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ECONOMIC GROWTH AND STRUCTURAL CHANGE – A SYNTHESIS OF THE WALRASIAN GENERAL EQUILIBRIUM, RICARDIAN DISTRIBUTION AND NEOCLASSICAL GROWTH THEORIES

Wei-Bin Zhang[1]

[1]*Ritsumeikan Asia Pacific University, Japan*

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ABSTRACT

This study proposes an economic growth model of structural change with heterogeneous households. It endogenously determine wealth accumulation and land value. Our model is based on three core theories in economics - Walrasian general equilibrium theory, Ricardian theory of distribution, and neoclassical growth theory. The paper is focused on effects of changes in determinants of the economic dynamics on income and wealth distribution and economic growth. We build an analytical framework for a disaggregate and microfounded general economic growth theory with endogenous wealth accumulation. We simulate the model. We find a unique equilibrium point and confirm stability. We plot the motion of the economy with three groups of households. We also conduct comparative dynamic analyses to get some insights into the complexity of economic growth and wealth and income distribution. For instance, we show that as the group with highest human capital increases its propensity to save, in transitory process the group's and the other two groups' wealth levels are enlarged, the national output experiences negative growth; while in the long term the group's and the other two groups' wealth levels continue to be enlarged, the national output experiences positive growth. We also demonstrate that if the population of the group whose human capital is lowest (highest) is increased, the wage rates and the wealth levels and consumption levels of consumer good of all the groups are reduced (enhanced).

Contribution/ Originality: The paper makes an original contribution to economic growth by integrating Walrasian general equilibrium theory, Ricardian theory of distribution, and neoclassical growth theory. The paper develops a model of dynamic interactions between economic structural changes, economic growth, wealth accumulation, and income and wealth distribution between heterogeneous households.

1. INTRODUCTION

Different economic theories have been proposed to study economic phenomena of different economic systems. The conventional distinction is between micro (household and firm behavior) and macro (nationally or globally aggregated models). There are also intermediate stages such as interactions between the agricultural sector and industrial sector. Explicit consideration of economic structures is necessary especially when we deal with dynamic phenomena. It is obviously important to develop an economic theory on the basis of microeconomic mechanism. In such a theory micro, intermediate and macro variables should be treated as an interdependent whole. This study builds an economic growth model with economic structure and heterogeneous households. The model determines land distribution and value. The model is based on three core theories in economics - Walrasian general equilibrium theory, Ricadian theory of distribution, and neoclassical growth theory.

It is well known that the Walrasian general equilibrium theory of pure exchange and production economies has played an import role in the development of contemporary general equilibrium theory. Walras initially proposed the general equilibrium theory (Walras, 1874). The theory was further formalized by Arrow (Arrow and Debreu, 1954; Arrow and Hahn, 1971; Arrow, 1974) Debreu and others (e.g., (Gale, 1955; Nikaido, 1956;1968; Debreu, 1959; McKenzie, 1959; Mas-Colell *et al.*, 1995)). The general equilibrium theory is essential for understanding economic mechanisms of production, consumption, and exchanges with heterogeneous industries and households. Nevertheless, this theory has not been further developed to explain modern economic phenomena. For instance, endogenous wealth has not been successfully introduced into the general equilibrium theory. Walras introduced saving and capital accumulation in his general equilibrium theory. But his treatments of capital accumulation are not proper, especially in the light of modern neoclassical growth model (Impicciatore *et al.*, 2012). Over years many attempts have been made to further develop Walras' capital accumulation within Walras' framework (e.g., (Morishima, 1964;1977; Diewert, 1977; Eatwell, 1987; Dana *et al.*, 1989; Montesano, 2008)). These approaches are not successful as they don't build the models on the basis of proper microeconomic foundation of wealth accumulation. Recently Impicciatore *et al.* (2012) proposed a model in which capital goods are saved by consumers in order to supply their services to the production sector in the next period. It is assumed that capital goods exiting in one period totally depreciate at the end of the period. The study does not properly explain household saving behavior. This study applies Zhang's alternative approach for modeling behavior of households with wealth accumulation. The model is a further development of the traditional Walrasian general equilibrium theory with wealth accumulation and heterogeneous households.

The Walrasian general equilibrium theory determines economic equilibrium. But it fails to address important issues about interdependent between economic growth, structural change, and wealth and income distribution. Ricardo dealt with income distribution. He tried to explain how a shift in this distribution could affect accumulation. His *On the Principles of Political Economy and Taxation* of 1817 made a very valuable contribution to economics. Applying the law of diminishing returns in agriculture, Ricardo made important development of the theory of rent. Ricardo's study show how that wages, interest rate, and rent can be determined within a compact theory. Ricardo considered three different production factors, labor, capital, and land. He provided a theory to determine the functional income shares of labor share, the capital, and the land rent in the total income. Ricardo (1821) stated: "The produce … is divided among three classes of the commodity, namely, the proprietor of land, the owners of the stock or capital necessary for its cultivation, and laborers by whose industry it is cultivated. But in different stages of the society, the proportions of the whole produce of the earth which will be allotted to each of these classes, under the names of rent, profits, and wages, will be essentially different; depending mainly on the actual fertility of the soil, on the accumulation of capital and population, and on the skill, ingenuity, and the instruments in agriculture." Since the publication of the *Principles*, many efforts have been made to generalize and extend the system. The beginning of modeling the Ricardian system perhaps dates back to 1833 with the model of Whewell (see Barkai (1959;1966)). Economists extended and generalized Ricardo's system in different ways (Cochrane, 1970; Caravale and Tosato, 1980; Morishima, 1989; Negishi, 1989). For instance, Samuelson (1959) studied Ricardo-like models to reveal validity and limitations of the Ricardian propositions in a wider setting than that assumed by Ricardo; Brems (1970) considered fixed capital in the Ricardian problem of machinery; Pasinetti (1960;1974) dealt with the validity of what Ricardo meant by making the necessary assumptions for Ricardian conclusions; Casarosa (1985) reformed the Pasinetti model to give a new interpretation of Ricardo with respect to the role of the market wages. What Ricardo (1821) observed long time ago is still proper to state the contemporary situation: "To determine the laws which regulate this distribution, is the principal problem in Political Economy: much as the science has been improved by the writings of Turgot, Stuart, Smith, Say, Sismondi, and others, they afford very little satisfactory information respecting the natural course of rent, profit, and wages." This study will make a further contribution to literature on the laws of wealth and income distribution with capital accumulation and land value in a general equilibrium framework.

Neither the Walrasian theory nor Ricardian theory provides a profound microeconomic mechanism of wealth accumulation. The neoclassical growth theory explicitly deals with endogenous wealth accumulation with microeconomic foundation. The representative model is the Ramsey model. We will apply an alternative approach to households by Zhang to examine dynamic interdependence between growth, wealth and income distribution, and economic structures. We synthesize the neoclassical growth theory, the Walrasian general equilibrium theory, and Ricardian theory of distribution into a single compact analytical framework. Although there are some studies on integrating the neoclassical growth theory with the general equilibrium analysis (e.g., Jensen and Larsen (2005)); Shoven and Whalley (1992) observed: "Most contemporary applied general models are numerical analogs of traditional two-sector general equilibrium models popularized by James Meade, Harry Johnson, Arnold Harberger, and others in the 1950s and 1960s." Only a few formal models have been developed to explicitly deal with distribution issues in the neoclassical growth theory (Solow, 1956; Burmeister and Dobell, 1970; Barro and Sala-i-Martin, 1995). From the history of analytical economics it is a well-known difficult problem to study economic growth and wealth and income distribution. As mentioned before, the Arrow-Debreu general economic theory studies economic equilibrium problems in economies of heterogeneous households and firms. It is desirable to integrate the economic mechanisms of the three core theories in economics into an integrated analytical framework. This study integrates the three theories by applying Zhang's approach to household behavior (Zhang, 1993). This study synthesizes the ideas in the three-sector model with economic structure and land (Zhang, 1996) and the growth model of heterogeneous groups by Zhang (2013). In Zhang's 1996 paper, the population is homogeneous, while in Zhang's 2013 paper neither land nor housing is considered. The rest of the paper is structured as follows. Section 2 builds the growth model of land distribution and housing with wealth and income distribution with. Section 3 studies analytical properties of the model and simulates the model for an economy of three types of households. Section 4 carries out comparative dynamic analysis with regard to the populations, the propensity to save, and propensity to consume housing. Section 5 concludes the study.

2. THE BASIC MODEL

This study deals with an economy with three - agricultural, capital goods and consumer goods - sectors. The agricultural sector supplies goods, such as vegetables, rice, and corn, for consumption. We follow the Uzawa two sector model in describing the capital goods and consumer goods sectors (Uzawa, 1961). In this study services are treated as consumer goods. We apply the neoclassical growth theory to describe the production sectors (Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). Capital goods are be employed as input factors in the three sectors. Capital is assumed to depreciates at a fixed rate δ_k, which is not related to the manner of use. The assets of the economy are owned by households. They use their incomes to consume and save. All the markets are assumed to be perfectly competitive. Factors are inelastically supplied. All factors are assumed to be fully utilized at every moment. Saving is not undertaken by firms.

The population is classified into multiple groups. Each group has a fixed population, $\bar{N}_j$, ($j = 1, \ldots, J$). In the Walrasian general equilibrium theory, $\bar{N}_j = 1$. We measure prices in terms of capital goods. We denote the wage rate of worker of type j and rate of interest by $w_j(t)$ and $r(t)$, respectively. Capital good is selected to serve as numeriare. Let $p_a(t)$ and $p_s(t)$ denote respectively the prices of agricultural commodity and consumer goods at time t. As all the markets are perfectly competitive and the land, capital, and labor force, are fully employed, the wage rate, $w_j(t)$, land rent $R(t)$, and $r(t)$, are identical in the economy. We denote the fixed land by L. The total capital stock $K(t)$ is allocated among the three sectors. We use subscript index, a, i, and s to represent agricultural sector, capital good sector, and consumer good sectors, respectively. Let $N_m(t)$ and $K_m(t)$ stand for the labor force and capital stocks employed by sector m. The total population $\bar{N}$ and total qualified labor supply N are

$$\bar{N} = \sum_{j=1}^{J} \bar{N}_j, \quad N = \sum_{j=1}^{J} h_j \bar{N}_j, \tag{1}$$

in which h_j is the human capital of group j. The assumption of labor force being fully employed implies

$$N_a(t) + N_i(t) + N_s(t) = N. \tag{2}$$

We introduce

$$k_m(t) \equiv \frac{K_m(t)}{N_m(t)}, \quad k(t) \equiv \frac{K(t)}{N(t)}, \quad m = a, i, s.$$

2.1. The Agricultural Sector

There are three input factors, land, labor force, and capital, in the agricultural production. We use $L_a(t)$ to denote the land used by the agricultural sector. Let $F_m(t)$ stand for the production function of sector m, $m = a, i, s$. We specify the agricultural sector's production function as follows

$$F_a(t) = A_a K_a^{\alpha_a}(t) N_a^{\beta_a}(t) L_a^{\varsigma}(t), \quad \alpha_a, \beta_a, \varsigma > 0, \ \alpha_a + \beta_a + \varsigma = 1, \tag{3}$$

where A_a, α_a, β_a, and ζ_a are parameters. The marginal conditions imply

$$r(t) + \delta_k = \frac{\alpha_a \, p_a(t) F_a(t)}{K_a(t)}, \quad w(t) = \frac{\beta_a \, p_a(t) F_a(t)}{N_a(t)}, \quad R(t) = \frac{\varsigma \, p_a(t) F_a(t)}{L_a(t)}, \tag{4}$$

2.2. The Capital Goods Sector

The production function of the industrial sector is specified as follows

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad \alpha_i, \beta_i > 0, \ \alpha_i + \beta_i = 1, \tag{5}$$

where A_i, α_i, and β_i are positive parameters. The capital goods sector employs two input factors, capital and labor force. Here, we omit land use possibly employed by the capital goods and consumer goods sectors. The marginal conditions for the capital goods sector are given by

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)}. \tag{6}$$

2.3. The Consumer Goods Sector

The production function of the consumer goods sector is specified as follows

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \quad \alpha_s + \beta_s = 1, \ \alpha_s, \beta_s > 0, \tag{7}$$

where A_s, α_s, and β_s are the technological parameter of the service sector. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_s p_s(t) F_s(t)}{K_s(t)}, \quad w(t) = \frac{\beta_s p_s(t) F_s(t)}{N_s(t)}. \tag{8}$$

2.4. Consumer Behaviors and Wealth Dynamics

We apply the model for describing behavior of households developed by Zhang (1993). We assume the public land ownership, which means that the revenue of land is distributed among the population equally. The total land revenue is given by $R(t)L$. Let $\bar{k}_j(t)$ and $\bar{r}(t)$ stand for per capita wealth of group j and per capita land revenue. We have

$$\bar{k}_j(t) = \frac{\bar{K}_j(t)}{\bar{N}_j}, \quad \bar{r}(t) = \frac{R(t)L}{\bar{N}},$$

where $\bar{K}_j(t)$ is the total wealth held by group j. Per capita current income from the interest payment $r(t)\bar{k}_j(t)$, the wage payment $h_j w(t)$, and land ownership $\bar{r}(t)$ is given by

$$w_j(t) = h_j w(t).$$

Per capita current income from the interest payment $r(t)\bar{k}_j(t)$, the wage payment $h_j w(t)$, and land ownership $\bar{r}(t)$ is given by

$$y_j(t) = r(t)\bar{k}_j(t) + h_j w(t) + \bar{r}(t).$$

The per capita disposable income is the current disposable income plus the value of wealth. That is

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t). \tag{9}$$

The household uses the disposable income for saving and consumption. The value, $\bar{k}_j(t)$, (i.e., $p(t)\bar{k}_j(t)$ with $p(t) = 1$) is a flow variable. We assume that selling wealth is conducted instantaneously and there is no transaction cost. We consider $\bar{k}_j(t)$ as the income that the household gets by selling the wealth. Accordingly, the household has the total amount of income $\hat{y}_j(t)$ to distribute between consumption and saving.

The representative household from group j would distribute the total available budget among savings $s_j(t)$, consumption of agricultural goods $c_{aj}(t)$, housing (measured by lot size $l_j(t)$), and consumption of consumer goods $c_{sj}(t)$. The budget constraint is as follows

$$R(t) l_j(t) + p_a(t) c_{aj}(t) + p_s(t) c_{sj}(t) + s_j(t) = \hat{y}_j(t). \tag{10}$$

In our model, at each point in time, consumers have four variables to decide. We assume that utility level $U_j(t)$ is a function of $l_j(t)$, $c_{aj}(t)$, $c_{sj}(t)$, and $s_j(t)$, as follows

$$U_j(t) = l_j^{\eta_{0j}}(t) c_{aj}^{\mu_{0j}}(t) c_{sj}^{\xi_{0j}}(t) s_j^{\lambda_{0j}}(t), \ \ \eta_{0j}, \mu_{0j}, \xi_{0j}, \lambda_{0j} > 0,$$

where η_{0j} is the propensity to consume housing, ξ_{0j} is the propensity to consume agricultural goods, ξ_{0j} is the propensity to consume consumer goods, and λ_{0j} the propensity to save. It should be remarked that heterogeneous households are taken into account in some growth models of endogenous wealth accumulation. But the heterogeneity in these approached is due to the differences in the initial endowments of wealth rather than in preferences (Chatterjee, 1994; Caselli and Ventura, 2000; Maliar and Maliar, 2001; Penalosa and Turnovsky, 2006; Turnovsky and Penalosa, 2006). Households in this approach are essentially homogeneous as all the households are described with the same preference utility function. Our approach considers the heterogeneity due to differences in utility functions.

Maximizing the utility subject to (10) yields

$$R(t) l_j(t) = \eta_j \hat{y}_j(t), \ \ p_a(t) c_{aj}(t) = \mu_j \hat{y}_j(t), \ \ p_s(t) c_{sj}(t) = \xi_j \hat{y}_j(t), \ \ s_j(t) = \lambda_j \hat{y}_j(t), \tag{11}$$

where

$$\eta_j \equiv \rho_j \eta_{0j}, \ \ \mu_j \equiv \rho_j \mu_{0j}, \ \ \xi_j \equiv \rho_j \xi_{0j}, \ \ \lambda_j \equiv \rho_j \lambda_{0j}, \ \ \rho_j \equiv \frac{1}{\eta_{0j} + \mu_{0j} + \xi_{0j} + \lambda_{0j}}.$$

We now define equation for capital accumulation. From the definition of $s_j(t)$, we have the household's change of wealth as follows

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t) = \lambda_j \hat{y}_j(t) - \bar{k}_j(t). \tag{12}$$

This equation implies that the change in wealth equals the saving minus dissaving.

2.5. Demand and Supply of the Three Sectors

The equilibrium of demand of and supply for the agricultural good is

$$\sum_{j=1}^{J} c_{aj}(t) \bar{N}_j = F_a(t). \tag{13}$$

The demand and supply equilibrium for the consumer goods sector is

$$\sum_{j=1}^{J} c_{sj}(t) \bar{N}_j = F_s(t). \tag{14}$$

The output of the capital goods sector equals the net savings and the depreciation of capital stock. We have

$$S(t) - K(t) + \delta_k K(t) = F_i(t), \tag{15}$$

where

$$S(t) \equiv \sum_{j=1}^{J} s_j(t) \bar{N}_j, \ \ K(t) = \sum_{j=1}^{J} \bar{k}_j(t) \bar{N}_j.$$

It should be noted that (15) is similarly held as in the traditional Uzawa two-sector model. As agricultural goods and services cannot be saved, the net saving in this model is $S(t) - K(t)$ which equals $F_i(t) - \delta_k K(t)$. The change in the physical capital is given by

$$\dot{K}(t) = F_i(t) - \delta_k K(t).$$

This is the capital accumulation in the Uzawa model.

2.6. Capital Being Fully Utilized

Total capital stock $K(t)$ is allocated to the three sectors

$$K_a(t) + K_i(t) + K_s(t) = K(t). \tag{16}$$

2.7. Land Being Fully Used

The land is fully used

$$L_a(t) + \sum_{j=1}^{J} l_j(t) \bar{N}_j = L. \tag{17}$$

We completed the model. Irrespective of the obvious strict assumptions in our model, the model structurally is general as some well-known models in economics are its special cases. For instance, if housing, agricultural sector, and land are omitted and the population is homogeneous, our model is structurally similar to the models by Solow (1956) and Uzawa (1961). Our model is structurally similar to the Ricardian models formed by Pasinetti and Samuelson (Samuelson, 1959; Pasinetti, 1960;1974; Caravale and Tosato, 1980; Casarosa, 1985). We can also demonstrate that our model is structurally similar to the growth models for the dual economy (Todaro, 1969; Kaiyama, 1973; Marino, 1975; Amano, 1980).

3. THE DYNAMICS AND ITS PROPERTIES

As the dynamic system consists of any (finite) number of households, it is nonlinear and highly dimensional. In general, it is difficult to analyze properties of such nonlinear dynamic systems. Nevertheless, we can simulate the motion of the economy. The following lemma shows that the dimension of the dynamical system is equal to the number of types of households. We also provide a computational procedure for calculating all the variables at any point in time. First, we define a new variable $z(t)$ by

$$z(t) \equiv \frac{r(t) + \delta_k}{w_j(t)/h_j}.$$

As demonstrated in the appendix, the introduction of the variable makes it much easier to find the dynamic equations.

Lemma

The motion of the economic system is determined by J differential equations with $z(t)$ and $\{\bar{k}_j(t)\}$, where $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \cdots, \bar{k}_J(t))$, as the variables

$$\dot{z}(t) = \Lambda_1(z(t), \{\bar{k}_j(t)\}),$$
$$\dot{\bar{k}}_j(t) = \Lambda_j(z(t), \{\bar{k}_j(t)\}), \quad j = 2, \ldots, J, \tag{18}$$

in which $\Lambda_j(t)$ are unique functions of $z(t)$ and $\{\bar{k}_j(t)\}$ defined in the appendix. The following procedure determines the other variables uniquely as functions of $z(t)$ and $\{\bar{k}_j\}$ as follows: $r(t)$ and $w_j(t)$ by (A3) $\rightarrow \bar{k}_1(t)$ by (A17) $\rightarrow N_i(t)$ by (A14) $\rightarrow N_a(t)$, $N_s(t)$, and $\bar{r}(t)$ by (A13) $\rightarrow R(t) = \bar{r}(t)\bar{N}/L \rightarrow w(t) = w_1(t)/h_1 \rightarrow \hat{y}_j(t)$ by (A6) $\rightarrow K_a(t)$, $K_s(t)$, and $K_i(t)$ by (A1) $\rightarrow F_i(t)$, $F_s(t)$ and $F_a(t)$ by the definitions $\rightarrow p_s(t)$ by (A4) $\rightarrow p_a(t)$ by (4) $\rightarrow l_j(t)$, $c_{sj}(t)$, $c_{aj}(t)$, and $s_j(t)$ by (11) $\rightarrow K(t)$ by (16).

The lemma gives a computational procedure for plotting the motion of the economy with any number of types of households. It is well known that calibration of general equilibrium involves solving high-dimensional nonlinear equations. With regard to the Arrow-Debreu concept of general equilibrium the final stage of analysis is to find a price vector at which excess demand is zero (Judd, 1998). There are numerical approaches for calculating equilibria (e.g., (Scarf, 1967; Scarf and Hansen, 1973)). We can apply these traditional methods to find how the prices and other variables are related to the variables in the differential equations. As it is difficult to interpret the analytical results, to study properties of the system the model is simulated with the following parameters:

$$A_i = 1.3,\ A_s = 1,\ A_a = 0.8,\ L = 10,\ \alpha_i = 0.34,\ \alpha_s = 0.3,\ \alpha_a = 0.17,\ \beta_a = 0.2,\ \delta_k = 0.05,$$

$$\begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 40 \\ 20 \end{pmatrix}, \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 1.5 \\ 0.6 \end{pmatrix}, \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.78 \\ 0.75 \\ 0.7 \end{pmatrix}, \begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.12 \\ 0.16 \\ 0.18 \end{pmatrix}, \begin{pmatrix} \eta_{10} \\ \eta_{20} \\ \eta_{30} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.16 \\ 0.14 \end{pmatrix}, \begin{pmatrix} \mu_{10} \\ \mu_{20} \\ \mu_{30} \end{pmatrix} = \begin{pmatrix} 0.06 \\ 0.08 \\ 0.09 \end{pmatrix}. \tag{19}$$

The population of group 2 is largest, while the population of group 3 is the next. The human capital level of group 1 is highest, while the human capital level of group 3 is lowest. The total factor productivities of the capital goods, agricultural, and consumer goods sectors are respectively 1.3, 1 and 0.8. We specify the values of the parameters, α_j, in the Cobb-Douglas productions for the capital goods and consumer goods sectors approximately equal to 0.3 (for instance, (Miles and Scott, 2005; Abel *et al.*, 2007)). The depreciation rate of physical capital is specified at 0.05. Group $1's$ propensity to save is 0.78 and group $3's$ propensity to save is 0.7. The value of group $2's$ propensity is between the two groups. We specify the initial conditions as follows

$$z(0) = 0.045,\ \bar{k}_2(0) = 15,\ \bar{k}_3(0) = 8.5.$$

The motion of the variables is plotted in Figure 1. In Figure 1, the national income is

$$Y(t) = F_i(t) + p_s(t)F_s(t) + p_a(t)F_a(t) + (L_0 - L_a(t))R(t).$$

The output level of the capital goods sector is enhanced and the output levels of the consumer goods and agricultural fall over time. The rate of interest rises. The wage rates of the three groups fall. The national capital stocks and output fall. The consumption levels of the three groups also vary over time. The economic structure varies over time. After confirming the existence of a unique equilibrium and its stability, we will examine how the dynamic patterns are affected by different variables.

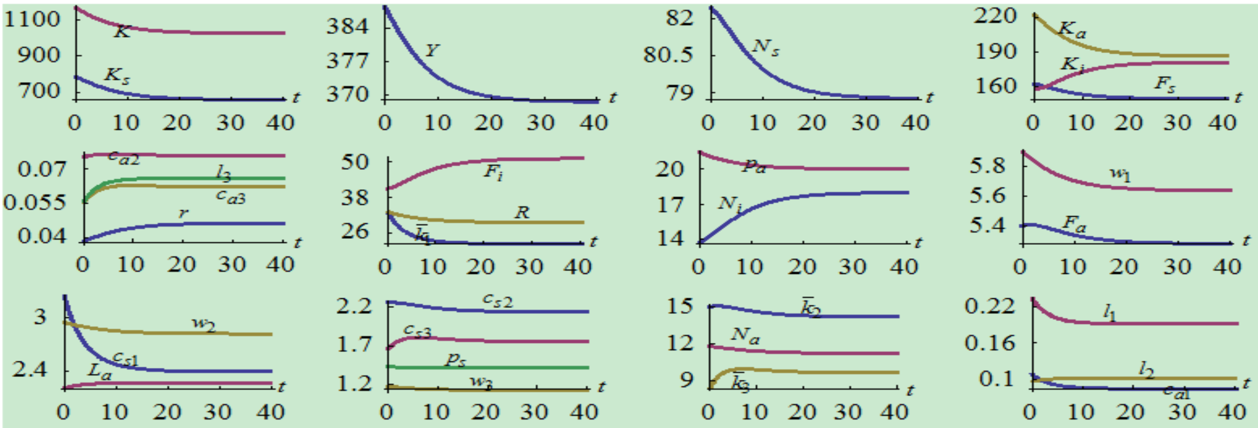

Figure-1. The Motion of the Economic System

It is straightforward to confirm that the variables become stationary. The simulation shows that the system has a unique equilibrium. We list the equilibrium values in (20)

$$Y = 596.2, \; K = 1025.6, \; F_i = 51.28, \; F_s = 148.9, \; F_a = 5.28, \; r = 0.046, \; R = 29.42,$$
$$p_a = 20.02, \; p_s = 1.42, \; w_1 = 5.64, \; w_2 = 2.82, \; w_3 = 1.13, \; L_a = 2.26, \; N_a = 11.25, \; N_i = 18.0,$$
$$N_s = 78.75, \; K_a = 186.5, \; K_i = 180.9, \; K_s = 658.2, \; \bar{k}_1 = 22.07, \; \bar{k}_2 = 14.18, \; \bar{k}_3 = 9.67,$$
$$c_{s1} = 2.39, \; c_{s2} = 2.13, \; c_{s3} = 1.75, \; c_{a1} = 0.085, \; c_{a2} = 0.075, \; c_{a3} = 0.062, \; l_1 = 0.192,$$
$$l_2 = 0.103, \; l_3 = 0.066. \tag{20}$$

It is straightforward to calculate the three eigenvalues as follows

$$\{-0.335, \; -0.303, \; -0.159\}.$$

The real parts of the eigenvalues are negative. The unique equilibrium is locally stable.

4. COMPARATIVE DYNAMIC ANALYSES

We already simulated the motion of the national economy under (19). We are now concerned with how the economic system is affected by exogenous changes. As the lemma gives the computational procedure to simulate the motion of the economic variables, it is straightforward to examine effects of change in any parameter on transitory processes as well stationary states of all the variables. We introduce a variable $\overline{\Delta} x_j(t)$ which stands for the change rate of the variable, $x_j(t)$, in percentage due to changes in the parameter value.

Group 1 Augmenting the Propensity to Save

Preferences of different households are important for understanding economic structures as demonstrate in the Walrasian general equilibrium theory. Nevertheless, economics has not yet an effective analytical framework for analyzing effects of changes in one type of households on national economic growth as well as wealth and income distribution among different households. As our analytical framework integrates the economic mechanism of the

Walrasian general equilibrium theory and neoclassical growth theory, in principle we can analyze effects a change in the preference of any people on the dynamic path of the economic growth. First, we examine the case that group 1 increases its propensity to save in the following way: $\lambda_{01}: 0.78 \Rightarrow 0.8$. The simulation results are given in Figure 2. Group 1's per capita wealth is increased. The levels of agricultural and service consumption and lot size are lessened initially ad subsequently increased. This occurs as the group saves more from the disposable income, the household consumes less. Nevertheless, as the household has more wealth and the wage rate is also increased, the disposable income is increased in the long term, which leads to the rises in the consumption levels. Hence, group 1 benefits from saving more from the disposable income in the long term. As far as the transitory effects are concerned, the effects on the other two groups are different. The two groups' per capita wealth and service consumption levels are slightly affected, initially being reduced and augmented in the long term. The lot sizes and consumption levels of agricultural goods are initially increased and subsequently lowered. Although the two groups' wage rates are increased, as the land rent and price of agricultural goods fall initially ad rise subsequently, the net effects on the two variables on the two groups are negative. As more capital is accumulated as a consequence of the rise in the propensity to save, the rate of interest falls. As more capital good is produced, labor force is shifted from the service and agricultural goods sectors to the capital good sector. The capital stock employed by the capital goo sector is increased. The capital stocks employed by the other two sectors fall initially but rise subsequently (as a consequence of the rise in the two capital stock). As group 1 (the rich group) reduces initially the demand for lot size and agricultural good, the land rent and price of consumer goods sector fall; subsequently as the group has more disposable income and its demand for lot size and agricultural good are increased, the land rent and price of consumer goods sector rise. The output level of the capital good sector is increased, while the output levels of the other two sectors are slightly reduced. The impact on the national output is negative initially and positive in the long term.

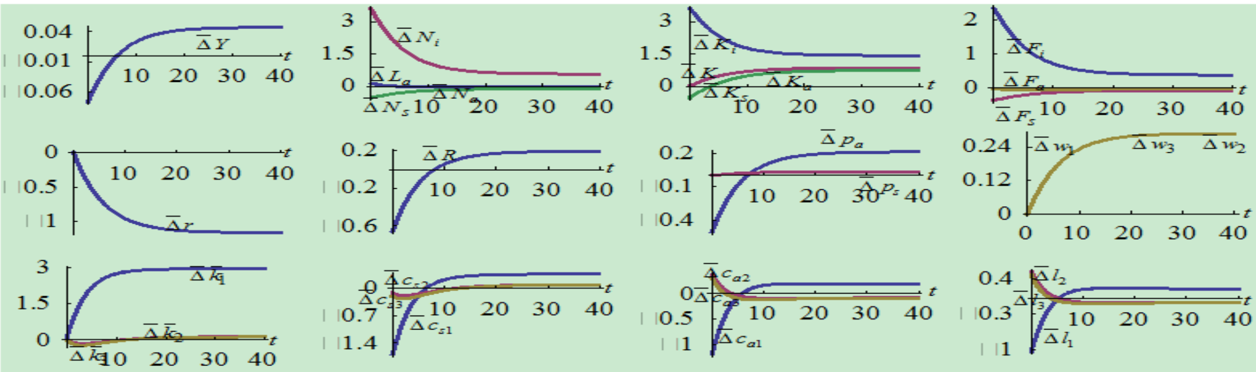

Figure-2. A Rise in Group 1' Propensity to Save

Group 1 Improving Human Capital

The impact of human capital is currently a main topic in economic theory and empirical research. In modern economies, one of the key determinants of economic growth is human capital (e.g., (Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; Castelló-Climent and Hidalgo-Cabrillana, 2012)). There are many studies on estimating the impact of education on earnings. Earlier studies (Tilak, 1989) demonstrate that inequality within countries can be reduced in association with spread education. Could *et al.* (2001) construct an economic model, concluding that increasing randomness is the primary factor for inequality growth among uneducated workers, but inequality growth within educated workers is mainly due to changes in the composition and return to ability (Tselios, 2008; Fleisher *et al.*, 2011). We now re-examine issues related to how differences in human capital affect wealth and income distribution among heterogeneous households during transitory processes and in long-term steady state. We now allow group 1 to improve its human capital as follows: $h_1: 3 \Rightarrow 3.1$. We plot the simulation results in

Figure 3. The rise in group 1's human capital leads to the rise in the group's wage rate, while the other two groups' wage rates are slightly affected. Hence, the wage gaps between group 1 and the other two groups are enlarged. It should be remarked that the wage gaps due to the change in the propensity to save just analyzed are not so strongly affected like in the case of change in the human capital. As group 1 increases its wage, its lot size, wealth level and the consumption levels of the two goods are all increased both in the short term and in the long term. The other two groups' lot sizes are reduced. The other two groups reduce their consumption levels of agricultural good. As the households accumulate more wealth, one observes falls in the rate of interest. The land rent and the price of agricultural good are increased, while the price of consumer good is almost not affected. The output levels of the three sectors and the national output are increased.

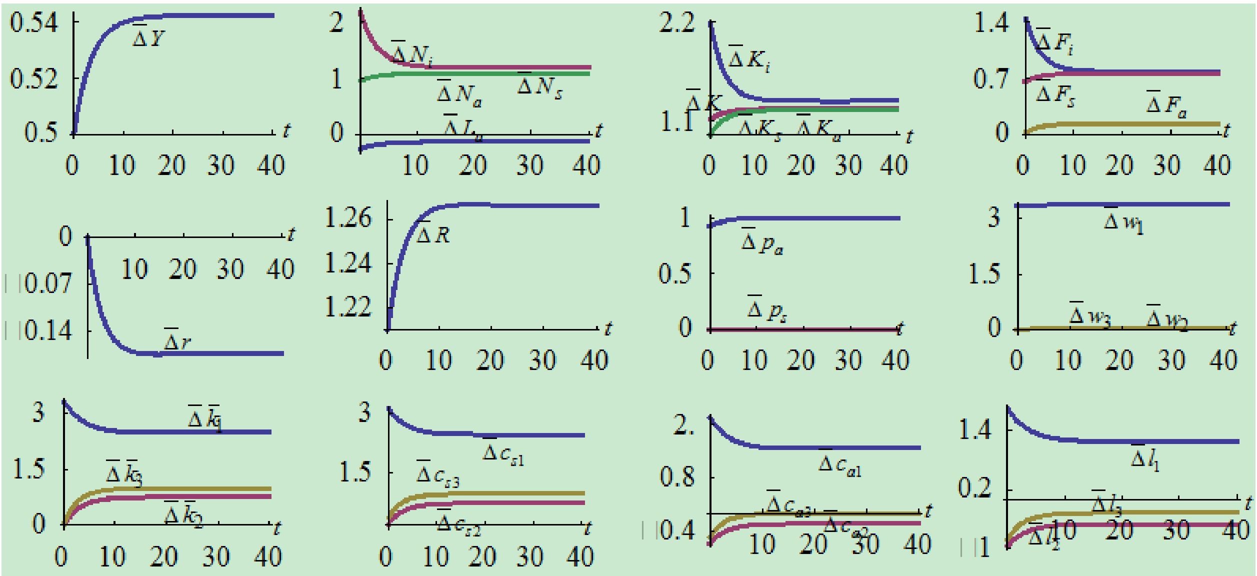

Figure-3. A Rise in Group 1's Human Capital

Group 3 Increasing Propensity to Consume Agricultural Good

We now examine how group 3's preference for agricultural good may affect the economic structure and growth. We allow group 3 to increase its propensity to consume agricultural good as follows: $\mu_{03}: 0.09 \Rightarrow 0.11$. The preference change leads to more consumption of agricultural good by the group. The increased demand results in the rise of supply in the agricultural sector. The agricultural sector employs more labor, capital and land. The land rent and the price of agricultural good are thus increased. The increased price of agricultural good make the other groups consume less agricultural good. As more land is employed by the agricultural sector and the land rent is increased, the other two groups use less lot sizes. Group 3 consumes less consumer good and has lower wealth, while each of the other two groups consumes more consumer good and has lower wealth. The price of consumer good is slightly lowered. The national capital and capital stocks employed by the capital good and consumer good sectors are reduced. The reduced national capital stock is associated with rises in the rate of interest. The national output is increased, even though the output levels of the consumer good and capital good sectors are reduced.

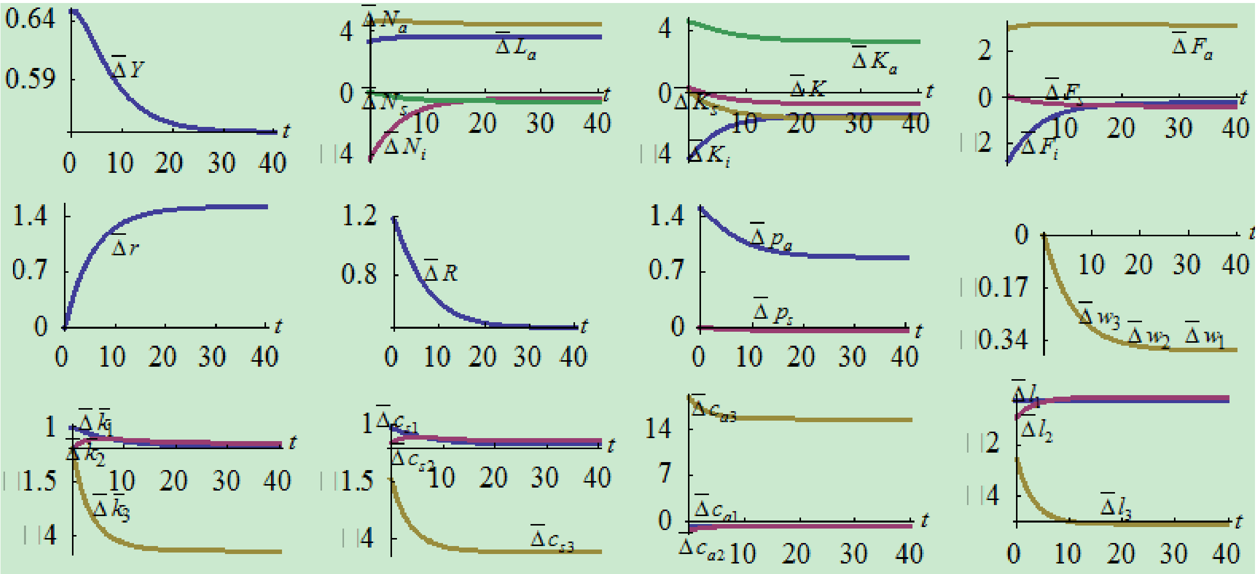

Figure-4. A Rise in Group 3' Propensity to Consume Agricultural Good

Group 3's Population Being Increased

The relationship between population change and economics is a challenging area. Although this study assumes the population fixed, it is important to examine effects of changes in the population sizes. As different countries have different levels of knowledge utilization efficiency and creativity, increases in the population sizes may have different effects upon the global economy. It has been observed that the effect of population growth varies with the level of economic pment and can be positive for some developed economies. Theoretical models with human capital predict situation-dependent interdependence of population and economic development (see, (Ehrlich and Lui, 1997; Galor and Weil, 1999; Boucekkine *et al.*, 2002; Bretschger, 2013)). One reads different mixed conclusions from empirical studies related to the issue (e.g., (Furuoka, 2009; Yao *et al.*, 2013)). We now allow group $3's$ population to be increase as follows: $N_3 : 20 \Rightarrow 22$. The results are plotted in Figure 5. We see that as far as the aggregate real variables are concerned, in the long term the national output, the total capital stock, the capital inputs, the labor inputs and land input, the output levels of the three sectors are all increased. We note that group 3 has the lowest level of human capital. Hence, a rise in the population lowers the average level of human capital of the population. It is expectable to see that the wage rates are reduced as a consequence of the fall in group $3's$ population. The rise in the unskilled population lowers the wealth levels, consumption levels of consumer and agricultural goods, and lot sizes for all the groups. The land rent and the prices of the two goods are enhanced.

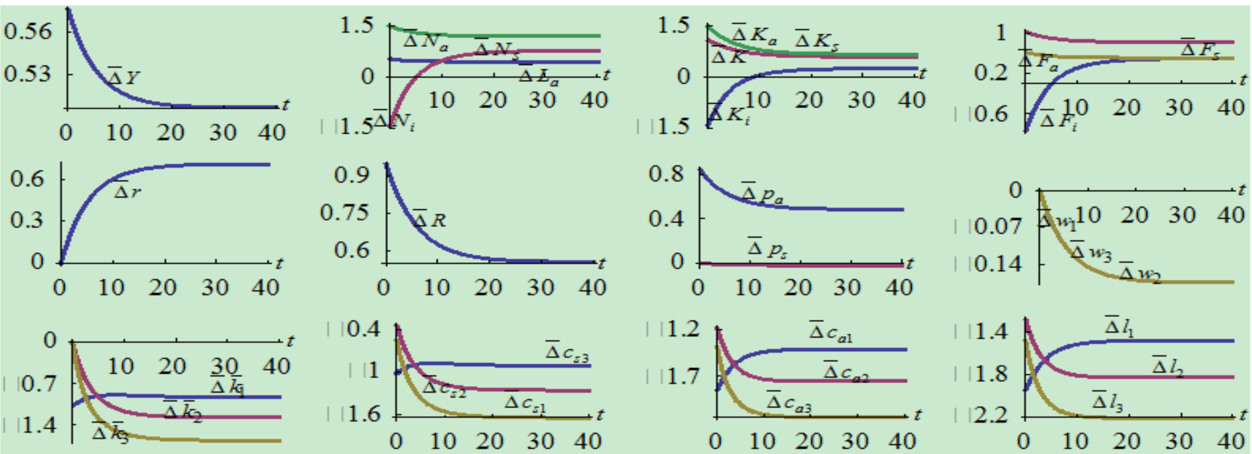

Figure-5. A Rise in Group 3' Population

Group 1's Population Being Increased

We analyzed the effects of change in group $3's$ population. It is interesting to compare effects of changes the populations with the highest and lowest levels of human capital. We now allow group $1's$ population to be increase as follows: $N_1 : 12 \Rightarrow 13.$ The results are plotted in Figure 6. We see that as far as the aggregate real variables are concerned, in the long term the national output, the total capital stock, the capital inputs, the labor inputs and land input, the output levels of the three sectors are all increased. Hence, the aggregated variables are affected similarly by the changes in the two group's population. In contrast to the rise in the rise in group $3's$ population, a rise in group $1's$ population enhances the wage rates. The rise in the skilled population lowers the consumption levels of agricultural good and lot sizes for all the groups as the rise in the unskilled population; but different from the case of the rise in unskilled population, the rise in the skilled population raises the wealth levels and consumption levels of consumer good for all the groups.

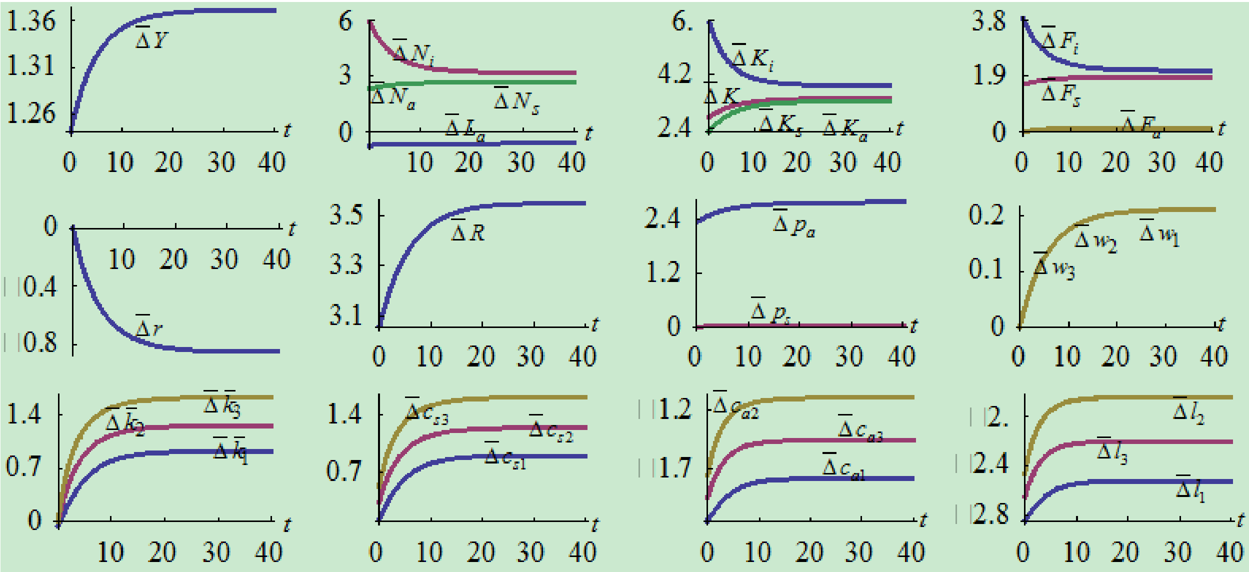

Figure-6. A Rise in Group 1's Population

5. CONCLUDING REMARKS

This study proposed an economic growth model with economic structure and heterogeneous households. The framework is influenced by the three core theories in economics - Walrasian general equilibrium theory, Ricadian theory of distribution, and neoclassical growth theory. The economic system of heterogeneous households consists of one consumer goods sector and one capital goods sector. We studied economic growth and structural change with exogenous land resource, population and human capital. The motion is described by a set of differential equations. For illustration, we simulated the motion of the economic system with three groups. We identified the existence of a unique stable equilibrium point. We also carried out comparative dynamic analysis. We can comprehensively discussed some important issues related to growth in a unique manner because our analytical framework contains not only the economic mechanisms for analyzing these issues, but also because we provide the computational procedure to follow the motion of the nonlinear dynamic system. By comparative analyses we reveal into the complexity of dynamic interdependence between economic growth, economic structural change, and wealth and income distribution. For instance, there are many studies on relations between wealth and income distribution and development. Kaldor (1956) holds that as income gap is larger, growth is faster because of more savings. The positive relation between income inequality and growth is tested by, for instance, Bourguignon (1981); Forbes (2000) and Frank (2009). There are other studies which find negative relations between income distribution and growth. There are some formal models which show negative relations (e.g., (Galor and Zeira, 1993; Benabou, 2002; Galor and Moav, 2004)). Some empirical studies find negative relations (e.g., Persson and Tabellini (1994)). From our simulation, we see that relations between inequality and economic growth are complicated in the sense that these relations are determined by many factors. For instance, from Figure 2 we observe that in initial transitory process the rich group's and the other two groups' wealth levels are enlarged, the national output experiences negative growth; while in the long term the rich group's and the other two groups' wealth levels continue to be enlarged, the national output experiences positive growth. Our comparative analysis also shows that in order to understand growth and development processes it is important to classify the population into different types according to their preferences and human capital. For instance, if the population of the group with the lowest (highest) level of human capital is increased, the wage rates and the wealth levels and consumption levels of consumer good of all the groups are reduced (enhanced).

Appendix: Proving the lemma

By (4), (6), and (8), we obtain

$$z \equiv \frac{r + \delta_k}{w} = \frac{N_a}{\bar{\beta}_a K_a} = \frac{N_i}{\bar{\beta}_i K_i} = \frac{N_s}{\bar{\beta}_s K_s}, \tag{A1}$$

where $\bar{\beta}_j \equiv \beta_m / \alpha_m$. From (A1) and (15), we obtain

$$\bar{\beta}_a K_a + \bar{\beta}_i K_i + \bar{\beta}_s K_s = \frac{N}{z}. \tag{A2}$$

Insert (A1) in (4)

$$r = \alpha_r z^{\beta_i} - \delta_k, \ \ w_j = \alpha_j z^{-\alpha_i}, \tag{A3}$$

where

$$\alpha_r = \alpha_i A_i \bar{\beta}_i^{\beta_i}, \ \ \alpha_j = h_j \beta_i A_i \bar{\beta}_i^{-\alpha_i}.$$

Hence, we determine the rate of interest and the wage rates as functions of z. From (7) and (8), we have

$$p_s = \frac{\bar{\beta}_s^{\alpha_s} z^{\alpha_s} w}{\beta_s A_s}. \tag{A4}$$

From (4), we have

$$L_a = \frac{w \varsigma N_a}{\beta_a R}. \tag{A5}$$

From (A3) and the definitions of $\hat{y}_j$, we have

$$\hat{y}_j = (1 + r)\bar{k}_j + h_j w + \bar{r}. \tag{A6}$$

Insert $p_s c_{sj} = \xi_j \hat{y}_j$ in (13)

$$\sum_{j=1}^{J} \xi_j \bar{N}_j \hat{y}_j = p_s F_s. \tag{A7}$$

Substituting (A6) in (A7) yields

$$\frac{wN_s}{\beta_s} - \bar{r}\,\tilde{g}_0 = \sum_{j=1}^{J} \tilde{g}_j\,\bar{k}_j + \tilde{g}\,w, \tag{A8}$$

where we use $p_s F_s = wN_s / \beta_s$ and

$$\tilde{g}_j(z) \equiv (1 + r)\xi_j\,\bar{N}_j,\ \ \tilde{g} \equiv \sum_{j=1}^{J} h_j\,\xi_j\,\bar{N}_j,\ \ \tilde{g}_0 \equiv \sum_{j=1}^{J} \xi_j\,\bar{N}_j.$$

Insert $p_a c_{aj} = \mu_j\,\hat{y}_j$ from (11) in (13)

$$\sum_{j=1}^{J} \mu_j\,\hat{y}_j\,\bar{N}_j = \frac{wN_a}{\beta_a}, \tag{A9}$$

where we use $p_a F_a = wN_a / \beta_a$. Insert (A6) in (A9)

$$\frac{wN_a}{\beta_a} - \tilde{h}_0\,\bar{r} = \sum_{j=1}^{J} \tilde{h}_j\,\bar{k}_j + w\tilde{h}, \tag{A10}$$

where

$$\tilde{h}_j(z) \equiv (1 + r)\mu_j\,\bar{N}_j,\ \ \tilde{h} \equiv \sum_{j=1}^{J} h_j\,\mu_j\,\bar{N}_j,\ \ \tilde{h}_0 \equiv \sum_{j=1}^{J} \mu_j\,\bar{N}_j.$$

Insert (A5) and $Rl_j = \eta_j\,\hat{y}_j$ from (11) in (17)

$$\frac{w\varsigma N_a}{\beta_a} + \sum_{j=1}^{J} \eta_j\,\hat{y}_j\,\bar{N}_j = RL. \tag{A11}$$

Insert (A6) in (A11)

$$-\frac{w\varsigma N_a}{\beta_a} + \left(\bar{N} - \bar{h}_0\right)\bar{r} = \sum_{j=1}^{J} \bar{h}_j\,\bar{k}_j + \bar{h}\,w, \tag{A12}$$

where

$$\bar{h}_j(z) \equiv (1 + r)\eta_j\,\bar{N}_j,\ \ \bar{h} \equiv \sum_{j=1}^{J} h_j\,\eta_j\,\bar{N}_j,\ \ \bar{h}_0 \equiv \sum_{j=1}^{J} \eta_j\,\bar{N}_j.$$

From (A8), (A10) and (A12), we see that the left-hand sizes of the three equations contain three variables, N_a, N_s, and $\bar{r}$. The three equations are linear in N_a, N_s, and $\bar{r}$. We also note that the right-hand sides of the three equations contain $\bar{k}_j$ which don't appear in the left-hand sides of the equations. These imply that we can get the solution of (A8), (A10) and (A12) with N_a, N_s, and $\bar{r}$ as the variables in the following form

$$N_a\left(z,\left(\bar{k}_j\right)\right)=\sum_{j=1}^{J}\varphi_{aj}(z)\bar{k}_j,$$
$$N_s\left(z,\left(\bar{k}_j\right)\right)=\sum_{j=1}^{J}\varphi_{sj}(z)\bar{k}_j, \tag{A13}$$
$$\bar{r}\left(z,\left(\bar{k}_j\right)\right)=\sum_{j=1}^{J}\varphi_{Rj}(z)\bar{k}_j.$$

We don't provide explicit expressions of $\varphi_{..}(z)$ as they are tedious. From (2) and (A13), we solve

$$N_i\left(z,\left(\bar{k}_j\right)\right)=\sum_{j=1}^{J}\varphi_{ij}(z)\bar{k}_j. \tag{A14}$$

From (A1), (16) and $K=\sum_{j=1}^{J}\bar{k}_j\,\bar{N}_j$, we get

$$\frac{N_a}{\bar{\beta}_a}+\frac{N_i}{\bar{\beta}_i}+\frac{N_s}{\bar{\beta}_s}=z\sum_{j=1}^{J}\bar{k}_j\,\bar{N}_j. \tag{A15}$$

Substituting N_a, N_s, and N_i in (A13) and (A14) yields

$$\sum_{j=1}^{J}\varphi_j(z)\bar{k}_j=0, \tag{A16}$$

where

$$\varphi_j(z)\equiv\frac{\varphi_{aj}(z)}{\bar{\beta}_a}+\frac{\varphi_{ij}(z)}{\bar{\beta}_i}+\frac{\varphi_{sj}(z)}{\bar{\beta}_s}-z\,\bar{N}_j.$$

Hence, solve (A16) with $\bar{k}_1$ as the variable

$$\bar{k}_1=\varphi\left(z,\left\{\bar{k}_j\right\}\right)\equiv-\frac{1}{\varphi_1(z)}\sum_{j=2}^{J}\varphi_j(z)\bar{k}_j, \tag{A17}$$

where $\left\{\bar{k}_j\right\}\equiv\left(\bar{k}_2,\,...,\,\bar{k}_J\right)$. It is straightforward to confirm that all the variables can be expressed as functions of z and $\left\{\bar{k}_j\right\}$ by the following procedure: r and w_j by (A3) $\rightarrow$ $\bar{k}_1$ by (A17) $\rightarrow$ N_i by (A14) $\rightarrow$ N_a, N_s, and $\bar{r}$ by (A13) $\rightarrow$ $R=\bar{r}\,\bar{N}/L$ $\rightarrow$ $w=w_1/h_1$ $\rightarrow$ $\hat{y}_j$ by (A6) $\rightarrow$ K_a, K_s, and K_i by (A1) $\rightarrow$ F_i, F_s and F_a by the definitions $\rightarrow$ p_s by (A4) $\rightarrow$ p_a by (4) $\rightarrow$ l_j, c_{sj}, c_{aj}, and s_j by (11) $\rightarrow K$ by (16). From this procedure, (A21), and (12), we have

$$\dot{\bar{k}}_1=\bar{\Omega}_1\left(z,\left\{\bar{k}_j\right\}\right)\equiv\lambda_1\,\hat{y}_1-\varphi, \tag{A18}$$

$$\dot{\bar{k}}_j=\Lambda_j\left(z,\left\{\bar{k}_j\right\}\right)\equiv\lambda_j\,\bar{y}_j-\bar{k}_j,\quad j=3,\,...,\,J. \tag{A19}$$

Taking derivatives of equation (A17) with respect to t and combining with (A18) implies

$$\dot{\bar{k}}_1 = \frac{\partial \varphi}{\partial z}\dot{z} + \sum_{j=2}^{J} \Lambda_j \frac{\partial \varphi}{\partial \bar{k}_j}. \tag{A20}$$

Equaling the right-hand sizes of equations (A18) and (A20), we get

$$\dot{z} = \left[\bar{\Omega}_1 - \sum_{j=2}^{J} \Lambda_j \frac{\partial \varphi}{\partial \bar{k}_j}\right]\left(\frac{\partial \varphi}{\partial z}\right)^{-1}. \tag{A21}$$

In summary, we proved the lemma.

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