

## Bitcoin and portfolio diversification during crises: Evidence from the French market with mean–variance and stochastic dominance analysis



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### ABSTRACT

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This study explores the diversification potential of Bitcoin in a French investment portfolio comprising oil, currency, and gold across three distinct market regimes: a pre-crisis stable period, the COVID-19 pandemic, and the Russia–Ukraine conflict. The purpose is to assess whether Bitcoin can enhance portfolio efficiency and provide hedging opportunities under varying market conditions. The analysis is conducted using daily data for Bitcoin, gold, oil, currency, and the CAC40 index from January 1, 2019, to April 22, 2022. Portfolio performance is evaluated through the Mean–Variance (MV) framework and Stochastic Dominance (SD) analysis, allowing for a robust comparison of risk–return trade-offs and investor preferences. The MV results show that including Bitcoin consistently improves the portfolio’s risk–return profile, evidenced by an upward shift in the efficient frontier across all sub-periods. However, the SD analysis yields more nuanced insights. Before and during the COVID-19 crisis, the portfolio excluding Bitcoin dominates the Bitcoin-inclusive portfolio under second- and third-order stochastic dominance criteria, suggesting that risk-averse investors would prefer the traditional asset mix. In contrast, during the Russia–Ukraine war, no clear stochastic dominance is detected between Bitcoin-inclusive and Bitcoin-exclusive portfolios. These findings emphasize that Bitcoin’s diversification role is highly context- and framework-dependent.

**Contribution/ Originality:** This study contributes to the existing literature by examining the role of Bitcoin in portfolio diversification within the French market. This study uses both Mean-Variance and Stochastic Dominance analyses. The paper’s primary contribution is finding that Bitcoin’s diversification benefits are framework-dependent and crisis-sensitive, offering novel insights for investors and policymakers.

## 1. INTRODUCTION

Diversifying portfolio risk is of interest to investors and their managers. Modern portfolio theory suggests that investors can decrease the overall risk of their portfolios by allocating funds to negatively correlated assets [1]. In other words, owning a diversified portfolio of assets allows investors to improve returns and reduce risk.

In this study, we contribute to the recent literature by examining whether Bitcoin provides diversification benefits in French portfolios, especially during periods of crisis such as COVID-19 and the Russia–Ukraine war. This research gap remains underexplored in the context of the French financial market, despite the growing global literature on Bitcoin as a hedging and diversification tool. Since its inception in 2009, Bitcoin has attracted the attention of the media and economic actors. Debates on this decentralized cryptocurrency have particularly increased

during the sovereign debt crisis of 2010–2013. Some practitioners have turned away from conventional currencies to use Bitcoin instead. The CFTC (US regulatory body) recognized Bitcoin as a commodity and financial product in September 2015, further consolidating its role in financial markets. Since Bitcoin was first proposed by Nakamoto [2] several empirical studies have been conducted on this cryptocurrency, focusing on its financial characteristics, such as the study by Brandvold, et al. [3] and Bouoiyour, et al. [4] which focused on price discovery in the Bitcoin market. Studies also show that Bitcoin price formation is subject to unique factors that differ significantly from those affecting traditional assets. Some key studies have examined certain properties of cryptocurrencies, such as market returns and volatility [5], speculation [6]. Other studies focus on the role of Bitcoin in portfolio diversification, Eisl, et al. [7]. Bouri, et al. [8] emphasized that Bitcoin gained traction as a shelter from uncertainty in traditional economic and banking systems. In search of portfolio diversification, studies examine how Bitcoin is diversified with traditional financial assets and alternative investments [9, 10] global and emerging stock markets [11], and commodities [12]. Building on this literature, the objective of this paper is to analyze the benefits of including Bitcoin in a French portfolio diversified with traditional assets. Unlike most prior research, we apply both Mean-Variance optimization and Stochastic Dominance analysis across different crisis regimes to provide new insights for investors and portfolio managers. The remainder of the paper is structured as follows: Section 2 presents a literature review. Section 3 discusses the database and its descriptive statistics. Section 4 introduces the efficient frontier. Section 5 describes the approach of genetic algorithms and stochastic dominance, as well as their results. Section 6 concludes.

## 2. LITERATURE REVIEW

Since its inception, Bitcoin has attracted increasing attention from both academics and practitioners due to its unique characteristics compared to traditional assets. As a decentralized digital currency without a central authority, it exhibits persistently low correlations with conventional financial instruments but also extremely high volatility and returns. These distinctive features have fueled a vast literature investigating whether Bitcoin should be considered a diversifier, a speculative asset, or even a safe haven during periods of financial turmoil.

A large body of research highlights the diversification benefits of Bitcoin in portfolio construction. Bouoiyour, et al. [4]; Kristoufek [13] and Briere, et al. [14] demonstrate that Bitcoin exhibits very weak correlations with equities, bonds, and commodities, largely explained by its unique price formation mechanisms. Similar findings are reported by Dyrberg [12]; Carpenter [15] and Baur, et al. [16] who emphasize that even a marginal allocation to Bitcoin can enhance the risk–return profile of diversified portfolios. These results are further confirmed by Krause and Pham [17] and Kajtazi and Moro [11], who reaffirm Bitcoin’s role as a portfolio diversifier, while stressing its speculative nature and the instability of correlations over time. Ji, et al. [18] further point out that these correlations are not stable but subject to structural breaks, implying that diversification benefits are conditional and time-varying. Another strand of the literature investigates Bitcoin’s role within portfolio optimization frameworks. Wu, et al. [19] and Eisl, et al. [7] show that Bitcoin enhances portfolio efficiency when measured by Sharpe or Sortino ratios. More sophisticated approaches, such as mean-CVaR models [11] and DCC-GARCH models [20] reinforce these findings, though the extent of the benefits depends on time periods and market conditions. Along similar lines, Youssef, et al. [21] employ a bi-objective goal programming approach and demonstrate that the inclusion of Bitcoin in traditional portfolios shifts the Markowitz efficient frontier toward superior risk–return combinations, particularly for risk-seeking investors. More recently, Marinescu, et al. [22] provide strong evidence that the optimal allocation of crypto-assets is highly sensitive to prevailing economic uncertainty, highlighting the importance of adopting dynamic rather than static portfolio strategies. The debate is more contentious regarding Bitcoin’s role as a safe-haven asset. While early arguments described it as “digital gold” Popper [23]; Baur, et al. [16]; Bouri, et al. [8] and Bouri, et al. [24] showed that Bitcoin could hedge certain risks, empirical evidence from major crises paints a more nuanced picture. During the COVID-19 pandemic, Conlon and McGee [25]; Kristoufek [26] and Wen, et al. [27] conclude that Bitcoin failed to act as a safe haven and instead increased portfolio risk, while gold consistently outperformed it as a

hedge. Similar results are reported by Snene and Jeribi [28] in the context of the Russia–Ukraine conflict, confirming gold’s dominance as a safe-haven asset. In contrast, Ullah, et al. [29] provide evidence that, within the Russian financial market, Bitcoin occasionally served a hedging function. Consistently, Belkhir and Aoun [30] reveal that Bitcoin and Ethereum display strong interconnectedness and predominantly act as net transmitters of volatility, particularly in the short term. Likewise, Singh, et al. [31] document weak co-movement between Bitcoin and global equity markets, supporting its diversification potential even though its safe-haven status remains unproven.

### 3. DATA

This study relies on daily price data for Bitcoin, gold, oil, currency, and the CAC40 index over the period from January 1, 2019, to April 22, 2022, resulting in 864 daily observations. To account for heterogeneous market conditions, the sample is divided into three sub-periods. The first represents a stable phase (January 1, 2019–December 31, 2019). The second corresponds to the COVID-19 crisis (January 1, 2020–February 23, 2022). The third reflects the Russia–Ukraine conflict (February 24, 2022–April 22, 2022). All data are sourced from International DataStream and the financial portal investing.com. We focus on the French market because it represents one of the largest financial markets in Europe and remains relatively understudied in the Bitcoin diversification literature. The chosen asset mix (gold, oil, currency, CAC40) reflects the main categories of assets considered by French investors, allowing us to assess whether Bitcoin provides added value compared to traditional hedging instruments.

All series are expressed in US dollars (USD), ensuring consistency across assets. This choice aligns with the international quotation of Bitcoin, gold, and oil, while the CAC40 is also available in USD from DataStream, which guarantees homogeneity in the dataset.

Daily logarithmic returns were computed as follows:

$$R_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \text{ is a daily return.}$$

Where:

- $R_t$ : Return at time  $t$ .
- $P_t$ : Stock price at time  $t$ .
- $P_{t-1}$ : Stock price at time  $t - 1$ .

To ensure data comparability, we aligned trading days across assets and handled missing values using linear interpolation.

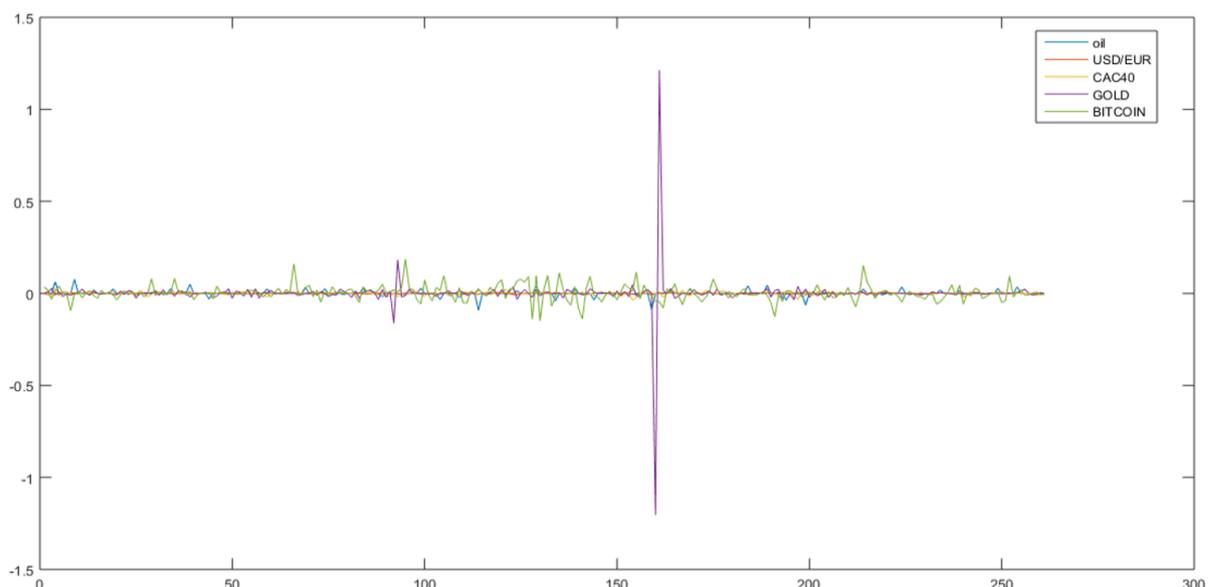


Figure 1. Patterns of Daily returns for the Five Portfolio Assets during the Stable Period

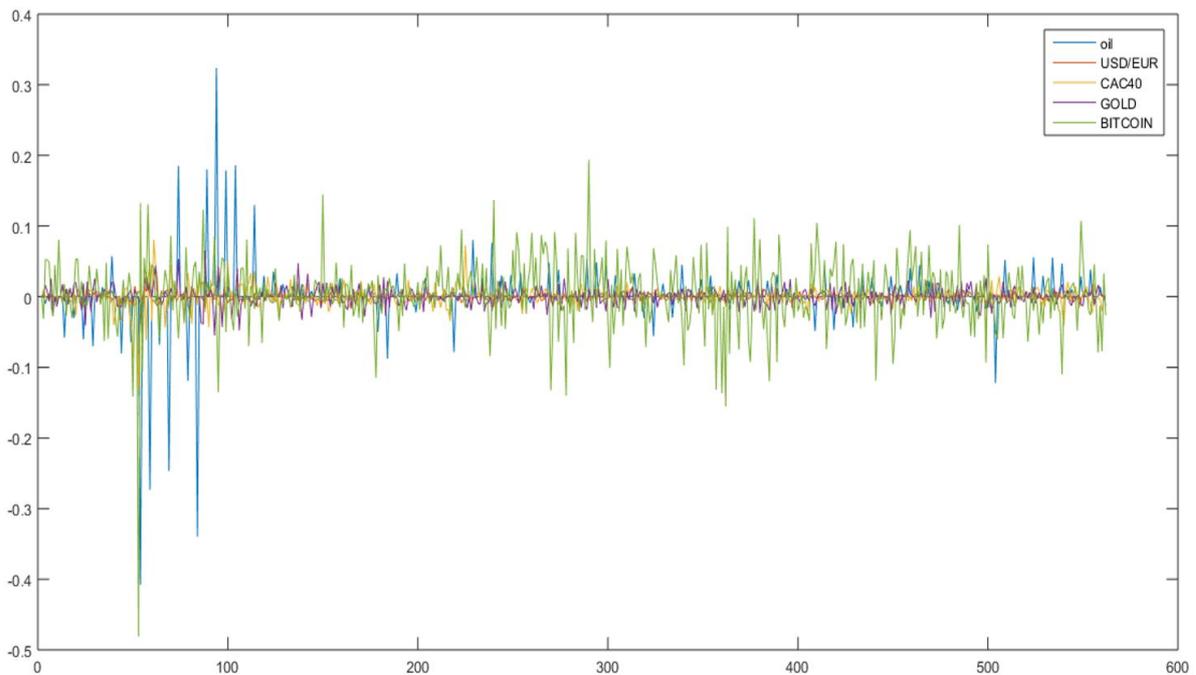


Figure 2. Patterns of daily returns for the five portfolio assets during the COVID-19 Crisis

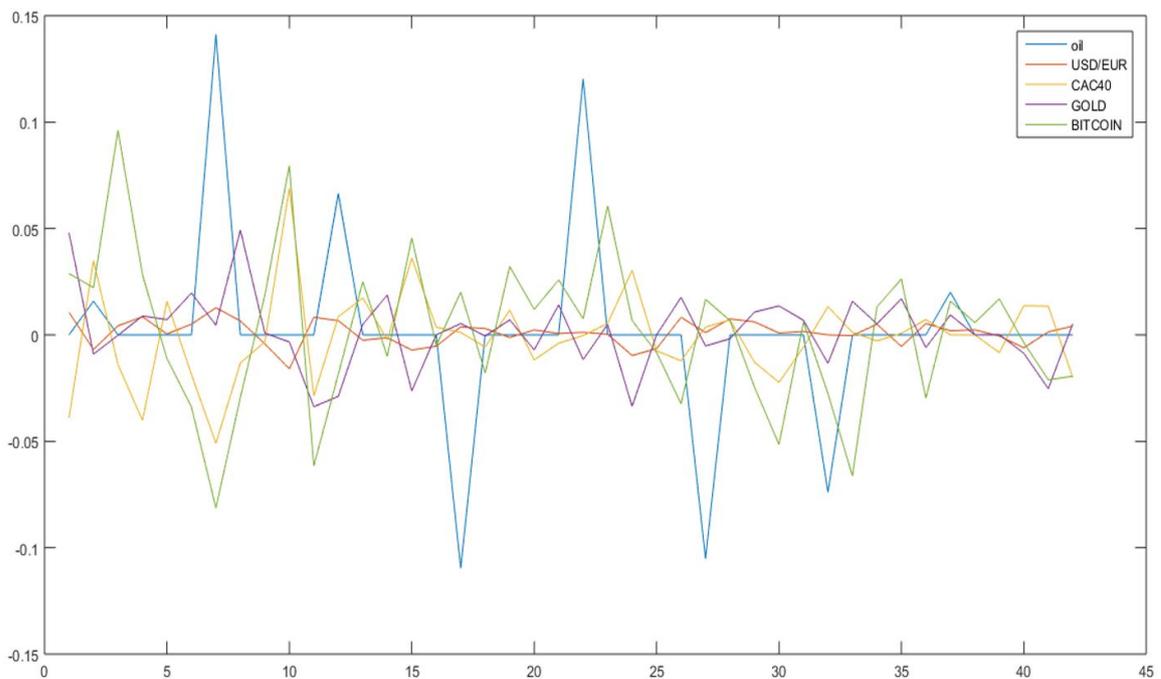


Figure 3. Patterns of daily returns for the five portfolio assets during the Russia–Ukraine conflict.

Figures 1, 2, and 3 illustrate the daily returns of the five assets studied across three distinct market regimes. Figure 1 depicts the stable period, showing relatively low volatility and moderate fluctuations in all assets. Figure 2 illustrates the COVID-19 crisis period, highlighting increased volatility and more pronounced swings in returns, particularly for Bitcoin and oil. Figure 3 presents the Russian–Ukrainian conflict period, where the returns exhibit moderate volatility, with patterns differing from both the stable and pandemic periods, reflecting the unique market conditions during this geopolitical event.

The descriptive statistics of the time series are presented in Tables 1, 2, and 3. According to the average returns (Table 1), Bitcoin is the best-performing asset (0.24%). On the other hand, the currency has the lowest average return with 0.0086%. Regarding the standard deviation, gold has the highest risk exposure (10.78%), followed by bitcoin

(4.18%). Table 2 shows that Bitcoin's average return is the highest (0.29%) and highlights that Bitcoin's return series has the highest volatility (4.83%) compared to other assets. From Table 3, we notice that Bitcoin's average return has decreased compared to previous periods. The deviation has also reduced.

We use the Jarque-Bera test along with the coefficient of skewness and the coefficient of kurtosis to test the normality of the returns of Bitcoin and other traditional assets. A high value of Jarque-Bera indicates the rejection of the hypothesis of normality of the returns. However, the coefficient of skewness is equal to 0, and the coefficient of kurtosis is equal to 3; the distribution is said to be normal. Indeed, if the skewness statistics are positive, that is to say that the distribution is asymmetric to the right, while if it is negative, we say that the distribution is skewed to the left. For the coefficient of kurtosis, if it is less than 3, this implies that the distribution is platykurtic; however, if it is greater than 3, this implies that the distribution is leptokurtic. According to Tables 1 and 2, the returns of all variables do not follow the normal distribution. According to Table 3, the distribution of oil and the CAC40 index is not normal.

**Table 1** Descriptive statistics of daily asset returns during the stable period

Variables	BTC	CAC40	GOLD	OIL	USD/EUR
Mean	0.24%	0.089%	0.075%	0.114%	0.008%
Std. Dev.	4.18%	0.82%	10.78%	1.48%	0.30%
Skewness	0.395	-0.727	0.104	-0.967	0.073
Kurtosis	6.782	5.645	121.764	18.724	3.772
Jarque-Bera	162.352	99.044	153391.9	2729.658	6.718
Probability	0.000	0.000	0.000	0.000	0.035

**Table 2.** Descriptive statistics of daily asset returns during the COVID-19 period

Variables	BTC	CAC40	GOLD	OIL	USD/EUR
Mean	0.29%	0.022%	0.033%	0.062%	-0.001%
Std. Dev.	4.83%	1.55%	1.36%	3.85%	0.41%
Skewness	-1.821	-1.366	-0.071	-2.739	0.194
Kurtosis	21.202	16.603	5.827	54.562	4.572
Jarque-Bera	8054.274	4500.312	187.2834	62849.00	61.286
Probability	0.000	0.000	0.000	0.000	0.000

**Table 3.** Descriptive statistics of monthly asset returns during the Russia-Ukraine conflict

Variables	BTC	CAC40	GOLD	OIL	USD/EUR
Mean	0.15%	-0.071%	0.19%	0.17%	0.11%
Std. Dev.	3.59%	2.12%	1.72%	4.06%	0.58%
Skewness	0.133	0.476	0.277	0.590	-0.547
Kurtosis	3.516	4.949	4.355	8.298	3.456
Jarque-Bera	0.590	8.244	3.752	51.561	2.458
Probability	0.744	0.016	0.153	0.000	0.292

#### 4. EFFICIENT FRONTIER AND RESULTS

We focus our study on the impact of including Bitcoin in a French portfolio composed of traditional assets, namely oil, currency (USD/EUR), CAC40, and gold, during three sub-periods: calm period, COVID-19 crisis, and the Ukrainian war. We try to answer this question: what are the consequences of adding Bitcoin to traditional portfolios on their returns and risks? To answer this question, we will build efficient frontiers for two types of portfolios, P1 (without Bitcoin) and P2 (with Bitcoin), and then we will try to compare them.

- P1 is composed of traditional assets: oil, currency, CAC40, and gold.
- P2 is composed of traditional assets: oil, currency, CAC40, and gold, plus Bitcoin.

According to Markowitz, a portfolio is said to be efficient if, for a given level of return, it has the lowest level of risk, and for a given level of risk, it has the highest return. The set of efficient portfolios constitutes the Markowitz efficient frontier.

In our application, we estimate the efficient frontiers of P1 and P2 in each sub-period to determine whether the inclusion of Bitcoin shifts the frontier upward, indicating an improvement in the risk–return trade-off. By comparing the two frontiers, we can directly evaluate whether Bitcoin provides diversification benefits for French investors during stable times and crisis periods.

This model is described as follows:

$$\text{Min } \sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \quad (1)$$

Under the constraints:

$$\begin{aligned} \sum_{i=1}^n w_i E(R_i) &= E(R_p) \\ \sum_{i=1}^n w_i &= 1, w_i \geq 0 \\ &\text{or} \\ \text{Max } E(R_p) &= \sum_{i=1}^n x_i E(R_i) \quad (2) \end{aligned}$$

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij}$$

$$\sum_{i=1}^n w_i = 1, w_i \geq 0$$

Where

n: Is the number of different assets making up the portfolio.

$\sigma_{ij}$  : Is the covariance between the returns of assets i and j.

$w_i$  : Is the weight of each asset in the portfolio.

$r_i$  : Is the average return of asset i.

$\sigma_p^2$  : The variance of the portfolio.

And R: Is the desired average return of the portfolio.

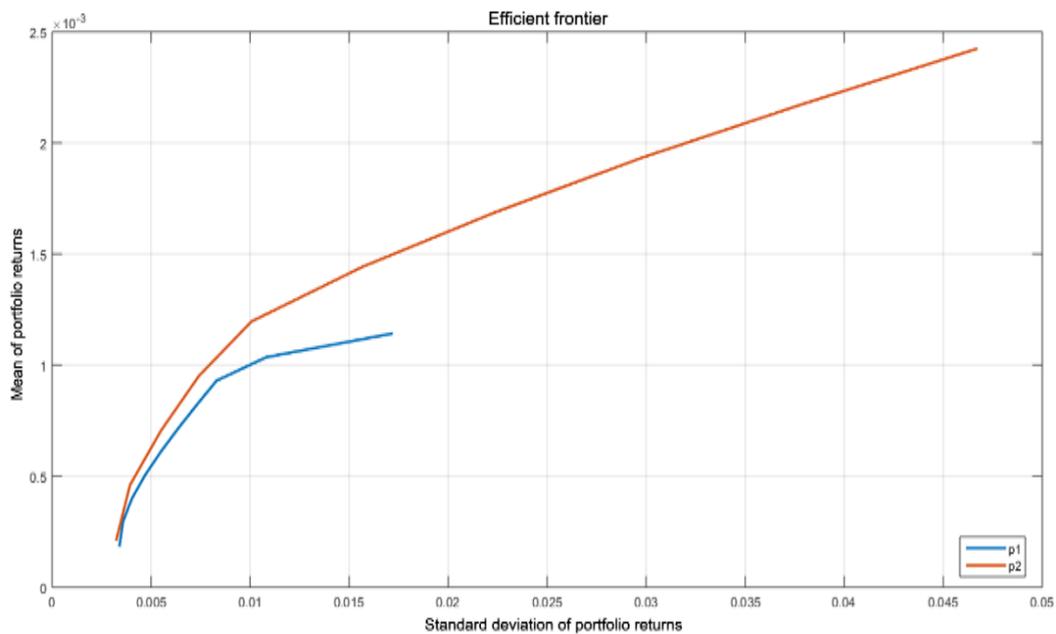


Figure 4. Efficient frontiers' comparison for two portfolios P1 and P2 during the stable period.

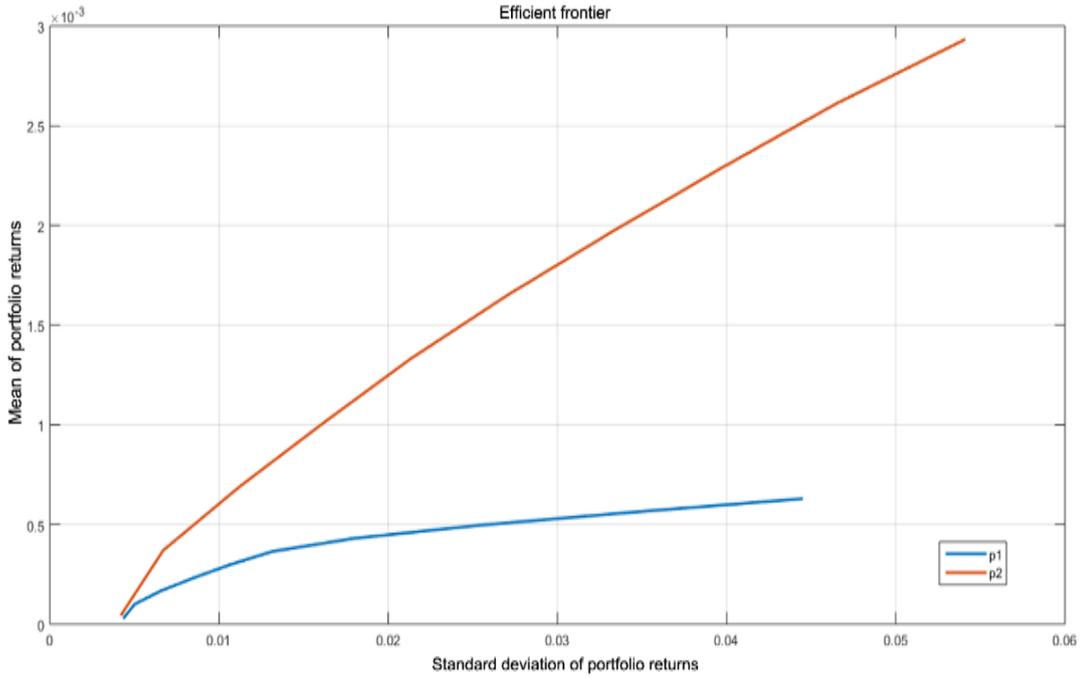


Figure 5. Efficient Frontiers' comparison for two portfolios, P1 and P2, during the COVID-19 crisis.

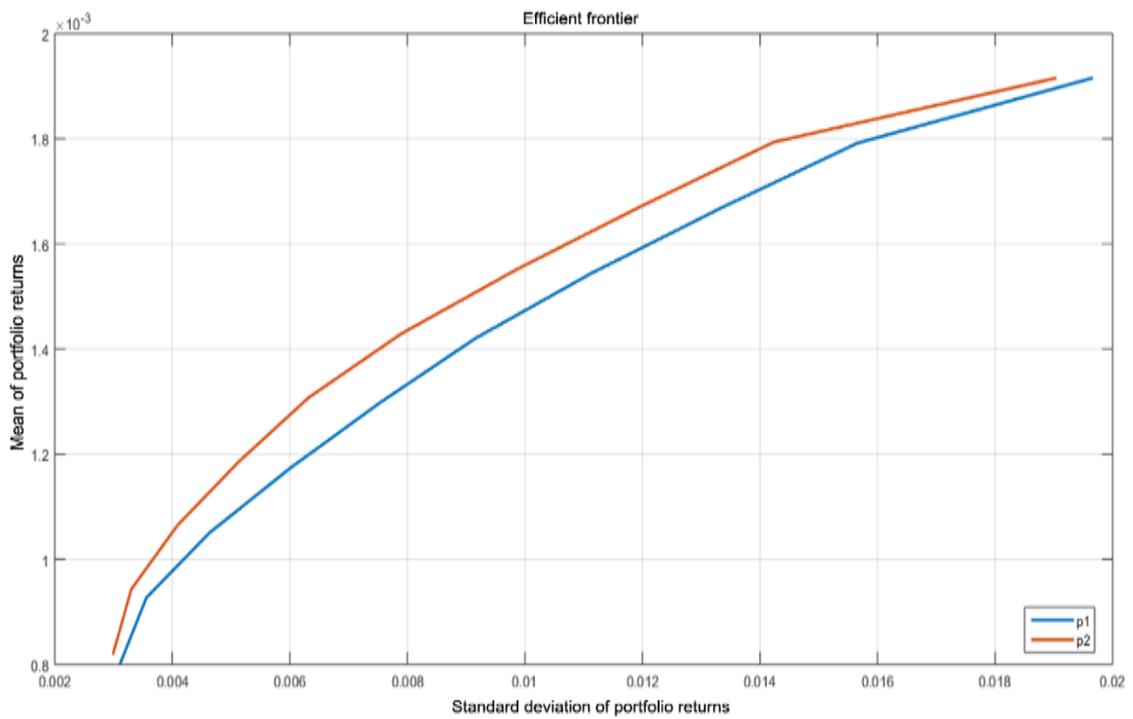


Figure 6. Efficient Frontiers' comparison for two portfolios, P1 and P2, during the Russian-Ukrainian conflict.

We construct the efficient frontiers of the daily returns for the two portfolios, P1 and P2, across the three sub-periods, and present them in Figures 4, 5, and 6. We now move on to comparing the frontiers of the two portfolios, P1 and P2, for each sub-period. Figures 4, 5, and 6 plot two efficient frontiers. The efficient brown frontier represents the efficient frontier that includes Bitcoin, which corresponds to portfolio P2. The blue efficient frontier describes the efficient frontier that does not include Bitcoin, which corresponds to portfolio P1. We notice, from Figures 4, 5, and 6, that the portfolio P2 with bitcoin moves the efficient frontier upwards and exceeds the other frontiers. Especially during the COVID-19 crisis, we notice that Bitcoin widens the frontier and largely exceeds the other frontiers. This means that the portfolio with Bitcoin has a higher return than the one without Bitcoin, for the same level of risk. For example, for daily data (Figure 4),

at a risk level of 1%, the portfolio without bitcoin (P1) has a return of 0.1%. The portfolio composed of bitcoin (P2) earns a return higher than 0.1%. We can conclude that adding Bitcoin to a portfolio increases its efficiency. The same interpretations apply to Figures 5 and 6. Our results are confirmed by the study of Youssef, et al. [21]. These results imply that French investors could significantly improve their risk-return trade-off by including Bitcoin in their portfolios, especially during crisis periods such as COVID-19. For policymakers, the findings highlight the growing role of cryptocurrencies as alternative assets, which may affect financial stability and regulation debates in the French market.

## 5. GENETIC ALGORITHMS, STOCHASTIC DOMINANCE

For the methodology, we start by building a base portfolio (P1) representative of the French market through the CAC 40 index. This portfolio is optimized via the genetic algorithms method and diversified with traditional assets such as gold, oil, and currencies. Then, we construct a second portfolio (P2) by adding Bitcoin to the basic portfolio P1, while maintaining the same genetic algorithm optimization process. Indeed, the genetic algorithm method makes it possible to optimize both types of portfolios by minimizing risk and maximizing return simultaneously. Then, we apply the stochastic dominance (SD) method to compare the financial performance of the optimal portfolios.

### 5.1. Genetic Algorithm (GA)

Genetic Algorithms (GA), introduced by Holland [32] are heuristic optimization methods inspired by natural selection. They operate through iterative processes of selection, crossover, and mutation to improve candidate solutions. Unlike traditional optimization techniques, GA performs a global and multidirectional search, making it well suited for complex financial problems such as portfolio selection. In this study, we apply GA to construct optimal portfolios (P1 without Bitcoin and P2 with Bitcoin) under different market conditions (calm period, COVID-19 crisis, and Ukrainian war). The algorithm generates successive populations of portfolios, where each candidate solution is represented by asset weights subject to the constraint that they are non-negative and sum to one. At each iteration, the fittest portfolios those with the best risk-return trade-offs are selected and combined to produce new portfolios. This process converges toward an efficient frontier for each portfolio type. Several studies have confirmed the efficiency of GA in financial applications, such as portfolio optimization [30, 33, 34]. By employing GA, we ensure a robust optimization process capable of handling the nonlinearities and non-normality observed in our dataset.

#### 5.1.1. Optimization by Genetic Algorithm

A genetic algorithm is an iterative optimization technique that operates on a fixed-size population of candidate solutions, known as chromosomes. Each chromosome represents a potential solution to the problem at hand and is composed of a series of elements called genes, which can take on multiple values. The algorithm simulates a process of competition among chromosomes, where those better adapted to the environment are more likely to be selected for reproduction. At each generation, a new population of the same size is generated, comprising chromosomes that exhibit higher fitness according to the objective function. Over successive generations, this evolutionary process leads the population to converge toward the optimum solution, guided by three key genetic operators: Selection, crossover, and mutation. The process begins with the random generation of an initial population, denoted as generation 'k'. The three genetic operations are then applied to this population to produce the next generation, 'k + 1'. The first operator, selection, identifies and retains the most relevant chromosomes that best optimize the fitness function. Crossover, the main genetic operator, works by pairing two parent chromosomes to produce two offspring through the combination of their genetic material. In the context of a portfolio optimization problem, the crossover corresponds to the exchange of asset weights between the parent portfolios. Various crossover techniques exist, such as single-point, two-point, multipoint, and uniform crossover. Finally, mutation introduces random modifications in the chromosomes, serving as a background operator that maintains genetic diversity within the population. It helps prevent premature convergence by allowing the algorithm to explore new areas of the search space. While crossover

creates significant changes by mixing genetic material from two parents, mutation introduces small, random perturbations that enhance the exploration capability of the algorithm.

### 5.1.2. The Mathematical Formulation of the Objective Function in a GA Application

This subsection presents the problem of multi-objective portfolio optimization and outlines how the Multi-Objective Genetic Algorithm (MOGA) can be employed to address it.

The evaluation process relies on an objective function, which is defined based on the specific characteristics of the problem and the optimization goals set by the genetic algorithm [35]. The main objective is to determine the optimal allocation of weights across the assets in the portfolio so as to minimize risk while maximizing expected returns.

The mathematical formulation used in this context is an extended version of the classical Markowitz Mean-Variance (MV) model and is expressed as follows:

$$\text{Min } \delta_p^2(w) = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{ij} \quad (3)$$

$$\text{Max } r_p(w) = \sum_{i=1}^n \mu_i w_i \quad (4)$$

Under the constraints:  $\sum_{i=1}^n w_i = 1$  et  $w_i \geq 0, i = 1, \dots, n$

$\delta_p^2$  : Portfolio variance.

$r_p$  : the return of the portfolio.

$\sigma_{ij}$  : the covariance between asset returns  $i$  and  $j$ .

$w_i$  : the weight of each asset in the portfolio.

$\mu_i$  : the average yield of assets  $i$ .

In general, single-objective optimization aims to find an optimal global solution; however, multi-objective optimization aims to find a set of Pareto optimal global solutions. Since then, there have been two contradictory objectives to optimize, namely the maximization of return and the minimization of risk. In this study, the problem of optimizing multi-objective portfolios is maintained by the following:

$$\text{Min } H(w) = \delta_p^2(w) - r_p(w) \quad (5)$$

Subject to:  $\sum_{i=1}^n w_i = 1$  et  $w_i \geq 0, i = 1, \dots, n$

### 5.1.3. Fitness Function

Fitness function is another important aspect of GA for solving optimization problems. In optimizing asset allocation, the fitness function must make a rational compromise between risk reduction and maximizing returns. Thus, it can be designed as follows:

$$\text{Min } H(w) = \delta_p^2(w) - r_p(w) \quad (6)$$

Such that:  $\sum_{i=1}^n w_i = 1$  et  $w_i \geq 0, i = 1, \dots, n$

The fitness function of each chromosome is the indicator that allows GA to make the selection.

## 5.2. Stochastic Dominance

The Mean-Variance (MV) framework offers useful insights, but it relies on restrictive assumptions such as normally distributed returns and variance as a symmetric risk measure. Since our results in Table 1 reject normality for most assets, MV findings may be biased. To address this limitation, we adopt the Stochastic Dominance (SD) approach. Stochastic Dominance, originally developed by Hadar and Russell [36] provides a general framework for portfolio comparison without assuming any particular return distribution. Unlike the MV approach, which only considers the first two moments (mean and variance), SD accounts for the entire distribution of returns and is therefore consistent with a broad class of investor utility functions. In our study, we apply the Davidson and Duclos [37] test (DD test) to examine whether the portfolio without Bitcoin (P1) dominates the portfolio including Bitcoin (P2), or vice versa, across the three sub-periods. This allows us to directly assess investors' preferences and determine

whether Bitcoin provides diversification benefits under different market conditions. Previous research has applied the SD approach in portfolio studies. For instance, Abid, et al. [38] compared national and international portfolios, Meyer, et al. [39] examined diversification gains from foreign assets, and Thi-Hong-Van, et al. [40] investigated the role of gold in French portfolios. By extending this line of research, we provide new evidence on Bitcoin's diversification role in the French market

Let  $X$  and  $Y$  be two real random variables, with their cumulative distribution functions (CDFs)  $F_x$  and  $F_y$  and their probability density functions (PDFs)  $f_x$  and  $f_y$  respectively, defined on the common support  $[n, m]$  with  $n < m$ . We define:

$$H_0 = h \quad \text{and} \quad H_j(a) = \int_n^a H_{j-1}(t) dt \quad (7)$$

For  $h = f_x, f_y$ ,  $H = F_x, F_y$  and  $j=1,2,3$ .

The most widely used stochastic dominance (SD) rules are: first-order stochastic dominance (FSD), second-order stochastic dominance (SSD), and third-order stochastic dominance (TSD). Under FSD, all investors are non-satisfied (i.e., they prefer higher return to less). Under SSD, investors are non-satisfied and risk-averse. Under TSD, investors are non-satisfied, risk-averse, and possess decreasing absolute risk aversion (DARA).

We define the fact that  $X$  is stochastically dominated by  $Y$  at order 1 noted,  $X <^{st1} Y$ , as follows:  $X <^{st1} Y \leftrightarrow F_{x1} \geq F_{y1} \leftrightarrow F_{x1}(a) \geq F_{y1}(a)$  for all possible returns  $a \in [n, m]$  with a strict inequality for some  $a$ . Stochastic dominance at order 1 will only be valid if the cumulative distribution functions of the alternatives do not intersect. We can say that if  $X$  is stochastically dominated by  $Y$  at order 1, if there is an arbitrage opportunity between  $X$  and  $Y$  so that investors will increase their expected wealth, as well as their expected utility, if their investments are spent from  $X$  to  $Y$ . On the other hand, if FSD does not exist between  $X$  and  $Y$ , we can say that the markets are efficient, and the investors are rational.

We define the fact that  $X$  is stochastically dominated by  $Y$  at order 2 noted,  $X <^{st2} Y$ , as follows:  $X <^{st2} Y \leftrightarrow F_{x2} \geq F_{y2} \leftrightarrow F_{x2}(a) \geq F_{y2}(a)$  for all possible returns  $a \in [n, m]$  with a strict inequality for some  $a$ . In this case, the two-distribution function of  $X$  and  $Y$  intersect. Indeed, for any possible value of  $a$ , the area under  $F_{x2}$  is larger than that under  $F_{y2}$ .

We define the fact that  $X$  is stochastically dominated by  $Y$  at order 3 noted,  $X <^{st3} Y$ , as follows:  $X <^{st3} Y \leftrightarrow F_{x3} \geq F_{y3} \leftrightarrow F_{x3}(a) \geq F_{y3}(a)$  for all possible returns  $a \in [n, m]$  with a strict inequality for some  $a$ .

We note that there is a hierarchical relationship in stochastic dominance. FSD implies SSD, which in turn implies TSD. However, the opposite is not true. The existence of SSD does not imply the existence of FSD. Similarly, the existence of TSD does not imply the existence of SSD or FSD.

There are two main classes of Stochastic Dominance tests: one is the minimum/maximum statistic [41, 42] and the other is based on distribution values calculated over a set of grid points (DD) [37]. The DD test is one of the most powerful tests; we use it in our analysis.

For two assets  $X$  et  $Y$  with their cumulative distribution functions  $F_x$  and  $F_y$ , respectively, and for a grid of pre-selected points  $a_1, a_2, \dots, a_n$ , the order- $j$  DD statistic,  $T_j(a)$  ( $j=1,2$  et  $3$ ), is:

$$\hat{T}_j(a) = \frac{\hat{F}_{xj}(a) - \hat{F}_{yj}(a)}{\sqrt{\hat{V}_j(a)}} \quad (8)$$

Where:

$$\hat{V}_j(a) = \hat{V}_x^j(a) + \hat{V}_y^j(a) - 2\hat{V}_{x,y}^j(a),$$

$$\hat{H}_j(a) = \frac{1}{N(j-1)!} \sum_{i=1}^N (a - h_i)_+^{j-1},$$

$$\hat{V}_H^j(a) = \frac{1}{N} \left[ \frac{1}{N(j-1)!^2} \sum_{i=1}^N (a - h_i)_+^{2(j-1)} - \hat{H}_j(a)^2 \right], \quad H = F_x, F_y \quad \text{and} \quad h = x, y,$$

$$\hat{V}_{x,y}^j(a) = \frac{1}{N} \left[ \frac{1}{N(j-1)!^2} \sum_{i=1}^N (a - x_i)_+^{j-1} (a - y_i)_+^{j-1} - \hat{F}_{xj}(a) \hat{F}_{yj}(a) \right].$$

In which  $F_x$  and  $F_y$  are defined in (1) and  $(a)_+ = \max\{a, 0\}$ .

It is empirically impossible to test the null hypothesis for the total support of the distributions. Thus, we test the null hypothesis for a preconceived finite number of values. Specifically, the following hypotheses are tested:

H0:  $F_{xj}(a_i) = F_{yj}(a_i)$  for all  $a_i, i=1,2,\dots,k$ .

HA:  $F_{xj}(a_i) \neq F_{yj}(a_i)$  for some  $a_i$ .

HA1:  $F_{xj}(a_i) \leq F_{yj}(a_i)$  for all  $a_i, F_{xj}(a_i) < F_{yj}(a_i)$  for some  $a_i$ .

HA2:  $F_{xj}(a_i) \geq F_{yj}(a_i)$  for all  $a_i, F_{xj}(a_i) > F_{yj}(a_i)$  for some  $a_i$ .

To control the probability of rejecting the null hypothesis, following Bishop, et al. [43] (BFT), we use the Student's Maximum Modulus (SMM) distribution with  $m$  and infinite degrees of freedom, noted  $M_{\infty}^k$ . The percentile  $1-\alpha$  of  $M_{\infty}^k$  noted  $M_{\infty,\alpha}^k$ , is tabulated by Stoline and Ury [44] and the following decision rules are adopted:

If  $|T_s(a_i)| < M_{\infty,\alpha}^k$  for  $i=1, \dots, k$ , 'accept H0'.

If  $T_s(a_i) < M_{\infty,\alpha}^k$  for all  $i$  et  $-T_s(a_i) > M_{\infty,\alpha}^k$  for some  $i$ , 'accept HA1'.

If  $-T_s(a_i) < M_{\infty,\alpha}^k$  for all  $i$  et  $T_s(a_i) > M_{\infty,\alpha}^k$  for some  $i$ , 'accept HA2'.

If  $T_s(a_i) > M_{\infty,\alpha}^k$  for all  $i$  et  $-T_s(a_i) > M_{\infty,\alpha}^k$  for some  $i$ , 'accept HA'.

The DD test compares distributions to a finite number of grid points  $\{a_k, k = 1, 2, \dots, k\}$ . The choice of these points is guided by the results of various studies. Tse and Zhang [45] demonstrated that the appropriate choice of  $k$  for reasonably large samples is between 6 and 15. In this case, too few grids will lack information on the distributions between any two consecutive grids [41]. It is important to note that under the above assumptions, the general alternative hypothesis (HA) is excluded from both HA1 and HA2. This means that accepting either HA1 or HA2 does not imply that HA is accepted. Accepting either the null hypothesis (H0) or the general alternative (HA) indicates the absence of stochastic dominance (SD) relationships and the lack of arbitrage opportunities between the two portfolios. In such cases, neither portfolio is strictly preferred over the other. However, if HA1 or HA2 is accepted at the first order, it implies that portfolio P1 stochastically dominates portfolio P2 at the first-order level. This situation reflects the existence of an arbitrage opportunity, where investors can increase their expected wealth by shifting from the dominated portfolio to the dominant one. Conversely, if HA1 or HA2 is accepted at the second or third order, then P1 is said to stochastically dominate P2 at that respective order. In such cases, no arbitrage opportunity exists; instead, moving from one portfolio to the other may enhance the investor's expected utility, but not their expected wealth [46].

### 5.3. Empirical Results

Tables 4 and 5 present the optimal asset weights for portfolios P1 and P2, obtained using the genetic algorithm method. According to Table 4, the portfolio P1 is based exclusively on traditional assets, represented by the CAC 40 (French market), oil, gold, and currencies. Optimization assigns a majority weighting to the CAC 40 (57.54%), followed by oil (28.41%), gold (9.13%), and currencies (4.83%). The portfolio has a modest daily expected return of 0.02857% and very low volatility of 0.01323%. The P2 portfolio (Table 5) is obtained by integrating Bitcoin into the basic P1 portfolio, with a balanced weighting of 49.61% for Bitcoin and 50.49% for P1. This new composition generates a daily expected return of 0.14%, well above that of the P1 portfolio. However, this improvement is accompanied by a marked increase in risk, with a standard deviation of 0.0656%, almost five times that of the initial portfolio. The optimal allocation assigns significant weight to Bitcoin, highlighting its potential contribution to the portfolio's overall performance. Nevertheless, this increased performance comes at the cost of much higher volatility. These results reflect the typical profile of Bitcoin in a diversification strategy: an asset with a high expected return, but whose inclusion considerably increases overall risk. The analysis, therefore, highlights the risk-return trade-off that investors must accept when choosing to add a digital asset such as Bitcoin to a traditional diversified portfolio.

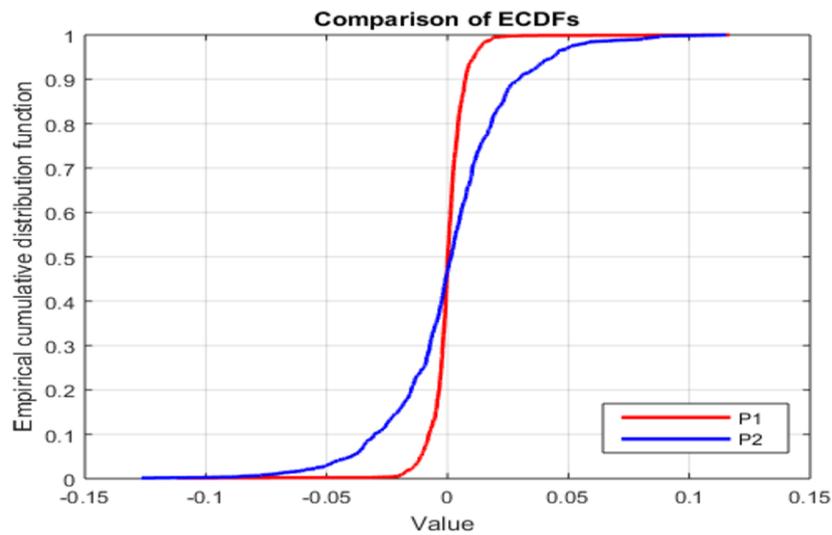
**Table 4.** The optimal weights of the different assets in portfolio P1.

Portfolio	Portfolio 1
CAC40	57.54%
OIL	28.41%
GOLD	9.13%
DEWISE	4.83%
Portfolio return	0.028%
Variance	0.013%

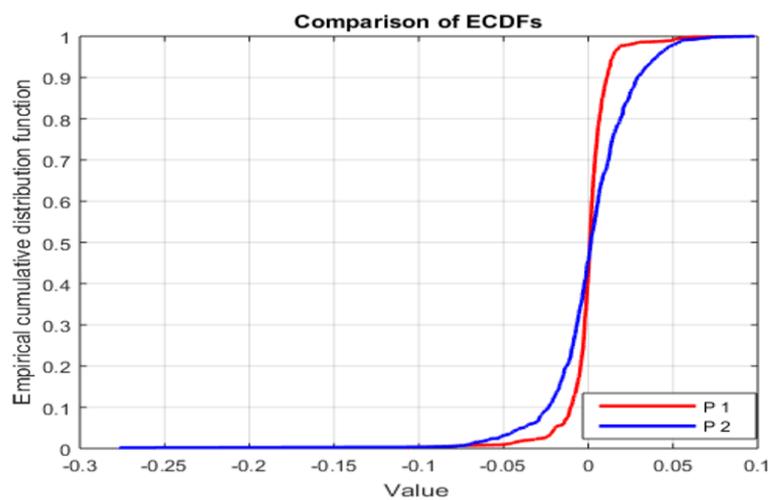
**Table 5.** The optimal weights of the different assets in portfolio P2.

Portfolio	Portfolio 2
Portfolio P1	50.49%
BITCOIN	49.61%
Portfolio return	0.001
Variance	0.065%

From Figures 7, 8, and 9, we see that the empirical distribution functions of the two optimal portfolios intersect for all three sub-periods, implying that it is very likely that there is no first-order standard deviation between the portfolios. This means that there is no arbitrage opportunity between the two portfolios.



**Figure 7.** Plot of the cumulative distribution functions of the two optimal portfolios P1 and P2 before the crisis.



**Figure 8.** Plot of the cumulative distribution functions of the two optimal portfolios P1 and P2 during the COVID-19 crisis.

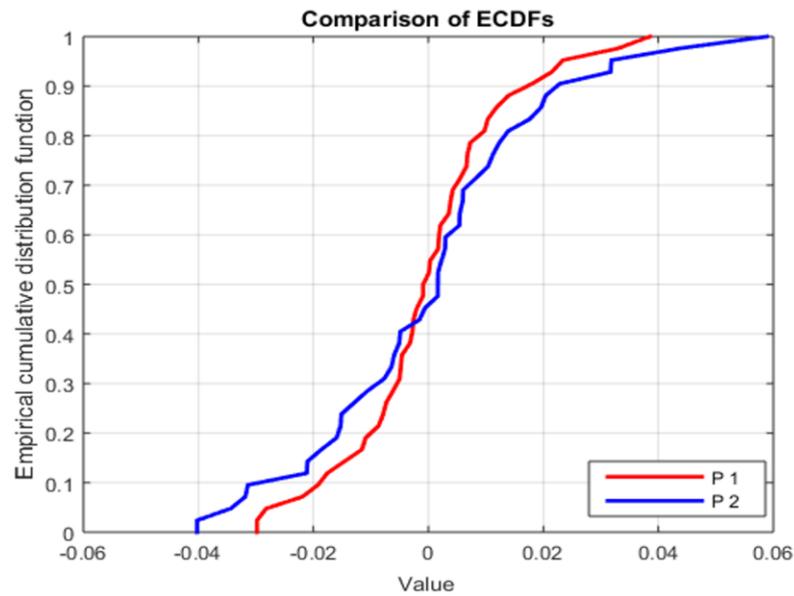


Figure 9. Plot of the cumulative distribution functions of the two optimal portfolios, P1 and P2, during the Russian-Ukrainian conflict.

To better understand the results of the DD test, we present the corresponding graphs in Figures 10, 11, and 12. Figures 10 and 11 show that the T2 and T3 statistics are significantly negative, which suggests that the null hypothesis of no stochastic dominance can be rejected, implying that P1 dominates P2 according to the second and third orders. On the other hand, Figure 12 reveals an absence of SD between the two portfolios. This result supports the idea that investors would not be interested in including Bitcoin in their portfolios during periods of crisis.

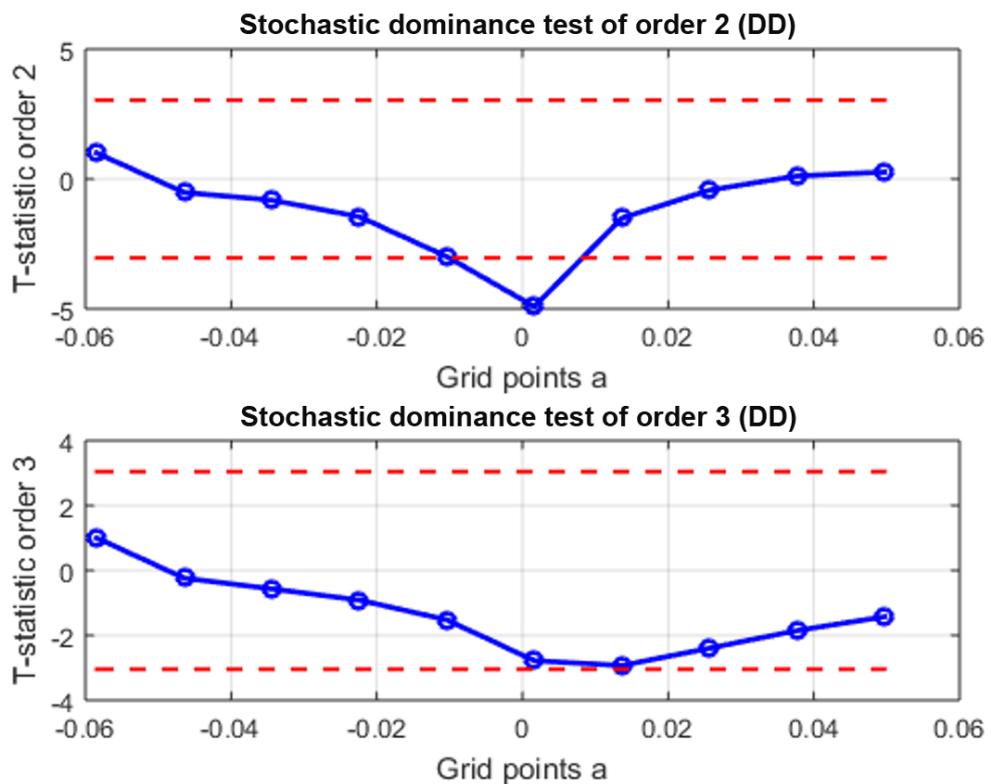


Figure 10. Plot DD statistics before the crisis.

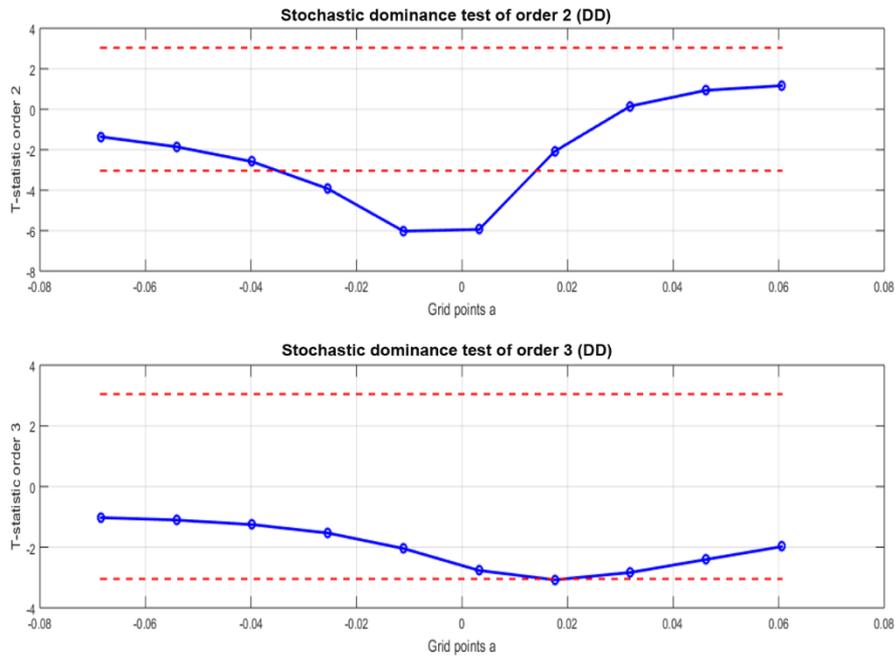


Figure 11. Plot DD Statistics during the COVID-19 crisis.

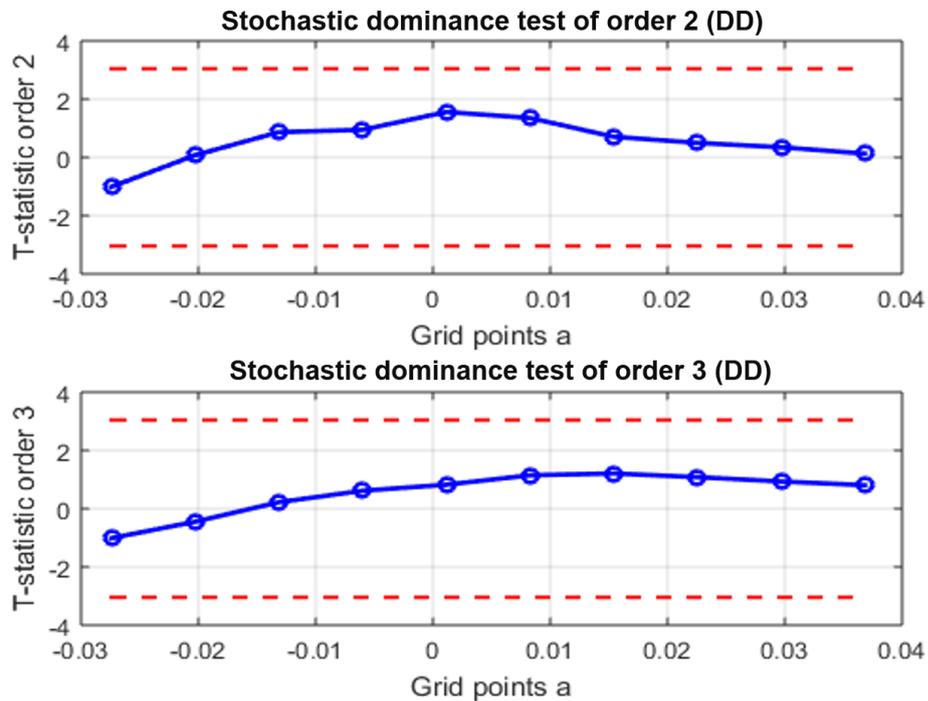


Figure 12. Plot DD during the Russian-Ukrainian conflict.

Table 6 reveals that the portfolio without Bitcoin dominates the portfolio with Bitcoin according to second and third-order stochastic dominance, before and during the COVID-19 crisis. However, during the Russo-Ukrainian crisis, neither portfolio dominated the other. This indicates that, from a risk-averse investor's perspective, Bitcoin did not provide additional diversification benefits and cannot be considered a safe haven asset in these periods. These results are confirmed by Wen, et al. [27] who reject the safe haven property of Bitcoin during the COVID-19 crisis.

These results contradict those obtained from Markowitz's efficient frontiers, which show that the frontier of the portfolio with Bitcoin exceeds that of the portfolio without Bitcoin. This contradiction is explained by the fact that stochastic dominance analyzes the full distribution of returns, including extreme risks, while Markowitz only considers the mean and variance, which can mask certain aspects of risk. In practical terms, this suggests that

although Bitcoin can enhance portfolio performance on average, it also exposes investors to higher downside risks during crises. For regulators and policymakers, this highlights the importance of monitoring cryptocurrency markets, as their integration into traditional portfolios may increase systemic vulnerability rather than provide stability.

**Table 6.** Stochastic dominance test between P1 and P2.

Periods	Stable period	Covid19 crisis	Ukrainian war
	P1	P1	P1
P2 (With Bitcoin)	< <sup>2,3</sup>	< <sup>2,3</sup>	ND

**Note:** <<sup>2,3</sup> means that the portfolio with bitcoin stochastically dominates the portfolio without bitcoin. <sup>2,3</sup> mean SSD and TSD. ND means that there is no SD. The significance level of all our SD tests is the conventional one, which is 5%.

## 6. CONCLUSION

This study provides comprehensive evidence on the diversification role of Bitcoin in a French investment portfolio across three distinct periods: a stable pre-crisis period, the COVID-19 pandemic, and the Russia–Ukraine conflict. By employing both the mean-variance (MV) framework and stochastic dominance (SD) analysis, the results reveal that Bitcoin's contribution to portfolio performance is highly dependent on the methodological approach and market conditions. While the MV results suggest that Bitcoin consistently enhances portfolio efficiency by improving the risk-return trade-off, the SD analysis, which is more robust in the presence of non-normal return distributions, shows that Bitcoin does not act as a safe haven, particularly during turbulent periods. In fact, during the stable period and the COVID-19 crisis, traditional portfolios without Bitcoin dominate, whereas in the Russia–Ukraine war period, no clear dominance is observed, highlighting Bitcoin's conditional and unstable diversification role. From a practical perspective, the results imply that Bitcoin can be considered a speculative asset in stable markets, where it may improve returns. However, in times of turbulence, its lack of safe-haven properties means that investors should be cautious in allocating significant weights to it. For asset managers, this highlights the need to adjust portfolio strategies dynamically depending on market conditions. For policymakers, the findings emphasize the importance of monitoring cryptocurrencies' growing integration into traditional markets, given their potential to amplify risks during crises.

Future research could extend this analysis by including other digital assets or by adopting dynamic allocation frameworks to better capture the evolving role of cryptocurrencies in portfolio diversification under uncertainty.

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