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FRACTAL DIMENSION OF S&P CNX NIFTY STOCK RETURNS

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ABSTRACT

A fractal is a geometrical structure that is self-similar when scaled. By assessing the fractal dimension of asset returns, the retail investors with recent innovation in financial prediction and computational power, can exploit these prices. A branch of a tree is often used as an example in Fractal. The branch is similar to the whole tree and if we break a twig off the branch, the twig is similar to the branch. In a true, mathematical, fractal dimension, this scaling goes on forever but in all real systems, there is the largest and smallest scale which exhibit fractal behavior. The main aim of this study is to predict the prices in the stock market and thus give profitable opportunities to the investors and financial analysts. This paper examined the fractal dimension of returns of top five companies - ITC Limited, Reliance Industries Limited, Infosys Limited, ICICI Bank Limited and HDFC Limited, listed in the S&P CNX Nifty Index. The Rescaled Range (R/S) Analysis was used to detect the long-term memory. The analysis of results found that fractal dimension existed in the returns of stocks in the market.

Keywords: Fractal dimension, Indian stock market, Long memory, S & P CNX Nifty companies

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INTRODUCTION

A fractal dimension is an index for characterizing the fractal patterns or sets by quantifying their complexity as a ratio of the change in detail to the change in scale. Fractal dimension is used to characterize a broad spectrum of objects ranging from the abstract to practical phenomena, including turbulence, river networks, urban growth, human physiology, medicine and market trends[6]. Fractal Analysis consists of several methods to assign a fractal dimension and other fractal characteristics to a dataset which may be a theoretical dataset or a pattern or signal extracted from phenomena, including natural geometric objects, sound, market fluctuations, heart rates, digital images, molecular motion, networks, etc. In assessing the fractal dimension of asset returns, the investors may better understand the systematic pattern of price returns and consequently the investors may adjust their pricing strategies to buy or sell their stock accordingly (Thomas, 2007). Rescaled Range Analysis is a statistical instrument generally applied to estimate the Hurst exponent and fractal dimension of price returns. A Fractal Dimension of 1.5 or a Hurst of 0.5 indicates a random walk where there is no long memory process between the data. This type of series is hard to predict (Dhari, 2011).

The article is organized as follows. The brief results of various studies in fractal analysis provided in second section. The third section enumerates database and research methodology. The fourth section explores the fractal dimension exists in S&P CNX Nifty companies. The final section summarizes empirical findings and provides conclusions and implications.

LITERATURE REVIEW

Lots of research studies in India and abroad have been conducted to test the fractal dimension of various stock return indices using Rescaled Range analysis (Dhari, 2011; Thomas, 2007). These studies have also been conducted to examine the fractal dimension of various indices using other techniques like modified Rescaled Range**,** Multifractality Detrended Fluctuations Analysis, neural networks to uncover the long range dependence (Dhari, 2011; Mandelbrot *et al.,* 1997); Hsing *et al.* 1994; Jonathan, 2008). A few studies that have tried to explore this prominent recent issue are briefly discussed below. Andrew, (1991) analyzed the daily and monthly stock returns indexes, Centre for Research in Security Prices (CRSP) of U.S. stock market from1962-1987 using Monte Carlo simulations and modified Rescaled Range test. The study found the little evidence of long-term memory in historical U.S. stock market returns for both daily and monthly. Singh and Parikshit (2002) examined the efficacy of the statistical measures of risk of stock market using the Rescaled range analysis and Hurst's exponent, Fractal dimensions and the Lyapunov exponents. The results found that the stock market has nonlinearities. The study also suggested that the fractal

[6] The terms fractal and fractal dimension were coined by mathematician Benoit Mandelbrot in 1975.

dimension depict the risk of stock prices more rationally than the standard deviation. Enrico and Goddard (2005) examined the daily log returns series of the Mitbel, Dow-Jones Industrial Average and FTSE 100, for a sample period from 1995-2005 using Rescaled Range Analysis and Multifractality Detrended Fluctuations Analysis. The study found the log return series of the Mitbel, Dow Jones Industrial Average and FTSE was antipersistence. Although the article provided a strong idea about multifractality represents volatility clustering in a more natural manner, through variation in scaling behavior of the higher moments of the returns distribution. Bio and Khaled, (2005) enumerated the daily index returns of Dow-Jones Industrial average from 1930-2004 with the help of Monte Carlo Simulation, Scramble test and neural networks. The results found that the periods with large Hurst Exponents could be predicted more accurately than those with Hurst Exponents values close to random series. Prashanta, (2010) scrutinized the daily time series data viz., S&P 500 Index, RTH(1950-2010), Dow Jones Industrial Average (1928-2010) and DAX (1990-2010). Tools like Monte Carlo Simulation, Auto Regressive Residuals and Rescaled Range Analysis were used in this study to find out the long range dependence. The study found that markets have persistence, bias and demonstrated long memory effects. Gayathri and Selvam (2011) studied the monthly index returns of Nifty from 2007 to 2010 using the tools Descriptive Statistics, Rescaled Range Analysis to analyze fractal structure exists in the market. The results found that Fractal Structure existed in the National Stock Exchange of India and the stock prices did not reflect the information in the past series of stock prices. Murugesan *et al.* (2011) described the daily index returns of BSE Sensex from 2005-2009. The stationarity in the market was found using Augmented Dickey-Fuller Test and fractal structure by using Rescaled Range Analysis. The study found that the fractal structure existed in the BSE Sensex. The empirical results suggested that there was a stronger degree of long range dependence in the daily index returns of Sensex. Further, the study suggested using Heterogeneous Market Hypothesis (HMH) and Coherent Market Hypothesis (CMH) to estimate fractal dimension in future studies. Gayathri *et al.* (2012) evaluated the daily index returns of BSE Sensex returns from the year 2002-2011. The article found that Persistence and Long Range Dependence existed in BSE Sensex Returns[7].

Carlos *et al.* (2013) examined the time series behavior of housing prices series for 69 cities in China. The general housing price index, the index of newly constructed buildings and the price index of second hand buildings from 2005:7 to 2010:12 are examined. In order to determine whether shocks to the variables have permanent effects, the univariate fractionally integrated models are employed. They found that persistence was accepted for the general housing price index and for the newly constructed buildings and the mean reversion was accepted in most of the second hand building price indices. All these studies have been conducted with different companies, different indices with different periods. But no comprehensive study has been made so far in India

[7] The study also stated that there was persistence in stock market returns during 20 months, 50 months, 80 months and 120 months, as all the measures of Hurst Exponent value was 0.5 to 1.

to test the fractal dimension in the stock market for a particular period. Hence the present study is an attempt to test again the fractal model in India upon the top five companies of S&P CNX Nifty over a five year period from 2007 to 2012.

Statement of the problem

The predicting of equity markets and stock price movements is important for every investor. Equity analysts use many methods and indicators to predict price movements in the market. The stock market is one among the most unpredictable financial system[8]. Almost any reason whether it is real or imagined, can cause these extreme price fluctuations that often affect the stock market's credibility. Therefore, there have been numerous attempts at modelling a reliable price predictor. The varying awareness of investors across markets makes it difficult to predict the price. This study describes the prediction of price through fractal model which may prove to be the answer to the problem of prediction.

DATA AND METHODOLOGY

A stock index is the best method of measuring the value of a section of the stock market. It is computed from the prices of selected stocks. It is a tool used by investors and financial managers to describe the market and economy and to compare the return on specific investments. S&P CNX Nifty is a well diversified 50 stock indices, representing all important sectors of the Indian Economy. There were fifty companies listed on S&P CNX Nifty. The sample of the study constitutes 5 Indian companies selected from S&P CNX Nifty index[9]. On the basis of combined weightage, ITC Limited, Reliance Industries Limited, Infosys Limited, ICICI Limited and HDFC Limited were selected as sample for this study. To examine the fractal dimension among selected companies from S&P CNX Nifty, daily returns of each company were examined over a period of five years, form 1st September, 2007 to 31st August, 2012. These returns were downloaded from NSE (National Stock Exchange) website. The required data were collected from the yahoo finance website, www.yahoofinance.com and the PROWESS, a corporate database. The other required data were collected from reputed journals and various websites.

Hypotheses of the study

Two null hypotheses were developed and tested in the light of the above objectives:

NH_1 – The daily returns of sample companies are not normally distributed

NH_2 – The daily returns of sample companies have no long range dependence

[8] The unpredictable stock market tends to be the biggest problem for the investors and affects their expected returns.

[9] The combined weightage of the five sample companies was 36.33% in the Nifty as on August 2012.

Statistical tools

The investigation of fractal analysis of sample companies was made by using the following statistical tools:

- **Descriptive Analysis**

The descriptive analysis includes Mean, Median, Standard Deviation, Skewness and Kurtosis.

-Rescaled Range Analysis

Hurst (1965) developed the Rescaled Range Analysis to analyze long records of natural phenomena. There are two factors used in this analysis: firstly, the range R, which is the difference between the minimum and maximum 'accumulated' values or cumulative sum of X (t, *tau*) of the natural phenomenon at discrete integer-valued time t over a time span *tau* and secondly, the standard deviation S, which is estimated from the observed values *Xi*(t). Hurst found that the ratio R/S is described a large number of natural phenomena[10].

VARIABLES

This research article covered only two types of variables. The dependent variable represented Logarithmic Returns of companies' prices. The independent variable represented the Time Period and was measured by daily intervals. The purpose of this study is to determine the fractal dimension existed in S&P CNX Nifty companies' returns.

ANALYSIS OF NORMALITY AND FRACTAL DIMENSION IN THE S&P CNX NIFTY COMPANIES' RETURNS

For the purpose of this study, the analysis was made as follows:

1. Analysis of Normality using Descriptive Statistics (all sample companies)
2. Estimation of Rescaled Range value of sample companies
3. Analysis of Fractal Dimension of sample companies

Analysis of normality using descriptive statistics (all sample companies)

The results of descriptive statistics in S&P CNX Nifty Companies Returns for the period from September 2007 to August 2012, are exhibited in Table 1. The returns of sample companies are calculated using the equation-1. Table 1 clearly reveals that there was positive mean return in ITC Ltd (0.0004) and Infosys Ltd (0.0002) during the study period and negative mean return in the case of Reliance Industries Ltd (-0.0007), ICICI Bank Ltd (-4.4975) and HDFC Bank Ltd (-0.0004). The maximum and minimum return values were recorded for ICICI Bank Ltd (0.2073) and ITC Ltd (0.1055) during the study period. It is to be noted that the high value of standard deviation (0.0513)

[10] See Thomas (2007) for detailed description about Rescaled Range Analysis

was earned by HDFC Bank Ltd during the study period. It is to be noted that companies' returns distribution was negatively skewed for all the five companies. The negative skewness indicates a distribution with an asymmetric tail extending towards more negative values. The results of kurtosis measure of all the five companies were above the value of three and therefore, the return distributions were Leptokurtic during the study period. The presence of Leptokurtosis in the data provided the evidence for systematic bias in the data generating process. It describes how concentrated data were around a single value, usually the mean. Thus, kurtosis assesses how peaked or flat is the data distribution. More peaked or flat the distribution, less normally distributed the data.

The daily returns of five sample companies were not normally distributed. Hence there was no normality in the daily returns of all the sample companies and hence the results accept the null hypothesis. The findings did support existence of study by Kang and Seong-Min, 2008. Thus, the investors could easily assess the risk and return in the markets and accordingly they could develop investment strategy to earn adequate returns as expected.

Table 1: The Results of Descriptive Statistics for S&P CNX Nifty companies from 2007 to 2012

Companies / Descriptive Statistics	ITC Limited	Reliance Industries Limited	Infosys Limited	ICICI Bank Limited	HDFC Bank Limited
Mean	0.0004	-0.0007	0.0002	-4.4975	-0.0004
Maximum	0.1055	0.1937	0.1223	0.2073	0.1510
Minimum	- 0.7060	-0.7242	-0.1438	-0.2214	-1.6055
Standard Deviation	0.0282	0.0332	0.0222	0.0330	0.0513
Skewness	- 12.67	-8.37	-0.27	-0.09	-24.66
Kurtosis	318.81	183.43	4.54	4.77	777.34
Number of Observations	1241	1241	1241	1241	1241

Estimation of rescaled range value of sample companies

The data of daily returns for NSE Nifty sample companies over the period from 1st September 2007 to 31st August 2012, a total of 1,240 observations, were used to study the fractal dimension of NSE company returns using the rescaled range analysis. These returns are the log price relative of a composite representative Nifty stock market index. To test the nature of systematic bias, Rescaled Range Analysis was applied. In the first step, the sample period was divided into sub periods of length n. For financial time series, Peter (1994) suggests n=10 as a starting point as values of n<10 have been found to produce unstable estimates when sample sizes are small. Thus, the data may be split into 340 contiguous sub periods of the whole sample of 1,240 observations. The log n, log

R/S, log E(R/S) and V-Statistic were calculated using Matlab R2012 software, refer Appendix-I[11]. The estimation of rescaled range value for all sample companies was made as follows:

a) Estimation of rescaled range value for ITC Limited
b) Estimation of rescaled range value for Reliance Industries Limited
c) Estimation of rescaled range value for Infosys Limited
d) Estimation of rescaled range value for ICICI Limited, and
e) Estimation of rescaled range value for HDFC Limited

$$R / S = (a*N)^{H} \qquad (1)$$

where, R / S = Rescaled Range; a = constant (number of intervals); N = Number of observations; H = Hurst Exponent

The V-Statistic takes the following format:

$$V_n = \frac{R_n / S_n}{\sqrt{n}} \qquad (2)$$

where, V_n = V-Statistic; R/S = Rescaled Range; n = Increment Time

a) Estimation of rescaled range value for ITC limited

Table 2 shows the results of rescaled range value for various time vectors during the study period from 2007 to 2012. The log R/S represents the empirical rescaled ranges of the sample data sets while the log E(R/S) denotes the expected rescaled range analysis. The highest value (1.2188) of log R/S was recorded during a 330 day period while the lowest value (0.4570) of log R/S was registered at the starting period (N=10). The highest value (1.3423) and the lowest value (0.4805) of log E(R/S) were found for the 340 day period and the starting period of n=10 respectively. From the above Table 2, it is to be noted that the log R/S values and log E(R/S) values were closer from N=10 to N=90 but it did not continue after the 90 day period. In the case of V-statistics, the highest value of 1.0656 was recorded for the 90 day period and the lowest value of 0.8301 was recorded for the 290 day period. More specifically, the value for n≤90 was not easily distinguishable from a random walk. The V-statistic values diverge until n=130. The values in between the region (n=100 to n=130) increased at a faster rate. After the 130 day period, the V-statistic recorded moderate values till the end of the study period. It is to be noted that there were not more differences in the values after the 130 day period except the mere decrease in value in the 290 period. That is, the return patterns behavior in the last five year period (2007-2012) of ITC limited influenced the

[11] Available upon request from authors

returns of subsequent five year period. The above analysis reveals the significant fact that the market returns followed a biased random walk, that is, the return values tended to trend in one direction until some exogenous event occurred in the market to change their bias. Consequently, the stock price movements of ITC limited might possibly decrease in the subsequent period (2013-2017).

Table 2: Rescaled range analysis of ITC limited

N	Log n	Log R/S	Log E(R/S)	V- Statistic (R/S)	V- Statistic E(R/S)
10	1.0000	0.4570	0.4805	0.9058	0.9561
20	1.3010	0.6152	0.6638	0.9219	1.0311
30	1.4771	0.7231	0.7667	0.9651	1.0670
40	1.6021	0.7922	0.8382	0.9798	1.0893
50	1.6990	0.8316	0.8928	0.9596	1.1050
60	1.7782	0.8862	0.9370	0.9935	1.1167
70	1.8451	0.9251	0.9741	1.0058	1.1260
80	1.9031	0.9682	1.0060	1.0392	1.1336
90	1.9542	1.0047	1.0340	1.0656	1.1399
100	2.0000	0.9835	1.0589	0.9627	1.1453
110	2.0414	1.0015	1.0814	0.9568	1.1500
120	2.0792	1.0208	1.1018	0.9576	1.1541
130	2.1139	1.0644	1.1206	1.0172	1.1578
140	2.1461	1.0427	1.1379	0.9326	1.1610
150	2.1761	1.0568	1.1540	0.9306	1.1640
160	2.2041	1.0711	1.1690	0.9313	1.1666
170	2.2304	1.0835	1.1831	0.9295	1.1691
180	2.2553	1.1009	1.1963	0.9403	1.1713
190	2.2788	1.0928	1.2088	0.8983	1.1734
200	2.3010	1.0929	1.2207	0.8758	1.1753
210	2.3222	1.1437	1.2319	0.9607	1.1771
220	2.3424	1.1143	1.2426	0.8772	1.1788
230	2.3617	1.1538	1.2529	0.9395	1.1803
240	2.3802	1.1862	1.2626	0.9910	1.1818
250	2.3979	1.1985	1.2720	0.9989	1.1832
260	2.4150	1.1877	1.2810	0.9555	1.1845
270	2.4314	1.1964	1.2896	0.9566	1.1857
280	2.4472	1.1681	1.2980	0.8801	1.1868
290	2.4624	1.1503	1.3060	0.8301	1.1880
300	2.4771	1.1700	1.3137	0.8540	1.1890
310	2.4914	1.2030	1.3212	0.9065	1.1900
320	2.5051	1.2115	1.3285	0.9097	1.1910
330	2.5185	1.2188	1.3355	0.9111	1.1919
340	2.5315	1.2182	1.3423	0.8963	1.1927

Source: Computed from yahoo finance website using MATLAB R2012a Software

Therefore, the market returns of ITC limited were not influenced by past data during the study period. In other words, there was no long range dependence in the daily returns of ITC limited and thus there was little support for short term memory in the ITC company's returns. Therefore, the null hypothesis is accepted in the case of ITC limited. Hence the investors have to choose better stocks in better times by using this information. The previous study by Debasish and Mulligan,

2010 suggested that the market participants would over react to new information, behavior which imposes greater price volatility than would be consistent with market efficiency**.** In their study, the results of three information technology equities explored long range dependence.

Figure-1 represents a plot of log(R/S) and log E (R/S) against log (n). It is understood that the log R/S plot scale was close to the log E(R/S) plot until the period 90. During the period 130, the log R/S series (1.0644) was more close to the log E(R/S) series (1.1206), but the closeness of the same did not continue for the whole study period. After the point 13, the deviation of log R/S slope did not go close to the log E(R/S) until the last period (n=330). Therefore, it is clearly understood from the above Figure-1that there was absence of long memory in the case of ITC Limited. Thus the results evidenced that the returns of the ITC Limited recorded quite different characteristics. Hence the investors have to choose the unrisky markets and the best time to invest their hard earned money in the market.

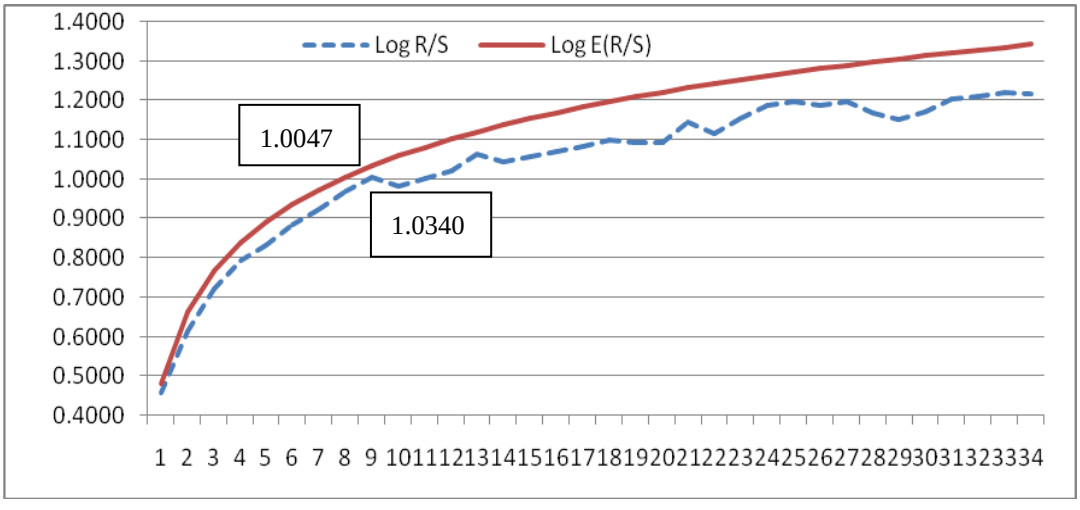

Figure 1: Chart showing Log (R/S) and Log E(R/S) values for Log n period of ITC Ltd

Source: Computed from table 2 using microsoft excel 2007

A plot of the V-Statistic against log (n) is presented in Figure-2, to display the deviation of the log R/S and E(R/S) series. According to the above Figure-2, the slope of V-statistic of R/S values increased to 1.0656 around the 90 day period from the period n=10 (the value of 0.9058). After this point (90), the further increase in the V-statistic value (1.0172) was recorded on the 130 day period. The growth movement of V-statistic slowed down after the 130 day period. Thereafter, the V-statistic values decreased throughout the sample period till n=340 (0.8963). It is to be noted that the V-statistic R/S values were clearly distinguishable from the E(R/S) throughout the study period. The plot of V-statistic for the ITC limited indicates antipersistence in the returns generating process. The values of 0.9989 after point 25 had decreased throughout the period. This indicates the

fact that the time series was up in the previous period and it is more likely that the value would be down in the next period. This indicates the fact that there was a mean reversion during the study period. According to the results, the time series distribution length of ITC limited returns showed the absence of long memory. Therefore, the prediction of share price movements was difficult for the ITC limited during the study period. The investors can easily identify the risks in the particular market and time.

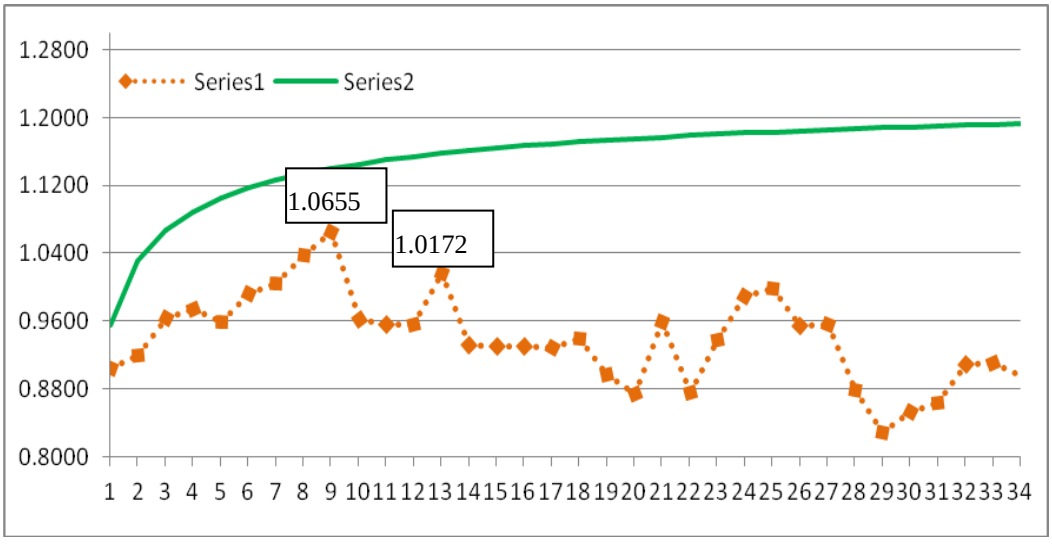

Figure 2: Chart showing V-statistic of ITC limited company for log N period

Source: Computed from Table 2 using Microsoft Excel 2007

b) Estimation of rescaled range value for reliance limited

Table 3 depicts the outcome of rescaled range value for reliance industries during the study period from 2007 to 2012. The highest value (1.3151) of log R/S was registered on the 340 day period while the lowest value (0.4739) of log R/S was recorded on the starting period (N=10). The highest value (1.3423) and the lowest value (0.4805) of log E(R/S) were found on 340 day period and starting period (N=10) respectively. It is to be noted from the above Table 3 that the log R/S values and log E(R/S) values (from N=10) were closer to N=180 and after that, the value diverged from the point N=190 up to N=210. Again the log R/S value diverged from E(R/S) during the period from N=260 to N=320. Later, the log R/S values were closer to E(R/S) until the end period. In the case of V-statistics, the highest value of 1.1832 was witnessed on the 80 day period and the lowest value of 0.9416 was found on the starting period (N=10). It implies the interesting fact that the stock returns tended to be increase in future. It is significant that the V-statistic values diverged until n=80. The values (0.9416) to (1.1832) were in between the region n=10 to n=80 and increased at a faster rate. After the 80 day period, the V-statistic values recorded fluctuation trend till the end of the study period. After the 80 day period, the values of V-statistics moved at moderate level till

n=180. That was the return patterns behavior in last five year period of Reliance industries limited, which influenced the return patterns for subsequent five year period (2013-2017).

Table 3: Rescaled ranges of reliance industries limited

N0	Log n	Log R/S	Log E(R/S)	V-Statistic (R/S)	V-Statistic E(R/S)
10	1	0.4739	0.4805	0.9416	0.9561
20	1.3010	0.6476	0.6638	0.9933	1.0311
30	1.4771	0.7574	0.7667	1.0444	1.0670
40	1.6020	0.8303	0.8382	1.0696	1.0893
50	1.6989	0.8905	0.8928	1.0990	1.1050
60	1.7781	0.9312	0.9370	1.1018	1.1167
70	1.8450	0.9745	0.9741	1.1271	1.1260
80	1.9030	1.0246	1.0060	1.1832	1.1336
90	1.9542	1.0127	1.0340	1.0853	1.1399
100	2	1.0557	1.0589	1.1368	1.1453
110	2.0413	1.0512	1.0814	1.0728	1.1500
120	2.0791	1.0974	1.1018	1.1423	1.1541
130	2.1139	1.0999	1.1206	1.1040	1.1578
140	2.1461	1.1091	1.1379	1.0866	1.1610
150	2.1760	1.1445	1.1540	1.1387	1.1640
160	2.2041	1.1575	1.1690	1.1361	1.1666
170	2.2304	1.1880	1.1831	1.1825	1.1691
180	2.2552	1.1908	1.1963	1.1565	1.1713
190	2.2787	1.1274	1.2088	0.9728	1.1734
200	2.3010	1.1778	1.2207	1.0648	1.1753
210	2.3222	1.1990	1.2319	1.0912	1.1771
220	2.3424	1.2112	1.2426	1.0964	1.1788
230	2.3617	1.2374	1.2529	1.1389	1.1803
240	2.3802	1.2242	1.2626	1.0816	1.1818
250	2.3979	1.2652	1.2720	1.1647	1.1832
260	2.4149	1.2392	1.2810	1.0757	1.1845
270	2.4313	1.2388	1.2896	1.0546	1.1857
280	2.4471	1.2200	1.2980	0.9919	1.1868
290	2.4623	1.2619	1.3060	1.0732	1.1880
300	2.4771	1.2477	1.3137	1.0212	1.1890
310	2.4913	1.2433	1.3212	0.9945	1.1900
320	2.5051	1.2963	1.3285	1.1060	1.1910
330	2.5185	1.3021	1.3355	1.1036	1.1919
340	2.5314	1.3151	1.3423	1.1204	1.1927

Source: Computed from yahoo finance website using MATLAB R2012a Software

Therefore, it is clear from the analysis that market returns followed a biased random walk, that is, the return values tended to trend in one direction until some exogenous event occurred to change their bias. Accordingly, the stock price movements of Reliance industries limited would possibly increase in the subsequent five year period. The overall analysis shows the fact that the market returns of Reliance industries limited were influenced by the past period during the study period. There was long range dependence in the daily returns of Reliance Industries**,** because there was strong evidence of long term memory in the returns of Reliance Industries. The null hypothesis is

rejected. A study done by Michael, (2001) studied the Australian stock market data and established the presence of long range dependence. He revealed that rescaled range analysis is a robust nonparametric statistical technique which can discern the presence of fractal structure in financial time series. Figure 3 represents a plot of log(R/S) and log E(R/S) against log (N) for Reliance Industries. It is understood from the above Figure-3 that the log R/S plot scale was close to that of the log E(R/S) plot until the period n=180 from the starting period. Again, the log R/S series (1.2652) was close to the log E(R/S) series (1.2720) during n=250. After that, the R/S and E(R/S) values diverged until the period 320. In other words, after the point 25, the deviation of log R/S slope was not close to the log E(R/S) until the period n=320. Therefore, from the above Figure-3, it is clearly noted that there was presence of long memory in the case of Reliance Industries Limited. Thus, the outcome evidenced from this study is that the Reliance industries limited returns registered long range dependence. Hence, the investors are advised to buy the shares in the market at the right time because in next five year period, the stock prices may be expected to rise.

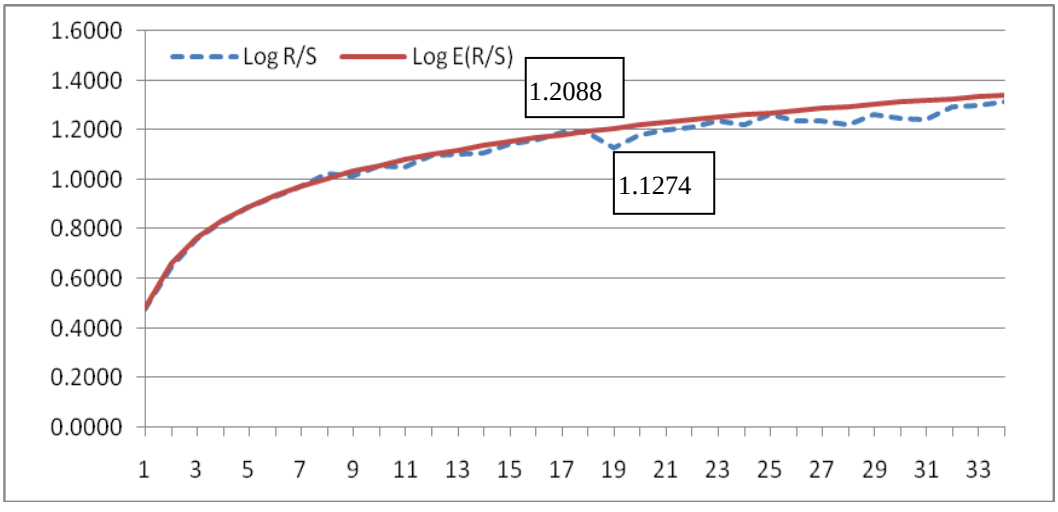

Figure 3: Chart showing Log (R/S) and Log E(R/S) values for Log N period of Reliance Ltd

Source: Table 3 using Microsoft Excel 2007

A plot of the V-statistic against log (N) is presented in Figure-4 to identify the deviation of the log R/S and E(R/S) series. It is clear from the above Figure-4 that the slope of V-statistic increased to 1.1832 around the 80 day period from the period n=10, with the value of 0.9416. After this point, further increase of the V-statistic value (1.1825) was recorded on the 170 day period. In other words, the growth movement of V-statistic declined after the 170 day period till n=190. Thereafter, the V-statistic values increased throughout the sample period till n=340 (1.1204). The analysis of plot of V-statistic for the Reliance industries limited indicates the persistence in the returns generating process as the values were on the increase throughout the period. As stated earlier, if a time series has been up in the previous period, it is more likely that the value will be up in the next

period also. This indicates the fact that there was an evidence of long term effect during the study period. The results of the time series distribution length of Reliance industries limited returns recorded the presence of long memory. Therefore, the prediction of share price movements was easily identified in the Reliance industries limited during the study period. The investors may use this information to their advantage.

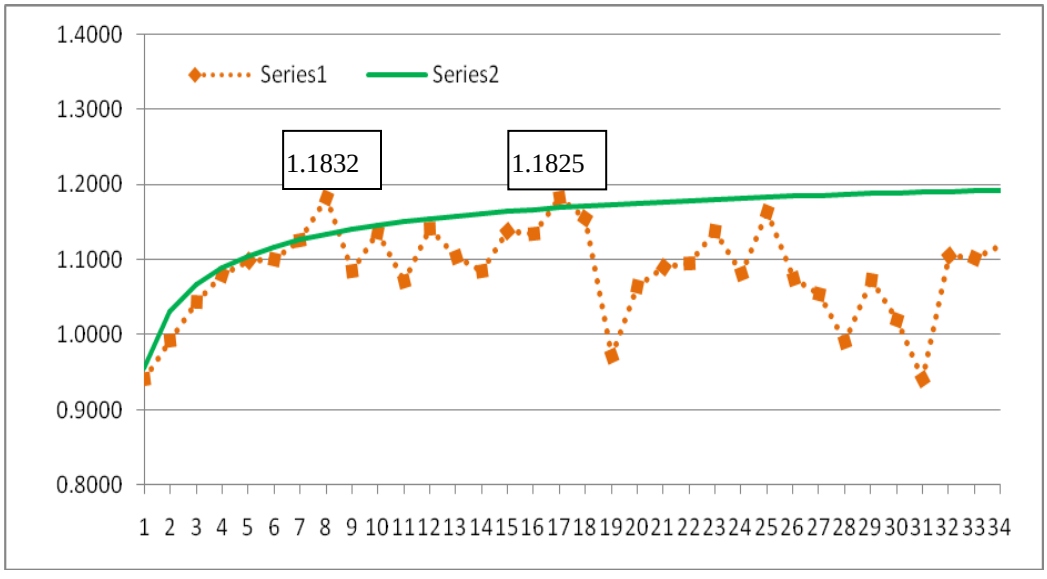

Figure 4: Chart showing V-statistic of reliance limited for log N period

Source: Table 3 using Microsoft Excel 2007

c) Estimation of rescaled range value of infosys limited

Table 4 illustrates the results of Rescaled range value for Infosys limited for the study period from 2007 to 2012. The highest value (1.3345) of log R/S was evidenced on the final period (N=340) of the study while the lowest value (0.4591) of log R/S was recorded on the period N=10 (log (n) =1). The highest value (1.3423) and the lowest value (0.4805) of log E(R/S) were found on the last day period (N=340) and starting period (N=10) respectively. It is significant to note from the above Table 4 that the log R/S values and log E(R/S) values of N=10 were close to N=80 and after that, the values at the point N=90 diverged up to N=210. Again for the second time during the study period, the log R/S values of 1.2406 from the period N=220 diverged till N=230. Even at a later period, the values were not closer to each other until the last period. However, the R/S and E(R/S) values were somewhat close during the periods N=260 and N=290. The analysis of V-Statistics shows that the highest value of 1.1732 was witnessed on 220 day period and the lowest value of 0.9102 was found on the starting period N=10. The R/S and E(R/S) values of V-statistics did not diverge from N=10 to N=70. More changes in the V-statistic values was recorded during the period N=10 to N=100 and after that, the changes in value proceeded at a moderate pace until the period N=200. Later, there was an increasing trend in the value of V-statistics up to the period N=220.

However, there were major changes at the point N=220 which continued till the end of the study period. The V-statistic during the study period ended with a value of 1.1715 (N=340). It is significant to note that the R/S and E(R/S) of V-statistic values did not get close to each other during N= 90 to N=210 and N=230 to N=330. From this it is clear that there was long range dependence in the daily returns of Infosys limited and hence the null hypothesis is rejected.

Table 4: Rescaled ranges of infosys limited

N	Log n	Log R/S	Log E(R/S)	V- Statistic R/S	V-Statistic E(R/S)
10	1	0.4591	0.4805	0.9102	0.9561
20	1.3010	0.6393	0.6638	0.9745	1.0311
30	1.4771	0.7439	0.7667	1.0124	1.0670
40	1.6020	0.8273	0.8382	1.0624	1.0893
50	1.6989	0.8767	0.8928	1.0648	1.1050
60	1.7781	0.9465	0.9370	1.1413	1.1167
70	1.8450	0.9811	0.9741	1.1443	1.1260
80	1.9030	0.9868	1.0060	1.0846	1.1336
90	1.9542	1.0006	1.0340	1.0557	1.1399
100	2	1.0095	1.0589	1.0222	1.1453
110	2.0413	1.0523	1.0814	1.0756	1.1500
120	2.0791	1.0502	1.1018	1.0248	1.1541
130	2.1139	1.0697	1.1206	1.0296	1.1578
140	2.1461	1.0708	1.1379	0.9948	1.1610
150	2.1760	1.1137	1.1540	1.0609	1.1640
160	2.2041	1.0871	1.1690	0.9661	1.1666
170	2.2304	1.1305	1.1831	1.0359	1.1691
180	2.2552	1.1424	1.1963	1.0347	1.1713
190	2.2787	1.1523	1.2088	1.0302	1.1734
200	2.3010	1.1564	1.2207	1.0137	1.1753
210	2.3222	1.2049	1.2319	1.1062	1.1771
220	2.3424	1.2406	1.2426	1.1732	1.1788
230	2.3617	1.2288	1.2529	1.1168	1.1803
240	2.3802	1.2170	1.2626	1.0639	1.1818
250	2.3979	1.2261	1.2720	1.0645	1.1832
260	2.4149	1.2479	1.2810	1.0976	1.1845
270	2.4313	1.2277	1.2896	1.0280	1.1857
280	2.4471	1.2051	1.2980	0.9583	1.1868
290	2.4623	1.2647	1.3060	1.0803	1.1880
300	2.4771	1.2361	1.3137	0.9943	1.1890
310	2.4913	1.2434	1.3212	0.9948	1.1900
320	2.5051	1.2646	1.3285	1.0281	1.1910
330	2.5185	1.2940	1.3355	1.0833	1.1919
340	2.5314	1.3345	1.3423	1.1715	1.1927

Source: Computed from yahoo finance website using MATLAB R2012a

The market returns of Infosys limited were influenced by past returns during the study period. The long memory was evidenced in the returns and has major implications for the stock pricing models. Seong-Min and Sang, (2008) suggested that the test for long memory in the stock returns provides an important guideline of market efficiency. The business cycles were identified using the rescaled

range analysis and also suggested that identification of market cycles gives a potential opportunity to earn abnormal returns in the stock returns. Plot of log(R/S) and log E(R/S) against log (N) period for Infosys limited are given in Figure-5. It is clear from Figure-5 that the plot scale of log (R/S) (0.9868) was close to the scale of log E(R/S) (1.0060) until the period n=80. After that, the plot of log (R/S) and E(R/S) (1.2426) diverged until the period n=220. But from the period N=90, (log R/S = 1.0340) they did not converge for a long period of time. Again, the plot of log R/S deviated until 330 day period. The ranges of plot increased throughout the study period and this reveals the presence of long term memory as the range of plot for log (R/S) and log E(R/S) did not converge during the study period in the case of Infosys limited. This proved that the returns of Infosys limited experienced long term memory. Since the share prices of Infosys limited may increase in the future, investors may use this information in such a way to gain benefits to them.

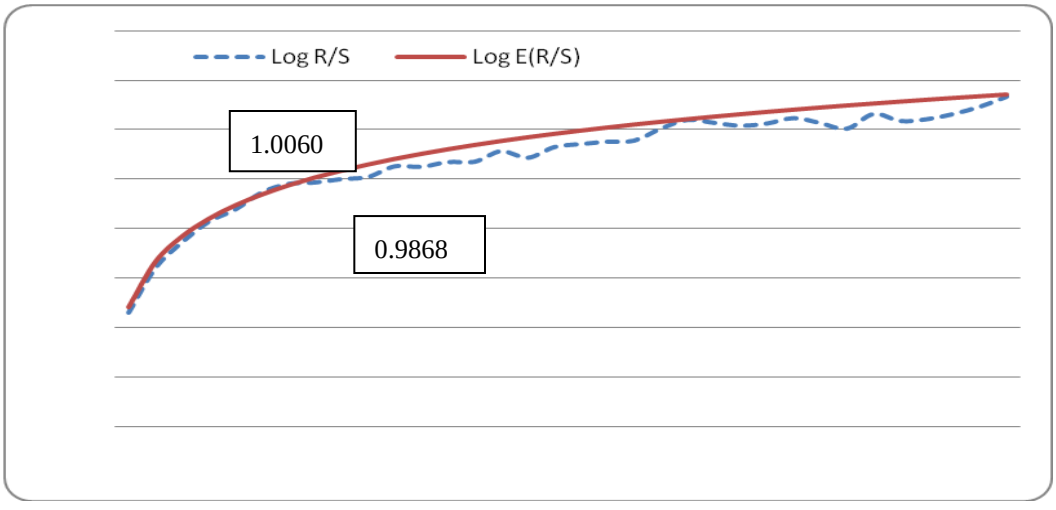

Figure 5: Chart showing log (R/S) and log E(R/S) values for log N period of infosys Ltd
Source: Computed from Table 4 using Microsoft Excel 2007

Figure 6 shows **t**he V-statistic plot of Infosys limited against log (N) to record the deviation of the log R/S and E(R/S) series. According to Figure 6**,** there was an increase in the plot of V-statistic value (1.1443) on the 70 day period. After that, there was an increase in the trend of the V-statistic slope during 220 day period with a value of 1.1732. It is significant to note that the growth movement of V-statistic for Infosys Limited declined after the 220 day period. The deep downward value (0.9583) was also recorded during the period n=280. Thereafter, the V-statistic values continued to climb up throughout the study period (till n=340). The plots of R/S and E(R/S) V-Statistic were not close to each other for all the periods and this indicates the fact that there was persistence in the returns of Infosys limited. From this, the investors may infer that the return value was up in the previous period and it is more likely that the value will be up in the subsequent period also. It implies that the stock returns would tend to increase in future. It establishes that there was evidence of long term effect during the study period. Based on the results of Infosys limited, the forecasting of share price is possible in the case of Infosys limited.

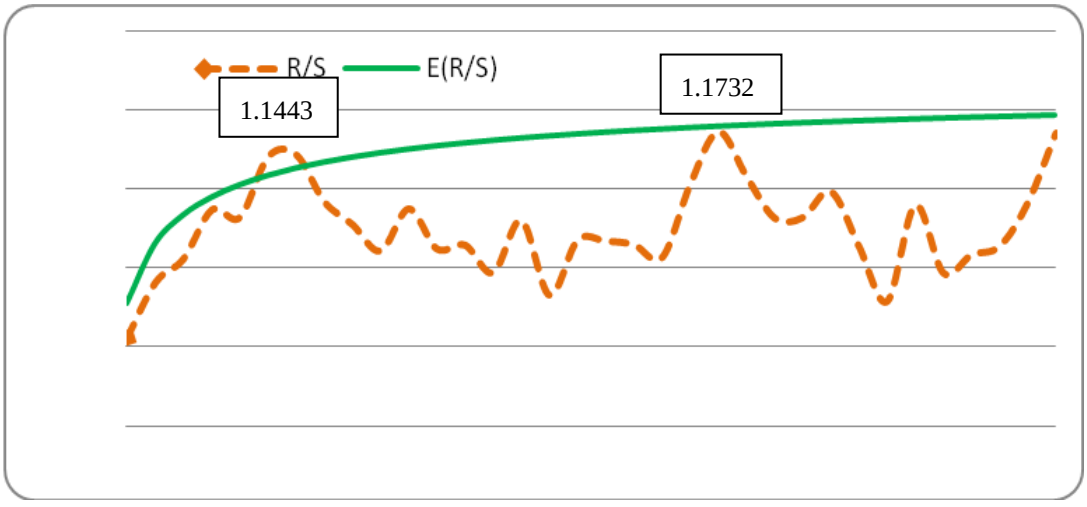

Figure 6: Chart showing v-statistic of infosys limited for log n period

Source: Computed from Table 4 using Microsoft Excel 2007

d) Estimation of rescaled range value of ICICI limited

Table 5 displays the results of rescaled range value for various time vectors during the study period from 2007 to 2012. The log R/S represents the empirical rescaled ranges of the sample data sets while the log E(R/S) denotes the expected rescaled range analysis. The highest value (1.4288) of log R/S was found on 340 day period while the lowest value (0.4774) of log R/S was listed on the starting period (N=10). The lowest value (0.4805) and the highest value (1.3423) of log E(R/S) were found on the starting period of N=10 and N=340 day period respectively. According to Table 5, the V-statistic value of R/S was recorded at moderate level from N=10 to N=130 day period. The V-statistic for R/S values diverged from the period N=130 till the end of study period. It is to be noted that the values of log R/S and the values of log E(R/S) were closer from N=10 to N=310 but it was not close after the period N=310 till the last period. In the case of V-statistics for R/S, the highest value of 1.4555 was recorded during 340 day period and the lowest value of E(R/S) 0.9494 was recorded on the starting day period. More specifically, the value for $n \leq 310$ was not identifiable from a random walk. It is to be noted that the values in between the region N=310 to N=340, increased at a faster rate. The analysis of the Table 5 reveals the fact that ICICI limited did have persistence of market returns whereas the values of V-statistic for R/S were not close to the E(R/S). But at the end of the period, the values of 1.3226 increased and deviated from that of E(R/S). Therefore, the stock returns of ICICI limited would have been influenced by past data during the study period. The analysis shows that there was long range dependence in the daily returns of ICICI limited and hence the null hypothesis is rejected. Hence, investors have to note this information and develop their investing strategy in a better way. A study by Sharad and Bhattacharya, (2012) studied the ten emerging stock markets and pointed out that, exploring long memory property is appealing to derivative market participants, risk managers and asset allocation decision makers, whose interest is to reasonably forecast stock market movements.

Table 5: Rescaled ranges of ICICI bank limited

N	Log n	Log R/S	Log E(R/S)	V- Statistic R/S	V-Statistic E(R/S)
10	1	0.4774	0.4805	0.9494	0.9561
20	1.3010	0.6590	0.6638	1.0198	1.0311
30	1.4771	0.7601	0.7667	1.0507	1.0670
40	1.6021	0.8243	0.8382	1.0550	1.0893
50	1.6990	0.8903	0.8928	1.0987	1.1050
60	1.7782	0.9488	0.9370	1.1475	1.1167
70	1.8451	0.9559	0.9741	1.0797	1.1260
80	1.9031	1.0168	1.0060	1.1621	1.1336
90	1.9542	1.0400	1.0340	1.1558	1.1399
100	2	1.0552	1.0589	1.1356	1.1453
110	2.0414	1.0843	1.0814	1.1576	1.1500
120	2.0792	1.1131	1.1018	1.1843	1.1541
130	2.1139	1.1182	1.1206	1.1514	1.1578
140	2.1461	1.1551	1.1379	1.2078	1.1610
150	2.1761	1.1764	1.1540	1.2256	1.1640
160	2.204	1.1899	1.1690	1.2243	1.1666
170	2.2304	1.2209	1.1831	1.2755	1.1691
180	2.2553	1.1963	1.1963	1.1712	1.1713
190	2.2788	1.2022	1.2088	1.1556	1.1734
200	2.3010	1.1903	1.2207	1.0958	1.1753
210	2.3222	1.2469	1.2319	1.2184	1.1771
220	2.3424	1.2848	1.2426	1.2990	1.1788
230	2.3617	1.2773	1.2529	1.2487	1.1803
240	2.3802	1.2714	1.2626	1.2060	1.1818
250	2.3979	1.2664	1.2720	1.1679	1.1832
260	2.4150	1.2461	1.2810	1.0931	1.1845
270	2.4313	1.2767	1.2896	1.1508	1.1857
280	2.4472	1.2969	1.2980	1.1839	1.1868
290	2.4624	1.3189	1.3060	1.2237	1.1880
300	2.4771	1.3261	1.3137	1.2233	1.1890
310	2.4914	1.3293	1.3212	1.2124	1.1900
320	2.5052	1.3740	1.3285	1.3226	1.1910
330	2.5185	1.4084	1.3355	1.4097	1.1919
340	2.5315	1.4288	1.3423	1.4555	1.1927

Source: Computed from yahoo finance website using MATLAB R2012a

A plot of log(R/S) and log E(R/S) against log(N) for ICICI Limited are presented in Figure 7. It is inferred from the Figure-7that the plot scale of log R/S plot scale was close to the log E(R/S) plot until the period N=140 from the starting day period. After that, the R/S plot deviated from N=150 to N=170 for a short period of time. Again the log R/S series (1.1963) was close to the E(R/S) series (1.1963) from n=180 till n=190. It is to be noted that the R/S and E(R/S) values diverged after the period n=310. After the point 34, the log R/S slope did not converge with the log E(R/S) until the last study period. From the Figure 7, it is inferred that there was long memory effect on the return data of ICICI limited. Thus the long term dependence was found in this stock returns and hence the investors are advised to buy the shares in ICICI limited using this information.

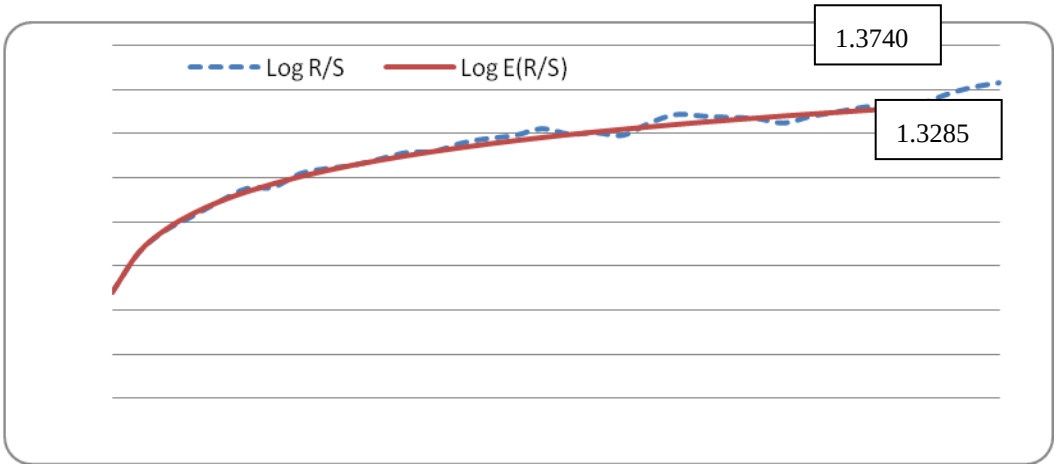

Figure 7: Chart showing log (R/S) and log E(R/S) values for log N period of ICICI Ltd

Source: Table 5 using Microsoft Excel 2007

Figure 8 displays the V-statistic plot of ICICI limited against log (N) and the movements of the log R/S and E(R/S) series of stock market returns were evaluated. As per the information given in Figure-8, the V-statistic slope moved up to 1.4555 around the point 34 during the last day of the study period. It is to be noted that before this plot, the increase in the V-Statistic of R/S plot (1.2990) could be portrayed on a 220 day period. The highest plot was recorded on the last period (n=340). The growth movement of V-statistic for R/S (1.2755) declined after the point 17. The deep decreasing trend in value (1.0931) was found during the point 26. After that, the V-statistic for R/S plot increased throughout the study period. The plot of V-statistic for R/S and E(R/S) did not converge for the whole study period and this implies that the ICICI limited returns showed long range dependence. As a result, the stock returns of ICICI limited would tend to increase in the future too. Hence investors may use this information in their favor.

e) Estimation of rescaled range value of HDFC limited

The outcome of Rescaled Range Value for HDFC limited for the study period from 2007 to 2012 is presented in Table 6. It is evident that the highest value (1.3368) of log R/S was evidenced on last period of the study while the lowest value (0.4772) of log R/S was recorded on the period N=10 (log (n) = 1). The highest value (1.3423) and the lowest value (0.4805) of log E(R/S) were found on the last day period (N=340) and the first period (N=10) respectively. It is to be noted from the Table 6 that the log R/S values and log E(R/S) values were closer at N=10 to N=60. After that, the value diverges from the point N=70 up to N=80. Again for the second time, the log R/S value diverged from E(R/S) during the period from N=180 to N=210. Later, the values were closer till the last day period. However, the R/S and E(R/S) values were not close during the period from N=250 to N=280.

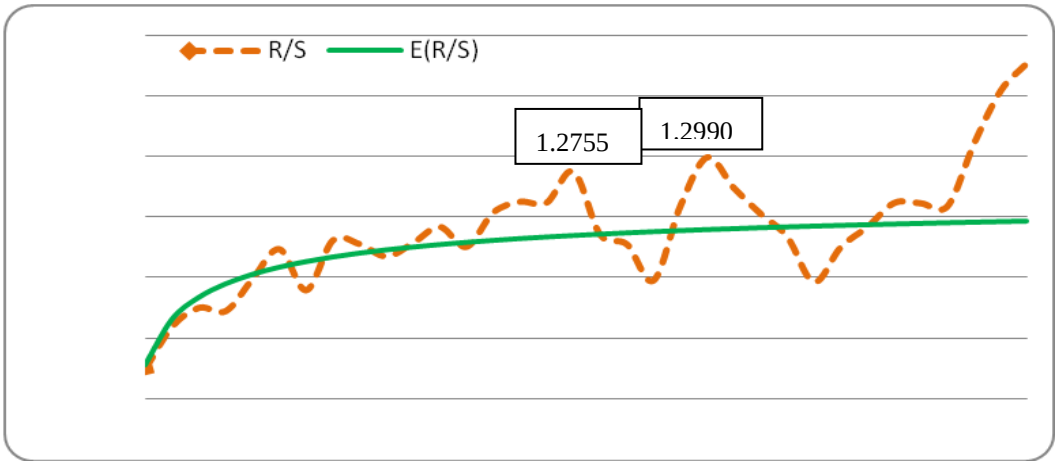

Figure 8: Chart showing V-statistic of ICICI limited for log N period

Source: Computed from Table 5 using Microsoft Excel 2007

In the case of V-statistics for R/S, the lowest value of 0.9489 was found on the starting period (N=10) and the highest value of 1.2660 was observed on the 160 day period. The wide fluctuation in the V-statistic values was recorded during the period from N=16 to N=20 and after that, fluctuation trend was recorded at moderate level until the last period. Later, the V-statistic moved up till the period N=260 where the value was 1.2655. However, there was no major change from N=260 to N=340 during the study period. It is to be noted that the V-statistic ended with a value of 1.1778 (N=340). The significant feature is that the V-statistic for R/S and E(R/S) values were not close together from log n=2.2728 to log n=2.301 and log n=2.3979 to log n=2.4472. Hence there was long range dependence in the daily returns of HDFC limited. The market returns of HDFC limited were influenced by past data during the study period. In other words, there was evidence of long term effect on the daily returns of HDFC limited. Investors may note this information and use the same to devise their investment strategy. A previous study by Prashanta, (2010) found the long memory in sectoral stock indices as the analysis showed that all three sectoral markets showed the persistence.

Figure 9 shows the log-log plot of the empirical R/S values and the expected R/S values against log (n) for the daily returns of HDFC limited. It is to be noted that the log R/S values were indistinguishable from E(R/S) values until the period 180 and after that, the slope of log R/S was discernible and shows two break points, one at 200 day period (log(n)= 2.301) and another at 260 day period (log(n)=2.4150). It is significant that after the 260 day period, the slope of log R/S came closer to log E(R/S) till the end of the study period. Figure-9 brings out the presence of long term memory. The returns of ICICI limited experienced long term dependence and the prices of ICICI Limited may increase in the future. Investors may note this information of ICICI limited.

Table 6: Rescaled ranges of HDFC bank limited

N	Log n	Log R/S	Log E(R/S)	V- Statistic R/S	V-Statistic E(R/S)
10	1	0.4772	0.4805	0.9489	0.9561
20	1.3010	0.6497	0.6638	0.9982	1.0311
30	1.4771	0.7555	0.7667	1.0397	1.0670
40	1.6021	0.8274	0.8382	1.0625	1.0893
50	1.6990	0.8734	0.8928	1.0566	1.1050
60	1.7782	0.9387	0.9370	1.1210	1.1167
70	1.8451	0.9454	0.9741	1.0541	1.1260
80	1.9031	1.0074	1.0060	1.1371	1.1336
90	1.9542	1.0346	1.0340	1.1415	1.1399
100	2	1.0658	1.0589	1.1636	1.1453
110	2.0414	1.0874	1.0814	1.1661	1.1500
120	2.0792	1.1078	1.1018	1.1701	1.1541
130	2.1139	1.1368	1.1206	1.2018	1.1578
140	2.1461	1.1487	1.1379	1.1902	1.1610
150	2.1761	1.1607	1.1540	1.1820	1.1640
160	2.204	1.2045	1.1690	1.2660	1.1666
170	2.2304	1.1927	1.1831	1.1952	1.1691
180	2.2553	1.1929	1.1963	1.1622	1.1713
190	2.2788	1.1654	1.2088	1.0618	1.1734
200	2.301	1.1598	1.2207	1.0215	1.1753
210	2.3222	1.2443	1.2319	1.2112	1.1771
220	2.3424	1.2467	1.2426	1.1898	1.1788
230	2.3617	1.2455	1.2529	1.1605	1.1803
240	2.3802	1.2594	1.2626	1.1729	1.1818
250	2.3979	1.2889	1.2720	1.2301	1.1832
260	2.4150	1.3097	1.2810	1.2655	1.1845
270	2.4314	1.3144	1.2896	1.2553	1.1857
280	2.4472	1.3083	1.2980	1.2153	1.1868
290	2.4624	1.2938	1.3060	1.1549	1.1880
300	2.4771	1.3072	1.3137	1.1713	1.1890
310	2.4914	1.3135	1.3212	1.1690	1.1900
320	2.5052	1.3246	1.3285	1.1803	1.1910
330	2.5185	1.3354	1.3355	1.1918	1.1919
340	2.5315	1.3368	1.3423	1.1778	1.1927

Source: Computed from yahoo finance website using MATLAB R2012a

The plot of V-statistic for HDFC limited to evaluate the movements of the log R/S and E(R/S) series of stock market returns against log (N) is displayed in Figure-10. The V-statistic of R/S slope moved up to 1.2660 (in around point 16). After this, there was an increase in the V-statistic for R/S plot (1.2655) on the 260 day period. It is to be noted that the growth movement of V-statistic for R/S (1.2755) declined after the point 17. The low value (1.0215) of V-statistic for R/S was found during the point 20. After that, the V-statistic for R/S plot increased throughout the study period. It is significant that the plot of V-Statistic for R/S and E(R/S) was not close during the whole study period and this implies that long range dependence did exist in HDFC limited returns. As a result, the stock returns of HDFC limited would tend to increase in the future too. Hence the investors may use this information.

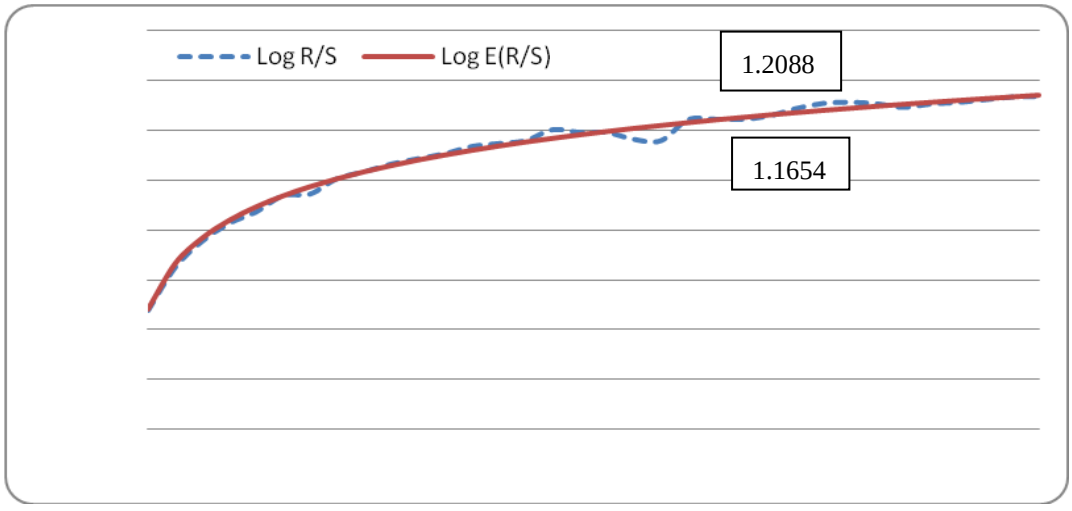

Figure 9: Chart showing log (R/S) and log E(R/S) values for log N period of HDFC Ltd

Source: Computed from Table 6 using Microsoft Excel 2007

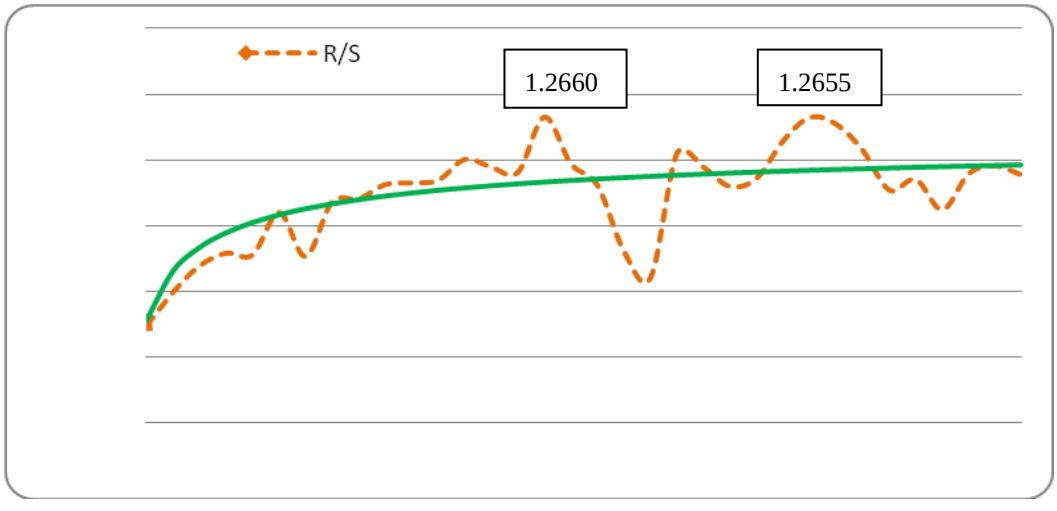

Figure 10: Chart showing V-statistic of HDFC limited for log N period

Source: Computed from Table 6 using Microsoft Excel 2007

Analysis of fractal dimension of sample companies

The results of Hurst exponent estimated initially over the period from 2007 to 2012 for all the sample companies, are depicted in Table 7. The values of Hurst exponent (0.5333) and (0.5630) were found for the Reliance industries limited and Infosys limited respectively while the HDFC Bank limited and ICICI bank limited seemed to exhibit more persistent behavior, with the Hurst exponent at values 0.6336 and 0.8390 respectively. Thus the actual value for Hurst exponent was greater than 0.5, indicating that the return series showed persistence and the value of returns would increase in the future. For ITC limited, the Hurst recorded a value of 0.4769, which indicates anti-

persistence. The Hurst exponent value was lesser than that of 0.5, specifying that the return series showed anti-persistence and hence the value of returns would decrease in the future. It was observed that the ITC limited recorded the highest Fractal Dimension (1.5231) while the ICICI Bank limited registered the lowest Fractal Dimension (1.161). Therefore, it is inferred that ITC limited was less risky than ICICI Bank limited during the study period.

Table 7: Hurst exponent values for S&P CNX nifty companies

Statistic	ITC Limited	Reliance Industries Limited	Infosys Limited	ICICI Bank Limited	HDFC Bank Limited
Hurst Values	0.4769	0.5333	0.5630	0.8390	0.6336
Fractal Dimension	1.5231	1.4667	1.437	1.161	1.3664

Source: Computed from yahoo finance website using equation (3) and (4):

$$H = \frac{\ln (R_n / S_n) - \ln (c)}{\ln (n)} \quad (3)$$

where, R / S = Rescaled Range; c = constant (number of intervals); n = time increment; H = Hurst exponent:

$$\text{Fractal Dimension} = 2 - H \quad (4)$$

Table 7 also denotes the Fractal Dimension values for all the sample companies. The values were viz., 1.4667, 1.437, 1.161 and 1.3664 for Reliance Industries Limited, Infosys Limited, ICICI bank limited and HDFC bank limited respectively. Since the values of fractal dimension for all these companies are closer to 1, except for the company ITC limited (1.5231), there was the persistent behavior with fractal dimension at $1 < D < 1.5$. Srikanth, (2013) found long memory in Indian stock market. It reveals the fact that if the share price had been increasing for a current period it was expected to continue into the subsequent period. The companies' return was predictable in one direction. Therefore, in all the daily returns of all firms, except ITC limited, the fractal dimension exists.

DISCUSSION AND CONCLUSIONS

This study has investigated the possibility of a fractal exponent in the process of daily returns of sample companies. The obtained results confirmed the presence of fractal structure with long memory in the share price series in India. The long memory was found for Reliance industries limited, Infosys limited, ICICI bank limited and HDFC bank returns series but not found for the

ITC limited returns series. In the daily returns of sample companies, the anti-persistent was not constant and the returns of sample companies indeed displayed periods of persistent behavior. The strongest inference that could be drawn from the above analysis is that the persistence of sample companies indicated upward trends for the next period as the value of Hurst exponent was greater than 0.5.

It is to be noted that some previous studies using R/S analysis did find evidence of long term memory in various financial markets. Andrew W. Lo (1991) tested the behavior of the daily and monthly stock returns indexes. The study found that there was little evidence of long-term memory in historical U.S. stock market returns. A similar study was conducted by Bio and Khaled, (2005) who found that the periods with large Hurst exponents could be predicted more accurately than those with Hurst exponents values close to random series. The study conducted by Prashanta, (2010) found that the markets have persistence, bias and demonstrated long memory effects. The study by Murugesan *et al.* (2011) found that the fractal structure existed in the BSE Sensex. A study illustrated by Bhattacharya and Bhattacharya, (2013) found that all the tests were consistent with long range dependence. In the case of Malaysia (KLSE), Lo statistic could not show long memory. Absence of long memory in return series of the indices shows no evidence against the weak form of market efficiency in stock returns. Enrico and Goddard (2005) examined the daily log returns series of the Mitbel, Dow-Jones Industrial Average and FTSE 100. The study found that the log return series of the Mitbel, Dow Jones Industrial Average and FTSE experienced antipersistence. Gayathri and Selvam (2011) found that there was Fractal Structure in the National Stock Exchange of India and the stock prices did not reflect the information in the past series of stock prices. The findings of the present study did not confirm the findings of Enrico and Goddard and, Gayathri and Selvam.

The Hurst exponent can be a powerful measurement of current market conditions and in the technical analysis of predicting the share price in the market. The returns series of sample companies have a fractal structure and these returns are predictable in the long term period. A similar study may be conducted in future is to investigate the long memory in various stock market indices with different time periods. A comparative study of stock returns between weekly and monthly returns may also be made to analyze the long range dependence using various methods like AFRIMA, Spectral Density or Wavelet.

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