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An investigation into non-normality of stock returns

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ABSTRACT

This paper proposes a new methodology to investigate the non-normality of stock returns, such as to do with what insights emerge from making use of the Mixture of Distribution Hypothesis. Typically, researchers check for non-normality using standardized residuals of GARCH, Jarque-Bera test or by making use of the built-in functions for descriptive statistics offered by various software. The major limitation of the above-mentioned methods is that those methods work under the assumption of parameters being constant over the time period under study. We propose a new method, called the K-month analysis to overcome the limitation due to this unrealistic assumption. This method is tested using data from five different stock indices. What we find is that in general, when parameters are held constant over a longer period, the kurtosis becomes increasingly significant indicating that the long term stock returns are not normally distributed but remain a mixture of normal.

Contribution/ Originality

We hereby declare that the paper titled "An investigation into non-normality of stock returns" is an orginal work of research carried out under the guidance and contribution of Dr. S. Maheswaran. This paper proposes a new method to analyze the non-normality of stock returns without relying on the unrealistic assumption of parameters being held constant throughout the time period under study.

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1. INTRODUCTION

The motivation behind this paper is the basic idea provided by the earlier researches that the stock price changes are independently and identically distributed but does not follow normal distribution through the capital asset pricing models makes an important underlying assumption of normality of stock returns.

In general the kurtotic nature of the distribution is analyzed by finding the kurtosis of the entire sample under study. However, we make use of the insights that emerge from mixture of distribution hypothesis to study the kurtotic nature of distribution of long term stock returns. This paper is an investigation into the non-normality of stock returns, such as to do with what insights emerge from making use of the Mixture of Distribution Hypothesis.

Typically, researchers check for non-normality using standardized residual of GARCH. This method could be misleading in the sense that the distribution of standard residuals depend heavily on the specification of the model. Other prominent methods to analyze the presence of non-normality are to rely on a set of descriptive statistics like mean, skewness, kurtosis, Jarque-Bera statistic etc. which could be derived using software such as E-Views. The limitation of depending on these statistics to check for deviation from normality is that all these methods consider the parameters constant over the entire sample period. This assumption of parameters being constant over the time period under study is unrealistic. Therefore, deviance from normality has to be checked under the conditions where the parameters are not held constant throughout the sample period.

We propose a new method to check the normality of the distribution. The technique is k-month analysis in which the parameters are considered constant over a k-month window with k ranging from 1,2,3,6,12..60 and whole sample period. The skewness, kurtosis as well as standard deviation of skewness and kurtosis for Indices is generated for each k-month window. Indices considered for the study are Nify (India), CAC (France), DAX (Germany), S&P 500 (U.S.A) and FTSE100 (U.K).
We observe that for all indices under study, the skewness is negative through-out the k-month window and for the whole sample and values of skewness are not statistically significant. Interestingly t-statistic for kurtosis (for all indices) is negative and significant for 1 month window. It remains insignificant for k = 2, 3 and gradually become increasingly significant as the parameters are allowed to be restricted over a longer period.

The analysis is explained in detail in following sections. Section 2 contains review of Literature. Section 3 consists of methodology, which consists of sub-sections 3.1, 3.2 which explains the theory, hypotheses respectively. The empirical analysis is described in Section 4 with sub-sections 4.1 containing data and analysis and 4.2 with empirical findings. Concluding remarks are incorporated in Section 5.

2. LITERATURE REVIEW

Prediction and distribution of stock returns are two important areas that have induced a large amount of research. Researches since early 1960s have pointed out that distribution of stock returns has bigger tails than a normal distribution (Fama, 1965; Mandelbrot, 1963; Mandelbrot and Taylor, 1967). Although financial theory rests heavily on the assumption of normality, series of daily stock returns display significant deviations from normality. A detailed analysis by Officer (1972) shows that the distributions appear to be "fat-tailed" when compared to the normal distribution. In addition, a number of properties that are not consistent with the stable hypothesis were observed. Mills (1995) provides an empirical analysis of the distribution of daily returns of the three London Stock Exchange indices in which empirical densities are fitted to each of the return distributions before their shapes are studied using Tukey's g- and h-distributions. The returns are characterized by highly non-Gaussian behavior. That is to say, the distributions are both skewed and extremely kurtotic.

The kurtotic nature of distribution of daily price changes can be well explained by Mixture of Distribution Hypothesis. Clark (1973) proposed the mixture of normal distributions to model the distribution of daily security prices. A study of the mixture of distributions hypothesis using transactions data has been carried out by Harris (1987) by assuming that the transactions take place at a uniform rate within a given time interval. Richardson and Smith (1994) proposed a direct test for the Mixture of Distributions Hypothesis for stock returns. It provides a framework in which the model's bivariate conditional normality can be tested under relatively weak assumptions regarding the daily flow of information. In particular, important parameters governing the distribution of information flow that cannot be observed are obtained. Linden (2001) developed a simple model of stock market returns in the context of Mixture of Distribution Hypothesis. The basic assumption of his study was that conditional return distribution is normal distribution for a stock i.e., Unconditional return distribution suffers from potential non-normality.

In this paper our focus is to analyze the non-normality of the distribution of stock returns making use of the insights emerging from the Mixture of Distribution Hypothesis.

3. METHODOLOGY

3.1. Theory

To investigate the non-normality of stock returns, we make use of the Mixture of Distribution Hypothesis.

Mixture of Distribution Hypothesis says that unconditionally, the returns on stock follows a mixture of normal distributions.

For a given variance, say $\sigma^2{}_t$ which is observed during time period 't', the stock returns follow normal distribution with mean zero and variance $\sigma^2{}_t$.

Let $\sigma^2{}_t = Y_t$
Return, $r_t \mid Y_t \sim N(0, Y_t)$
$\rightarrow r_t = Z_t \sqrt{Y_t}$ ……………………………….. (1)

In equation (1),
Z_t is independently and identically distributed ~ N(0,1)
Y_t itself is random.
Now, Consider the skewness of r_t,

$E[r_t^3] = E[Z_t^3] . E[Y_t^{3/2}]$
$E[Z_t^3] = 0$ { Z_t is standard normal}
So, $E[r_t^3] = 0$ ……………………………….. (2)

The Kurtosis of r_t is defined as,

$K = E[r_t^4] / [E(r_t^2)]^2$
$E[r_t^4] = E[E(Z_t^4 . Y_t^2 / Y_t)$
$= E(Z_t^4).E(Y_t^2)$
$= 3* E(Y_t^2)$
$Var(Y_t) = E(Y_t^2) – [E(Y_t)]^2$

Therefore,
$E[r_t^4] = 3[E(Y_t)]^2 + Var(Y_t)$
$K = 3\{1 + (Var(Y_t) / [E(Y_t)]^2)\}$ [Because $[E(r_t)^2] = E(Y_t)$] ……………………………….. (3)

Considering equations (1), (2) and (3), it can be concluded that

- Conditionally $r_t \mid Y_t \sim N(0, Y_t)$.
- Unconditionally $r_t = Zt \sqrt{Yt}$, ie. r_t follows mixture of normal distributions.
- Skewness of the series of stock returns would not be significantly different from zero.
- The sample kurtosis would be greater than or equal to 3 except when, variance of variance (var (Y_t)) is zero.

3.2. The k-month analysis

We propose a new procedure called k-month analysis to check for non-normality. Data is analyzed over k-month window ranging from 1,2,3,6,12,24…60..full sample is considered where the parameters are held constant within each k-month window. That is to say, when k = 1, we hold the parameters constant over a month which means the parameters are allowed to change after every k-month.

The formula for skewness and kurtosis (equations 2 & 3) holds the population. To use it effectively for the sample, we use the bootstrap technique in MATLAB. We consider 1000 bootstrap duplications of the sample with replacement. Using this method, we are able to generate random samples with same sample-size using random sampling with replacement. We generate 1000 random samples from the existing sample. The skewness and kurtosis for each sample generated is computed using k-month analysis. This technique enables us to observe the actual mean (for k-month), standard deviation (for k-month), naïve standard error (standard deviation/√total number of months) bootstrap mean and bootstrap standard error, t-statistic with respect to 0 and bootstrap means for skewness and kurtosis respectively. The bootstrap procedure helps to obtain reliable standard errors based on the finite sample distribution of the test statistics. This makes the procedure robust.

3.3. Hypotheses

The stock returns are calculated using the following formula.

r_t = *log (closing price at day t) – log (closing price at day t-1)*

We make use of the k-month analysis (explained in *Section 4.2*) to investigate the non-normality of the series of stock returns by finding skewness and kurtosis over a k-month window with k ranging from 1,2,3,6,12..60 and whole sample period.

We have 2 hypotheses to test for normality of the stock returns.

- Hypothesis I: For Skewness (α = 5%)
  Ho: Actual Mean of skewness = 0
  Ha: Actual Mean of skewness ≠ 0
- Hypothesis II: For Kurtosis (α = 5%)
  Ho: Actual Mean of kurtosis – 3 = 0
  Ha: Actual Mean of kurtosis - 3 ≠ 0

4. EMPIRICAL ANALYSIS

4.1. Data

Daily data of stock indices such as NIFTY, CAC, DAX, S&P500, FTSE100 from Jan 1996 to April 2015 is used. Log returns for the period are calculated using the closing price which is used for the study.

4.2. Analysis

We use the k-month analysis to check for non-normality. In this method, skewness and kurtosis are analyzed over k-month moving window ranging from 1,2,3,6,12,24…60..full sample is considered. The parameters are held constant within each k-month window.

Analyzing kurtosis for the entire sample may not provide evidence for deviation from normality but analyzing k-month window and slowly progressing to the full sample, helps understand the behavior of data better. Hence to test for non-normality this procedure is efficient relative to analysis of kurtosis of the full sample.

The skewness, kurtosis as well as standard deviation of skewness and kurtosis for each index is generated for each k-month window. To test the hypothesis stated in *Section 3.2*, we undertake t-tests, with the size of the test being α = 5%.

4.3. Empirical findings

The output of the analysis is summarized in the following tables. Each has four columns that represent k-month, time series means standard error and t-statistic. Table 1 to Table 5 contains output of k-month analysis of skewness whereas Table 6 to Table 10 contains output of k-month analysis of kurtosis for Nifty, CAC, DAX, S&P 500 and FTSE 100 respectively.

Table 1: Skewness of the daily returns of nifty index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	-0.016	0.057	-0.279
2	-0.089	0.085	-1.050
3	-0.123	0.106	-1.158
6	-0.174	0.148	-1.169
12	-0.220	0.195	-1.128
24	-0.285	0.240	-1.188
36	-0.298	0.255	-1.168
48	-0.291	0.279	-1.043
60	-0.280	0.275	-1.021
Full sample	-0.211	0.273	-0.756

Notes to Table 1: This table displays the k-month, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of Nifty Index.

Table 2: Skewness of the daily returns of CAC Index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	-0.011	0.054	-0.203
2	-0.055	0.083	-0.669
3	-0.081	0.099	-0.811
6	-0.102	0.121	-0.843
12	-0.106	0.139	-0.759
24	-0.082	0.148	-0.553
36	-0.062	0.155	-0.397
48	-0.039	0.163	-0.240
60	-0.006	0.163	-0.038
Full sample	-0.033	0.265	-0.123

Notes to Table 2: This table displays the k-month, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of CAC Index.

Table 3: Skewness of the daily returns of DAX Index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	-0.058	0.055	-1.046
2	-0.119	0.077	-1.557
3	-0.165	0.094	-1.764

6	-0.214	0.114	-1.876
12	-0.228	0.127	-1.795
24	-0.194	0.143	-1.358
36	-0.143	0.146	-0.984
48	-0.103	0.154	-0.669
60	-0.056	0.157	-0.356
Full sample	-0.124	0.147	-0.841

Notes to Table 3: This table displays the k-month, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of DAX Index.

Table 4: Skewness of the daily returns of S&P500 Index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	-0.037	0.064	-0.583
2	-0.113	0.098	-1.155
3	-0.151	0.125	-1.205
6	-0.185	0.170	-1.091
12	-0.191	0.212	-0.904
24	-0.192	0.240	-0.802
36	-0.172	0.246	-0.698
48	-0.163	0.266	-0.613
60	-0.137	0.257	-0.533
Full sample	-0.233	0.265	-0.879

Notes to Table 4: This table displays the k-month, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of S&P500 Index.

Table 5: Skewness of the daily returns of FTSE100 Index when K-months are considered at a time

K- Month	Time Series Mean	Standard Error	t-statistic
1	-0.0348	0.0598	-0.582
2	-0.0847	0.0832	-1.0181
3	-0.1126	0.1105	-1.0197
6	-0.1448	0.1413	-1.0244
12	-0.1619	0.1718	-0.9422
24	-0.1545	0.1882	-0.8211
36	-0.1487	0.1947	-0.7639
48	-0.1491	0.2072	-0.7195
60	-0.1439	0.2126	-0.6769
Full sample	-0.1611	0.1919	-0.8393

Notes to Table 5: This table displays the k-month, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of FTSE100 Index.

It can be observed that for all indices, the skewness is negative through-out the k-month window and for the whole sample. The values of t-statistic are not statistically significant at 95% confidence interval, therefore we do not reject the null hypothesis of actual skewness being zero. This result is in agreement with the theoretical value of skewness (Section 3.1) derived using the Mixture of Distribution Hypothesis. Therefore, the value of skewness does not indicate any deviation from normality. However, a conclusion cannot be arrived at without looking into the results of K-month analysis of kurtosis.

Table 6: Kurtosis of the daily returns of Nifty Index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	2.727	0.119	-2.289
2	3.372	0.261	1.423
3	3.792	0.397	1.995
6	4.707	0.771	2.214
12	5.733	1.209	2.261
24	7.330	1.527	2.836
36	8.096	1.780	2.863
48	8.853	1.870	3.130
60	9.399	2.031	3.151
Full sample	9.613	1.968	3.273

Notes to Table 2: This table displays the k-months, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of Nifty Index.

Table 7: Kurtosis of the daily returns of CAC Index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	2.668	0.1182	-2.8065
2	3.091	0.2184	0.4152
3	3.326	0.3053	1.0687
6	3.844	0.4473	1.8874
12	4.460	0.5615	2.6006
24	5.112	0.6095	3.4645
36	5.772	0.6276	4.4163
48	6.460	0.6704	5.1606
60	7.126	0.6780	6.0858
Full sample	7.441	0.6329	7.0171

Notes to Table 7: This table displays the k-months, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of CAC Index.

Table 8: Kurtosis of the daily returns of DAX Index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	2.664	0.113	-2.965
2	3.142	0.219	0.649
3	3.438	0.273	1.603
6	3.981	0.394	2.492
12	4.498	0.485	3.090
24	5.206	0.577	3.824
36	5.853	0.587	4.862
48	6.513	0.619	5.676
60	7.105	0.639	6.426
Full sample	7.099	0.591	6.937

Notes to Table 8: This table displays the k-months, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of DAX Index.

Table 9: Kurtosis of the daily returns of S&P Index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	2.816	0.133	-1.383
2	3.333	0.309	1.078
3	3.639	0.452	1.413
6	4.184	0.749	1.582
12	4.740	1.016	1.712
24	5.553	1.263	2.022
36	6.248	1.320	2.460
48	7.104	1.410	2.911
60	7.876	1.443	3.380
Full sample	10.750	1.304	5.944

Notes to Table 9: This table displays the k-months, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of S&P500 Index.

Table 10: Kurtosis of the daily returns of FTSE100 Index when K-months are considered at a time

K-Month	Time Series Mean	Standard Error	t-statistic
1	2.5721	0.1189	-3.5982
2	2.9952	0.2665	-0.0180
3	3.1996	0.3689	0.5412
6	3.6372	0.5836	1.0920
12	4.1602	0.7968	1.4560
24	4.8768	0.9059	2.0718
36	5.6691	0.9447	2.8254
48	6.4611	0.9696	3.5697
60	7.2697	0.9972	4.2816
Full sample	8.7658	0.9198	6.2688

Notes to Table 10: This table displays the k-months, time series mean skewness , standard error and t-statistic during the k-month window ranging from 1,2,3,6,12,24…60..full sample (Jan 1996 to April 2015) of FTSE Index.

Excess kurtosis for all indices is negative and significant for k-month equal to one. However the value of kurtosis remains insignificant for k-month = 2,3 and gradually become increasingly significant. In case of Nifty and DAX for k-month ≥ 6 (refer Table 6 and Table 8), the t-statistic is significant indicating that Hypothesis II (Section 3.2) can be rejected with 95% confidence interval. The t-statistic is significant from k-month ≥ 12 for CAC (refer Table 7) which provides evidence for non-normality. For S&P500 and FTSE100, Hypothesis II (Section 3.2) can be rejected for k-month ≥ 24 (refer Table 9 and Table 10). We find that when the parameters are held constant over a longer period, t-statistic becomes significant. Therefore the distribution of stock distribution exhibits excess kurtosis which indicates non-normality.

5. CONCLUSION

In this paper, we have undertaken a study of the non-normality stock returns in the Indian stock market as well as stock returns in developed markets like France, Germany, U.S.A and U.K using the insights from the Mixture of Distribution Hypothesis. We proposed a new procedure called k-month analysis in which we measure the skewness and kurtosis of the long-term stock returns over a k-month window ranging from 1,2,3,6,12,24…60..full sample, where the parameters are held constant within each k-month window. What we find is that in general, when parameters are held constant over longer period, the kurtosis becomes increasingly significant. In particular for k-month ≥ 6, the series of long-term returns of Nifty and DAX is leptokurtic, indicating non-normality. Whereas the point at which long-term returns exhibit deviation from normality is $k \geq 12$ for CAC and $k \geq 24$ for S&P500 and FTSE100. However, the value of skewness for the series of long-term stock returns is insignificant over the k-month window and full sample (for all indices). Considering the leptokurtic nature of the distribution of long-term stock returns we can say that long-term returns are not normally distributed but remain a mixture of normals.

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