\frac{dN_2}{dt} = -d_2 N_2 - a_{22} N_2^2 + a_{23} N_2 N_3 \quad (2)$$

(iii) Equation for the third species (N_3):

$$\frac{dN_3}{dt} = g_3 N_3 - a_{33} N_3^2 \quad (3)$$

The system under investigation has four equilibrium states given by $\frac{dN_i}{dt} = 0, i = 1, 2, 3$

(i) Fully washed out state.

$$E_1 : \bar{N}_1 = 0, \bar{N}_2 = 0, \bar{N}_3 = 0$$

(ii) Only the third species is washed out and the other two are not.

$$E_2 : \bar{N}_1 = 0, \bar{N}_2 = 0, \bar{N}_3 = k_3$$

(iii) Only the second species is washed out and the other two are not.

$$E_3 : \bar{N}_1 = 0, \bar{N}_2 = -e_2, \bar{N}_3 = 0$$

(iv) Only the first species is washed out and the other two are not.

$$E_4 : \bar{N}_1 = 0, \bar{N}_2 = \frac{a_{23} k_3}{a_{22}} - e_2, \bar{N}_3 = k_3$$

2.3. Stability of the Equilibrium States

Let us consider small deviations from the steady state

$$\text{i.e., } N_i(t) = \bar{N}_i + u_i(t), i = 1, 2, 3 \quad (4)$$

where $u_i(t)$ is a small perturbations in the species S_i.

The basic equations are quasi-linearized over the equilibrium state $\bar{N} = \left(\bar{N}_1, \bar{N}_2, \bar{N}_3\right)$ to obtain the equations for the perturbed state as

$$\frac{du_1}{dt} = \left(g_1 + a_{12}\bar{N}_2 + a_{13}\bar{N}_3\right)u_1 + \left(a_{12}\bar{N}_1\right)u_2 + \left(a_{13}\bar{N}_1\right)u_3 \quad (5)$$

$$\frac{du_2}{dt} = \left(-d_2 - 2a_{22}\bar{N}_2 + a_{23}\bar{N}_3\right)u_2 + \left(a_{23}\bar{N}_2\right)u_3 \quad (6)$$

$$\frac{du_3}{dt} = \left(g_3 - 2a_{33}\bar{N}_3\right)u_3 \quad (7)$$

The characteristic equation for the system is

$$|A - \lambda I| = 0 \quad (8)$$

The equilibrium state is stable, if all the roots of the equation (8) are negative in case they are real or have negative real parts, in case they are complex.

2.3.1. The Stability of

$E_1 : \bar{N}_1 = 0,\ \bar{N}_2 = 0,\ \bar{N}_3 = 0$

The basic equations are quasi-linearized to obtain the equations as

$$\frac{du_1}{dt} = g_1 u_1;\ \frac{du_2}{dt} = -d_2 u_2;\ \frac{du_3}{dt} = g_3 u_3 \quad (9)$$

The characteristic equation is $\left(\lambda - g_1\right)\left(\lambda + d_2\right)\left(\lambda - g_3\right) = 0$ (10)

The characteristic roots of (10) are $g_1, -d_2, g_3$. Since two of these three roots are positive. Hence the state is **unstable** and the solutions of the equations (9) are

$$u_1 = u_{10}\, e^{g_1 t};\ u_2 = u_{20}\, e^{-d_2 t};\ u_3 = u_{30}\, e^{g_3 t} \quad (11)$$

where u_{10}, u_{20}, u_{30} are the initial values of u_1, u_2, u_3 respectively.

Trajectories of Perturbations

The trajectories in $u_1 - u_2$ and $u_2 - u_3$ planes are

$$\left(\frac{u_1}{u_{10}}\right)^{\frac{1}{g_1}} = \left(\frac{u_2}{u_{20}}\right)^{-\frac{1}{d_2}} = \left(\frac{u_3}{u_{30}}\right)^{\frac{1}{g_3}}$$

2.3.2. The Stability of

$E_2 : \bar{N}_1 = 0,\ \bar{N}_2 = 0,\ \bar{N}_3 = k_3$

In this state, the basic equations can be quasi-linearized, we get

$$\frac{du_1}{dt} = a_{13}k_3 u_1;\ \frac{du_2}{dt} = (a_{23}k_3 - d_2)u_2;\ \frac{du_3}{dt} = -g_3 u_3 \quad (12)$$

The characteristic roots are $a_{13}k_3,\ a_{23}k_3 - d_2$ and $-g_3$. Since one of these three roots is positive, hence the state is **unstable.**

Case (i): When $a_{23}k_3 < d_2$

In this case, the solutions of (12) are

$$u_1 = u_{10}e^{a_{13}k_3 t}; u_2 = u_{20}\ e^{-(d_2+a_{23}k_3)\ t}; u_3 = u_{30}e^{-g_3 t} \quad (13)$$

Case (ii): When $a_{23}k_3 > d_2$

In this case, the solutions are given by

$$u_1 = u_{10}e^{a_{13}k_3 t}; u_2 = u_{20}\ e^{(d_2+a_{23}k_3)\ t}; u_3 = u_{30}e^{-g_3 t} \quad (14)$$

Case (iii): When $a_{23}k_3 = d_2$

In this case, the solutions are

$$u_1 = u_{10}e^{a_{13}k_3 t}; u_2 = u_{20}; u_3 = u_{30}e^{-g_3 t} \quad (15)$$

Trajectories of perturbations

The trajectories in the $u_1 - u_2$ and $u_2 - u_3$ planes are given by

$$\left(\frac{u_1}{u_{10}}\right)^{\frac{1}{a_{13}k_3}} = \left(\frac{u_2}{u_{20}}\right)^{\frac{1}{a_{23}k_3-d_2}} = \left(\frac{u_3}{u_{30}}\right)^{-\frac{1}{g_3}}$$

2.3.3. The Stability of

$E_3 : \bar{N}_1 = 0, \bar{N}_2 = -e_2, \bar{N}_3 = 0$

The basic equations can be quasi-linearized, we get

$$\frac{du_1}{dt} = (g_1 - a_{12}e_2)u_1; \ \frac{du_2}{dt} = d_2u_2 - a_{23}k_2u_3; \ \frac{du_3}{dt} = g_3u_3 \quad (16)$$

The characteristic roots are $g_1 - a_{12}e_2, d_2$ and g_3. Since two of these three roots are positive, hence the state is **unstable**.

Case (i): When $g_1 < a_{12}e_2$

In this case, the equations (16) yield the solutions,

$$u_1 = u_{10}e^{-(g_1+a_{12}e_2)\ t}; u_2 = (u_{20} - u_{30}A)e^{d_2 t} + Au_{30}e^{g_3 t}; u_3 = u_{30}e^{g_3 t} \quad (17)$$

where $A = \dfrac{a_{23}e_2}{d_2 - g_3}$, with $d_2 \neq g_3$ (18)

Case (ii): When $g_1 > a_{12}e_2$

In this case, the solutions of (16) are

$$u_1 = u_{10}e^{(g_1+a_{12}e_2)\ t}; u_2 = (u_{20} - u_{30}A)e^{d_2 t} + Au_{30}e^{g_3 t}; u_3 = u_{30}e^{g_3 t} \quad (19)$$

Case (iii): When $g_1 = a_{12}e_2$

In this case, the solutions are

$$u_1 = u_{10};\ u_2 = \left(u_{20} - u_{30}A\right)e^{d_2 t} + Au_{30}e^{g_3 t};\ u_3 = u_{30}e^{g_3 t} \quad (20)$$

Trajectories of Perturbations

The trajectories in the $u_1 - u_2$ and $u_2 - u_3$ planes are

$$u_2 = \left(u_{20} - Au_{30}\right)\left(\frac{u_1}{u_{10}}\right)^{\frac{d_2}{g_1 - a_{12}e_2}} + u_{30}A\left(\frac{u_1}{u_{10}}\right)^{\frac{g_3}{g_1 - a_{12}e_2}};\ u_2 = \left(u_{20} - u_{30}A\right)\left(\frac{u_3}{u_{30}}\right)^{\frac{d_2}{g_3}} + Au_3$$

2.3.4. The Stability of $E_4 : \bar{N}_1 = 0,\ \bar{N}_2 = \frac{a_{23}k_3}{a_{22}} - e_2,\ \bar{N}_3 = k_3$

In this state, the basic equations can be quasi-linearized,
We have

$$\frac{du_1}{dt} = \left(\delta_1 - a_{12}e_2\right)u_1;\ \frac{du_2}{dt} = \left(d_2 - a_{23}k_3\right)u_2 + \frac{a_{23}}{a_{22}}\left(a_{23}k_3 - d_2\right)u_3;\ \frac{du_3}{dt} = -g_3u_3 \quad (21)$$

where $\delta_1 = (g_1 + a_{13}k_3 + \frac{a_{12}a_{23}k_3}{a_{22}}) > 0$ (22)

The characteristic roots are $\delta_1 - a_{12}e_2$, $d_2 - a_{23}k_3$ and $-g_3$. The equations (21) yield the solutions.

$$u_1 = u_{10}e^{(\delta_1 - a_{12}e_2)t};\ u_2 = \left(u_{20} - Bu_{30}\right)e^{(d_2 - a_{23}k_3)t} + Bu_{30}e^{-g_3 t};\ u_3 = u_{30}e^{-g_3 t} \quad (23)$$

where $B = \frac{a_{23}(d_2 - a_{23}k_3)}{a_{22}(g_3 + d_2 - a_{23}k_3)}$ with $g_3 + d_2 \neq a_{23}k_3$ (24)

Case (i): When $\delta_1 < a_{12}e_2$ and $d_2 < a_{23}k_3$

In case all the three roots are negative, hence the state is **stable**. The solution (23) become

$$u_1 = u_{10}e^{-(\delta_1 + a_{12}e_2)t};\ u_2 = \left(u_{20} - Bu_{30}\right)e^{-(d_2 + a_{23}k_3)t} + Bu_{30}e^{-g_3 t};\ u_3 = u_{30}e^{-g_3 t} \quad (25)$$

Case (ii): When $\delta_1 = a_{12}e_2$ and $d_2 < a_{23}k_3$

In case the state is **neutrally stable** and the solution (23) become

$$u_1 = u_{10};\ u_2 = \left(u_{20} - Bu_{30}\right)e^{-(d_2 + a_{23}k_3)t} + Bu_{30}e^{-g_3 t};\ u_3 = u_{30}e^{-g_3 t} \quad (26)$$

Case (iii): When $\delta_1 < a_{12}e_2$ and $d_2 = a_{23}k_3$

In case the state is **neutrally stable** and the solutions are $u_1 = u_{10}e^{-(\delta_1 + a_{12}e_2)t};\ u_2 = u_{20};\ u_3 = u_{30}e^{-g_3 t}$ (27)

Case (iv): When $\delta_1 = a_{12}e_2$ and $d_2 = a_{23}k_3$

In case the state is **neutrally stable** and the solutions are given by

$$u_1 = u_{10};\, u_2 = u_{20};\, u_3 = u_{30}e^{-g_3 t} \tag{28}$$

Case (v): When $\delta_1 > a_{12}e_2$ and $d_2 > a_{23}k_3$

In case the state is **unstable** and the equations (21) yield the solutions.

$$u_1 = u_{10}e^{(\delta_1 + a_{12}e_2)t};\, u_2 = \left(u_{20} - Bu_{30}\right)e^{(d_2 + a_{23}k_3)t} + Bu_{30}e^{-g_3 t};\, u_3 = u_{30}e^{-g_3 t} \tag{29}$$

Case (vi): When $\delta_1 > a_{12}e_2$ and $d_2 > a_{23}k_3$

In case the state is **unstable** and solutions are

$$u_1 = u_{10}e^{(\delta_1 + a_{12}e_2)t};\, u_2 = \left(u_{20} - Bu_{30}\right)e^{(d_2 + a_{23}k_3)t} + Bu_{30}e^{-g_3 t};\, u_3 = u_{30}e^{-g_3 t} \tag{30}$$

Case (vii): When $\delta_1 < a_{12}e_2$ and $d_2 > a_{23}k_3$ or $\delta_1 = a_{12}e_2$ and $d_2 > a_{23}k_3$

When $\delta_1 > a_{12}e_2$ and $d_2 < a_{23}k_3$ or $\delta_1 > a_{12}e_2$ and $d_2 = a_{23}k_3$

In case the state is **unstable.**

Trajectories of perturbations

The trajectories in the $u_1 - u_2$ and $u_2 - u_3$ planes are given by

$$u_2 = \left(u_{20} - Bu_{30}\right)\left(\frac{u_1}{u_{10}}\right)^{\frac{d_2 - a_{23}k_3}{\delta_1 - a_{12}e_2}} + Bu_{30}\left(\frac{u_1}{u_{10}}\right)^{\frac{g_3}{a_{12}e_2 - \delta_1}};\; u_2 = \left(u_{20} - Bu_{30}\right)\left(\frac{u_3}{u_{30}}\right)^{\frac{a_{23}k_3 - d_2}{g_3}} + Bu_3$$

3. LAPUNOV'S FUNCTION FOR GLOBAL STABILITY

In section 5 we discussed the local stability of all four equilibrium states. From which only one state $E_4\left(0, \bar{N}_2, \bar{N}_3\right)$ is **stable** and rest of them are **unstable**. We now examine the global stability of dynamical system (1), (2) and (3) at this state by suitable Liapunov's function.

Theorem: *The equilibrium state* $E_4\left(0, \frac{a_{23}k_3}{a_{22}} - e_2, k_3\right)$ *is globally asymptotically stable.*

Proof: Let us consider the following Liapunov's function

$$L\left(N_2, N_3\right) = N_2 - \bar{N}_2 - \bar{N}_2 \ln\left(\frac{N_2}{\bar{N}_2}\right) + l_1\left[N_3 - \bar{N}_3 - \bar{N}_3 \ln\left(\frac{N_3}{\bar{N}_3}\right)\right] \tag{31}$$

where l_1 is a suitable constant to be determined as in the subsequent steps.

Now, the time derivative of L, along with solutions of (2) and (3) can be written as

$$\frac{dL}{dt} = \left(\frac{N_2 - \bar{N}_2}{N_2}\right)\frac{dN_2}{dt} + l_1\left(\frac{N_3 - \bar{N}_3}{N_3}\right)\frac{dN_3}{dt} \tag{32}$$

$$= \left(N_2 - \bar{N}_2\right)\left(-d_2 - a_{22}N_2 + a_{23}N_3\right) + l_1\left(N_3 - \bar{N}_3\right)\left(g_3 - a_{33}N_3\right)$$

$$= -a_{22}\left(N_2 - \bar{N}_2\right)^2 + a_{23}\left(N_2 - \bar{N}_2\right)\left(N_3 - \bar{N}_3\right) + l_1\left[-a_{33}\left(N_3 - \bar{N}_3\right)^2\right]$$

Choosing, $l_1 = \dfrac{a_{23}^2}{4a_{22}a_{33}} > 0$ and with some algebraic manipulation, we get

$$\frac{dL}{dt} = -\left[\sqrt{a_{22}}\left(N_2 - \bar{N}_2\right) + \frac{a_{23}}{2\sqrt{a_{22}}}\left(N_3 - \bar{N}_3\right)\right]^2 < 0 \qquad (33)$$

Hence, the steady state is **globally asymptotically stable.**

4. NUMERICAL EXAMPLES

The numerical solutions of the growth rate equations computed employing the fourth order Runge-Kutta method for specific values of the various parameters that characterize the model and the initial conditions. The results are illustrated in Figures from 1 to 6.

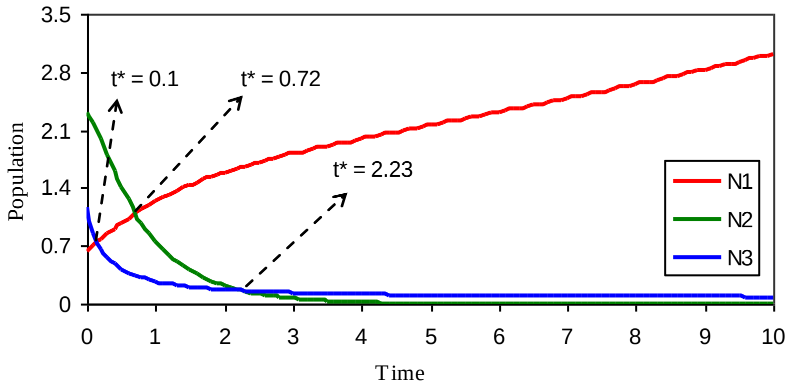

Figure-1. Variation of N_1, N_2, N_3 against time (t) for $g_1 = 0.02$, $a_{12} = 0.28$, $a_{13} = 0.52$, $d_2 = 1.46$, $a_{22} = 0.32$, $a_{23} = 1.64$, $g_3 = 0.28$, $a_{33} = 3.52$, $N_1 = 0.62$, $N_2 = 2.32$, $N_3 = 1.16$.
Source: MS-Excel by using Runge-Kutta method

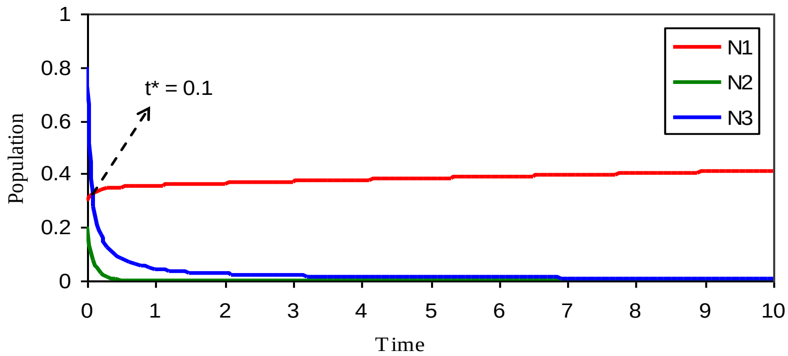

Figure-2. Variation of N_1, N_2, N_3 against time (t) for $g_1 = 0.01$, $a_{12} = 5.16$, $a_{13} = 0.44$, $d_2 = 8.64$, $a_{22} = 13.05$, $a_{23} = 0.45$, $g_3 = 0.17$, $a_{33} = 23.53$, $N_1 = 0.3$, $N_2 = 0.2$, $N_3 = 0.8$.
Source: MS-Excel by using Runge-Kutta method

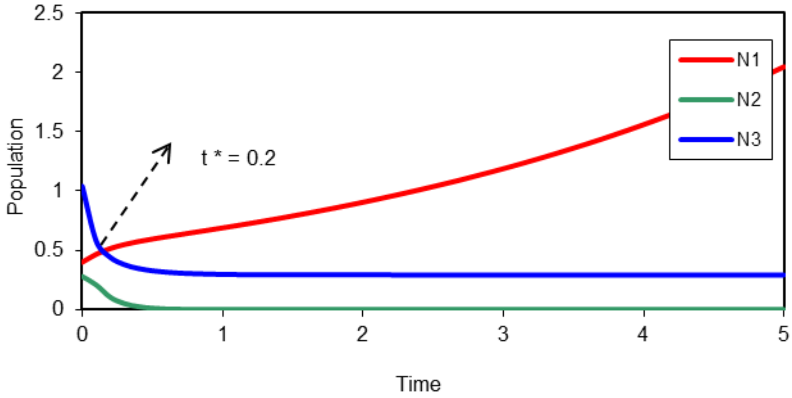

Figure-3. Variation of N_1, N_2, N_3 against time (t) for $g_1 = 0.12$, $a_{12} = 3.96$, $a_{13} = 0.52$, $d_2 = 17.76$, $a_{22} = 38$, $a_{23} = 33.24$, $g_3 = 3.8$, $a_{33} = 13$, $N_1 = 0.4$, $N_2 = 0.28$, $N_3 = 1.04$.
Source: MS-Excel by using Runge-Kutta method

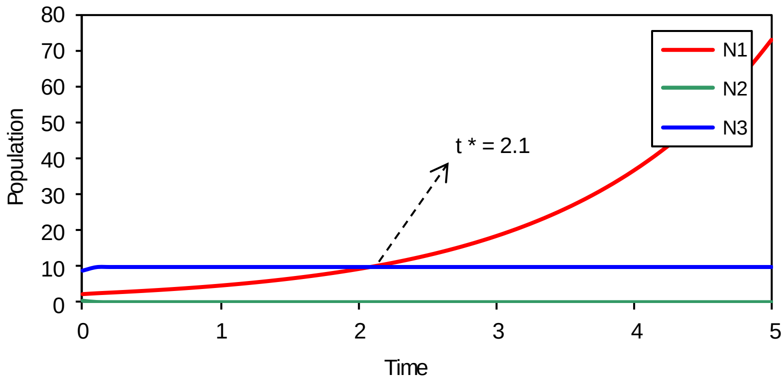

Figure-4. Variation of N_1, N_2, N_3 against time (t) for $g_1 = 0.68$, $a_{12} = 1.72$, $a_{13} = 0.001$, $d_2 = 24.52$, $a_{22} = 0.001$, $a_{23} = 0.4$, $g_3 = 32.44$, $a_{33} = 3.32$, $N_1 = 2.24$, $N_2 = 0.4$, $N_3 = 8.76$.
Source: MS-Excel by using Runge-Kutta method

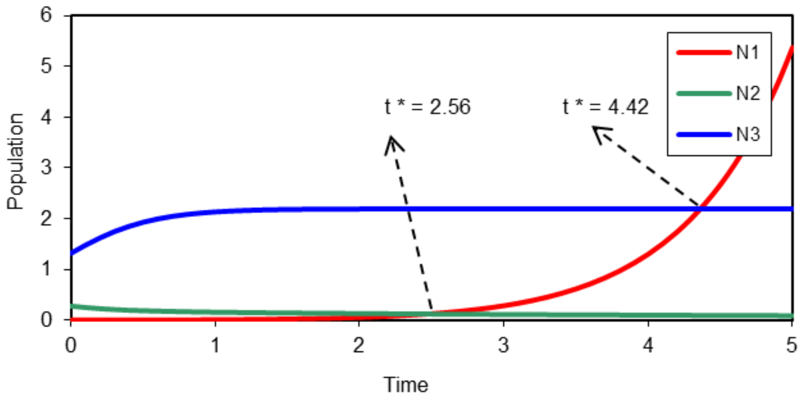

Figure-5. Variation of N_1, N_2, N_3 against time (t) for $g_1 = 0.67$, $a_{12} = 7.8$, $a_{13} = 0.001$, $d_2 = 2.3$, $a_{22} = 1.04$, $a_{23} = 1.04$, $g_3 = 3.16$, $a_{33} = 1.44$, $N_1 = 0.001$, $N_2 = 0.28$, $N_3 = 1.32$.
Source: MS-Excel by using Runge-Kutta method

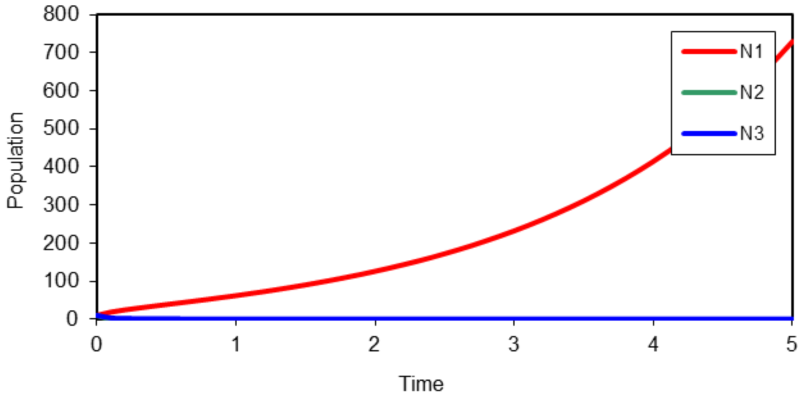

Figure-6. Variation of N_1, N_2, N_3 against time (t) for g_1 = 0.28, a_{12} = 0.8, a_{13} = 0.8, d_2 = 10.72, a_{22} = 15.48, a_{23} = 1.84, g_3 = 0.68, a_{33} = 2, N_1 = 10, N_2 = 10, N_3 = 10.
Source: MS-Excel by using Runge-Kutta method

5. OBSERVATIONS OF THE ABOVE GRAPHS

Case 1: In this case the first species has the least natural growth rate. Initially the second and third species dominates over the first till the time instant $t^* = 0.72$ and $t^* = 0.1$ respectively and thereafter the dominance is reversed. The second species dominates over the third initially up to the time $t^* = 2.23$ after which the dominance is reversed as shown in Figure 1.

Case 2: This is a situation at the self inhibition coefficient of the third species is highest. Initially the third species dominates over the first till the time instant $t^* = 0.1$ and thereafter the dominance is reversed. Further the coefficients a_{13} and a_{23} are almost equal. This is illustrated in Figure 2.

Case 3: In this case the second species has the highest self inhibition coefficient. The third species dominates over the first initially up to the time $t^* = 0.2$ after which the dominance is reversed as shown in Figure 3.

Case 4: In this case the coefficients a_{13} and a_{22} are identical. Initially the third species dominates over the first till the time instant $t^* = 2.1$ and thereafter the dominance is reversed. Further the second species has the least initial value. This is shown in Figure 4.

Case 5: In this case the initial values of S_1, S_2, S_3 are in increasing order. The coefficients a_{22} and a_{23} are identical. Initially the second and third species dominates over the first till the time instant $t^* = 4.42$ and $t^* = 2.56$ respectively and thereafter the dominance is reversed. (Figure 5).

Case 6: In this case the initial conditions of the three species are identical. This is a situation at the self inhibition coefficient of the second species is highest. Further we notice that the coefficient a_{12} is same as the coefficient a_{13}. (Figure 6).

6. CONCLUSION

The present paper deals with an investigation on the stability of a three species syn eco-system with mortality rate for the host. In this paper we established all possible equilibrium states. It is conclude that, in all four equilibrium states, only one state E_4 is conditionally stable. Further the global stability is established with the help of suitable Liapunov's function and the growth rates of the species are numerically estimated using Runge-Kutta fourth order method.

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