Asian Journal of Empirical Research 2(6): 219-227

Asian Journal of Empirical Research

journal homepage: http://aessweb.com/journal-detail.php?id=5004

A SEASONAL ARIMA MODEL FOR DAILY NIGERIAN NAIRA-US DOLLAR EXCHANGE RATES

Ette Harrison Etuk[1]

ABSTRACT

Time series analysis of daily Nigerian Naira-US Dollar Exchange Rates (DNDER) data is conducted. The time plot reveals a positive trend. Seasonality of order 7 is observed; troughs tend to appear on Mondays and peaks on Fridays. Seasonal differencing once produced a series SDDNDER with a slightly overall negative trend. A non-seasonal differencing of SDDNDER yielded a series DSDDNDER with no trend but with a correlogram revealing seasonality of order 7. Moreover, the correlogram reveals the involvement of a seasonal moving average component of order 1 and a nonseasonal autoregressive component of order 2. An adequate multiplicative seasonal autoregressive integrated moving average (ARIMA) model, (2, 1, 0)x(0, 1, 1)$_7$, is therefore fitted to the series.

Key Words: Naira-Dollar Exchange Rate, Seasonal Time Series, ARIMA model, Nigeria

INTRODUCTION

A time series is defined as a set of data collected sequentially in time. It has the property that neighboring values are correlated. This tendency is called *autocorrelation*. A time series is said to be stationary if it has a constant mean and variance. Moreover the autocorrelation is a function of the lag separating the correlated values and called the *autocorrelation function* (ACF).

A stationary time series $\{X_t\}$ is said to follow an *autoregressive moving average model of orders p and q* (designated ARMA(p, q)) if it satisfies the following difference equation

$$X_t - \alpha_1 X_{t-1} - \alpha_2 X_{t-2} - \dots - \alpha_p X_{t-p} = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_t \qquad (1)$$

$$A(L)X_t = B(L)\varepsilon_t \qquad (2)$$

[1] Department of Mathematics/Computer Science, Rivers State University of Science and Technology, Nigeria

E-mail: ettetuk@yahoo.com

where $\{\varepsilon_t\}$is a sequence of uncorrelated random variables with zero mean and constant variance, called *a white noise process,* and the α_i's and β_j's constants; $A(L) = 1- \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p$ and $B(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$ and L the backward shift operator defined by $L^k X_t = X_{t-k}$.

If p = 0, the model (1) becomes a *moving average model of order q* (designated MA (q)). If, however, q = 0 it becomes an *autoregressive process of order* p (designated AR (p)). An AR (p) model may be defined as a model whereby a current value of the time series X_t depends on the immediate past p values: X_{t-1}, X_{t-2}, … , X_{t-p}. On the other hand an MA(q) model is such that the current value X_t is a linear combination of immediate past values of the white noise process: ε_{t-1}, ε_{t-2}, … ,ε_{t-q}. Apart from stationarity, invertibility is another important requirement for a time series. It refers to the property whereby the covariance structure of the series is unique (Priestley, 1981). Moreover it allows for meaningful association of current events with past history of the series (Box and Jenkins, 1976).

An AR (p) model may be more specifically written as $X_t + \alpha_{p1} X_{t-1} + \alpha_{p2} X_{t-2} + \dots + \alpha_{pp} X_{t-p} = \varepsilon_t$. Then the sequence of the last coefficients $\{\alpha_{ii}\}$ is called the *partial autocorrelation* function (PACF) of {Xt}. The ACF of an MA (q) model cuts off after lag q whereas that of an AR(p) model is a combination of sinusoidals dying off slowly. On the other hand the PACF of an MA(q) model dies off slowly whereas that of an AR (p) model cuts off after lag p. AR and MA models are known to exhibit some duality relationships. These include:

1. A finite order AR model is equivalent to an infinite order MA model.
2. A finite order MA model is equivalent to an infinite order AR model.
3. The ACF of an AR model exhibits the same behaviour as the PACF of an MA model.
4. The PACF of an AR model exhibits the same behaviour as the ACF of an MA model.
5. An AR model is always invertible but is stationary if A(L) = 0 has zeros outside the unit circle.
6. An MA model is always stationary but is invertible if B(L) = 0 has zeros outside the unit circle.

Parametric parsimony consideration in model building entails preference for the mixed ARMA fit to either the pure AR or the pure MA fit. Stationarity and invertibility conditions for model (1) or (2) are that the equations A(L) = 0 and B(L) = 0 should have roots outside the unit circle respectively. Often, in practice, a time series is non-stationary. Box and Jenkins, (1976) proposed that differencing of appropriate order could render a non-stationary series {Xt} stationary. Let degree of differencing necessary for stationarity be d. Such a series {Xt} may be modelled as

$$A(L)\nabla^d X_t = B(L)\varepsilon_t \qquad (3)$$

where∇ = 1 - L and in which case $A(L)\nabla^d = 0$ shall have unit roots d times. Then differencing to degree d renders the series stationary. The model (3) is said to be an *autoregressive integrated moving average model of orders p, d and q* and designated ARIMA (p, d, q).

SEASONAL ARIMA MODELS

A time series is said to be seasonal of order d if there exists a tendency for the series to exhibit periodic behaviour after every time interval *d*. Traditional time series methods involve the identification, unscrambling and estimation of the traditional components: secular trend, seasonal component, cyclical component and the irregular movement. For forecasting purpose, they are reintegrated. Such techniques could be quite misleading. The time series $\{X_t\}$ is said to follow a multiplicative $(p, d, q)x(P, D, Q)_s$ *seasonal ARIMA model* if

$$A(L)\Phi(L^s)\nabla^d\nabla^D_sX_t = B(L)\Theta(B^s)\varepsilon_t \qquad (4)$$

whereΦ and Θ are polynomials of order P and Q respectively. That is,

$$\Phi(L^s) = 1 + \phi_1L^s + \ldots + \phi_PL^{sP} \qquad (5)$$

$$\Theta(L^s) = 1 + \theta_1L^s + \ldots + \theta_qL^{sQ} \qquad (6)$$

where the ϕ_i and the θ_j are constants such that the zeros of the equations (5) and (6) are all outside the unit circle for stationarity or invertibility respectively. Equation (5) represents the autoregressive operator whereas (6) represents the moving average operator. Existence of a seasonal nature is often evident from the time plot. Moreover for a seasonal series the ACF or correlogram exhibits a spike at the seasonal lag. Box and Jenkins, (1976) and Madsen, (2008) have written extensively on such models. Knowledge of the theoretical properties of the models provides basis for their identification and estimation. The purpose of this paper is to fit a seasonal ARIMA model to the daily Naira-Dollar exchange rates (DNDER). Etuk, (2012) fitted a $(0, 1, 1)x(1, 1, 1)_{12}$ model to the monthly exchange rates and on its basis obtained 2012 forecasts.

MATERIALS AND METHODS

The data for this work are eighty two daily Nigerian Naira-US Dollar exchange rates from 26 April to 16 July, 2012 obtainable from the daily publications of the Nation newspaper.

Determination of the orders d, D, P, q and Q:

Seasonal differencing is necessary to remove the seasonal trend. If there is secular trend non-seasonal differencing will be necessary. To avoid undue model complexity it has been advised that orders of differencing d and D should add up to at most 2 (i.e. d + D < 3). If the ACF of the differenced series has a positive spike at the seasonal lag then a seasonal AR component is suggestive; if it has a negative spike then a seasonal MA term is suggestive. As already mentioned above, an AR (p) model has a PACF that truncates at lag p and an MA (q) has an ACF that truncates at lag q. In practice $\pm 2/\sqrt{n}$ where n is the sample size are the non-significance limits for both functions.

Model Estimation

The involvement of the white noise process in an ARIMA model entails a nonlinear iterative process in the estimation of the parameters. An optimization criterion like least error sum of squares, maximum likelihood or maximum entropy is used. An initial estimate is usually used. Each iteration is expected to be an improvement of the last one until the estimate converges to an optimal one. However, for pure AR and pure MA models linear optimization techniques exist (Box and Jenkins, 1976; Oyetunji, 1985). There are attempts to adopt linear methods to estimate ARMA models (Etuk, 1987, 1998).

Diagnostic Checking

The model that is fitted to the data should be tested for goodness-of-fit. We shall do some analysis of the residuals of the model. If the model is correct, the residuals would be uncorrelated and would follow a normal distribution with mean zero and constant variance. The autocorrelations of the residuals should not be significantly different from zero.

RESULTS AND DISCUSSION

The analysis of the series shows a seasonal pattern, with troughs tending to appear on Mondays and peaks on Fridays. The time plot of the original series DNDER in Figure-1 shows an overall positive trend. Seasonal (7-day) differencing of the series produces a series SDDNDER with a slightly positive trend (Figure-2). Non-seasonal differencing of SDDNDER yields a series DSDDNDER with no trend. Its ACF in Figure-4 has a negative spike at lag 7 revealing a seasonality of lag 7 and a seasonal MA component of order one to the model. The PACF shows a spike at lag 2 suggesting a seasonal AR component of order two. We therefore propose the seasonal model

$$DSDDNDER_t = \alpha_1 DSDDNDER_{t-1} + \alpha_2 DSDDNDER_{t-2} + \beta_{12}\varepsilon_{t-12} + \varepsilon_t \qquad (7)$$

The estimation of the model is summarized in Table-1. The fitted model is given by

$$\underset{}{DSDDNDER_t} + \underset{(\pm 0.1141)}{0.2331 DSDDNDER_{t-1}} + \underset{(\pm 0.1152)}{0.3352 DSDDNDER_{t-2}} + \underset{(\pm 0.0409)}{0.9018\varepsilon_{t-7}} \qquad (8)$$

The fitted model is a (2, 1, 0)x(0, 1, 1)$_7$ model. The estimation involved 8 iterations.

All coefficients are significantly different from zero, each being larger than twice its standard error. There is close agreement between the actual and the fitted models as evident from Figure-5. The correlogram of the residuals in Figure-6 depicts the adequacy of the model. All the residual autocorrelations are not significantly different from zero. Moreover from the histogram of the residuals shown in Figure-7, they are normally distributed with zero mean indicating model adequacy.

CONCLUSION

A $(2, 1, 0)x(0, 1, 1)_7$ model has been fitted to the series NDER. It is found that the series DSDDNDER follows an ARMA (2, 7) model given by equation (8). By a variety of approaches the model has been found adequate for DNDER.

REFERENCES

Box, G. E. P. and Jenkins, G. M. (1976) Time series analysis, forecasting and control. *Holden-Day*, San Francisco.

Etuk, E. H. (1987) On the selection of autoregressive moving average models. PhD. Thesis, Department of Statistics, University of Ibadan, Nigeria.

Etuk, E. H. (1998) An autoregressive integrated moving average (ARIMA) simulation model: a case study, Discovery and Innovation, Vol.10, pp. 23–26.

Etuk, E. H. (2012) Forecasting Nigerian Naira-US Dollar exchange rates by a seasonal ARIMA model, American Journal of Scientific Research, Vol. 59, pp. 71–78.

Madsen, H. (2008) Time series analysis. *Chapman & Hall/CRC*, London.

Oyetunji, O. B. (1985) Inverse autocorrelations and moving average time series modelling, Journal of Official Statistics, Vol.1, pp. 315–322.

Priestley, M. B. (1981) Spectral analysis and time series. *Academic Press*, London.

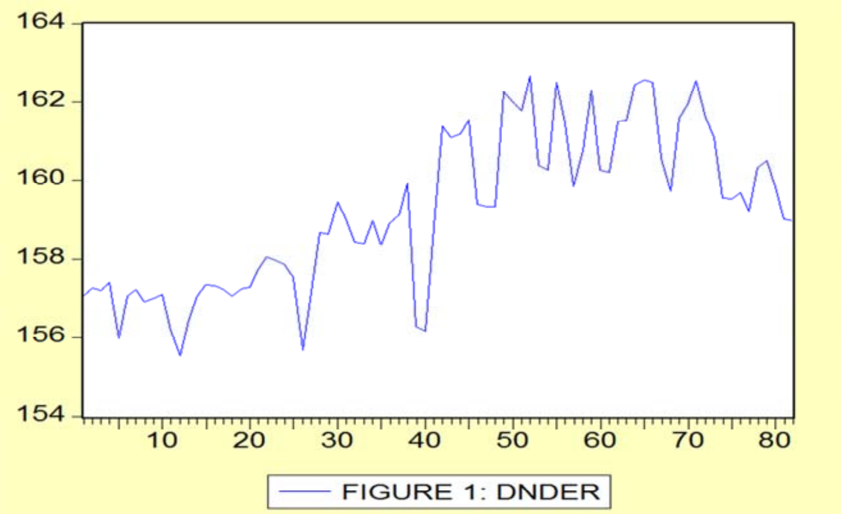


FIGURE 1: DNDER

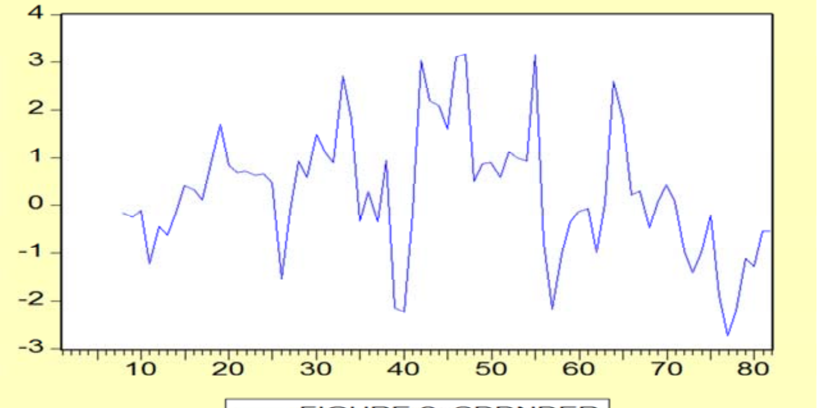


FIGURE 2: SDDNDER

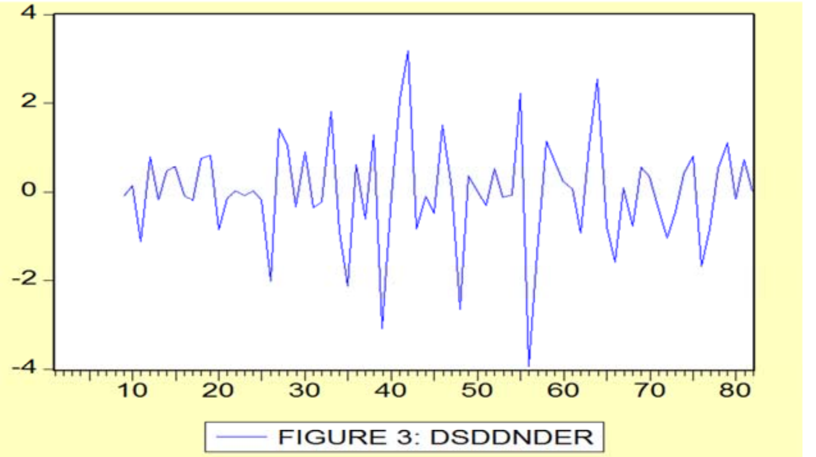


FIGURE 3: DSDDNDER

Table-1. Model Estimation

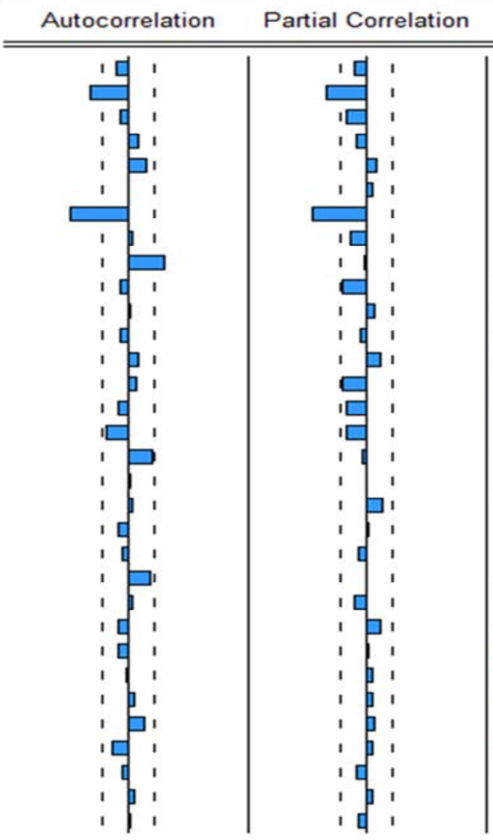
Dependent Variable: DSDDNDER
Method: Least Squares
Date: 08/15/12 Time: 17:26
Sample(adjusted): 11 82
Included observations: 72 after adjusting endpoints
Convergence achieved after 8 iterations
Backcast: 4 10

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	-0.233100	0.114092	-2.043096	0.0449
AR(2)	-0.335249	0.115150	-2.911399	0.0048
MA(7)	-0.901807	0.040903	-22.04740	0.0000

R-squared	0.518012	Mean dependent var	-0.006000
Adjusted R-squared	0.504041	S.D. dependent var	1.205398
S.E. of regression	0.848894	Akaike info criterion	2.551008
Sum squared resid	49.72283	Schwarz criterion	2.645870
Log likelihood	-88.83630	F-statistic	37.07850
Durbin-Watson stat	2.082386	Prob(F-statistic)	0.000000

Inverted AR Roots	-.12+.57i	-.12 -.57i		
Inverted MA Roots	.99	.61+.77i	.61 -.77i	-.22 -.96i
	-.22+.96i	-.89 -.43i	-.89+.43i	

Figure-4. Correlogram of Dsddnder



	AC	PAC	Q-Stat	Prob
1	-0.100	-0.100	0.7675	0.381
2	-0.333	-0.347	9.4518	0.009
3	-0.074	-0.176	9.8850	0.020
4	0.086	-0.082	10.481	0.033
5	0.157	0.087	12.479	0.029
6	0.004	0.054	12.480	0.052
7	-0.501	-0.473	33.509	0.000
8	0.033	-0.137	33.602	0.000
9	0.308	-0.017	41.811	0.000
10	-0.074	-0.217	42.288	0.000
11	0.012	0.081	42.300	0.000
12	-0.070	-0.047	42.749	0.000
13	0.084	0.126	43.400	0.000
14	0.070	-0.220	43.853	0.000
15	-0.098	-0.177	44.763	0.000
16	-0.187	-0.178	48.140	0.000
17	0.199	-0.025	52.050	0.000
18	0.021	-0.007	52.095	0.000
19	0.034	0.133	52.213	0.000
20	-0.089	0.022	53.030	0.000
21	-0.042	-0.080	53.219	0.000
22	0.183	0.004	56.835	0.000
23	0.027	-0.114	56.914	0.000
24	-0.089	0.123	57.809	0.000
25	-0.091	0.015	58.757	0.000
26	-0.012	0.042	58.773	0.000
27	0.046	0.057	59.022	0.000
28	0.136	0.064	61.282	0.000
29	-0.143	0.054	63.845	0.000
30	-0.048	-0.097	64.145	0.000
31	0.051	0.057	64.479	0.000
32	0.010	-0.071	64.492	0.001

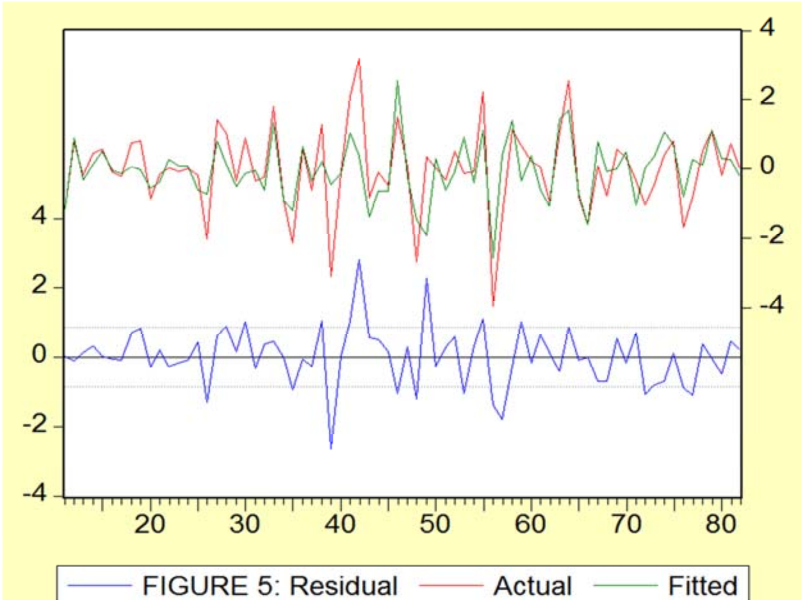


Figure-6. Correlogram of Residuals

Lag	AC	PAC	Q-Stat	Prob
1	-0.042	-0.042	0.1355	
2	-0.037	-0.039	0.2401	
3	-0.125	-0.129	1.4460	
4	-0.013	-0.026	1.4582	0.227
5	0.009	-0.003	1.4654	0.481
6	-0.079	-0.098	1.9636	0.580
7	-0.005	-0.020	1.9660	0.742
8	-0.108	-0.121	2.9458	0.708
9	0.117	0.084	4.1088	0.662
10	0.054	0.048	4.3582	0.738
11	0.058	0.044	4.6489	0.794
12	-0.005	0.021	4.6508	0.864
13	0.040	0.065	4.7936	0.905
14	-0.013	-0.008	4.8081	0.940
15	-0.093	-0.073	5.6203	0.934
16	-0.154	-0.162	7.8912	0.851
17	0.242	0.270	13.577	0.482
18	-0.085	-0.117	14.297	0.503
19	0.103	0.108	15.357	0.499
20	0.032	0.074	15.465	0.562
21	-0.016	-0.029	15.492	0.628
22	0.135	0.144	17.437	0.560
23	-0.053	-0.033	17.745	0.604
24	0.063	0.052	18.186	0.637
25	-0.200	-0.065	22.714	0.418
26	-0.014	-0.087	22.735	0.476
27	0.004	0.064	22.737	0.535
28	-0.006	-0.079	22.742	0.593
29	-0.017	-0.032	22.776	0.646
30	-0.057	-0.070	23.194	0.675
31	0.050	-0.062	23.522	0.706
32	-0.082	-0.077	24.409	0.709

Figure-7. Histogram of Residuals