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A NEW APPROACH ON NUMERICAL SOLUTIONS OF BURGER'S EQUATION USING PMEDG ITERATIVE METHOD

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ABSTRACT

Modified Explicit Decoupled Group (MEDG) scheme from the rotated finite difference discretization to the numerical solution of the nonlinear steady two dimensional Burgers' Equation introduced by Saeed [1]. The objective of this paper is to develop the MEDG method in combination with suitable preconditioned iterative scheme for solving the Burgers' Equation. Numerical experiments are carried out to confirm the effectiveness of the preconditioner in terms of number of iterations and execution timings. Comparison with its unpreconditioned counterpart will also be reported.

Contribution/ Originality: This study contributes in the existing literature about the foundation of fast group iterative schemes for solving Burgers' Equation. It is one of the few studies which combine a suitable splitting-type block preconditioner with the group iterative scheme as a way to further improve the convergence rate of the method in solving the this equation.

1. INTRODUCTION

Consider steady two dimensional Burgers' equation ([1]; [2]) as the following:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)=0 \qquad (1)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}-\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)=0 \qquad (2)$$

with Dirichlet boundary conditions on u and v. This equation is considered to be a simplified form of the Navier-Stokes equation, where the pressure term is neglected. Here, Re is the Reynolds number. Saeed [1] introduced MEDG scheme from the rotated finite difference discretization to the numerical solution of the nonlinear steady two dimensional Burgers' Equations (1) and (2) and this iterative scheme has shown improvements in the number of iterations and the execution time experimentally.

Several methods have been developed on the preconditioned iterative methods for the last 15 years, but this quest is still going on ([3]; [4]; [5]). The aim of this paper is to propose new preconditioned iterative scheme and apply it to the MEDG iterative method for solving the steady two dimensional Burgers' equation.

The paper is organized as follows. In Section 2, we briefly describe the formulation of the proposed preconditioned modified explicit decoupled group (PMEDG) for solving the steady two dimensional Burgers' equation. The numerical results are presented in Section 3 in order to show the efficiency of the new preconditioned method. Finally, the conclusion is given in Section 4.

2. THE PROPOSED PRECONDITIONED MEDG FORMULATION

Let n be a fixed positive integer. Determine the grid size h = 2/n so that a uniformly spaced square network ($\Delta x = \Delta y = h$) with $x_i = -1 + ih,\ y = jh, i, j = 0,1,2,\ldots,n,$ is imposed on S. By using the centred difference approximation and neglecting the error terms, equations (1) and (2) can be discretised at the grid points (x_i, y_j) by the following finite difference equations:

$$[\frac{h\,\mathrm{Re}(u_{i+1,j}-u_{i-1,j})+8}{2\ \mathrm{Re}\ h^2}]u_{ij} = \frac{(u_{i-1,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1})}{\mathrm{Re}\ h^2} - v_{ij}(\frac{u_{i,j+1}-u_{i,j-1}}{2h}) \quad (3)$$

$$[\frac{h\,\mathrm{Re}(v_{i,j+1}-v_{i,j-1})+8}{2\ \mathrm{Re}\ h^2}]v_{ij} = \frac{(v_{i-1,j}+v_{i+1,j}+v_{i,j-1}+v_{i,j+1})}{\mathrm{Re}\ h^2} - u_{ij}(\frac{v_{i+1,j}-v_{i-1,j}}{2h}) \quad (4)$$

It can be seen that if v is known, then we can solve (3) iteratively for u, while if u is known, we can solve (4) iteratively for v, and vice versa. By the same manner of the schemes presented for the Navier-Stokes problem, we can devise a similar algorithm by first making initial guesses u_{ij}^0 and v_{ij}^0, and then generate an alternating sequence of outer iterates. The iteration is continued until for some k such that $\left|u_{ij}^{(k+1)} - u_{ij}^{(k)}\right| < \varepsilon$ and $\left|v_{ij}^{(k+1)} - v_{ij}^{(k)}\right| < \varepsilon$ for some given tolerance (ε). The solutions $u_{ij}^{(k+1)}$ and $v_{ij}^{(k+1)}$ generated are then taken to be the numerical solutions of the given problem ([6]; [7]).

Another type of approximation that can represent the Burgers' equation under study is the cross orientation which can be obtained by rotating the i-plane axis and the j-plane axis clockwise by 45° as the following:

$$[\frac{h\,\mathrm{Re}(u_{i+1,j-1}-u_{i-1,j+1}+u_{i+1,j+1}-u_{i-1,j-1})+8}{2\ \mathrm{Re}\ h^2}]u_{ij} + (\frac{v_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})u_{i-1,j+1} + \quad (5)$$
$$(\frac{-v_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})u_{i+1,j-1} + (\frac{-v_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})u_{i-1,j-1} + (\frac{v_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})u_{i+1,j+1} = 0$$

$$[\frac{h\,\mathrm{Re}(v_{i+1,j+1}-v_{i-1,j-1}-v_{i+1,j-1}+v_{i-1,j-1})+8}{4\ \mathrm{Re}\ h^2}]u_{ij} + (\frac{-u_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})v_{i-1,j+1} + \quad (6)$$
$$(\frac{u_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})v_{i+1,j-1} + (\frac{-u_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})v_{i-1,j-1} + (\frac{u_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})v_{i+1,j+1} = 0$$

When i-plane axis and the j-plane axis are rotated clockwise 45^0 with grid spacing 2h, then equations (5) and (6) become as the following equations:

$$[\frac{h\,\mathrm{Re}(u_{i+2,j-2}-u_{i-2,j+2}+u_{i+2,j+2}-u_{i-2,j-2})+8}{2\ \mathrm{Re}\ h^2}]u_{ij} + (\frac{v_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})u_{i-2,j+2} + \quad (7)$$
$$(\frac{-v_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})u_{i+2,j-2} + (\frac{-v_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})u_{i-2,j-2} + (\frac{v_{ij}}{4h} - \frac{1}{2\,\mathrm{Re}\ h^2})u_{i+2,j+2} = 0$$

$$[\frac{h\,\mathrm{Re}(v_{i+2,j+2}-v_{i-2,j-2}-v_{i+2,j-2}+v_{i-2,j-2})+8}{4\,\mathrm{Re}\,h^2}]u_{ij}+(\frac{-u_{ij}}{4h}-\frac{1}{2\,\mathrm{Re}\,h^2})v_{i-2,j+2}+ \tag{8}$$
$$(\frac{u_{ij}}{4h}-\frac{1}{2\,\mathrm{Re}\,h^2})v_{i+2,j-2}+(\frac{-u_{ij}}{4h}-\frac{1}{2\,\mathrm{Re}\,h^2})v_{i-2,j-2}+(\frac{u_{ij}}{4h}-\frac{1}{2\,\mathrm{Re}\,h^2})v_{i+2,j+2}=0$$

The four-point MEDG for solving the problem (1)-(2) can now formulated by using the above rotated finite difference approximation (7)-(8). Without loss of generality, assume the generation of $v_{ij}^{(k+1)}$ is done first using equation (2) followed by the generation of $u_{ij}^{(k+1)}$ using equation (7).Using the equation (8) for V_{ij} any group of four points on a discretised solution domain can be solved resulting in a (4×4) system of equations as the following [2]:

$$A\times\begin{bmatrix} v_{ij} \\ v_{i+2,j+2} \\ v_{i+2,j} \\ v_{i,j+2}\end{bmatrix}=\begin{bmatrix} rhs_{ij} \\ rhs_{i+2,j+2} \\ rhs_{i+2,j} \\ rhs_{i,j+2}\end{bmatrix} \tag{9}$$

where

$$A=\begin{bmatrix} \frac{h\,\mathrm{Re}(v_{i+2,j+2}-v_{i-2,j-2}+v_{i+2,j-2}+v_{i-2,j+2})+8}{4\,\mathrm{Re}\,h^2} & cu_{ij}-d & 0 & 0 \\ -cu_{i+2,j+2}-d & \frac{h\,\mathrm{Re}(v_{i+4,j+4}-v_{ij}+v_{i+4,j}-v_{i,j+4})+8}{4\,\mathrm{Re}\,h^2} & 0 & 0 \\ 0 & 0 & \frac{h\,\mathrm{Re}(v_{i+4,j+2}-v_{i,j-2}-v_{i+4,j-2}+v_{i,j+2})+8}{4\,\mathrm{Re}\,h^2} & -cu_{i+2,j}-d \\ 0 & 0 & cu_{i,j+2}-d & \frac{h\,\mathrm{Re}(v_{i+2,j+4}-v_{i-2,j}-v_{i+2,j}+v_{i-2,j+4})+8}{4\,\mathrm{Re}\,h^2}\end{bmatrix},$$

$$\begin{aligned} rhs_{ij} &= (cu_{ij}+d)v_{i-2,j+2}+(-cu_{ij}+d)v_{i-2,j-2}+(cu_{ij}+d)v_{i-2,j-2}, \\ rhs_{i+2,j+2} &= (cu_{i+2,j+2}+d)v_{i,j+4}+(-cu_{i+2,j+2}+d)v_{i+4,j}+(-cu_{i+2,j+2}+d)v_{i+4,j+4}, \\ rhs_{i+2,j} &= (-cu_{i+2,j}+d)v_{i+4,j+2}+(-cu_{i+2,j}+d)v_{i+4,j-2}+(cu_{i+2,j}+d)v_{i,j-2}, \\ rhs_{i,j+2} &= (cu_{i,j+2}+d)v_{i-2,j+4}+(-cu_{i,j+2}+d)v_{i+2,j+4}+(cu_{i,j+2}+d)v_{i-2,j}, \\ c &= \frac{1}{4h}, \quad d=\frac{1}{2\,\mathrm{Re}\,h^2}. \end{aligned}$$

The system (7) leads to a decoupled system of (2×2) equations which can be made explicit as follows:

$$\begin{bmatrix} v_{ij} \\ v_{i+2,j+2}\end{bmatrix}^{(k+1)}=\frac{(4\,\mathrm{Re}\,h^2)^2}{[\mathrm{Re}\,h(v^k_{i+2,j+2}-v_{i-2,j-2}-v_{i+2,j-2}+v_{i-2,j+2})+8][\mathrm{Re}\,h(v^k_{i+4,j+4}-v^k_{i,j}-v_{i+4,j}+v_{i,j+4})+8]}$$
$$\times\begin{bmatrix} \frac{\mathrm{Re}\,h(v_{i+4,j+4}-v^k_{ij}-v_{i+4,j}+v_{i,j+4})+8}{4\,\mathrm{Re}\,h^2} & -cu_{ij}+d \\ cu_{i+2,j+2}+d & \frac{\mathrm{Re}\,h(v^k_{i+2,j+2}-v_{i-2,j-2}-v_{i+2,j-2}+v_{i-2,j+2})+8}{4\,\mathrm{Re}\,h^2}\end{bmatrix}\begin{bmatrix} rhs_{ij} \\ rhs_{i+2,j+2}\end{bmatrix}$$

and

$$\begin{bmatrix} v_{i+2,j} \\ v_{i,j+2} \end{bmatrix}^{(k+1)} = \frac{(4\,\mathrm{Re}\,h^2)^2}{[\mathrm{Re}\,h(v^k_{i+2,j+2} - v_{i-2,j-2} - v_{i+2,j-2} + v_{i-2,j+2}) + 8][\mathrm{Re}\,h(v_{i+4,j+4} - v^k_{i,j} - v_{i+4,j} + v_{i,j+4}) + 8] + (h\,\mathrm{Re}\,u_{i+2,j+2} + 2)(h\,\mathrm{Re}\,u_{ij} - 2)}$$
$$\times \begin{bmatrix} \dfrac{\mathrm{Re}\,h(v_{i+2,j+4} - v^k_{i+2,j} - v_{i-2,j} + v_{i-2,j+4}) + 8}{4\,\mathrm{Re}\,h^2} & c u_{i+2,j} + d \\ -c u_{i,j+2} + d & \dfrac{\mathrm{Re}\,h(v^k_{i,j+2} - v_{i,j-2} - v_{i+4,j-2} + v_{i+4,j+2}) + 8}{4\,\mathrm{Re}\,h^2} \end{bmatrix} \begin{bmatrix} rhs_{i+2,j} \\ rhs_{i,j+2} \end{bmatrix}$$

By the same manner, from the generation of u_{ij} using equation (8), a (4×4) system of equations can be formed as the following:

$$B \times \begin{bmatrix} u_{ij} \\ u_{i+2,j+2} \\ u_{i+2,j} \\ u_{i,j+2} \end{bmatrix} = \begin{bmatrix} \overline{r\,hs_{ij}} \\ \overline{r\,hs_{i+2,j+2}} \\ \overline{r\,hs_{i+2,j}} \\ \overline{r\,hs_{i,j+2}} \end{bmatrix} \qquad (10)$$

where

$$B = \begin{bmatrix} \dfrac{h\,\mathrm{Re}(u_{i+2,j-2} - u_{i-2,j+2} + u_{i+2,j+2} - u_{i-2,j-2}) + 8}{4\,\mathrm{Re}\,h^2} & c v_{ij} - d & 0 & 0 \\ -c v_{i+2,j+2} - d & \dfrac{h\,\mathrm{Re}(u_{i+4,j} - u_{i,j+4} + u_{i+4,j+4} - u_{ij}) + 8}{4\,\mathrm{Re}\,h^2} & 0 & 0 \\ 0 & 0 & \dfrac{h\,\mathrm{Re}(u_{i+4,j-2} - u_{i,j+2} + u_{i+4,j+2} - u_{i,j-2}) + 8}{4\,\mathrm{Re}\,h^2} & c v_{i+2,j} - d \\ 0 & 0 & -c v_{i,j+2} - d & \dfrac{h\,\mathrm{Re}(u_{i+2,j} - u_{i-2,j+4} + u_{i+2,j+4} - u_{i-2,j}) + 8}{4\,\mathrm{Re}\,h^2} \end{bmatrix},$$

$$\overline{r\,hs_{ij}} = (-c v_{ij} + d) u_{i-2,j+2} + (c v_{ij} + d) u_{i+2,j-2} + (c v_{ij} + d) u_{i-2,j-2},$$
$$\overline{r\,hs_{i+2,j+2}} = (-c v_{i+2,j+2} + d) u_{i,j+4} + (c v_{i+2,j+2} + d) u_{i+4,j} + (-c v_{i+2,j+2} + d) u_{i+4,j+4},$$
$$\overline{r\,hs_{i+2,j}} = (-c v_{i+2,j} + d) u_{i+4,j+2} + (c v_{i+2,j} + d) u_{i+4,j-2} + (c v_{i+2,j} + d) u_{i,j-2},$$
$$\overline{r\,hs_{i,j+2}} = (-c v_{i,j+2} + d) u_{i-2,j+4} + (-c v_{i,j+2} + d) u_{i+2,j+4} + (c v_{i,j+2} + d) u_{i-2,j}.$$

The system (10) leads to a decoupled system of (2×2) equations whose explicit forms can be obtained as follows:

$$\begin{bmatrix} u_{ij} \\ u_{i+2,j+2} \end{bmatrix}^{(k+1)} = \frac{(4\,\mathrm{Re}\,h^2)^2}{[\mathrm{Re}\,h(u_{i+2,j-2} - u_{i-2,j+2} + u^k_{i+2,j+2} - u_{i-2,j-2}) + 8][\mathrm{Re}\,h(u_{i+4,j} - u^k_{i,j} - u_{i,j+4} + u_{i+4,j+4}) + 8] + (\mathrm{Re}\,h\,v_{i+2,j+2} + 2)(\mathrm{Re}\,h\,v_{ij} - 2)}$$
$$\times \begin{bmatrix} \dfrac{\mathrm{Re}\,h(u_{i+4,j} - u_{i,j+4} + u_{i+4,j+4} - u_{ij}) + 8}{4\,\mathrm{Re}\,h^2} & -c v_{ij} + d \\ c v_{i+2,j+2} + d & \dfrac{\mathrm{Re}\,h(u^k_{i+2,j-2} - u_{i-2,j+2} + u_{i+2,j+2} - u_{i-2,j-2}) + 8}{4\,\mathrm{Re}\,h^2} \end{bmatrix} \begin{bmatrix} \overline{r\,hs_{ij}} \\ \overline{r\,hs_{i+2,j+2}} \end{bmatrix}$$

$$\begin{bmatrix} u_{i+2,j} \\ u_{i,j+2} \end{bmatrix}^{(k+1)} = \frac{(4\,\mathrm{Re}\,h^2)^2}{[h\,\mathrm{Re}(u_{i+4,j-2} + u_{i+4,j+2} - u^k_{i,j+2} - u_{i,j-2}) + 8][h\,\mathrm{Re}(u^k_{i+2,j} - u_{i-2,j+4} - u_{i-2,j} + u_{i+2,j+4}) + 8] + (h\,\mathrm{Re}\,v_{i,j+2} + 2)(h\,\mathrm{Re}\,u_{i+2,j} - 2)}$$
$$\times \begin{bmatrix} \dfrac{\mathrm{Re}\,h(u_{i+2,j} - u_{i-2,j+4} + u_{i+2,j+4} - u_{i-2,j}) + 8}{4\,\mathrm{Re}\,h^2} & -c v_{i+2,j} + d \\ c v_{i,j+2} + d & \dfrac{\mathrm{Re}\,h(u_{i+4,j-2} - u_{i,j+2} + u_{i+4,j+2} - u_{i,j-2}) + 8}{4\,\mathrm{Re}\,h^2} \end{bmatrix} \begin{bmatrix} \overline{r\,hs_{i+2,j}} \\ \overline{r\,hs_{i,j+2}} \end{bmatrix}$$

A preconditioner is a matrix that transforms the system into one that is equivalent in the sense that it has the same solution, but that has more favourable spectral properties. A good preconditioner should be constructed inexpensively and should be a good approximation to the inverse of coefficient matrix of the iterative method. By multiplication the following preconditioner matrix P_1 for both sides of equation (9):

$$P_1 = \begin{bmatrix} 0 & cu_{ij} - d & 0 & 0 \\ -cu_{i+2,j+2} - d & 0 & 0 & 0 \\ 0 & 0 & 0 & -cu_{i+2,j} - d \\ 0 & 0 & cu_{i,j+2} - d & 0 \end{bmatrix},$$

and multiply the following preconditioner matrix P_2 for both sides of equation (10):

$$P_2 = \begin{bmatrix} 0 & cv_{ij} - d & 0 & 0 \\ -cv_{i+2,j+2} - d & 0 & 0 & 0 \\ 0 & 0 & 0 & cv_{i+2,j} - d \\ 0 & 0 & -cv_{i,j+2} - d & 0 \end{bmatrix},$$

The effectiveness of this preconditioned MEDG method will be shown in the next section.

3. NUMERICAL EXPERIMENTATION AND RESULTS

In order to demonstrate the feasibility of the proposed method in solving steady two dimensional Burgers' equation, numerical experiments have been carried out to solve the Burgers' Equations (1)-(2) with the exact solution

$$u = \frac{-2(a_2 + a_4 y + \lambda a_5 \cos \lambda y (e^{\lambda(x-x_0)} - e^{-\lambda(x-x_0)}))}{\mathrm{Re}(a_1 + a_2 x + a_3 y + a_4 xy + a_5 (e^{\lambda(x-x_0)} + e^{-\lambda(x-x_0)}) \cos \lambda y)}$$

$$v = \frac{-2(a_3 + a_4 x - \lambda a_5 \sin \lambda y (e^{\lambda(x-x_0)} + e^{-\lambda(x-x_0)}))}{\mathrm{Re}(a_1 + a_2 x + a_3 y + a_4 xy + a_5 (e^{\lambda(x-x_0)} + e^{-\lambda(x-x_0)}) \cos \lambda y)}, \quad -1 \le x \le 1,\ 0 \le y \le 2$$

with the boundary conditions satisfying the exact solutions. Here, $a_1, a_2, a_3, a_4, a_5, \lambda$ and x_0 can be chosen to produce different behavior of exact solutions [8]. To compare the numerical results among EDG iterative method, MEDG iterative method with the previous work [1] we randomly chose $a_1 = a_2 = 1.0,\ a_3 = a_4 = 0.0,\ a_5 = x_0 = 1.0$ and $\lambda = 0.3$ for $\mathrm{Re} = 100$ and 1000. Throughout the experiment, a tolerance of $\delta = \varepsilon = 10^{-11}$ was used as the termination criteria for both the outer and inner iterations.

The software used to implement and generate the results was Developer C++ Version 4.9.9.2. Tables 1 and 2 list the iteration counts and timings for both the MEDG and PMEDG iterative methods respectively.

The results from PMEDG scheme portray similar behavior as the MEDG. However, it can be seen that the PMEDG requires only about 58-63% of the time required by the original MEDG system. Furthermore, the iteration count for the PMEDG system increases at a slower rate than the MEDG system. The best results were obtained when the model problem was solved using the four-point PMEDG inner iterative scheme. In conclusion, the new iterative schemes serve as viable alternatives in solving the two dimensional Burger's equation.

Table-1. Number of iterations and Elapsed Time for 4-points MEDG outer-inner scheme

N	Re	Ave-Abs. Error for u	Ave-Abs. Error for v	Number of outer iterates	Number of inner iteration for v	Number of inner iteration for u	Time (secs)
51	10	6.38E-08	3.92E-08	1 2 3 4	43 31 22 1	46 33 12 1	36.29
	100	5.78E-09	3.83E-09	1 2 3 4	29 19 12 1	33 11 3 1	27.69
	1000	5.26E-09	3.85E-09	1 2 3 4	23 13 6 1	25 15 1 1	25.14
87	10	6.36E-08	3.59E-08	1 2 3 4	54 39 21 1	56 33 12 1	43.56
	100	5.33E-09	3.91E-09	1 2 3 4	29 16 8 1	32 11 4 1	30.72
	1000	5.48E-10	3.63E-10	1 2 3 4	23 10 7 1	27 13 1 1	26.36
121	10	3.79E-09	1.98E-09	1 2 3 4	96 81 31 1	102 73 15 1	101.61
	100	3.66E-09	1.74E-09	1 2 3 4	71 43 14 1	78 39 1 1	99.33
	1000	3.55E-10	1.64E-10	1 2 3 4	63 37 14 1	70 31 1 1	82.79
141	10	2.97E-08	1.61E-08	1 2 3 4	113 89 38 1	119 78 13 1	116.38
	100	2.86E-09	1.57E-09	1 2 3 4	83 62 21 1	88 57 3 1	107.36
	1000	2.37E-10	1.49E-10	1 2 3 4	72 46 19 1	79 36 1 1	98.63

Source: software results

Table-2. Number of iterations and Elapsed Time for 4-points PMEDG outer-inner scheme

N	Re	Ave-Abs. Error for u	Ave-Abs. Error for v	Number of outer iterates	Number of inner iteration for v	Number of inner iteration for u	Time (secs)
51	10	5.71E-08	3.12E-08	1 2 3 4	31 23 17 1	37 28 9 1	21.87
	100	5.25E-09	3.63E-09	1 2 3 4	22 14 8 1	28 10 2 1	19.37
	1000	5.29E-09	3.71E-09	1 2 3 4	14 7 5 1	17 8 1 1	16.92
87	10	5.97E-08	3.34E-08	1 2 3 4	33 21 13 1	38 25 9 1	27.43
	100	5.33E-09	3.66E-09	1 2 3 4	21 14 6 1	26 11 4 1	30.72
	1000	5.29E-10	3.47E-10	1 2 3 4	19 10 7 1	24 8 1 1	18.17
121	10	3.47-09	1.38E-09	1 2 3 4	63 54 17 1	73 58 13 1	82.74
	100	3.53E-09	1.63E-09	1 2 3 4	52 36 11 1	57 29 1 1	72.75
	1000	3.67E-10	1.89E-10	1 2 3 4	47 31 12 1	51 37 1 1	67.84
141	10	1.98E-08	0.99E-08	1 2 3 4	79 42 27 1	84 46 11 1	92.47
	100	1.87E-09	1.04E-09	1 2 3 4	71 53 17 1	77 38 2 1	88.95
	1000	1.27E-10	1.18E-10	1 2 3 4	67 29 16 1	69 31 1 1	81.48

Source: software results

4. CONCLUSION AND FUTURE WORK

This study focused on the formulation of new Preconditioned Modified Explicit Decoupled Group (PMEDG) for solving two dimensional steady Burgers' Equation. The proposed PMEDG scheme has shown improvements in the number of iterations and the execution time experimentally. Hence, we conclude that the PMEDG iterative method is superior to the original MEDG method for solving the two dimensional Burgers' Equation. However the estimation of implementing the family of explicit group methods for solving general type of Partial Differential Equations such as the Fractional Partial Differential Equations (FPDEs) remains a challenging task and therefore it will be a worthwhile effort to venture more into this group iterative method.

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