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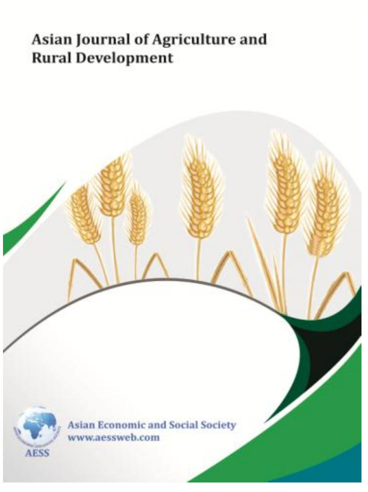

An Examination of the Crop Acreage Allocation Decision Process under Uncertainty

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Abstract

This study seeks to determine the workings of a system of acreage allocation given price and yield uncertainty so as to identify the role that uncertainty in market output has in acreage allocation decisions. This study adapts expected utility as developed by Chavas and Holt. The major findings of this study are as follows: 1) the effect of wealth in acreage decisions depends not only on the risk preference of the farm but also on the risk in and the structure of the output market, 2) violation of symmetry can come from (i) differing expectations regarding yield and price, (ii) risk differences in the price and yield for each crop, and/or (iii) different risk preferences among farm households, 3) the non-negativity of own compensated acreage effects would be satisfied if Proposition 1 or 2 holds, 4) production decisions would be affected by price and yield risk even when all input and output prices change proportionally, in which case homogeneity would not be satisfied without Proposition 1, and 5) symmetry of expected price and yield requires additional restrictions on the ratio of expected yields and prices and responsiveness of acreage to cross yield and price.

Keywords: Acreage Allocation, Uncertainty, Price, Yield, Risk Preference

Introduction

The acreage allocation decision is related to fixed input information and quasi-fixed output information. At planting time, a farm household allocates acreage under the premise of relating certain production factors (e.g., input costs) in some sort of relationship with assumptions regarding what market conditions will be at the time of harvest (e.g., market price given certain quantities). Uncertainty is directly related to price and yield in the output market. The randomness of price is represented by $p_i = \bar{p}_i + e_i$ where $e_i = h(x,\alpha)$, $E[e_i] = 0$ and $E[p_i] = \bar{p}_i$. The variable x represents a set of infinite demand shifting variables and α represents a set of passive parameters of the variables. Therefore, price will deviate from expected price $E[p_i]$ if $\sum_i \alpha_i \neq 0$ in a crop year. One example of price deviation could come from changes in consumer tastes and/or population diversity. Similarly, the randomness of yield is represented by $y_i = \bar{y}_i + \varepsilon_i$ where $\varepsilon_i = f(z,\beta), E[\varepsilon_i] = 0$ and $E[y_i] = \bar{y}_i$. The variable z represents a set of input factors uncontrolled by the farm household and β represents a set of passive parameters of the variables. Therefore, yield will deviate from expected yield $E[y_i]$ if $\sum_i \beta_i \neq 0$ in a crop year. One example of yield deviation could come from unusual and uncontrollable events of nature in a crop year (e.g., drought, pestilence, etc.).

Related to uncertainty, there are numerous studies that provide evidence showing that risk and risk preference are important factors in agricultural production decisions (Behrman, 1968; Just, 1974; Lin, 1977). Chavas and Pope (1985) examined expected utility maximizing conditions in allocating input factors under output price uncertainty for any risk preference and probability structure. In their study, they indicated that risk responsive behavior under uncertainty influences output supply and input factor demand and that those effects are critical if firms are not risk-neutral. Chavas and Holt (1990) developed an acreage supply response model using a specified expected utility function. In their study, they indicated that risk and wealth play an important role in determining acreage allocations.

Using these previous studies as a starting point, this study develops a system of acreage allocation under price and yield uncertainty to identify acreage response to changes in wealth and risk. As previous studies have indicated, wealth and risk effects on acreage response will depend on the risk preference of a particular farm household. Therefore, any such system should show how wealth and risk effects on acreage response depend on the farm household's risk preference. In addition, a system derived from expected utility maximizing procedure will be examined under the auspices of fundamental postulates of classical microeconomic theory. These include symmetry, positive semi-definiteness, and homogeneity. In order to achieve these objectives, this study will discuss model development in section two, properties of the acreage equations in section three, elasticities of net profit, price, and yield in section four, and present implications and concluding remarks in the final section.

Model Development

This analysis starts by defining a percentage (or share) for arable crop acreage. The total acreage of a farm household is defined as $L = \sum_{i=1}^{n} A_i$ where A_i is the number of acres devoted to crop *i* and *L* is the total acreage available for producing *n* crops. Individual farm size typically differs across farms so that the value of *L* is different depending upon the specifications for each individual farm household. To eliminate this difference among farm households, the acreage constraint can be modified into percentage (or share) form and expressed as $\sum_{i=1}^{n} a_i = 1$ where a_i is acreage share of crop $i\left(a_i = A_i / L\right)$. The sum of share acreage allocated to *n* crops will be equal to one regardless of differences in individual farm households. Also, the acreage share allows for the acreage constraint be defined as

(1) $$\left(\sum_{i=1}^{n} a_i\right)^k = 1.$$

Consider a farm household producing *n* crops where y_i is yield of the *i*th crop per acre and p_i is the corresponding market price, $i = 1, \ldots, n$, then the share of acreage revenue is given as

(2) $$r = \sum_{i=1}^{n} p_i y_i a_i .$$

Denoting the cost of production per acre of the *i*th crop as c_i, the total share acreage cost of agricultural production is expressed as

(3) $$c = \sum_{i=1}^{n} c_i a_i .$$

Since output prices $p = \left(p_1, \ldots, p_n\right)$ and crop yields $y = \left(y_1, \ldots, y_n\right)$ are not observed by a farm household when production decisions are made, share of acreage revenue (*r*) is a variable which we associate as being 'risky'. In contrast, total cost is fixed because input prices are known at the time crop acreages are allocated.

Now, let the budget constraint of a farm household be represented by:

(4) $$G = w + \sum_{i=1}^{n} \pi_i a_i ,$$

Where *G* represents all goods purchased by the farm household, *w* denotes exogenous income (or wealth), and $\pi_i = p_i y_i - c_i$ is net profit of crop *i*. *G*, *w*, and π_i (p_i and c_i) are assumed to be numéraire normalized by a consumer price index, *q*. Therefore, *G* also denotes the farm household's consumption expenditures. Equation (4) states that wealth (*w*) plus farm profit (π) is equal to consumption expenditures (*G*).[1]

If a farm household recognizes that the expected net profit for crop *i* is greater than that for crop *j* $\left(\bar{\pi}_i = E[\pi_i] > \bar{\pi}_j = E[\pi_j]\right)$ at planting, then a profit maximizing farm household will allocate all arable acreage to crop *i*. Therefore, we need an assumption in order to identify the effects of the random components of price and yield on acreage decisions that the expected net profit of crops are equal to each other, $\bar{\pi}_i = \bar{\pi}_j$. However, as has been alluded to, *real* net profit is different (

$\pi_i \neq \pi_j$) because of the random nature of the components of price (e) and yield (ε).

In addition, let us assume that a farm household's preferences are represented by a von Neumann-Morgenstern utility function (denoted $U(G)$) satisfying the necessary condition of concavity. If a farm household maximizes expected utility under competition, then the decision model is then expressed as

(5) $$L = EU(\Pi) + \lambda\left[1 - \left(\sum_{i=1}^{n} a_i\right)^k\right],$$

Where E is the expectation operator over the random variables and $\Pi = w + \sum_{i=1}^{n} \pi_i a_i$ represents wealth.

This formulation illustrates that an acreage decision is made under uncertainty because of the random nature attributed to the variables of *yield* and *price* with given probability distributions. Consequently, the expectation, E, in (5) is taken over the uncertain variables p and y and is based on the information available to a farm household at the time of planting. The utility function $U(\cdot)$ is assumed to be monotonically increasing in wealth at a decreasing rate. Also, under the assumption of competitiveness, the decision variable (a) does not influence the probability distributions of p and y.

The economic optimization problem (5) for the acreage allocation decision explains (*i*) risk and the attitude to risk as having a tangible effect on the acreage allocation process, and (*ii*) it examines the system of acreage allocation equations under the postulates of classical microeconomic theory which include symmetry, positive semi-definiteness, and homogeneity. In brief, we let a^* denote the optimal acreage choice in (5), such a choice depends on the expected wealth and the distribution of expected profit. In other words, the optimal acreage decision can be expressed as $a^*(w;\bar{\pi};\sigma)$.

From equation (5), the n first-order conditions to a farm household expected utility maximization problem are given by

(6.1) $$L_{a_i} = \frac{\partial L}{a_i} = E(U_\Pi \Pi_{a_i}) - \lambda k a_i \left(\sum_{i=1}^{n} a_i\right)^{n-1} = 0,$$

and

(6.2) $$L_\lambda = \frac{\partial L}{\partial \lambda} = 1 - \left(\sum_{i=1}^{n} a_i\right)^k = 0.$$

Then, n optimal acreage equations are defined as follows:

(7) $$a_i^* = \frac{E(U_\Pi \Pi_{a_i})}{E\left(\sum_{j=1}^{n} U_\Pi \Pi_{a_j}\right)}, \qquad i = 1,\dots,n.$$

Properties of the System of Acreage Share Equations

Wealth Effect

Sandmo (1971) and Chavas and Holt (1990) have examined the relationship between wealth effects, $\partial a^* / \partial w$, and the nature of risk preference. Their studies indicate that a wealth effect of zero implies constant absolute risk aversion, while a non-zero wealth effect corresponds to non-constant absolute risk aversion. In order to examine their findings, equation (7) can be differentiated in terms of wealth (w),

(8) $$\frac{da_i^*}{dw} = \frac{E(U_{\Pi\Pi} \Pi_{a_i} \Pi_w) E\left(\sum_{j\neq i}^{n} U_\Pi \Pi_{a_j}\right)}{E\left(\sum_{j=1}^{n} U_\Pi \Pi_{a_j}\right)^2},$$

where $E(U_{\Pi\Pi} \Pi_{a_i} \Pi_w)$
$= (\bar{p}_i \bar{y}_i - c_i)\bar{U}_{\Pi\Pi} + (\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)(U_{\Pi\Pi} - \bar{U}_{\Pi\Pi})$

- See Appendix I. $\bar{U}_{\Pi\Pi}$ represents an expected change in the marginal utility of wealth (Π) when w changes and $U_{\Pi\Pi}$ represents a real change in the marginal utility of wealth (Π) when w changes. The probability of w having a specific value in $\Re^+$ is $0 \le \phi(h) \le 1$ and $\int_0^h \phi(w)dw = 1$.

The properties of wealth's effect on the acreage allocation decision can be summarized in the following *Propositions*.

Proposition 1. *A zero wealth effect will be satisfied if a farm household is constantly absolute risk averse.*

Proposition 2. *A zero wealth effect will be satisfied under non-constant absolute risk averse if disturbances of price and yield are zero. Conversely, a zero effect of wealth will not be satisfied under non-constant absolute risk aversion if disturbances of price and yield are not zero.*

Proof

If a farm household is constant absolute risk averse, then $E\left(U_{\Pi\Pi}\Pi_{a_i}\right)$ will be zero regardless of risk of price, yield, and market condition. Thus, $\partial a_i^* / \partial w = \partial a_j^* / \partial w = 0$ in equation (8). Also, when *e* and *ε* are zero with $py = c$ in the competitive market, $E\left(U_{\Pi\Pi}\Pi_{a_i}\Pi_w\right)$ will be zero regardless of whether a farm household is constant or non-constant absolute risk averse. That is, $\partial a_i^* / \partial w = \partial a_j^* / \partial w = 0$ in equation (8). As a result, a zero effect of wealth (*w*) on acreage decisions will depend on that farm household's risk preference, risk preference with respect to the output market, and risk preference with respect to output market structure.

Farm Effect

In equation (7), $E\left(U_\Pi \Pi_{a_i}\right)$
$= \overline{U}_\Pi \overline{\pi}_i + \left(U_\Pi - \overline{U}_\Pi\right)\overline{p}_i \varepsilon_i + \left(U_\Pi - \overline{U}_\Pi\right)\overline{y}_i e_i + \left(U_\Pi - \overline{U}_\Pi\right) e_i \varepsilon_i$

See Appendix I. $\overline{U}_\Pi$ represents the expected marginal utility of wealth, (Π), which is expressed as $\overline{U}_\Pi = U_\Pi \int_0^h \phi(w) dw$. U_Π represents the real marginal utility of wealth (Π) as *w* changes from h_0 to h_1. Therefore, *n* differential acreage allocation equations can be obtained from differentiating equation (7) in terms of $\overline{\pi}$, e, ε, and $e\varepsilon$. This is illustrated mathematically as

$$da_i^* = \sum_{j=1}^n \gamma_{ij} d\overline{\pi}_j + \sum_{j=1}^n \phi_{ij} de_j + \sum_{j=1}^n \varphi_{ij} d\varepsilon_j + \sum_{j=1}^n \psi_{ij} de_j \varepsilon_j \tag{9}$$

.

The own- and cross-parameters of γ in equation (9) are

$$\gamma_{ii} = \frac{\overline{U}_\Pi \left(\sum_{j=1}^n \Phi_j - \overline{U}_\Pi \overline{\pi}_i \right)}{\left(\sum_{j=1}^n \Phi_j \right)^2}, \text{ and} \tag{10.1}$$

$$\gamma_{ij} = \frac{-\overline{U}_\Pi \overline{\pi}_i \overline{U}_\Pi}{\left(\sum_{j=1}^n \Phi_j \right)^2}, \tag{10.2}$$

where $\Phi_j = \overline{U}_\Pi \overline{\pi}_j + \left(U_\Pi - \overline{U}_\Pi\right)\overline{p}_j \varepsilon_j + \left(U_\Pi - \overline{U}_\Pi\right)\overline{y}_j e_j + \left(U_\Pi - \overline{U}_\Pi\right) e_j \varepsilon_j$.

The parameters of ϕ, φ, and ψ are presented in Appendix II.

Symmetry and positive semi-definite restrictions with respect to optimization for equation (5) are related to the compensated wealth acreage effect, assuming that one holds utility constant. The compensated wealth acreage effect takes the form

$$\frac{da_i^c}{d\overline{\pi}_j} = \frac{da_i^*}{d\overline{\pi}_j} - \frac{da_i^*}{dw} \cdot a_i^*, \quad i, j = 1, \ldots, n. \tag{11}$$

where $\partial a_i^c / \partial \overline{\pi}_j$ is the wealth compensated acreage effect of crop *j* on crop *i* maintaining constant utility of a farm household. The matrix of compensated effects is symmetric and positive semi-definite (Chavas, 1987). As Chavas and Holt (1990) indicated, equation (11) also implies that the slope of the uncompensated function $\partial a_i^* / \partial \overline{\pi}_j$ can be decomposed into the sum of two terms: the compensated slope (or substitution effect) $\partial a_i^c / \partial \overline{\pi}_j$ which maintains a given level of utility and the wealth effect $\left(\partial a_i^* / \partial w \cdot a_i^*\right)$.

Symmetry

The cross compensated acreage effects are derived using equations (7), (8), and (10.2) as follows:

(12)

$$\frac{da_i^c}{d\bar{\pi}_j} = \frac{-\overline{U}_\Pi \bar{\pi}_i \overline{U}_\Pi}{\left(\sum_{j=1}^{n} \Phi_j\right)^2} - \frac{(\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)(U_{\Pi\Pi} - \overline{U}_{\Pi\Pi})\left(\sum_{j \neq i}^{n} \Phi_j\right)\Phi_i}{\left(\sum_{j=1}^{n} \Phi_j\right)^3}.$$

Equation (12) indicates that cross-compensated acreage effects will depend upon the level of uncertainty and the expected values for price and yield. Therefore, the general condition of symmetry under uncertainty cannot be satisfied because disturbances are different for price and yield. However, if *Proposition 1* or *2* holds, then the symmetry condition will be satisfied.

Proof

Since the expected net profit of crop i is equal to the expected net profit of crop j,

$$\gamma_{ij} = \frac{-\overline{U}_\Pi \bar{\pi}_i \overline{U}_\Pi}{\left(\sum_{i=1}^{n} \Phi_i\right)^2} = \frac{-\overline{U}_\Pi \bar{\pi}_j \overline{U}_\Pi}{\left(\sum_{i=1}^{n} \Phi_i\right)^2} = \gamma_{ji}$$

Given condition, $\frac{da_i^c}{d\bar{\pi}_j} = \frac{da_j^c}{d\bar{\pi}_i}$ because $(\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)$ will be zero when *Proportion 1* holds or Φ_i will be zero when *Proportion 2* holds.

Therefore, any violation of the symmetry condition related to uncertain output market at planting time stems from 1) a farm household that has a different expectation with respect to crop yield and price, 2) price and yield risk for each crop is different, and/or 3) an individual farm household has a different/unique risk preference.

Positive Semi-definite

Own compensated acreage effects are derived by using equations (7), (8), and (10.1) as

$$\frac{da_i^c}{d\bar{\pi}_i} = \frac{\overline{U}_\Pi\left(\sum_{j \neq i}^{n} \Phi_j + (\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)\right)}{\left(\sum_{j=1}^{n} \Phi_j\right)^2} + \frac{(\bar{y}_i e_i + \bar{p}_i \varepsilon_i + e_i \varepsilon_i)(\overline{U}_{\Pi\Pi} - U_{\Pi\Pi})\left(\sum_{j \neq i}^{n} \Phi_j\right)\Phi_i}{\left(\sum_{j=1}^{n} \Phi_j\right)^3}. \quad (13)$$

Therefore, the general condition of positive semi-definiteness under uncertainty indicates that the non-negativity of own-compensated acreage effects will depend upon 1) price and yield risk (direction and magnitude), 2) expected values of price and yield, and 3) the relative strength between $U_{\Pi\Pi}$ and $\overline{U}_{\Pi\Pi}$. Regardless of a farm household's risk preference, violation of the non-negativity condition implies that the acreage allocated for crop i can decrease even when the expected net profit for crop i increases. This occurs because price and yield risks are high (in absolute terms) and a change in the real marginal utility of wealth of a farm household is relatively small compared to a change in expected marginal utility of wealth. However, if *Proposition 1* or *2* holds, then the non-negativity condition of the own-compensated acreage effect will be satisfied.

Proof

Under *Proposition 1* or *2*, the own compensated acreage effects will be reduced into the form $\frac{da_i^c}{d\bar{\pi}_i} = \frac{\overline{U}_\Pi\left(\sum_{j \neq i}^{n} \overline{U}_\Pi \bar{\pi}_j\right)}{\left(\sum_{j=1}^{n} \overline{U}_\Pi \bar{\pi}_j\right)^2}$. We know that expected marginal utility is positive $\left(\overline{U}_\Pi = U_\Pi \int_0^h \phi(\pi_i) d\pi_i \geq 0\right)$. Also, allocation of acreage to crop j requires that expected revenue in producing crop j be greater than or equal to the total cost of producing crop j, implying $\bar{\pi}_j \geq 0$.

Therefore, $\frac{da_i^c}{d\bar{\pi}_i} \geq 0$ when *Proposition 1* or *2* holds.

Homogeneity

Regardless of a farm household's risk preference, the homogeneity condition is defined as:

(14)

$$\sum_{j=1}^{n}\frac{da_i^c}{d\bar{\pi}_j}=\frac{\bar{U}_\Pi\sum_{j=1}^{n}\left(\bar{y}_j e_j+\bar{p}_j\varepsilon_j+e_j\varepsilon_j\right)}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}=0\cdot$$

The general condition of homogeneity depends, therefore on e_j, ε_j, and $e_j\varepsilon_j$. This implies that production decisions can be affected by risks associated with price and yield even when all input and output prices change proportionally. In this case, homogeneity would not be satisfied without *Proposition 1*.

Acreage Elasticities

To study the relation between acreage elasticity of expected net profit and acreage elasticity of expected price and yield under uncertainty, this study will consider the shift in $\bar{\pi}_i$ brought about by changes in $\bar{p}_i$ and $\bar{y}_i$. Such a change in $\bar{\pi}_i$ is defined as follows:

(15) $$d\bar{\pi}_i=\bar{y}_i d\bar{p}_i+\bar{p}_i d\bar{y}_i\,.$$

Substituting (15) into (7), we obtain *n* differential acreage equations defined in terms of $\bar{p}$ and $\bar{y}$ as follows:

(16)

$$da_i^*=\sum_{j=1}^{n}\xi_{ij}d\bar{p}_j+\sum_{j=1}^{n}\zeta_{ij}d\bar{y}_j+\sum_{j=1}^{n}\phi_{ij}de_j+\sum_{j=1}^{n}\varphi_{ij}d\varepsilon_j+\sum_{j=1}^{n}\psi_{ij}de_j\varepsilon_j\,.$$

The own- and cross-coefficients of expected price elasticity (ξ) in equation (16) are

(17.1) $$\xi_{ii}=\frac{\left[\bar{U}_\Pi\bar{y}_i+\left(U_\Pi-\bar{U}_\Pi\right)\varepsilon_i\right]\sum_{j\neq i}^{n}\Phi_j}{\left(\sum_{j=1}^{n}\Phi_j\right)^2},$$

(17.2) $$\xi_{ij}=\frac{-\left[\bar{U}_\Pi\bar{y}_i+\left(U_\Pi-\bar{U}_\Pi\right)\varepsilon_i\right]\Phi_j}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}.$$

The own- and cross-coefficients of expected yield elasticity (ζ) in equation (16) are expressed as

(18.1) $$\zeta_{ii}=\frac{\left[\bar{U}_\Pi\bar{p}_i+\left(U_\Pi-\bar{U}_\Pi\right)e_i\right]\sum_{j\neq i}^{n}\Phi_j}{\left(\sum_{j=1}^{n}\Phi_j\right)^2},$$

(18.2) $$\xi_{ij}=\frac{-\left[\bar{U}_\Pi\bar{y}_i+\left(U_\Pi-\bar{U}_\Pi\right)\varepsilon_i\right]\Phi_j}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}.$$

Since the coefficients of expected price (ξ) and yield (ζ) elasticity are weighted by expected yield $(\bar{y}_i)$ and price $(\bar{p}_i)$, respectively, symmetry conditions do not hold even when *Propositions* 1 and 2 hold because the expected yield of crop *i* could be different from the expected yield of crop *j* ($\bar{y}_i\neq\bar{y}_j$) and the expected price of crop *i* could be different from the expected price of crop *j*($\bar{p}_i\neq\bar{p}_j$). However, this relationship can identify the cross-effects of expected price and yield on acreage allocation when the condition $\frac{da_i^*}{d\bar{\pi}_j}=\frac{da_j^*}{d\bar{\pi}_i}$ is satisfied in which case *Propositions* 1 or 2 hold.

The cross effects of expected price and yield on acreage allocation can then be expressed by the following equations,

(19.1)

$$\frac{da_i^*}{d\bar{p}_j}=\frac{\bar{y}_j}{\bar{y}_i}\frac{da_j^*}{d\bar{p}_i}+\frac{\bar{y}_j}{\bar{p}_i}\frac{da_j^*}{d\bar{y}_i}-\frac{\bar{y}_j}{\bar{p}_j}\frac{da_i^*}{d\bar{y}_j}$$ and

(19.2)

$$\frac{da_i^*}{d\bar{y}_j}=\frac{\bar{p}_j}{\bar{p}_i}\frac{da_j^*}{d\bar{y}_i}+\frac{\bar{p}_j}{\bar{y}_i}\frac{da_j^*}{d\bar{p}_i}-\frac{\bar{p}_j}{\bar{y}_j}\frac{da_i^*}{d\bar{p}_j}.$$

The mathematical procedure to derive equations (19.1) and (19.2) is discussed in Appendix III.

As seen in the above equations, the symmetry conditions of expected price and yield would not be satisfied without additional restrictions with respect to the ratios of expected yield and prices of crops *i* and *j* as well as cross-responsiveness of acreage to changes in expected yield and price.

For example, the cross-effect of expected price of crop *j* on acreage of crop *i* would be equal to the cross-effect of expected price of crop *i* on the acreage of crop *j* only when the expected yield of crop *i* is equal to the expected yield of crop *j* and the cross effects of expected yields of crops *i* and *j* on the acreage allocations of crops *j* and *i* are zero.

Conclusions

This study developed a system of acreage allocation under price and yield uncertainty to identify the role that output market uncertainty plays in acreage allocation. The major findings of this study are as follows. First, a zero effect of wealth in acreage decisions would be satisfied regardless of uncertainty with regards to price, yield and market condition, if a farm household is constant absolute risk averse. Undoubtedly, when price and yield are determined in a competitive market (i.e., e and ε are zero with $py = c$), a zero effect of wealth in acreage decisions would be satisfied regardless whether a farm household is constant or non-constant absolute risk averse. These results imply that a zero effect of wealth on acreage decisions would depend on 1) risk preference and/or 2) the degree of uncertainty and structure of the output market.

The pre-requisite symmetric condition of the Hessian matrix of expected utility would not be satisfied because disturbances of price and yield are different. However, the symmetric condition would be satisfied if *Proposition 1* or *2* holds. This implies that a violation of symmetry might come from 1) different expectation of a farm household on yield and price, 2) different acceptable risk levels of yield and price for each crop, and 3) different risk preference of individual farm households.

The non-negativity of the diagonal elements of the Hessian matrix of expected utility would depend on 1) risk of price and yield (direction and magnitude), 2) expected values of price and yield, and 3) relative strength of changes in real and expected marginal utility $\left(U_{\Pi\Pi}\right.$ and $\left.\overline{U}_{\Pi\Pi}\right)$. Regardless of a farm household's risk preference, the violation of the non-negativity constraint under uncertainty implies that acreage of crop *i* can decrease even when the expected net profit of crop *i* increases. This is because price and yield risks are disproportionately skewed in a negative direction if it happens and thus a change in the real marginal utility of wealth of a farm household is relatively small (in magnitude) as compared to a change in the expected marginal utility of wealth. If *Propositions 1* or *2* hold however, the non-negativity condition of the diagonal elements of the Hessian matrix of expected utility would be satisfied.

Homogeneity under uncertainty would depend upon the variation (individually) and co-variation of both price and yield. This implies that production decisions would be affected by risk associated with the variability of price and yield even when all input and output prices change proportionally. In this case, homogeneity would not be satisfied without *Proposition 1*.

Unlike the conditions needed to achieve symmetry for expected profit, the symmetric conditions of expected price and yield will not be satisfied without imposing additional restrictions regarding the ratios of expected prices and yields of crops *i* and *j* as well as the cross responsiveness of acreage to changes in expected price and yield because the cross effects of expected price and yield on acreage allocation would depend upon the ratio of values of expected yields and prices as well as cross-responsiveness of acreage on changes in expected price and yield.

Footnote

1. See Chavas and Holt (1990)

References

Arnade, C. and D. Kelch (2007). Estimation of Area Elasticities from a Standard Profit Function. *American Journal of Agricultural Economics*, 89(3). 727-737.

Arrow, K. J. (1965). *Aspects of the Theory of Risk Bearing*, Helsinki: Johnsonin Saaatie.

Behrman, J. R. (1968). *Supply Response in Underdeveloped Agriculture*, New York: North-Holland Publishing Co.

Berndt, R. E. and N. E. Savin. (1975). Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances. *Econometrica,* 43(5/6). 937-958.

Chambers, G. R. and R. E. Just (1989). Estimating Multioutput Technologies. *American*

Journal of Agricultural Economics, 71(4). 980-995.

Chavas, Jean-Paul (1987). Constrained choices Under Risk. *Southern Economic Journal,* 53(3). 662-676.

Chavas, Jean-Paul and M. T. Holt (1990). Acreage Decisions Under Risk: The Case of Corn and Soybeans. *American Journal of Agricultural Economics,* 72(3). 529-538.

Chavas, Jean-Paul and R. D. Pope (1985). Price Uncertainty and Competitive Firm Behavior: Testable Hypotheses from Expected Utility Maximization. *Journal of Economics and Business,* 37(3). 223- 235.

Christopher, Ackello-Ogutu, Q. Paris. and W. A. Williams (1985). Testing a von Liebig Crop Response Function against Polynomial Specifications. *American Journal of Agricultural Economics,* 67(4). 873-880.

Coyle, B. (1999). Risk Aversion and Yield Uncertainty in Duality Models of Production: A Mean-Variance Approach. *American Journal of Agricultural Economics*, 81(3). 553-567.

Just, E. R. (1974). An Investigation of the Importance of Risk in Farmer's Decisions. *American Journal of Agricultural Economics,* 56(1). 14-25.

Just, R. and R. D. Pope (1978). Stochastic Specification of Production Functions and Economics Implications. *Journal of Econometrics*, 7. 67-86.

Lin, W. (1977). Measuring Aggregate Supply Response Under Instability. *American Journal of Agricultural Economics*, 59(5). 903-907.

Lin, W. and R. Dismukes (2006). Supply Response under Risk: Implications for Counter-Cyclical Payments' Production Impact. *Review of Agricultural Economics,* 29(1). 64-86.

Shumway, C. R. (1983). Supply, Demand, and Technology in a Multiproduct Industry: Texas Field Crops. *American Journal of Agricultural Economics,* 65(4). 748-760.

Sandmo, A. (1971). On the Theory of the Competitive Firm under Price Uncertainty. *American Economic Review,* 61, 65-73.

Appendix I: Expected Value of First and Second Derivative Utility Functions

Expected Value of First Derivative Utility Function

$$E\left(U_\Pi \Pi_{a_i}\right)$$
$$= E\left[U_\Pi\left((\bar{p}_i + e_i)(\bar{y}_i + \varepsilon_i) - c_i\right)\right]$$
$$= E\left[U_\Pi\left(\bar{\pi}_i + \bar{p}_i\varepsilon_i + \bar{y}_i e_i + e_i\varepsilon_i\right)\right]$$
$$= \bar{\pi}_i E\left[U_\Pi\right] + \bar{p}_i E\left[U_\Pi \varepsilon_i\right] + \bar{y}_i E\left[U_\Pi e_i\right] + E\left[U_\Pi e_i \varepsilon_i\right]$$
$$= \bar{\pi}_i E\left[U_\Pi\right] + \bar{p}_i U_\Pi \varepsilon_i - \bar{p}_i\varepsilon_i E\left[U_\Pi\right] + \bar{y}_i U_\Pi e_i - \bar{y}_i e_i E\left[U_\Pi\right] + U_\Pi e_i \varepsilon - e_i\varepsilon_i E\left[U_\Pi\right]$$
$$= \left(\bar{\pi}_i - \bar{p}_i\varepsilon_i - \bar{y}_i e_i - e_i\varepsilon_i\right) E\left[U_\Pi\right] + \left(\bar{p}_i\varepsilon_i + \bar{y}_i e_i + e_i\varepsilon_i\right) U_\Pi$$
$$= \left(\bar{\pi}_i - \bar{p}_i\varepsilon_i - \bar{y}_i e_i - e_i\varepsilon_i\right) \overline{U}_\Pi + \left(\bar{p}_i\varepsilon_i + \bar{y}_i e_i + e_i\varepsilon_i\right) U_\Pi$$

where $\overline{U}_\Pi = E\left[U_\Pi\right] = U_\Pi \int_0^h \phi(a_i) da_i$.

$$= \overline{U}_\Pi \bar{\pi}_i + \left(U_\Pi - \overline{U}_\Pi\right)\bar{p}_i\varepsilon_i + \left(U_\Pi - \overline{U}_\Pi\right)\bar{y}_i e_i + \left(U_\Pi - \overline{U}_\Pi\right) e_i\varepsilon_i .$$

Expected Values of Second Derivative Utility Function

$$E\left(U_{\Pi\Pi} \Pi_{a_i} \Pi_w\right)$$

$= E\left(U_{\Pi\Pi} \Pi_{a_i}\right)$ because of $\Pi_w = \dfrac{d\Pi}{dw} = 1.$

$$= E\left(U_{\Pi\Pi}\left((\bar{p}_i + e_i)(\bar{y}_i + \varepsilon_i) - c_i\right)\right)$$
$$= E\left(U_{\Pi\Pi}\left(\bar{p}_i\bar{y}_i - c_i + \bar{y}_i e_i + \bar{p}_i\varepsilon_i + e_i\varepsilon_i\right)\right)$$

$$= (\bar{p}_i\bar{y}_i - c_i)E(U_{\Gamma\Pi}) + \bar{y}_i E(U_{\Gamma\Pi} e_i) + \bar{p}_i E(U_{\Gamma\Pi}\varepsilon_i) + E(U_{\Gamma\Pi} e_i\varepsilon_i)$$
$$= (\bar{p}_i\bar{y}_i - c_i)E(U_{\Gamma\Pi}) + \bar{y}_i (U_{\Gamma\Pi} e_i - e_i E(U_{\Gamma\Pi})) + \bar{p}_i (U_{\Gamma\Pi}\varepsilon_i - \varepsilon_i E(U_{\Gamma\Pi})) + U_{\Gamma\Pi} e_i\varepsilon_i - e_i\varepsilon_i E(U_{\Gamma\Pi})$$

$$= (\bar{p}_i\bar{y}_i - c_i)E(U_{\Gamma\Pi}) + \bar{y}_i e_i U_{\Gamma\Pi} - \bar{y}_i e_i E(U_{\Gamma\Pi}) + \bar{p}_i\varepsilon_i U_{\Gamma\Pi} - \bar{p}_i\varepsilon_i E(U_{\Gamma\Pi}) + e_i\varepsilon_i U_{\Gamma\Pi} - e_i\varepsilon_i E(U_{\Gamma\Pi})$$

$$= (\bar{p}_i\bar{y}_i - c_i - \bar{y}_i e_i - \bar{p}_i\varepsilon_i - e_i\varepsilon_i)E(U_{\Gamma\Pi}) + (\bar{y}_i e_i + \bar{p}_i\varepsilon_i + e_i\varepsilon_i)U_{\Gamma\Pi}$$
$$= (\bar{p}_i\bar{y}_i - c_i)E(U_{\Gamma\Pi}) + (\bar{y}_i e_i + \bar{p}_i\varepsilon_i + e_i\varepsilon_i)U_{\Gamma\Pi} - (\bar{y}_i e_i + \bar{p}_i\varepsilon_i + e_i\varepsilon_i)E(U_{\Gamma\Pi})$$
$$= (\bar{p}_i\bar{y}_i - c_i)E(U_{\Gamma\Pi}) + (\bar{y}_i e_i + \bar{p}_i\varepsilon_i + e_i\varepsilon_i)(U_{\Gamma\Pi} - E(U_{\Gamma\Pi}))$$
$$= (\bar{p}_i\bar{y}_i - c_i)\bar{U}_{\Gamma\Pi} + (\bar{y}_i e_i + \bar{p}_i\varepsilon_i + e_i\varepsilon_i)(U_{\Gamma\Pi} - \bar{U}_{\Gamma\Pi})$$

where $\bar{U}_{\Gamma\Pi} = E(U_{\Gamma\Pi}) = U_{\Gamma\Pi}\int_0^h \phi(a_i)da_i$.

Appendix II: The parameters of ϕ, φ, and ψ in Equation (11)

In equation (9), $a_i^* = \dfrac{\bar{U}_\Pi\bar{\pi}_i + (U_\Pi - \bar{U}_\Pi)\bar{p}_i\varepsilon_i + (U_\Pi - \bar{U}_\Pi)\bar{y}_i e_i + (U_\Pi - \bar{U}_\Pi)e_i\varepsilon_i}{\sum_{j=1}^{n}\left[\bar{U}_\Pi\bar{\pi}_j + (U_\Pi - \bar{U}_\Pi)\bar{p}_j\varepsilon_j + (U_\Pi - \bar{U}_\Pi)\bar{y}_j e_j + (U_\Pi - \bar{U}_\Pi)e_j\varepsilon_j\right]}$.

$$\frac{da_i^*}{de_i} = \phi_{ii} = \frac{(U_\Pi - \bar{U}_\Pi)\bar{p}_i\left(\sum_{j=1}^{n}\Phi_j - (U_\Pi - \bar{U}_\Pi)\bar{p}_i\varepsilon_i\right)}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}.$$

$$\frac{da_i^*}{de_j} = \phi_{ij} = \frac{-(U_\Pi - \bar{U}_\Pi)\bar{p}_j}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}.$$

$$\frac{da_i^*}{d\varepsilon_i} = \varphi_{ii} = \frac{(U_\Pi - \bar{U}_\Pi)\bar{y}_i\left(\sum_{j=1}^{n}\Phi_j - (U_\Pi - \bar{U}_\Pi)\bar{y}_i e_i\right)}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}.$$

$$\frac{da_i^*}{d\varepsilon_j} = \varphi_{ij} = \frac{-(U_\Pi - \bar{U}_\Pi)\bar{y}_j}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}.$$

$$\frac{da_i^*}{de_i\varepsilon_i} = \psi_{ii} = \frac{(U_\Pi - \bar{U}_\Pi)\left(\sum_{j=1}^{n}\Phi_j - (U_\Pi - \bar{U}_\Pi)e_i\varepsilon_i\right)}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}.$$

$$\frac{da_i^*}{de_j\varepsilon_j} = \psi_{ij} = \frac{-\left(U_\Pi - \overline{U}_\Pi\right)}{\left(\sum_{j=1}^{n}\Phi_j\right)^2}.$$

Appendix III: Price and Yield Effects on Acreage

If $\frac{da_i^*}{d\overline{\pi}_j} = \frac{da_j^*}{d\overline{\pi}_i}$ given *Proposition* 1 or 2, then

$\frac{da_i^*}{d\overline{\pi}_j} = \frac{1}{\overline{y}_j}\frac{da_i^*}{d\overline{p}_j} + \frac{1}{\overline{p}_j}\frac{da_i^*}{d\overline{y}_j}$ and

$$\frac{da_i^*}{d\overline{\pi}_j} = \frac{1}{\overline{y}_j}\frac{da_i^*}{d\overline{p}_j} + \frac{1}{\overline{p}_j}\frac{da_i^*}{d\overline{y}_j} = \frac{1}{\overline{y}_i}\frac{da_j^*}{d\overline{p}_i} + \frac{1}{\overline{p}_i}\frac{da_j^*}{d\overline{y}_i} = \frac{da_j^*}{d\overline{\pi}_i}.$$

Therefore, price effect of crop *j* on acreage of crop *i* is

$$\frac{da_i^*}{d\overline{p}_j} = \frac{\overline{y}_j}{\overline{y}_i}\frac{da_j^*}{d\overline{p}_i} + \frac{\overline{y}_j}{\overline{p}_i}\frac{da_j^*}{d\overline{y}_i} - \frac{\overline{y}_j}{\overline{p}_j}\frac{da_i^*}{d\overline{y}_j},$$

and yield effect of crop *j* on acreage of crop *i* is