



## A MULTIVARIATE ANALYSIS FOR RISK CAPITAL ESTIMATION IN INSURANCE INDUSTRY: VINE COPULAS



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### ABSTRACT

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This paper deals with the risks aggregation issue and adequate risk capital modeling within a multivariate setting. Focusing on the non-life insurance risk module, we examine the sensitivity of capital requirement to the dependence among risks for a multi-line Tunisian insurance firm. Such a context entails a nonlinear dependence of risks problem whose resolution may be intended by means of multivariate copulas. The relevant analysis relies profoundly on the dependence modeling by the means of vine copulas which are a flexible technique to model multivariate distributions constructed using a cascade of bivariate copulas. Under various confidence levels in VaR and TailVaR, the reached findings reveal the advantages of D-Vine copula in modeling inhomogeneous structures of dependency due to its flexibility of use in a simulation context. Practitioners and regulators can explore our conclusions for the assessment of risk capital under Solvency 2, which is based on stochastic models.

**Contribution/ Originality:** This study uses new methodology based on CD-vine copulas estimation to investigate the sensitivity of solvency capital requirement to the dependence pattern between the losses derived from non-life business lines of a Tunisian insurance company.

### 1. INTRODUCTION

Financial institutions are required by regulators to withhold a minimum level of capital, referred as risk capital, to be able to keep a risky position. Under the new regulatory framework Solvency 2, the standard approach recommends insurances to marginally calculate the risk capital by aggregating stand-alone economic capitals per type of risk (insurance, market, credit ...) and per business line (Non-Life insurance, Life insurance...) using the linear correlations existing among different risks. Conceptually, the risk capital is defined as the capital cushion against unexpected losses in a worst-case scenario calculated using the risk measure VaR at a specified confidence  $\alpha$  over a given time period. Nevertheless, several authors have criticized the use of VaR to compute the minimum capital requirement and have recommended the use of an alternative measure, namely the TailVaR (Kaas *et al.*, 2009; Goovaerts *et al.*, 2010; Cossette *et al.*, 2013).

Although risk capital can restraint the insolvency risk of a firm, the recent events have exhibited that quantitative models can particularly lead to misspecification of total risk capital. For instance, the Solvency II Directive has adopted a standard approach based on the risk variance-covariance matrix allows for a richer arrangement of interactions across different risk types using linear correlation matrix (CEIOPS 09<sup>1</sup>). However, this approach is criticized because linear correlation is not invariant under non-linear strictly increasing transformation. Yet, it is an appropriate measure of dependence only in the elliptical framework, in other situations, it may be biased (Embrechts *et al.*, 2003).

For an adequate losses' aggregation deriving from various risk classes and, therefore, an accurate calculation of total risk capital, existing stochastic dependencies between risk-specific losses have to be adopted by integrated risk management approaches (Rosenberg and Schuermann, 2006; Grundke, 2010; Kretzschmar *et al.*, 2010).

Indeed, researchers and practitioners underscore that dependence modeling for insurance risk is an evolving field, where many approaches are currently used by insurance with probable impact on the capital requirements. One approach to model dependence is copula functions, which witnessed a tremendous growth in the field of finance and risk management (McNeil *et al.*, 2005; Embrechts, 2009).

The copula function permits separating the dependence structure from the marginal distributions, which is useful for constructing multivariate stochastic models. Within the actuarial literature, several applications of copulas to insurance framework have been dealt with (Frees and Valdez, 1998; Frees and Wang, 2006; Eling and Toplek, 2009; Zhao and Zhou, 2010; Shi and Frees, 2011; Cossette *et al.*, 2013; Fang and Madsenb, 2013). The recent advances for high-dimensional copula models tend towards hierarchical structures based on the building blocks, known as pair copula construction (PCC). This structure specially related to a kind of copula, called vine copulas, has simplified the construction of multivariate distributions. In this paper, we extend this flexibility to insurance framework and we make the first application of multivariate copulas by fitting CD-Vine copulas to empirical losses' insurance data.

Focusing on the non-life insurance risk module solely, the aim of this paper is to examine the sensitivity of capital requirement to the dependence among risks for a multi-line Tunisian insurance firm in multivariate settings. With respect to the literature that uses copulas in risk capital estimate, our research is broadly linked to the contributions of Tang and Valdez (2009) and Brechmann and Schepsmeier (2013). We suggest a flexible approach of dependence modeling for aggregating heterogeneous risks. This paper makes two contributions to the related literature. First and foremost, this study explores the dependence modeling in a realistic environment using the database relating to incurred claims of the Tunisian insurance company. We set up multivariate CD-vine copula among four losses from non-life business lines, estimate its parameters and perform goodness-of-fit tests to conclude the optimal copula. The multivariate distribution is the outcome of a mix of univariate marginal distributions and copulas simulated. Second, the risk capital forecast is deducted by applying VaR and TailVaR risk measures on aggregate loss simulated. Finally, we accomplish a comparative study stipulating the independence assumption and thus determine the diversification benefit in the two hypotheses of dependence and independence. Then, we display that a static approach of ignoring the real dependencies between different risks can systematically lead to an overestimation of the total capital requirement. The strategic management in an insurance industry involves a multi-year time horizon for economic decision-making, providing the motivation for this paper. Up to now, few researchers have focused on the multivariate dependence modeling for non-life insurance due to the paucity of data and the reluctance of the insurers to deliver their private and tricky information.

The remainder of the paper is organized as follows. In the upcoming section, a survey of the relevant literature is exposed, while Section 3 describes the methodological approach underlying the research in dependence modeling and the computation of the capital requirement by forecasting VaR and TailVaR for solvency purpose. In addition,

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<sup>1</sup> CEIOPS.Cp-74. SCR standard formula: Article 109 correlations. Technical report, CEIOPS, 2009.

we describe the multivariate distributions using pair copula constructions (PCC) and the estimation procedures of their parameters. In section 4, we present the empirical analysis and results, followed by a conclusion in Section 5.

## 2. LITERARY REVIEW

With the upsurge of integrated risk management as an outstanding discipline in banking and insurance sector, the issue of risk aggregation through copulas theory has recently become a focal point of research. This paper surveys the related empirical literature on using copulas for the risk capital estimate.

Financial literature has strongly emphasized on the relationship between market, credit, and operational risks in the banking sector (Alexander and Pezier, 2003; Dimakos and Aas, 2004). For instance, a detailed application of the modular approach is Rosenberg and Schuermann (2006) which displays how copulas can be used instead of correlations to consider better risk tail dependencies. Besides, Grundke (2009) assesses the accuracy of the total economic capital based on the top-down approach. The author accentuates that it is relatively difficult to select the copula function that should be used in a top-down approach in order to catch the risk integration simulated through a bottom-up model. Liang *et al.* (2013) derive the integration process of Chinese commercial banks' credit risk and market risk with factor copula. The results are compared with elliptical and Archimedean copulas models. The authors find that factor copula leads to a more cautious result in risk integration. Indeed, Li *et al.* (2015) review the state-of-the-art of risk aggregation methodologies applied to credit, market and operational risk aggregation of the Austrian banking industry. The total risks and diversification benefits from different approaches are compared to further investigate their relative magnitudes and tail behaviors. The results reveal that the *t*-copula is an adequate choice to capture tail dependence while Gaussian copula is not recommended. Moreover, the suggested mixture copulas consisting of *t*-copula and Gumbel copula exhibit heavier right tail dependence than the single *t*-copula.

A more comprehensive review of recent approaches unveils the need to explore the aggregated risk figure of a financial institution by examining the dependence of operational risks through the use of multivariate copulas (Guégan and Hassani, 2013; Brechmann *et al.*, 2014).

For solvency purposes, the recent literature on insurance risk has looked at a one-year perspective and, more recently, at a multi-period perspective using standard approaches. Empirical studies that focus on cross-risk aggregation using copula functions for the insurance institutions are less common. Using the concept of comonotonicity, an alternative aggregation approach is propounded by Dhaene *et al.* (2009) who assume that risk distributions are either normal or lognormal distributed, to compute the so-called total economic capital. The authors argue that risk aggregation process leads to a diversification effect. Operating in a multivariate elliptical context, Tang and Valdez (2009) investigate the sensitivities of the capital requirement to the choice of the copula and other modeling assumptions using numerical illustrations based on Australian general insurance data. The authors also explore the related issue of the diversification benefit from operating multiple business lines in the context of risks aggregation through copulas. They conclude that there is a large difference in the capital requirement as well as diversification benefit under diverse copula assumptions.

Under Solvency 2 project, Arbenz *et al.* (2012) provide a rigorous mathematical foundation for the risk aggregation method for the solvency capital requirements estimate. They suggest a hierarchical risk aggregation approach, which is flexible method in high dimensions. They perform an algorithm based on the Iman-Conover approach for numerical approximation, which make dependence between originally independent marginal samples through reordering. Moreover, they apply normal and lognormal distributions for the margins. Compared to classical dependence modeling approaches, this permits for more flexible dependence structures.

Similarly, Diers *et al.* (2012) explore the demonstrating of dependence structures of non-life insurance risks by means of the Bernstein copula in a risk management and solvency context. The advantage of Bernstein copula is its applicability in higher dimensions data and the use of all available information. They precede a goodness-of-fit analysis based on the Cramèr-Von-Mises statistics and compare the Bernstein copula with other widely used

copulas. Then, they illustrate the application of the Bernstein copula in a value-at-risk and tail-value-at-risk simulation study to determining solvency capital requirements purpose. Their study was performed on non-life insurance German market on six-dimensional data set. Their results display that the Bernstein copula is a supply approach due to its representation as a mixture of independent Beta densities, but not the solution to all modeling problems. [Bermúdez et al. \(2014\)](#) propose a flexible approach of solvency capital requirement based on Monte Carlo simulation by integrating different copula families to stylize the dependence structure among losses. They used Pareto distributions for margins. By definition, capital requirements are typically equal to, or proportional to, a risk measure value that defines the minimum cushion of economic liquidity. The illustration is carried out using data from the Spanish insurance market. Finally, they address the implications on the capital risk estimates of choosing the random behaviors of risk factors, the copula that determines the dependence between them and the risk measure that is chosen to assess total risk.

[Côté and Genest \(2015\)](#) consider a flexible approach for risk aggregation based on a tree structure, bivariate copulas, and marginal distributions. A procedure for picking the tree structure is advanced using hierarchical clustering techniques, along with a distance metric based on Kendall's tau. The approach is illustrated using data from a portfolio of insurance risks held by a large Canadian insurance company. The data available consist of monthly-earned premiums and incurred claim amounts for eight different risks from January 2004 to June 2012, inclusively. The results highlight the use of vine copulas and TailVaR to the insurance risks allocation problem.

More recently, [Araichi et al. \(2016\)](#) deal with claim reserving modeling and risks aggregation in the field of insurance sector. The authors suggest an advanced methodology to assess the aggregated amount of reserves and solvency capital of various lines of business. This approach attempts to handle temporal dependence, both between a line of business claim's amounts and between the two lines of business claims. In doing so, they apply a Generalized Autoregressive Conditional Sinistrality model to study the variation of dependence in time. Furthermore, applying numerical illustrations based on a French insurance company, they construct time varying copula functions with the aim of risk aggregation. Then, they carry out the impact on reserves and Solvency Capital Requirement in a simulation context. Their results reveal that a diversification benefits could be earned on the Solvency Capital requirement when considering time varying dependence structures.

Finally, [Bølviken and Guillen \(2017\)](#) suggest a simple scheme based on the log-normal distribution, which is proved to be superior to the standard formula and to adjustments of the Cornish–Fisher type for risk aggregation in Solvency 2. Relying on the Monte Carlo simulation, this approach considers the tail-dependence by mixing hierarchical Clayton copulas in order to approve the accuracy of the lognormal approximation and to illustrate the importance of encompassing tail dependence.

### 3. METHODOLOGY

#### 3.1. Copulas Approach

The dependence structure modeling provides a specific treatment of separating the marginal distributions functions to their joint distribution. The copula functions that have proposed firstly by [Sklar \(1959\)](#) associates univariate marginal distributions to their multivariate distribution. An  $n$ -dimensional copula refers to a multivariate distribution,  $C$ , with uniformly distributed marginal distributions  $\mathbf{U}(0, 1)$  on  $[0, 1]$ .

Consider a random vector  $(x_1, \dots, x_n)^T \sim F$  with marginal distribution functions  $F_1, \dots, F_n$ . In light of Sklar theorem, there is a unique copula function  $C$  to explain the joint distribution  $F$  of margins such that:

$$F(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n)) \quad (1)$$

From Eq.(1), for continuous random vector  $X$  with strictly increasing continuous margins  $F_1, \dots, F_n$ , the copula is expressed as:

$$c(u_1, \dots, u_n) = F(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)) \quad (2)$$

where the  $F_i^{-1}$  are the inverse functions of the marginal distributions.

The joint probability density function  $f$  of a copula C is defined by the following formula:

$$f(x_1, \dots, x_n) = C(F_1(x_1), \dots, F_n(x_n)) \times \left[ \prod_{i=1}^n f_i(x_i) \right] \quad (3)$$

Where  $f$  is expressed as the product of both the copula density and the univariate marginal densities. In fact, the joint density can be split into two blocks. The first block  $C(F_1(x_1), \dots, F_n(x_n))$  provides information's on the dependence structure between random variables. The second block is the product of the marginal densities<sup>2</sup>.

Our empirical analysis applies various families of copula with different tail dependence specifications. These families, which display different upper and lower tail dependence, include Gumbel (positive dependence and the upper tail), Clayton (positive dependence and the lower tail), Frank (both positive and negative dependence and tail independence), Joe (positive dependence and the upper bound) and BB copulas (see Appendix A).

Whereas most of the literature on dependence modeling of insurance losses focus on elliptical copulas, this study apply vine copula model, known as pair-copula constructions (PCC) to forecast the risk capital of an insurer. For high-dimensional distributions, vine copulas are a flexible technique to model multivariate distributions constructed using a cascade of bivariate copulas. Originally introduced by Joe (1997) and further explored in more detail in Bedford and Cooke (2001;2002); Aas *et al.* (2009); Brechmann and Schepsmeier (2013); Brechmann and Joe (2015) the vine model consists of isolation of the multivariate density into (conditional) bivariate copula densities.

The vine copulas, which are built from a tree sequence of bivariate copulas, consolidate  $n(n-1)/2$  pairs of copulas  $n$ -dimensional PCC to  $(n-1)$  linked trees (with nodes and edges). Aas *et al.* (2009) introduce two special cases of vine copulas, namely the canonical vine (C-vine) and the drawable vine (D-vine). Each model yields a specific way of decomposing the density. Figure 1 depicts the specifications corresponding to a four-dimensional C- and D- Vine respectively. The dependence in C- vines is modeled with respect to one specific variable, playing the role of pivot (factor) in every tree. Broadly, a root node is designated in each tree and all pair-wise dependencies with respect to this node are modeled conditioned on all previous root nodes.

Likewise, D-vines are also assembled by selecting a specific order of the variables. Afterwards, in the first tree, the dependence of the first and second variable, of the second and third, of the third and fourth, and so on, is modeled using pair-copulas.

Having used pair-copula constructions, the joint copula density can be expressed as a product of several bivariate pair-copulas. Since we attempt to analyze the impact of dependence structure among insurance losses on

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<sup>2</sup> The theoretical properties and applications of copulas have recently attracted considerable attention in the academic literature. Thorough analysis of this subject can be found in the publications of Joe (1997). and Nelsen (2006).

capital requirement, we considered a vine copula with four variables. Let  $f$  be the joint density function of  $n$  random variables  $X = (X_1, \dots, X_4)$ , we consider the following decomposition:

$$f(X_1, \dots, X_4) = f(X_4) \cdot f(X_3|X_4) \cdot f(X_2|X_3, X_4) \dots f(X_1|X_1, \dots, X_4) \tag{4}$$

With  $f(\cdot|\cdot)$  denotes the conditional density.

In general, the density formula can be written as follows:

$$f(x|v) = c_{(xv_j|v_{-j})} (F(x|v_{-j}), F(v_j|v_{-j})) f(x|v_{-j}) \tag{5}$$

with  $v$  is a vector of  $n$  dimensions,  $v_j$  is the  $j$ th component of the vector, and  $v_{-j}$  is the vector  $v$  that excludes the components  $v_j$ .

Using a formula of Joe (1997) the conditional density is built iteratively and depends on a specific factorization:

$$F(x|V) = \frac{\delta C_{(x,v_j|v_{-j})}(F(x|v_{-j}), F(v_j|v_{-j}))}{\delta F(v_j|v_{-j})} \tag{6}$$

where  $C_{(ij|k)}$  is a bivariate copula.

According to Aas *et al.* (2009) the decomposition of a multivariate density in C-Vine structure with  $n$  dimensions, is expressed as follows,

$$f(x) = \prod_{k=1}^n f_k(x_k) \prod_{i=1}^{n-1} \prod_{j=1}^{n-i} C_{i,i+j|1:(i-1)} [F(x_i|x_1, \dots, x_{i-1}), F(x_{i+j}|x_1, \dots, x_{i-1}|\theta_{i,i+j|1:(i-1)})] \tag{7}$$

Where  $f_k; k = 1, \dots, d$ ; denotes the marginal densities and  $C_{i,i+j|1:(i-1)}$  bivariate copula densities with parameter(s)  $\theta_{i,i+j|1:(i-1)}$ ; and the outer product runs over the  $d-1$  trees and root nodes  $i$ , while the inner product refers to the  $d - i$  pair-copulas in each tree  $i = 1, \dots, d - 1$ .

In D-Vine structure with  $n$  dimensions, there exists  $(n!)$  possible combinations to the root of the tree. For a D-Vine, the density  $f(x_1, \dots, x_n)$  is given by:

$$f(x) = \prod_{k=1}^n f_k(x_k) \prod_{i=1}^{n-1} \prod_{j=1}^{n-i} C_{j,j+i|(j+i+1)} [F(x_j|x_{j+1}, \dots, x_{i+j-1}), F(x_{i+j}|x_{i+1}, \dots, x_{i+j-1}|\theta_{j,j+i|j+1:(j+i-1)})] \tag{8}$$

where index  $j$  identifies the trees, while  $i$  run over the edges in each tree.



The simulation algorithms for CD-vines are simple to apply (see Aas *et al.* (2009)). For the selection of the variables orders, we follow the principle of Brechmann and Schepsmeier (2013) who present a sequential selection of trees CD-vine such as maximum recovery trees. The empirical rule for the first tree selection indicates an order of the variables that intends to capture as much dependence as possible. Hereafter, one chooses the variable as root node so that the sum of the empirical Kendall's absolute values of all resulting couples is maximized for this tree. The more the Kendall's Tau is higher, the more dependency is assumed to be strong between the underlying random variables. The idea is that the first tree should contain the pair copulas with the strongest dependency. Note that for each node in the tree, it is possible to implement a copula belonging to a different family without asking for a particular assumption on the structure and the degree of dependence between each node and the tree level.

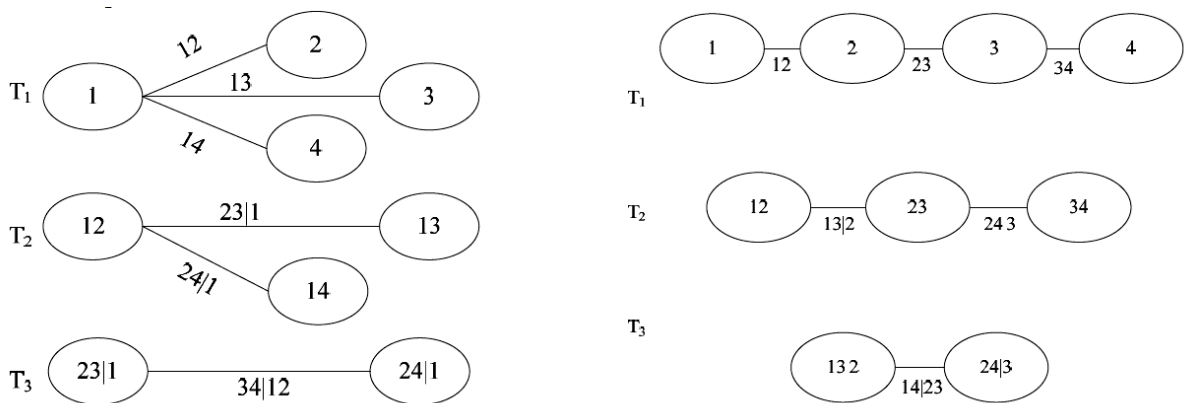


Fig-1. Trees C-Vine (left panel) and D-vine (right panel) with indices edges.

Source: Brechmann and Schepsmeier (2013)

To validate the choice of the best vine copula, we consider in a first step a bivariate analysis to examine the robustness of the choice of the bivariate copula in the first tree. For this reason, the issue of goodness-of-fit (GoF) testing is considered. A GoF test verifies if the copula is misspecified, i.e., different probably from the unknown best copula. The Cramér-von Mises (CvM) test is a broadly used test of copula models based on comparing the fitted copula CDF to the empirical copula (see, Rémillard (2010)). In this study, we conduct a goodness-of-fit analysis relying on Kendall process as indicated by Genest *et al.* (2006;2009) who compare the robustness of the various tests for the choice of copulas and they propose an alternative test based on the Cramér-Von Mises statistic.

The hypothesis to be tested can be expressed as follows:  $H_0: c \in \mathcal{C} = \{C_0; \theta \in \Theta\}$ . The test is based on the

Cramer-von-Mises statistic  $S_n$ , which measures the distance between empirical copula  $\hat{C}_n$  and the candidate parametric copula  $C_n$ . It is given by:

$$S_n = \int_{[0,1]^d} C_n(u)^2 d\hat{C}_n(u) \tag{9}$$

where  $C_n$  is Kendall's process given by:  $\sqrt{n} (C_n - \hat{C}_n)$ . Empirical copula  $\hat{C}_n$  of the uniform data  $U_1, \dots, U_d$  can

be expressed as:  $\hat{C}_n(u) = \frac{1}{n} \sum_{i=1}^n \mathbf{1}_{(U_{i1} \leq u_1, \dots, U_{id} \leq u_d)}$ .  $U$  signifies the empirical  $n$ -dimensional marginal

distributions. Since, we ignore the distribution of  $S_n$ , p-values are estimated using a bootstrap approach based on the algorithm from Genest *et al.* (2009).

The null hypothesis of stability is rejected when the observed value of  $S_n$  is larger than the  $(1 - \alpha)^{\text{th}}$  percentile of its distribution under the null hypothesis.

In second step, we choose the most appropriate C- or D- vine copula in the multivariate framework. We consider the information criterion's namely the Akaike Information (AIC) and the Bayesian Information (BIC).

### 3.2. Copula estimation method

For parameter estimation, we rely on the canonical maximum likelihood method (CML) proposed by Genest *et al.* (1995). We select this method because it does not impose any assumption on the parametric form of the margins. Indeed, a really specification can affect marginal parameter copula estimation. Specifically, the CML method uses the empirical probability integral transform to obtain uniform margins  $[0, 1]$ . This method is summarized in two steps:

- Transform margins  $\{(x_1^t, \dots, x_n^t)\}_{t=1}^T$  in uniform variables  $\{(u_1^t, \dots, u_n^t)\}_{t=1}^T$  using the empirical Cumulative Distribution Function (CDF);
- Estimate the parameters of the copula:  $\hat{\theta} = \arg \max \sum_{t=1}^T \ln C(\hat{u}_1^t, \dots, \hat{u}_n^t; \theta)$ .

### 3.3. Simulation Procedure

A main purpose of a multivariate model for non-life insurance losses is to accomplish an accurate evaluation of the risk capital that must be held by a company to cover unexpected losses. This section is devoted to the demonstration for risk capital estimate, which depends, in a critical manner, on the nature of marginal distributions as well as on the dependence structure of losses.

Let  $X = (X_1, \dots, X_d) \sim F_{1, \dots, d}$  with marginal distributions  $F_1, \dots, F_d$ , and  $X_i$  are non-negative random variables representing individual risks  $i$  for a given period  $t$ . As pointed out by Bernard *et al.* (2014) risk aggregation is identified as the behavior of a global position  $S(X)$  associated with a risk vector  $X = (X_1, \dots, X_d)$ . Therefore, the aggregate loss  $S$  ( $S \geq 0$ ) is generated by a multivariate random vector of dependent losses, that is,  $S = X_1 + \dots + X_d$  depend on the relationship between these risks.

In the basic framework, we have considered an insurance company with a portfolio of  $d$  lines of business. Now, we suppose that we need to generate the distribution of the aggregate loss  $S$  of  $d$  lines of business. In fact, the total loss depends on the relationship between these risks (Dhaene *et al.*, 2009) where  $S$  is generated by a bivariate random vector of dependent losses, i.e.  $S = \sum X_i$ .

The provided data in the current article are aggregated by risk class  $i$  for period  $t$  (quarter). Indeed, for each observation  $\{it\}$ , responses are made up of the total claims  $S_{it}$ . Thus, the total loss of the non-life risk portfolio is indicated as follows:

$$S = \sum_{t=1}^n S_{it} \tag{10}$$



where,  $S_{it}$  refers to claims yielded from individual risk  $i$  and obtained per quarter  $t$ ;  $S$  is the aggregated loss of non-life risk insurance portfolio.

To determine the risk capital of an insurance company in the context of Solvency 2, Value-at-Risk (VaR) is a measure of risk, which can be applied in this model. It is defined as the maximum potential loss being able to support a firm at a given confidence level  $\alpha$  over the horizon  $T$ . Indeed, we use in this study the Tail-Value-at-Risk (TailVaR) that describes the behavior of the tail of the distribution.

Let  $\Omega$  be a set of defined scenarios on  $\mathbb{R}$  and  $P$  a set indicating all the positions that provide real values on  $\Omega$ . A risk measure  $\rho$  is described as a mapping from the entire random variables indicating the risks for real numbers  $\rho: P \rightarrow \mathfrak{R}$ . Where,  $\rho(X)$  is a measure of risk which is defined as the  $\alpha$ -percentile of the distribution function of  $X$  and is expressed in monetary terms. In general, the Value-at-Risk at a level confidence  $\alpha \in [0,1]$ , is defined by:

$$VaR_{\alpha}(S) = F_S^{-1}(\alpha) \tag{11}$$

where  $F$  is the *cdf* continuous and strictly increasing.

For a continuous loss distribution function, *TailVaR* is interpreted as the conditional expected loss beyond VaR.

*Cossette et al. (2013)* define *TailVaR* as:  $\frac{1}{1-\alpha} \int_{\alpha}^1 VaR_{\alpha}(S).dp$ . Otherwise, TailVaR can be written as follows:

$$TVaR_{\alpha}(S) = VaR_{\alpha}(S) + \frac{1}{1-\alpha} E[(S - VaR_{\alpha}(S))^+] \tag{12}$$

To obtain VaR and TailVaR estimates combining copulas, we proceed to simulate  $N$  losses using multivariate model as follows:

**Step 1:** Fitting marginal distributions of losses.

**Step 2:** Transform each variable into uniform  $u_i \in (0;1)$  according to cumulative distribution function. We

denote  $u_1 = \hat{F}_1(X_1), \dots, u_n = \hat{F}_n(X_n)$  for  $i = 1, \dots, 4$ ;

**Step 3:** Fitting the appropriate copula  $\hat{C}$  for each pair of transformed data vectors and Concluding estimate parameters  $\hat{\theta}$  by maximum likelihood function method (MLE);

**Step 4:** Generate  $N$  iterations from the fitted copula and calculate simulated aggregate losses  $\hat{S}_j = \sum \hat{S}_{ij}$ ; form  $N$

portfolios and then conclude the estimate  $\widehat{VaR}_{\alpha}(S)$  and  $\widehat{TVaR}_{\alpha}(S)$ .

#### 4. NUMERICAL ANALYSIS

Using real-world data from the Tunisian database of insurance losses, we have run numerical study to examine the impact of the dependence structure on risk capital using different families of copula. Until fairly recently, the number of researches using data connecting to real-world insurance risk has remained negligible due to the scarceness of datasets and the reluctance of the insurance companies to publicize their private information.

#### 4.1. Data

This contribution was originally motivated by the risk aggregation issue for a portfolio of insurance risks owned by a large Tunisian insurance company. Having successfully overcome the effects of the global financial crisis of 2008, the Tunisian insurance and reinsurance sector resisted, until 2011, the post-revolution socioeconomic upheavals. Consequently, at the national level, the year 2011 was marked with instability at all levels which was due to the popular movements and riots that the country has experienced since the beginning of the year. Undoubtedly, the consequences of these events were immediately experienced causing a recession of the economy that the insurance sector has not spared. Furthermore, the costs of damage have been covered with optional warranties embodied in standard appendices, i.e., funds arranged by the State as exceptional measures. Indeed, the beneficiaries are the insured (policyholders) who have subscribed this guarantee qualified as "riots and popular movements". For instance, in 2012, the sector recorded a loss of equal importance to that of 2011 if not more. The regulatory framework within which insurance companies in Tunisia operate has been inspired from international standards. In this respect, several prudential standards have been improved with regard to risk division, monitoring of commitments and adequacy of capital. To keep the reference point with the reform within the framework of the internal model, we study, in this paper, the sensitivity of capital adequacy to dependencies between risks using copula applied to a typical insurance company.

At this regard, the data set applied to estimate the model consists of incurred claim amounts net of reinsurance, derived from four risks classified into four distinct groups: Motor branch (Mot), fire and various technical risks branch (FT), transport and aviation branch (TA) and finally, other branches (OB). Auto insurance industry mainly covers loss caused by car damage. Home insurance industry covers fire, theft and other property damage. Third party liability covers financial loss due the transport and aviation. The remaining amount falls under the coverage of the group health insurance, industrial accident and acceptance. The data that consist of the severity of incurred claims, but not the absolute frequency, are expressed in thousand Tunisian Dinars and are inflation-adjusted. Data are quarterly collected from 2000 (Q1) to 2016 (Q4) and are extracted from the financial statements which are published by the Tunisian Federation of Insurance Companies. In addition, we were able to retrieve our data from published annual reports and from indicators of quarterly company's technical operations. In picking these period scenarios, we follow [Tang and Valdez \(2009\)](#) which employed similar scenarios. Besides [Diers et al. \(2012\)](#) make use similar time periods in their study.

In the following, Table 1 reports some summary descriptions corresponding to the four lines of business. We note that the four variables are somewhat scattered. The data are not symmetrical supported by a positive coefficient of asymmetry greater than zero for risks FT, TA and OB in the range of 1.11, 0.45 and 0.23, which confirms an asymmetry distribution on the right. In contrast, the variable Mot exhibits a negative skewness coefficient in the range of -0.41. Kurtosis coefficients for variables Mot, TA and OB display values below zero while the variable FT has a positive one (leptokurtic). Given the heterogeneity of risks, the marginal distributions expose different types of tails<sup>3</sup>.

**Table-1.** Descriptive statistics of quarterly aggregate claims by line of business

	<b>Mot</b>	<b>FT</b>	<b>TA</b>	<b>OB</b>
<b>Mean</b>	20365	2152	565	13190
<b>Sd</b>	3667	961,7	322,1	2113
<b>Max</b>	27633	4998	1466	17260
<b>Min</b>	12014	921	79,0	9079
<b>kurtosis</b>	-0.323	0.848	-0.307	-0.942
<b>Skewness</b>	-0.418	1.112	0.451	0.231

**Note:** Mot (Motor branch), FT (fire and various technical risks branch), TA (transport and aviation) and OB (other branches).  
**Source:** Author's calculation.

<sup>3</sup> According to the results of the KPSS and PP tests, all data series are stationary at the conventional levels 1%.

**4.2. Marginal Distributions and their Parameterization**

In this paper, we have to decide for each line of Business the best fitting distribution and parameters estimate. Fitting loss insurance data is a delicate step because of their characters non-negativity, asymmetry and heavy tails. Among the main challenges in insurance risk modeling, one confronts the issue of the presence of very heterogeneous losses, scarcity of data, short time series with extreme tails and the entailment to estimate quantiles at very high confidence levels. In practice, the regulator and most financial institutions argue that the lognormal distribution is the best candidate. Theoretically, all continuous distributions with positive field definition are presented as candidates for the loss distribution modeling<sup>4</sup>.

We suggest to our dataset a variety range of distributions to be tested, widely proposed in the actuarial science, namely gamma distribution, logistic distribution, lognormal distribution, Weibull distribution, Pareto distribution, exponential distribution, generalized Pareto distribution (GPD), generalized error distribution (GED), skewed generalized error distribution (SGED), and generalized hyperbolic distribution (GHD). These distributions exhibit altered thicknesses tails.

**Table-2.** The marginal distributions calibration, parameter estimates, K-S and AD statistics

	<b>Mot</b>	<b>FT</b>	<b>TA</b>	<b>OB</b>
Distribution	Logistic	Log-normal	Weibull	Log-normal
Parm.1	<b>20539.73</b>	<b>7.585</b>	<b>1.825</b>	<b>9.450</b>
Parm.2	<b>2059.05</b>	<b>0.415</b>	<b>636.11</b>	<b>0.163</b>
KS	0.059***	0.091***	0.183**	0.119***
AD	0.310***	0.380***	0.391***	0.303***

(\*\*) and (\*\*\*) Significance level at the 5% and 1% respectively.

Source: Author's calculation.

Notably also, all the distributions parameters in this study are estimated by maximum likelihood method. In this case, the likelihood functions of a set of *n* independent observations *x* is estimated by:

$$L(\theta) = \prod_{j=1}^n f(X_j; \theta).$$

Where *f* is the density function whose parameters are represented by the vector  $\theta$ .

The results of the distributions parameter estimates of four risks are exhibited in Table 2. The parameter values of each distribution are deemed reasonable. The four loss variables demonstrate thicker tails at the ends of the fitted distributions. Once parameter estimation is performed, it is important to restrict our choice in one model.

In what follows, we will set up goodness-of-fit tests to test the acceptability of the different models and to classify them so that we could select among them the best one. A hypothesis test will examine the acceptability of the distribution. In fact, the null hypothesis can be formulated as follows:  $H_0: F_x(X) = F(X; \theta)$ . Where:  $F_x(X)$  is the empirical distribution function. Therefore, the model provides a good fit if the null hypothesis is not rejected. Then, we construct a Kolmogorov-Smirnov and Anderson Darling goodness of fit tests to ensure graphical results. The tests consist in measuring the absolute maximum deviation between the empirical distribution function and that of the model. Unlike the conventional test, these methods consist in calculating p-value and critical values by Monte Carlo simulation. The p-value allows us to decide whether we reject the null hypothesis that states the adequate fit of the distribution data. Thus, the critical values will be calculated from the generated samples. Table 2 summarizes the statistics KS and AD of each candidate distributions and their p-value respectively. The p-values designate that the two branches FT and OB have the most adequate distribution lognormal. However, the logistic

<sup>4</sup> Klugman, Panjer and Willmot (2009).

distribution provides a better fit for the Motor branch. Similarly, Weibull distribution seems adequate for adjusting TA branch.

The Appendix B exhibits the QQ-Plots of the four variables. It is worth noting that the Q-Q plots of Mot, VT, TA and OB versus respectively the Logistic (20539.73, 2059.05), Log-normal (7.585, 0.415), Weibull (1.825, 636.11) and Lognormal (9.450, 0.163) distributions are very close. Hence, the four lines of business are well specified.

The closing modeling of marginal distributions leads us to wonder about the nonlinear dependence between the variables. In doing so, we make use a correlation coefficient of ranks, called the Kendall tau which is established by Kendall (1938). The Kendall tau is useful for the modeling of copulas C and D vine. It is expressed as following:

$$\tau = \frac{(\text{number of concordant pair}) - (\text{number of discordant pair})}{\frac{1}{2}n(n - 1)}$$

Table 3 reveals Kendall's rank correlation. We notice that there is a significant correlation among the motor lines and OB lines (0.77), as well FT lines (0.50). The correlations between the pair of lines of business (FT, OB) are slightly important and they have positive correlations that equal to 0.49. However, the rest pairs of lines of business have weak correlations. Therefore, a car accident can affect the line "Fire and various technical risks branch" as well as the line "other branch".

Table-3. Empirical Kendall's matrix between empirical data.

Kendall's tau matrix				
	Mot	TA	FT	OB
Mot	1.000			
TA	0,190	1.000		
FT	0,509	0,165	1.000	
OB	0,774	0,175	0,492	1.000

Source: Author's calculation.

### 4.3. Results

In this section, we attempt to explore the dependence structure among losses data using copulas functions. As pointed out, we proceed to use various families of copulas with different specifications of tail dependence, which allows us to identify the tail-dependence between different variables. We pick out a copula model among the Gaussian, t-Student, Clayton, Gumbel, Frank, Joe, BB1, BB6, BB7, BB8 copulas.

For multivariate dependence modeling, we had to decide the choice of the best copula among C- or D- vine copula, which is built using a cascade of bivariate copulas. In doing so, we sort the order of the variables (losses) in the first tree when specifying C- or D-vine copulas. A rule to select the first root node consists of joining the most dependent pairs in the first tree in terms of absolute empirical values of pairwise Kendall's (Di β mann et al., 2013).

As reported in Table 3, the variable Mot displays a degree of dependence strongest with others. Therefore, this variable is the first root node for the C-vine copula. Applying the rest of the rule, the order of the first tree in C-vine copula is determined as the following: (Mot, FT), (Mot, TA), (Mot, OB). However, in the case of D-vine, the order of the variables in the first tree is taken as the following: Mot, FT, TA and OB. Having indicated which vine form we are working with, we specify the copula families and parameters as vectors of length d(d-1)/2, where d refers to the number of variables. In this respect, six pair copulas families related with C- or D- vine structure designated in the previous step have to be identified.

According to Brechmann and Schepsmeier (2013) the specification of the tree structure C- and D- vine is fitted sequentially using the AIC and BIC information criterions, which are served as benchmark criterion. To validate the choice of the bivariate copulas selected in the first tree which has a significant impact on the fitted copula, we conduct a goodness of fit test based on Kendall's process which calculate the Cramér-von-Mises statistic  $\hat{S}_n$ .

Having calibrated a vine to the data set, we conduct Monte-Carlo simulation to generate samples for each loss distributions  $\hat{u}_{i,j}$  from the fitted copula. Using the simulation algorithm for a C-or D- vine in Aas *et al.* (2009) we replicate 5,000 four-dimensional samples respecting the different initial marginal distributions.

**Table-4.** Estimated parameters for four-dimensional C-Vine.

	<i>Bivariate Copulas</i>	$\Theta_1$	$\Theta_2$	$\bar{S}_n$
<b>Level-1</b>				
Cop12(Mot,FT)	Gauss	0.2017	-	0.1588 (0.66)***
Cop 23 (Mot,TA)	t-Student	0.1662	10.278	0.1274 (0.52)***
Cop 34(Mot,OB)	t-Student	0.7401	5.004	0.0385 (0.15)***
<b>Level-2</b>				
Cop23/1	Clayton	0.3416	-	
Cop24/1	Clayton	0.2443	-	
<b>Level-3</b>				
Cop 34/12	t-Student	0.05108	8.204	
AIC [CVine]	2.1485			
BIC [CVine]	1.7468			
LL	<b>1578.81</b>			

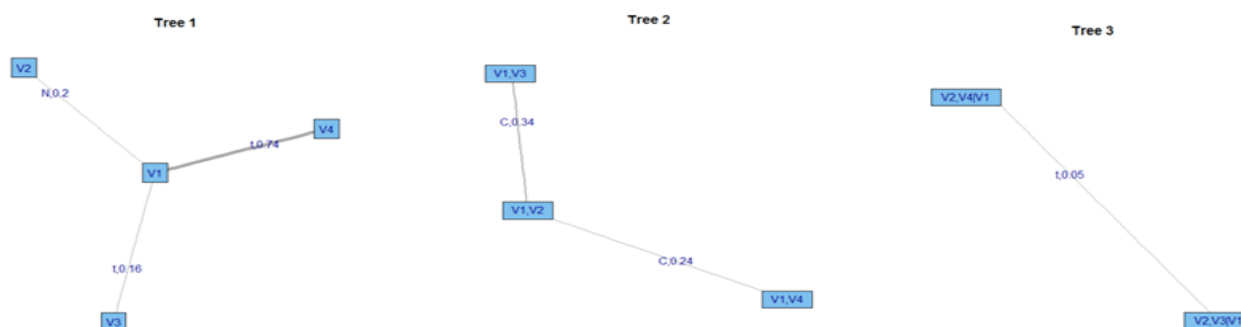
Note: the estimates obtained using the sequential estimation procedure.

The numbers in brackets are the p-values.

\*\*\* The statistics are significant at the 1% level.

Source: Author's calculation.

In the case of C-vine, the root nodes for each tree (three levels) are determined and the estimated dependence parameters are exhibited in Table 4. The C-Vine obtained with six bivariate copulas (conditional or unconditional) is partially depicted in a vine plot<sup>5</sup> in Figure 2. The first conclusion of the plot-vine stipulates the mixing of different bivariate copula families, which justifies the flexibility of the Vine copula. According to Table 4, we find that the best-fitted C-Vine copula is composed of Gaussian copula (1 time), t-Student copula (3 times) and Clayton copula (2 times). In the first tree, we find two different copulas with different tails: one Normal copula (0.20), two t-Student copulas (0.16; 0.74). The second tree assimilates two Clayton copulas (0.34; 0.24). Finally, the t-Student copula (0.05) is installed at last level.



**Fig-2.** The optimal C-Vine copulas in 4-dimensions

Source: Author's calculation.

Similarly, Table 5 reports the estimates of the pair-copulas fitted to compose the D-vine copula. Figure 3 delineates the D-vine plot obtained with six bivariate copulas. Accordingly, the empirical results reveal that four

<sup>5</sup> We denote that the vector (V1, V2, V3, V4) refers to (Motor, FT, TA, OB)

bivariate copulas have negative parameters including, three Frank copulas and a Gaussian copula. The two other copulas, Joe and Clayton, have positive parameters. The first tree includes three different bivariate copulas that connect loss variables according to the degree of dependence structure namely, the copula Frank (-0.26), the normal copula (-0.18) and Joe copula (1.4). The level two of the tree assimilates two Frank copulas (-0.5, 1.4). Finally, the Clayton copula (0.5) is at level three.

**Table-5.** Estimated parameters for four-dimensional pair-copula decomposition. AIC and BIC value's. Cramer-Von-Mises statistics.

	<i>Bivariate Copulas</i>	$\Theta_1$	$\Theta_2$	$\hat{\Sigma}_n$
<b>Level-1</b>				
Cop12(Mot-FT)	Frank	-0.2618	-	0.0764 (0.78)***
Cop 23 (FT-TA)	Gauss	-0.1828	-	0.1876 (0.24)***
Cop 34(TA-OB)	Joe	1.4860	-	0.2358 (0.7)***
<b>Level-2</b>				
Cop13/2	Frank	-0.5482	-	
Cop24/3	Frank	-1.4253	-	
<b>Level-3</b>				
Cop 14/23	Clayton	0.5655	-	
AIC [CVine]	-9.3232			
BIC [CVine]	1.1488			
LL	<b>1635.11</b>			

**Note:** the estimates obtained using the sequential estimation procedure.

The numbers in brackets are the p-values.

\*\*\* The statistics are significant at the 0.01 level.

**Source:** Author's calculation.

To verify the selected bivariate copulas for all pairs in the first tree, we apply a copula goodness-of-fit test based on Kendall's process which determine the Cramèr-Von Mises statistics. Then, we provided 10,000 bootstrap runs and exam for the validity of this choice. Looking at Tables 4 and 5 once more, only Guassian and t-student copulas yield a better fit in the first tree for C-vine type. However, according to CvM statistic, we perceived that the p-values of CvM statistic fail to reject Frank, Gauss and Joe copulas in the first tree for D-vine type. We can perceive that all estimated parameters of the conditional copulas in the first tree are significant at 1% significance level. Considering the Gaussian and Frank copulas, the estimated parameters (negative) are significant, indicating that the two lines of business display either an absence of dependency or a weak negative dependence between the small amounts of claims. However, the Joe copula show left-positive tail dependence between variables.

Having fitted the CD-vine copulas types to our data set, we attempted to decide the accuracy of Vine copula structure in terms of one or more criteria. Thence, we computed the information criteria AIC, BIC at the fitted models. Furthermore, we also looked at the log-likelihood values (LL) for each models so as to confirm our choice of the appropriate model. According to the results from Tables 4 and 5, we can see that the log-likelihood of C-vine is 1578.81, while the log-likelihood of D-vine is 1635.11. Comparing the empirical results in Table 4 and 5, we can conclude that the D-vine copula model reveals a higher LL value and lower AIC and BIC values. Hence, this model (D-vine) provides a better fit for data than the other model and it provides a more accurate description of claim amounts dependence. Therefore, we select as the best appropriate permutation for the first level of the D-vine under analysis  $(V_1, V_2, V_3, V_4) = (\text{Motor}, \text{FT}, \text{TA}, \text{OB})$ , since it comprises the largest possible dependencies. Their corresponding densities are shown in Figure 4.



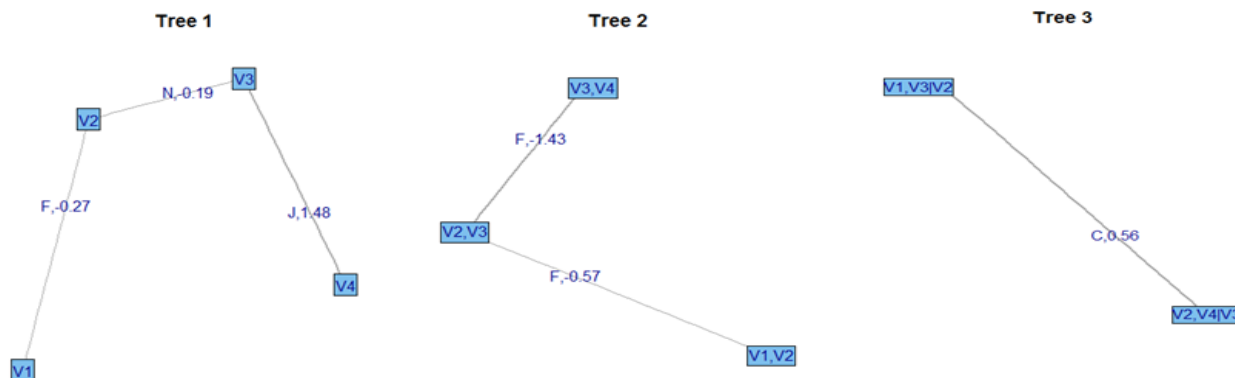


Fig-3. The optimal D-Vine copulas in 4-dimensions

Source: Author's calculation.

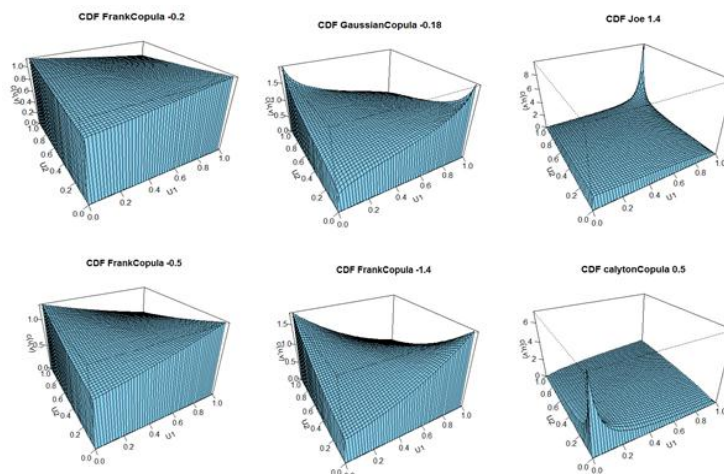


Fig-4. 3D surface plot of bivariate density for Frank, Gaussian, Joe and Clayton copulas with different parameter settings.

Source: Author's calculation.

#### 4.4. Sensitivity Analysis and Implications

The aim of this paper is to examine the levels of the capital requirement, which permit insurers to be solvent in upcoming periods at a certain probability threshold. At this stage of analysis, it is necessary to investigate the impact of dependency structure by aggregating losses using copulas on the risk capital estimation. Notably, also we highlight the sensitivity of the attained results to the choice of the used risk measures. In a first step, we simulate the random losses data from the fitted copulas: 5,000 four-dimensional samples are drawn from the D-vine copulas fitted to the four-dimensional dataset and inverted in terms of the original distributions of margins. By referring to the aforementioned advanced analysis in section 3, the simulated loss variables derived from D-vine are then summed up so as to establish an aggregated loss distribution as in Eq. (10).

In the second step, the capital requirement can be then inferred using the risk measures VaR and TailVaR for different levels  $\alpha\%$ . These measures of risk are calculated in terms of the aggregated loss distribution establish in the previous section with a specified level of probability. For our purpose, we choose arbitrary confidence levels  $\alpha=(95.5\%, 97.5\%, 98\%, 99\%, 99.5\%)$ . Especially,  $\alpha = 99.5\%$  is taken to simplify the coherent comparison of our results with the prescribed level in Solvency 2 in order to reply the research question of this paper. Given the distribution of prospective losses from the output of the R language (*actuar Package*), we can straightforwardly compute the VaR and TailVaR at the chosen levels of  $\alpha$ .

To make suitable assessment, Table 6 reveals the results of the simulated VaR and TailVaR of the aggregate loss for the base scenario for D-vine copula model (*model 1*) for different confidence levels. For comparison purposes, we propose a second model for capital requirement assessment, which stipulates the hypothesis of independence between losses (*model 2*). Continuing with the procedure outlined earlier, the simulated loss variables for each

business line are aggregated to produce a distribution of the aggregate loss under independence hypothesis. Through this method, we examine the switching of the funds required amount for the same confidence levels. The estimation Results of the two proposed approaches are compared numerically in Table 6. As we can see from the table, we exploited the models of risk capital assessment to isolate the effects of taking into account the structures of dependence. At first sight, the attained findings provide insights on the practical effects of dependence modeling.

In the following, we first focus on VaR for the two scenarios and then compare the results for VaR and TailVaR. By comparing to D-vine copula (*model1*), the case of independence, which provides a multivariate distribution according to Monte Carlo approach leads to the higher VaR. As first finding, we can notice the primary effect of considering the dependence structure between risks. Thus, the higher the level of confidence taken by an insurer, more the VaR will be and gives important values of risk capital. Obviously, we recognized that the VaR is slightly lower than that obtained in the case of TailVaR. This explained by the fact that the VaR is not coherent measure while the TailVaR do. In fact, the TailVaR stipulates a more conservative amount of capital requirement since it is a coherent measure. Therefore, the regulator should reconsider the use of the risk measure.

Now, by examining models in terms of dependency consideration, we notice immediately that the amounts of capital requirement calculated with vine copula are inferior to capital requirement evaluated with model 2. The estimation results show that the way in which dependence is modeled has a paramount impact on the estimation of risk capital. This impact is even more accentuated for the TailVaR. Moreover, we perceive that when a  $\alpha=99,5\%$  as under Solvency 2, differences between the D-vine are further underscored compared to the independence assumption. This impact is driven primarily by the amount of tail dependence that the vine copula tolerates. Therefore, the extent of the impact fluctuates depending on the risk measure used and according to the chosen copula.

**Table-6.** Results of VaR and TVaR estimates.

Methods		95.5%	97.5%	98%	99%	99.5%
<i>Model 1 D-Vine</i>	<i>VaR</i>	17423.67	18079.67	18325.14	19208.71	20131.48
	<i>TailVaR</i>	19079.80	19404.35	19985.48	20928.76	20974.89
<i>Model 2 Indep</i>	<i>VaR</i>	18341.55	19264.00	19657.96	20592.11	21364.61
	<i>TailVaR</i>	19452.79	19410.99	20404.22	20596.17	21729.36

Source: Author's calculation.

The second interest of this study is to investigate the impact of diversification benefit resulting from writing multiple lines of business assuming to be dependent. A main implication of diversification effect is to mitigate the riskiness of the insurer's portfolio by combining potentially correlated risks.

**Table-7.** Diversification Benefit

<i>Div<math>\alpha</math></i>	95.5%	97.5%	98%	99%	99.5%
<i>VaR</i>	2.90%	3.16%	4.20%	5.21%	5.77%
<i>TailVaR</i>	1.91%	1.98%	2.05%	2.85%	3.47%

Source: Author's calculation.

One has to realize that risk capital consists of quantifying the capital required to absorb losses in an adverse scenario. In fact, each line of business corresponds to a stand-alone business having to provide its own capital requirement (see also Embrechts (2009)). From the insurer's perception, the diversification benefit is defined as the difference (savings) between the risk capital on the aggregate loss and the sum of the amounts of risk capital for

each business line as if each is a stand-alone business (Tang and Valdez, 2009). In other words, this difference denotes the gained in risk capital required due to the diversification of adding lines of business to the portfolio.

According to Merton and Perold (1993) the total portfolio risk will be lower than the sum of independent risks if these risks are not perfectly correlated with each other. The authors stipulate that if each business is supposed to be organized as a stand-alone business, the capital requirements are increasing due to the loss of diversification. Therefore, the diversification gain refers to the gain in the capital when combining all risk factors in one company as opposed to running with the risk factors individually (see Eq. (13)).

In general, the diversification benefit can be quantified by the following expression:

$$Div_{\alpha} = \frac{\sum_{j=1}^n VaR_{\alpha}(S_j) - VaR_{\alpha}(\sum_{j=1}^n S_j)}{\sum_{j=1}^n VaR_{\alpha}(S_j)} \quad (13)$$

if  $Div_{\alpha} > 0$ , there is a diversification benefit.

The diversification impact for the vine copula model considered is displayed in Table 7. Thus, we closely observe there is a positive diversification benefit ranging from 1.91% to 5.77%, when explicitly modeling the dependence. This explained by the fact that the capital required to cover the four non-life risks, calculated with the copula D-Vine, is less than the risk capital assessed with the *model 2*. Moreover, insofar as the dependency structure between the lines of business is overlooked, the required capital becomes more accentuated than in the case where this dependence is considered. This emphasizes that the diversification effect is largely driven by the general level of dependence modeled using copulas. It is caused otherwise by the amount of tail dependence among losses that copula permits. The more tail dependence allowed by a copula, the lower the risk capital as if losses are aggregated under this copula. The attained findings confirm the theoretical analysis found by Tang and Valdez (2009) which states that there is always a positive diversification benefit by aggregating lines of business in a multi-line case set up rather than running the stand-alone individuals affairs. Consequently, the choice of the dependence modeling of non-life insurance risks seems to be of main importance once the marginal distributions are fitted. We notice also that the choice of risk measurement significantly affects the level of risk capital and diversification benefit. Insurance companies and regulators can explore our conclusions for the assessment of risk capital under Solvency 2, which is based on stochastic models.

## 5. CONCLUSION

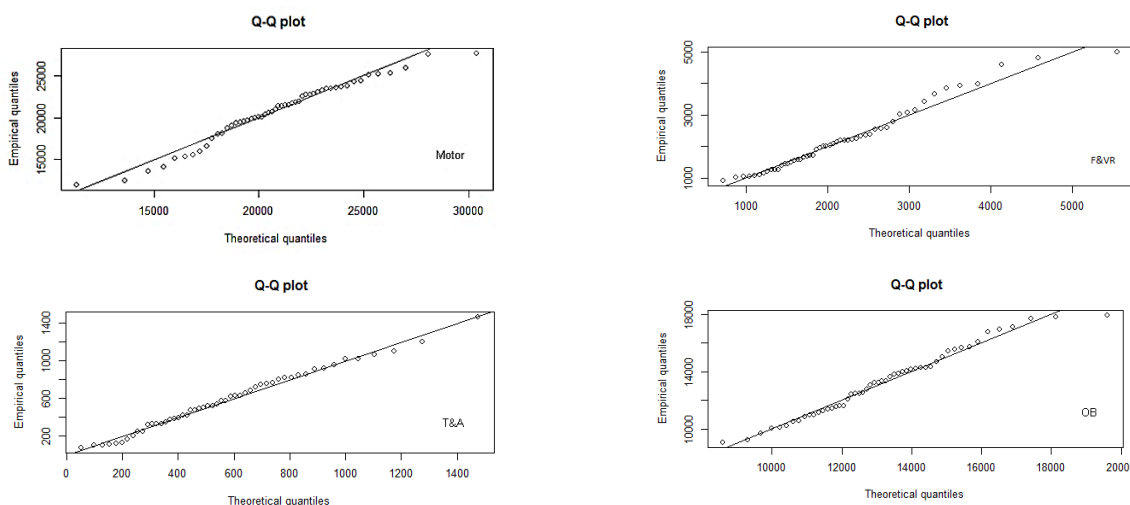
The standard approach of the Solvency Committee on Insurance Supervision recommends insurance firm to determine the risk capital, which is proportional to a measure of its global risk. In this paper, we investigated the sensitivity of risk capital estimation to dependence structure among the losses from four non-life business lines of a Tunisian insurance company. We conduct the empirical analysis in two stages. In a first step, we examined the modeling dependence between losses using the multivariate vine copulas. Explicit dependence modeling is discussed critically by considering two copula classes, namely C-Vine and D-Vine Copulas, which consist of elliptical and Archimedean families. To confirm the choice of the fitted copula, we conducted goodness-of-fit analysis based on AIC and BIC criteria as well as the Cramer-von Mises statistic. It turns out that from a theoretical perspective, the D-Vine copula is probably the most appealing model for non-life insurance losses, while still being simply interpretable in its parameters and structure and authorizing for the presence of varied pairwise and tail dependencies. In a second step, we focused on the impact of realistic dependence structure of risks on the total capital requirement of four-dimension portfolio using VaR and TVaR measures in the simulation context. To achieve interpretable and accurate results, we considered a model for capital requirement forecasting, which stipulated the hypothesis of independence between losses derived from simulated data. By means of this model, we scrutinized the switching of the funds required amount for the same confidence levels.

At first sight, we notice immediately that the capital requirement assessed with vine copula is less than the capital requirement evaluated under the independence assumption. The estimation results show that the way in which dependence is modeled has a paramount impact on the estimation of risk capital. Thus, the attained findings provide insights on the practical effects of dependence modeling. In addition, the TailVaR stipulates a more conservative amount of capital requirement since it is a coherent measure. Therefore, the regulator should reconsider the use of the risk measure. Therefore, the extent of the impact fluctuates depending on the risk measure used and according to the chosen copula. Several research perspectives appear to be promising, such as the integration of the extreme values theory (EVT) with the multivariate R-Vine copulas. Some limitations confront this analysis including sample size and data availability. The technique of D-Vine copula is a flexible approach when used with homogeneous databases, but it is not the appropriate solution to all dependency modeling problems.

Appendix-A. copula functions and its parameters.

Copula Function	Formula	Parameter
Gaussian (N)	$C_N(u, v, \rho) = \Phi(\Phi^{-1}(u), \Phi^{-1}(v))$	$\rho \in [-1; 1]$
Student-t	$C_t(u, v, \rho, \nu) = t(t_\nu^{-1}(u), t_\nu^{-1}(v))$	$\rho \in [-1; 1]$
Clayton	$C(u, v, \delta) = \max((u^{-\delta} + v^{-\delta} - 1)^{-\frac{1}{\delta}}, 0)$	$\delta \in [-1; \infty) \setminus \{0\}$
Gumbel	$C(u, v, \delta) = \exp(-((( -\log u)^\delta + (-\log v)^\delta - 1)^{\frac{1}{\delta}}))$	$\delta \in [1; \infty)$
Joe	$C(u, v, \theta) = 1 - (1 - \exp(-t))^{1/\theta}$	$t \in [0, \infty]$

Source: Brechmann and Schepsmeier (2013)



Appendix-B. Q-Q plots for the univariate distributions of four distributions of loss data

Source: Author's calculation.

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**Competing Interests:** The authors declare that they have no competing interests.  
**Contributors/Acknowledgement:** Both authors contributed equally to the conception and design of the study.

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