

INEQUALITY AND EDUCATION SUBSIDIES IN GENERAL EQUILIBRIUM GROWTH MODEL FOR A SMALL OPEN ECONOMY



 Wei-Bin Zhang¹

¹Ritsumeikan Asia Pacific University, Japan

Email: wbz1@apu.ac.jp



ABSTRACT

Article History

Received: 19 February 2018

Revised: 17 April 2018

Accepted: 20 April 2018

Published: 24 April 2018

Keywords

Government education subsidy

Income and wealth distribution

Small-open economy

Heterogeneous household

Learning by consuming

Education

Learning by doing

JEL Classification:

O41, F11

The paper is concerned with dynamic interactions between physical capital, human capital, income and wealth inequalities between different households with government subsidy to education. The model is developed on the basis of Solow-Uzawa's neoclassical growth theory, Uzawa-Lucas model, Arrow's learning by doing, Zhang's creative leisure, and Walrasian general equilibrium theory. The capital accumulation and economic structure are based on the neoclassical growth theory. The human capital accumulation is due to Uzawa's education, Arrow's learning by doing, and Zhang's creative leisure. The model explains income and wealth inequality between groups with government education subsidy policy in a small-open economy. The model reveals a complicated nonlinear dynamic interdependence between wealth accumulation, human capital accumulation, economic structural change, division of labor, and time distribution under perfect competition and government education subsidy policy. We simulate the economy composed of three groups of households. We carry out comparative dynamic analysis and demonstrated how a change in a parameter affects the path of economic growth.

Contribution/ Originality: This study is one of few theoretical studies, which model dynamic interactions between physical capital, human capital, and income and wealth inequalities. It integrates the main determinants of economic growth in the Solow-Uzawa's neoclassical growth theory, Uzawa-Lucas model, Arrow's learning by doing, Zhang's creative leisure, and Walrasian general equilibrium theory.

1. INTRODUCTION

This paper is concerned with dynamic relationships between economic growth and income and wealth inequalities. Forbes (2000) argued for the necessity of a new analytical framework as follows: "careful reassessment of the relationship between these two variables (growth rate and income inequality) needs further theoretical and empirical work evaluating the channels through which inequality, growth, and any other variables are related." This study emphasizes the role of government subsidy policy on human capital and inequalities. As emphasized by Zhang (2013) it is difficult to properly deal with issues related to income and wealth distribution with the current mainstream analytical economics. To overcome problems of the lack of a proper analytical framework for analyzing economic dynamics with heterogeneous households with microeconomic foundation, Zhang apply an alternative approach to household behavior. By applying

the new tool for analyzing household decision, we can effectively deal with many important issues in dynamic economics. This study applies the approach to the complicated issue of growth with government subsidy policy.

This study is based on a few economic theories. As far as economic structure at a point in time is concerned, the model framed within the Walrasian general equilibrium theory. The analytical framework of general equilibrium theory was initially constructed by Walras (1874). The theory is further developed by many other economists (e.g., (Arrow and Debreu, 1954; Gale, 1955; Nikaido, 1956;1968; Debreu, 1959; McKenzie, 1959; Arrow and Hahn, 1971; Arrow, 1974; Mas-Colell *et al.*, 1995)). The theory deals with equilibrium of pure economic exchanges. However, the theory failed to properly include endogenous wealth (and other dynamic factors such as changes in environment, resources, human capital and knowledge). This study introduces endogenous physical capital and human capital into the general equilibrium theory. Our model is to introduce the neoclassical growth theory into the Walrasian general equilibrium. The traditional neoclassical growth theory is not successful in examining economic growth with heterogeneous households. Most of the neoclassical growth models are developed for economies of homogenous population. In some neoclassical growth models the heterogeneity is the differences in the initial endowments of wealth among different types of households rather than in preferences (e.g., (Chatterjee, 1994; Caselli and Ventura, 2000; Maliar and Maliar, 2001; Penalosa and Turnovsky, 2006; Turnovsky and Penalosa, 2006)). In this approach different households are essentially homogeneous as all the households have the same preference utility function in the traditional Ramsey approach.

Human capital is essential for contemporary economic growth (Hanushek and Kimko, 2000; Barro, 2001; Krueger and Lindahl, 2001; Castelló-Climent and Hidalgo-Cabrillana, 2012). Education is commonly considered an essential way of accumulating human capital. The first formal modeling of education and economic growth is carried out by Uzawa (1965). Another popular work on the similar topic is done by Lucas (1988). There are many further works on growth and education based on the Uzawa-Lucas model (e.g., (Jones *et al.*, 1993; Stokey and Rebelo, 1995; Mino, 1996;2001; Zhang, 2003; Alonso-Carrera and Freire-Sere, 2004; De Hek, 2005; Chakraborty and Gupta, 2009; Sano and Tomoda, 2010)). As far as education is concerned, this study takes account of government subsidy within a comprehensive analytical framework. Moreover, this study takes account of another two sources of human capital accumulation: Arrow's learning by doing (Arrow, 1962) and Zhang's creative leisure (Zhang, 2007). The model is a synthesis of two models recently proposed by Zhang (2013;2016). Zhang (2013) proposed a heterogeneous-household growth model with endogenous physical and human capital. Zhang (2016) deals with the impact of education subsidy on economic growth. Nevertheless, this model does not deal with issues related to inequality between different people. We examine the impact of education subsidies. Another difference from the two models by Zhang is that this study is concerned with a small-open economy. As an important branch of economic growth, there are many studies on growth and trade of small open-economies (e.g., (Obstfeld and Rogoff, 1996; Lane, 2001; Kollmann, 2001;2002; Benigno and Benigno, 2003; Gali and Monacelli, 2005; Zeng and Xiwei, 2011)). We follow this tradition in determining trade pattern with free trade and the prices of tradable goods fixed in global markets. The rest of the paper is structured as follows. In Section 2 we develop the small-open growth model with economic structural change, endogenous physical and human capital accumulation. In Section 3 we examine properties of the dynamic model and conduction simulation of the model. In Section 4 deal with comparative dynamic analysis with regard to changes in some parameters. In Section 5 we conclude the study. The results of Section 3 are checked in the appendix.

2. THE BASIC MODEL

We refer to Zhang (2013;2016) for modelling economic structure, wealth and human capital accumulation, and government's taxation. Most aspects of the production sectors are developed within the framework of the standard growth models (Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). The economy is composed of capital good, consumer good and education sectors. The three sectors are perfectly competitive and are taxed by the government. The tax income is fully expended on subsidizing students. Assets of the economy belong to

households. The households' incomes are distributed to consume and to save. Saving is carried out only by households. Firms employ labor and physical capital inputs to produce goods and services. All markets are perfectly competitive. Input factors are always fully employed. All earnings of firms are paid to factors of production, labor, managerial skill and capital ownership. The population is grouped into J groups, indexed by $j = 1, \dots, J$. We measure prices in terms of the commodity. The price of the commodity be unit. As the economy is small, the rate of interest r^* is determined in global markets and is constant. We introduce variables as follows:

Subscript index i, s and e - capital good sector, consumer good sector, and education sector;

$N_m(t)$ and $K_m(t)$ - labor force and capital stocks employed by sector $m = i, s, e$ at t ;

$F_m(t)$ - the production function of sector m ;

$p_s(t)$ and $p_e(t)$ - the price of consumer good and the price of education per unit of time;

$\tau(t)$ and $\bar{\tau}(t)$ - the tax rate on each sector and $\bar{\tau}(t) \equiv 1 - \tau(t)$;

$K(t)$ and $\bar{K}(t)$ - physical capital employed by and wealth owned by the country;

\bar{N}_j and $H_j(t)$ - group j 's fixed population and level of human capital;

$T_j(t)$, $\bar{T}_j(t)$ and $T_{je}(t)$ - the work time, leisure time, and study time of a typical worker in group j ;

$w_j(t)$ and $\bar{k}_j(t)$ - group j 's wage and per capita wealth of group j ;

δ_k and r_0 - the fixed depreciation rate of capital and $r_0 \equiv r^* + \delta_k$.

The labor service is $T_j(t)H_j^{m_j}(t)$, where we call m_j utilization efficiency of human capital by group j . The labor input is the work time by the effective human capital. The total labor input by a group is the sum of labor inputs of the group population, $T_j(t)H_j^{m_j}(t)\bar{N}_j$. The total labor input $N(t)$ is the sum of all the groups' labor inputs

$$N(t) = \sum_{j=1}^J T_j(t)H_j^{m_j}(t)\bar{N}_j, \quad j = 1, \dots, J. \quad (1)$$

2.1. Full Employment of Input Factors

The three sectors employ all the labor force

$$N_i(t) + N_s(t) + N_e(t) = N(t). \quad (2)$$

The three sectors employ all the national capital

$$K_i(t) + K_s(t) + K_e(t) = K(t). \quad (3)$$

2.2. National Wealth Owned the Population

The national wealth is the sum of all the households' wealth

$$\bar{K}(t) = \sum_{j=1}^J \bar{k}_j(t) \bar{N}_j. \quad (4)$$

2.3. The Capital Good Sector

We take the production function of the capital good sector on the following form

$$F_i(t) = A_i K_i^{\alpha_i}(t) N_i^{\beta_i}(t), \quad \alpha_i, \beta_i > 0, \quad \alpha_i + \beta_i = 1, \quad (5)$$

Where A_i , α_i , and β_i are positive parameters. The marginal conditions imply

$$r_0 = \frac{\alpha_i \bar{r}(t) F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i \bar{r}(t) F_i(t)}{N_i(t)}. \quad (6)$$

2.4. The Consumer Goods Sector

The production function of the consumer good sector is taken on the following form

$$F_s(t) = A_s K_s^{\alpha_s}(t) N_s^{\beta_s}(t), \quad \alpha_s + \beta_s = 1, \quad \alpha_s, \beta_s > 0, \quad (7)$$

Where A_s , α_s , and β_s are technological parameters. The marginal conditions are

$$r_0 = \frac{\alpha_s \bar{r}(t) p_s(t) F_s(t)}{K_s(t)}, \quad w(t) = \frac{\beta_s \bar{r}(t) p_s(t) F_s(t)}{N_s(t)}. \quad (8)$$

2.5. The Education Sector

We follow Zhang (2013) in modelling the education sector. Teachers and capital input are paid according to the market rates. We measure the total education service by the total education time received by the population. The production function of the education sector is taken on as follows

$$F_e(t) = A_e K_e^{\alpha_e}(t) N_e^{\beta_e}(t), \quad \alpha_e, \beta_e > 0, \quad \alpha_e + \beta_e = 1, \quad (9)$$

Where A_e , α_e and β_e are positive parameters. The marginal conditions imply

$$r_0 = \frac{\alpha_e \bar{r}(t) p_e(t) F_e(t)}{K_e(t)}, \quad w(t) = \frac{\beta_e \bar{r}(t) p_e(t) F_e(t)}{N_e(t)}. \quad (10)$$

2.6. Current and Disposable Incomes

The variables chosen by a consumer include the leisure time, education time, consumption level of consumer good as well as on how much to save. The wage rate of the representative household in group j is given by

$$w_j(t) = w(t) H_j^{m_j}(t), \quad j = 1, \dots, J. \quad (11)$$

We define per capita current income from the interest payment $r(t) \bar{k}_j(t)$ and the wage payment $T_j(t) w_j(t)$ as follows

$$y_j(t) = r(t)\bar{k}_j(t) + T_j(t)w_j(t).$$

It is assumed selling and buying wealth can be conducted instantaneously without any barriers and transaction cost. We define the per capita disposable income as

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t) = (1 + r(t))\bar{k}_j(t) + T_j(t)w_j(t). \quad (12)$$

The total available budget is fully spent on saving $s_j(t)$, consumption of consumer goods $c_{sj}(t)$ and education $T_{ej}(t)$. As the representative household from group j receives τ_j units of subsidy per unit of time from the government. This is obviously a simplified subsidy policy. In the literature of growth with education subsidies, different ways of subsidies on education are specified (e.g., (Blankenau and Simpson, 2004; Bovenberg and Jacobs, 2005; Booth and Coles, 2010)). The education cost of the representative household equals the education price charged by the education sector minus the subsidy from the government

$$\bar{p}_j(t) \equiv p_e(t) - \tau_j(t).$$

The budget constraint is

$$p_s(t)c_{sj}(t) + \bar{p}_j(t)T_{ej}(t) + s_j(t) = \hat{y}_j(t) = (1 + r(t))\bar{k}_j(t) + w_j(t)T_j(t). \quad (13)$$

The representative household spends the time available on working, leisure and education

$$T_j(t) + \bar{T}_j(t) + T_{ej}(t) = T_0, \quad (14)$$

Where T_0 is the total available time. Substitute (14) into (13)

$$p_s(t)c_{sj}(t) + p_{ej}(t)T_{ej}(t) + w_j(t)\bar{T}_j(t) + s_j(t) = \bar{y}_j(t), \quad (15)$$

Where

$$p_{ej}(t) \equiv \bar{p}_j(t) + w_j(t), \quad \bar{y}_j(t) \equiv (1 + r(t))\bar{k}_j(t) + w_j(t)T_0.$$

If the household spends all the available time on work, then $\bar{y}_j(t)$ is the disposable income.

2.7. Utility Function and Optimal Decision

As stated by Lazear (1977) "education is simply a normal consumption good and that, like all other normal goods, an increase in wealth will produce an increase in the amount of schooling purchased. Increased incomes are associated with higher schooling attainment as the simple result of an income effect." (see also, (Heckman, 1976; Lazear, 1977; Malchow-Møller *et al.*, 2011)). This study treats education as normal good. As in Zhang (2013) the utility function is dependent on the following four variables, $T_{ej}(t)$, $\bar{T}_j(t)$, $c_{sj}(t)$, and $s_j(t)$. The utility function is taken on the following form

$$U_j(t) = T_{ej}^{\kappa_{0j}}(t)\bar{T}_j^{\sigma_{0j}}(t)c_{sj}^{\xi_{0j}}(t)s_j^{\lambda_{0j}}(t), \quad \kappa_{0j}, \sigma_{0j}, \xi_{0j}, \lambda_{0j} > 0,$$

In which κ_{0j} , σ_{0j} , ξ_{0j} , and λ_{0j} are the household's elasticities of utility with regard to education, leisure time, consumer good, and saving. The parameters κ_{0j} , σ_{0j} , ξ_{0j} , and λ_{0j} are called propensities to receive education, to enjoy leisure, to consume consumer good, and to hold wealth, respectively. There are other ways in describing consumption of education (Becker, 1981; Behrman *et al.*, 1982; Cox, 1987; Fernandez and Rogerson, 1998; Banerjee, 2004; Florida *et al.*, 2008; Galindez, 2011).

Maximize $U_j(t)$ subject to (15)

$$p_{ej}(t)T_{ej}(t) = \kappa_j \bar{y}_j(t), \quad w_j(t)\bar{T}_j(t) = \sigma_j \bar{y}_j(t), \quad p_s(t)c_{sj}(t) = \xi_j \bar{y}_j(t), \quad s_j(t) = \lambda_j \bar{y}_j(t), \quad (16)$$

Where

$$\kappa_j \equiv \rho_j \kappa_{0j}, \quad \sigma_j \equiv \rho_j \sigma_{0j}, \quad \xi_j \equiv \rho_j \xi_{0j}, \quad \lambda_j \equiv \rho_j \lambda_{0j}, \quad \rho_j \equiv \frac{1}{\kappa_{0j} + \sigma_{0j} + \xi_{0j} + \lambda_{0j}}.$$

2.8. Change in the Household Wealth

According to the definitions of $s_j(t)$, we describe the wealth accumulation of the representative household in group j as follows

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \quad (17)$$

This equation implies that the change in wealth is equal to saving minus dissaving.

2.9. Dynamics of Human Capital

As in Zhang (2013) there are three sources of human capital accumulation. The first is due to learning by producing suggested by the literature of technological change and economic growth by Arrow (1962). The basic idea that people have new ideas and accumulate skills when they produce goods and supply services. As pointed out by Zhang (2013) this idea has narrow implications as there are many other sources of accumulating skills and knowledge. Uzawa (1965) introduced another way of human capital accumulation. In the Uzawa model it is through formal education that human capital is accumulated. The Uzawa model assumes that education uses resources and there is a trade-off between education efficiency and economic growth. But the Uzawa model omits the role of learning by doing in human capital accumulation. Zhang (2007) introduced another source of accumulating human capital into growth theory. He called this source as the creative leisure. This source of learning is taken account neither in formal education approach nor in learning through producing approach. Zhang models human capital accumulation by synthesizing the three sources of learning in a single analytical framework. As leisure time is gradually increasing in many economies, learning through playing or leisure activities seems to become increasingly important. Leisure activities such as sports clubs, computer games, social parties, living in a safe and decent social environment, and touring different parts of the world, are obviously important for accumulating human capital. According to Zhang (2013) human capital accumulates according to the following equation

$$\dot{H}_j(t) = \frac{v_{ej} (F_e(t)/N_e(t))^{a_{ej}} (H_j^{m_j}(t) T_{ej}(t))^{b_{ej}}}{H_j^{\pi_{ej}}(t)} + \frac{v_{ij} (F_i(t)/N_i(t))^{a_{ij}}}{H_j^{\pi_{ij}}(t)} + \frac{v_{hj} C_{sj}^{a_{hj}}(t)}{H_j^{\pi_{hj}}(t)} - \delta_{hj} H_j(t), \quad (18)$$

Where $\delta_h (> 0)$ is the depreciation rate of human capital, $v_e, v_i, v_h, a_e, b_e, a_i$, and a_h are non-negative parameters. We don't specify the signs of the parameters π_{je}, π_{ji} , and π_{jh} in this stage of modelling.

2.10. Demand of and Supply for Consumer Goods

The households consume all what the consumer good sector supplies. The total demand equaling the total supply implies

$$\sum_{j=1}^J c_{sj}(t) \bar{N}_j = F_s(t). \quad (19)$$

2.11. Demand of and Supply for Education

The demand for education is the sum of $T_{ej}(t) \bar{N}_j$ for all j . The total demand equaling the total supply implies

$$\sum_{j=1}^J T_{ej}(t) \bar{N}_j = F_{ej}(t). \quad (20)$$

2.12. The Government Budget

The subsidies that all the students receive from the government is equal to the government's tax income

$$\sum_{j=1}^J \tau_j T_{ej}(t) \bar{N}_j = \tau(t)(F_i(t) + p_s(t)F_s(t) + p_e(t)F_e(t)). \quad (21)$$

We completed the model. The modelling structure is general. For instance, if we neglect taxation and subsidy and fix wealth and human capital and allow the number of types of households equal the population, then the model is a Walrasian general equilibrium model. If the population is homogeneous, our model is structurally similar to the neoclassical growth model by Solow (1956) and Uzawa (1961). The modelling structure includes to the multi-class models by Pasinetti and Samuelson (e.g., (Samuelson, 1959; Pasinetti, 1960;1974)) as special cases. The model contains some ideas in the literature of growth with education subsidy. We now examine dynamics of the model.

3. THE DYNAMIC PROPERTIES OF THE MODEL

The economy is composed of any number of households and three sectors. The dynamic system is nonlinear and may be highly dimensional. We may deal with this kind of nonlinear dynamic systems with computer. The following lemma provides a procedure to follow the motion of the economic system.

3.1. Lemma

The dynamics of the economy with J types of households is governed by the following $2J$ dimensional differential equations system with $\tau(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$, where $\{\bar{k}_j(t)\} \equiv (\bar{k}_2(t), \dots, \bar{k}_J(t))$ and $(H_j(t)) \equiv (H_1(t), \dots, H_J(t))$, as the variables

$$\begin{aligned} \dot{\tau}(t) &= \Lambda_1(\tau(t), (H_j(t)), \{\bar{k}_j(t)\}), \\ \dot{\bar{k}}_j(t) &= \Lambda_j(\tau(t), (H_j(t)), \{\bar{k}_j(t)\}), \quad j = 2, \dots, J, \\ \dot{H}_j(t) &= \Omega_j(\tau(t), (H_j(t)), \{\bar{k}_j(t)\}), \quad j = 1, \dots, J, \end{aligned}$$

in which Λ_j and Ω_j are unique functions of $\tau(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$ at any point in time, defined in the appendix. For given $\tau(t)$, $\{\bar{k}_j(t)\}$, and $(H_j(t))$, we decide the other variables as follows: $\bar{\tau}(t) = 1 - \tau(t) \rightarrow w(t)$ by (A3) $\rightarrow w_j(t)$ by (A4) $\rightarrow p_s(t)$ by (A5) $\rightarrow p_e(t)$ by (A6) $\rightarrow \bar{k}_1(t)$ by (A18) $\rightarrow N_i(t)$ by (A15) $\rightarrow N_e(t)$ by (A11) $\rightarrow N_s(t)$ by (A9) $\rightarrow \bar{y}_j(t)$ by (A7) $\rightarrow T_j(t)$ by (A12) $\rightarrow N(t)$ by (2) $\rightarrow K_m(t)$, $j = i, s, e$, by (A1) $\rightarrow F_m(t)$ by the definitions $\rightarrow \bar{T}_j(t)$, $T_{ej}(t)$, $c_j(t)$, and $s_j(t)$ by (16) $\rightarrow K(t)$ by (3) $\rightarrow \bar{K}(t)$ by (4).

The lemma gives a computational program to simulate the motion of the dynamic system with computer. We simulate the economy with three groups of households by specifying the parameters as follows

$$\begin{aligned} \begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} &= \begin{pmatrix} 0 \\ 0.2 \\ 0.4 \end{pmatrix}, \quad \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 69 \\ 20 \end{pmatrix}, \quad \begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.15 \\ 0.1 \end{pmatrix}, \quad \begin{pmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \end{pmatrix} = \begin{pmatrix} 0.07 \\ 0.18 \\ 0.2 \end{pmatrix}, \quad \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.9 \\ 0.65 \\ 0.6 \end{pmatrix}, \\ \begin{pmatrix} \sigma_{10} \\ \sigma_{20} \\ \sigma_{30} \end{pmatrix} &= \begin{pmatrix} 0.14 \\ 0.2 \\ 0.2 \end{pmatrix}, \quad \begin{pmatrix} \kappa_{10} \\ \kappa_{20} \\ \kappa_{30} \end{pmatrix} = \begin{pmatrix} 0.03 \\ 0.02 \\ 0.02 \end{pmatrix}, \quad \begin{pmatrix} v_{e1} \\ v_{e2} \\ v_{e3} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.7 \\ 0.6 \end{pmatrix}, \quad \begin{pmatrix} a_{e1} \\ a_{e2} \\ a_{e3} \end{pmatrix} = \begin{pmatrix} 0.3 \\ 0.25 \\ 0.2 \end{pmatrix}, \quad \begin{pmatrix} b_{e1} \\ b_{e2} \\ b_{e3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.15 \\ 0.1 \end{pmatrix}, \\ \begin{pmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \end{pmatrix} &= \begin{pmatrix} 0.2 \\ 0.15 \\ 0.1 \end{pmatrix}, \quad \begin{pmatrix} a_{i1} \\ a_{i2} \\ a_{i3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.15 \\ 0.1 \end{pmatrix}, \quad \begin{pmatrix} v_{h1} \\ v_{h2} \\ v_{h3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.15 \\ 0.1 \end{pmatrix}, \quad \begin{pmatrix} a_{h1} \\ a_{h2} \\ a_{h3} \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.15 \\ 0.1 \end{pmatrix}, \quad \begin{pmatrix} \delta_{h1} \\ \delta_{h2} \\ \delta_{h3} \end{pmatrix} = \begin{pmatrix} 0.04 \\ 0.06 \\ 0.08 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} r^* &= 0.053, \quad A_i = 1, \quad A_s = 0.9, \quad A_e = 0.7, \quad \alpha_i = 0.32, \quad \alpha_s = 0.34, \quad \alpha_e = 0.33, \quad T_0 = 24, \\ \delta_k &= 0.05, \quad \pi_{m1} = 0.2, \quad \pi_{m2} = 0.4, \quad \pi_{m3} = 0.5, \quad m = e, i, h. \end{aligned}$$

(20)

We specify group 1, 2 and 3's populations respectively 1, 69 and 20. The total factor productivities of the capital goods sector and consumer goods sector's are respectively 1 and 0.9. We specify group 1, 2 and 3's utilization efficiency parameters, m_j , respectively 0.7, 0.15 and 0.1. The three groups are respectively called as the rich, the middle, and the poor group. The rich's higher propensity to receive education is highest. The rich learn more effectively than the other two groups. The returns to capital α_j in the Cobb-Douglas productions are closely 0.3. The returns to scale parameters π_j are specified positive. This means that there are decreasing returns to scale in human capital accumulation. We assume that the subsidies to the rich, the middle and the poor are respectively 0, 0.2, and 0.4. We simulate the model with the following initial conditions

$$\tau(0) = 0.007, \bar{k}_2(0) = 66, \bar{k}_3(0) = 50, H_1(0) = 32, H_2(0) = 9, H_3(0) = 5. \tag{21}$$

Figure-1. describes the motion of the system. The national output Y in Figure 1 is

$$Y(t) = F_i(t) + p_s(t)F_s(t) + p_e(t)F_e(t).$$

We simulate the economy with different initial conditions not far from (21). It can be shown that the system converges. Under (21), the tax rate rises and the national output falls. The national wealth rises and the national capital employed experiences negative growth. The rich's and the middle's human capital levels fall slightly over time. The work hours of the rich and the middle also fall slightly. The changes in the other variables are described in Figure 1.

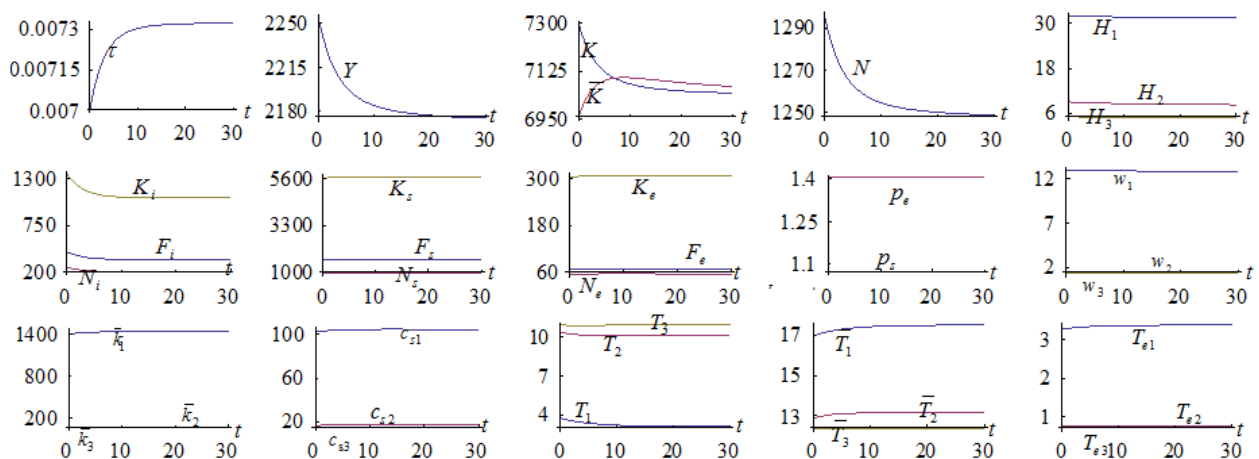


Figure-1. The Motion of the Economic System

It is identified that the system has an equilibrium point. The equilibrium values are as in (22).

$$\tau = 0.0073, Y = 2172.4, K = 7041.4, \bar{K} = 7053.8, N = 1248, F_i = 351.4, F_s = 1605.6,$$

$$F_e = 68.8, K_i = 1083.8, K_s = 5650.2, K_e = 307.4, N_i = 206.8, N_s = 985, N_e = 56,$$

$$p_s = 1.07, p_e = 1.41, w_1 = 12.75, w_2 = 1.57, w_3 = 1.34, H_1 = 31.2, H_2 = 8.15, H_3 = 4.83,$$

$$\bar{k}_1 = 1432, \bar{k}_2 = 67, \bar{k}_3 = 49.8, c_{s1} = 103.7, c_{s2} = 17.3, c_{s3} = 15.5, T_1 = 3.16, T_2 = 10.1, T_3 = 10.9, \bar{T}_1 = 17.5, \bar{T}_2 = 13.1, \bar{T}_3 = 12.4, T_{e1} = 3.4, T_{e2} = 0.74, T_{e3} = 0.71.$$

It is straightforward to calculate the six eigenvalues as follows

$$-0.38, -0.35, -0.18, -0.12, -0.08, -0.04.$$

The negative real eigenvalues imply that the equilibrium point is locally stable.

4. COMPARATIVE DYNAMIC ANALYSIS

The previous section simulated the motion of the dynamic system and found a stable equilibrium point. This section conducts comparative dynamics analysis with regard to different parameters. We use a variable $\bar{\Delta x}_j(t)$ to represent for the change rate of the variable $x_j(t)$ in percentage due to the change in a parameter.

4.1. A Rise in the Subsidy of Education to the Poor

First, we allow the government subsidy to the poor to be increased in the following way: $\tau_3 : 0.4 \Rightarrow 0.41$.

Figure 2 plots the transitional processes from the old path to the new path. The tax rate is increased and the education cost is almost not affected. Initially, the national output and the national capital employed are increased and in the long term they are slightly affected. The national wealth falls initially and rises in the long term. The poor's human capital is enhanced, the rich's human capital is reduced, and the middle's human capital is almost not affected. The poor spends more time on education. The national labor force rises initially and is slightly affected in the long term. All the groups' wage rates are reduced. As shown in Figure 2, the economic structure is also affected.

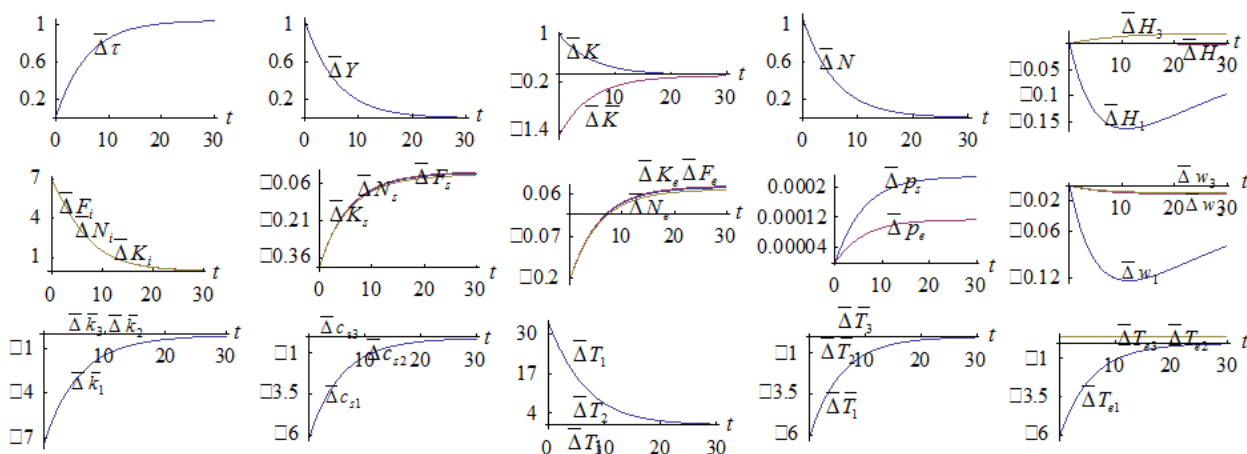


Figure-2. A Rise in the Subsidy of Education to the Poor

4.2. The Rich's Propensity to Receive Education Being Enhanced

We now study how the motion of the economic system is affected if the rich's propensity to receive education is increased as follows: $\kappa_{01} : 0.03 \Rightarrow 0.032$. We plot the simulation results in Figure 3. The rich increase the education time and shorten the leisure time and work time. The other two groups' time distributions are almost not affected. The rich's human capital is enhanced. The human capital levels of the other two groups are almost not influenced. The prices of education and consumer good are almost not affected. The three sectors are expanded. The wage rates are increased.

The rich's wealth and consumption of consumer good fall. This occurs as the rich shifts the available more to education and human capital is not much increased.

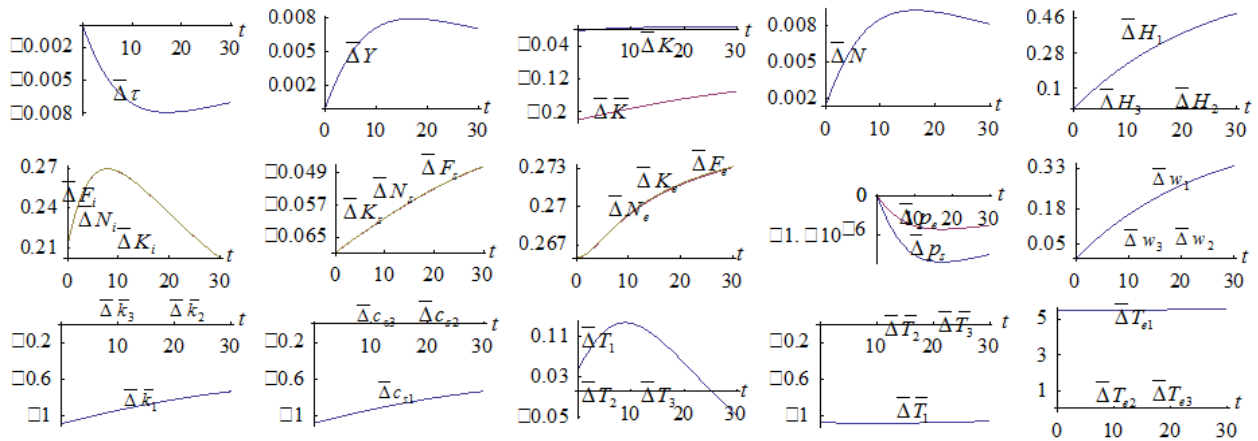


Figure-3. The Rich's Propensity to Receive Education Being Enhanced

4.3. The Rich's Propensity to Enjoy Leisure Being Increased

We now allow the rich's propensity to enjoy leisure to be increased as follows: $\sigma_{10} : 0.14 \Rightarrow 0.15$. The simulation results are plotted in Figure 4. The national wealth, national capital employed and national output are all reduced. The national labor force falls. The tax rates on all the sectors are increased as the rich spend more hours on leisure and less hours work and education. The capital good sector is expanded and the other two sectors are shrunken. The effects on the and The price of education, and price of consumer good are augmented. The output levels of the three sectors are reduced. The household from any group works less hours, consumes less goods, and owns less wealth.

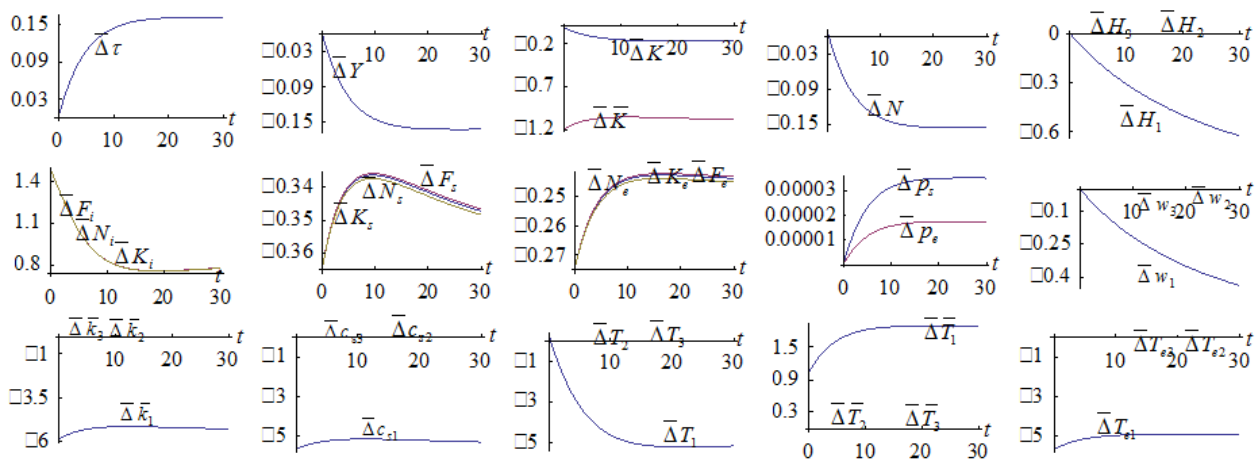


Figure-4. The Rich's Propensity to Enjoy Leisure Being Increased

4.4. The Poor's Population Being Increased

We now allow the poor's population to be increased as follows: $\bar{N}_3 : 20 \Rightarrow 21$. The simulation results are plotted in Figure 5. As more money for subsidizing students is needed, the tax rate is increased. The national labor force, national capital employed and national output are all increased. The national wealth falls initially and rises in the long term. The rich's human capital falls and the other two groups' human capital levels are slightly affected. All the groups' wage rates are reduced. The three sectors are expanded. As shown in the figure, the microeconomic variables are slightly affected due to the population growth in the long term, even though the macroeconomic variables are increased.

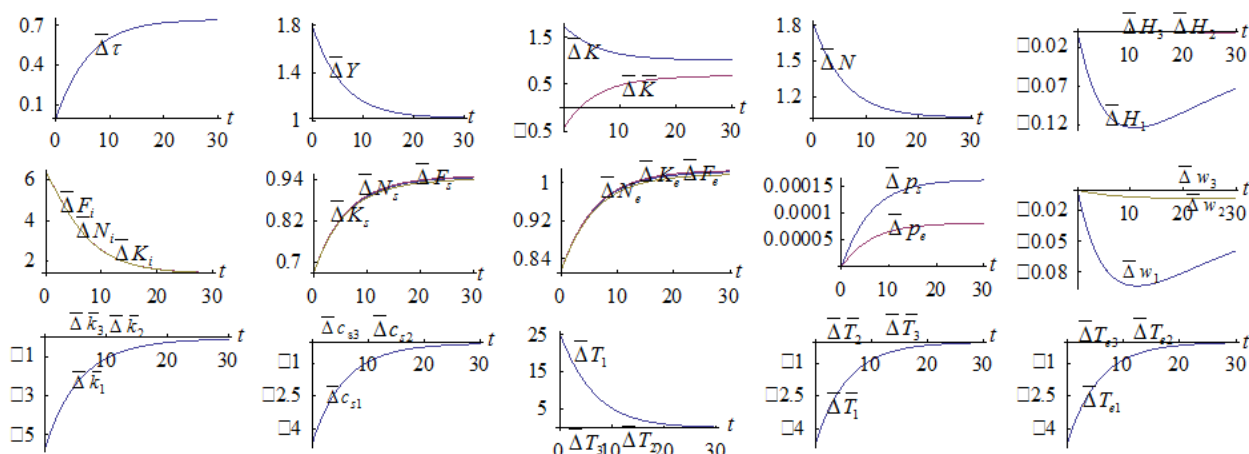


Figure-5. The Poor's Population Being Increased

4.5. The Total Productivity Factor of the Education Sector Being Enhanced

We now study what happen to the economic system if the total factor productivity of the education sector is increased as follows: $A_e : 0.7 \Rightarrow 0.71$. We describe the simulation results in Figure 6. The rise in the education sector's productivity increases the tax rate and lowers the price of education. As the price of education is reduced and the subsidies are not changed, the opportunity costs for the middle and poor are reduced. The two groups increase their education hours. In the long term the group 1's time distribution is slightly affected. The human capital levels of the three groups are enhanced in the long term. The output of education is enhanced and the inputs of the education sector are reduced.

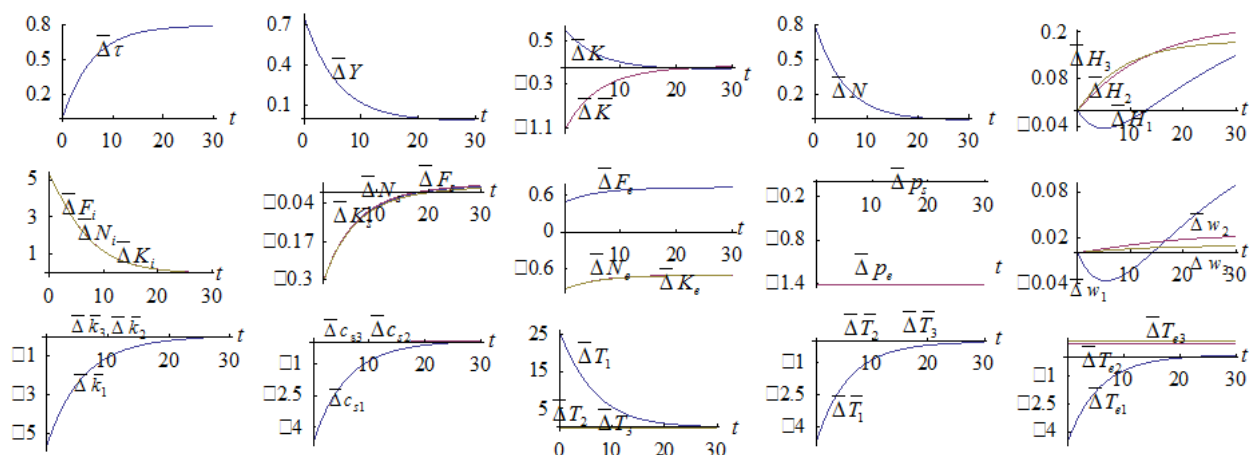


Figure-6. The Total Productivity Factor of the Education Sector Being Enhanced

4.6. The Poor Applying Human Capital More Effectively

We now examine what happen to the economic system if the poor's human capital utilization efficiency is enhanced as follows: $m_1 : 0.1 \Rightarrow 0.11$. The simulation results are plotted in Figure 7. The tax rate rises initially and falls in the long term. The national capital employed, national output and national labor force initially and rise in the long term. The national wealth is increased. The human capital levels of the three groups are enhanced. The consumer good and education sectors are expanded. The capital good sector is shrunken initially and expanded in the long term. The poor's wealth and consumption levels, wage rate and human capital are enhanced.

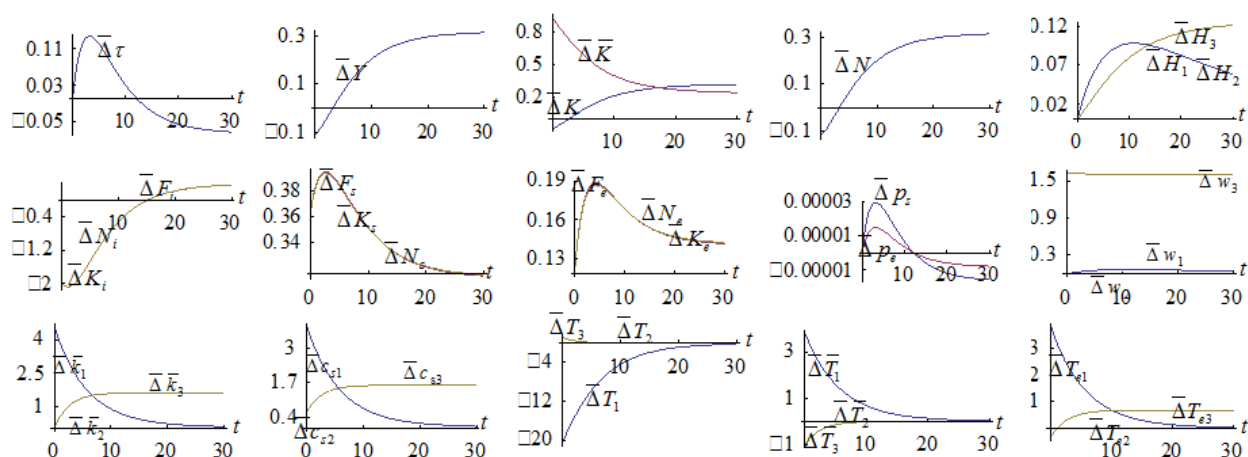


Figure-7. The Poor Applying Human Capital More Effectively

5. CONCLUDING REMARKS

This paper proposed an endogenous growth model of a small-open economy. The paper dealt with dynamic interactions between physical capital, human capital, income and wealth inequalities between different households with government subsidy to education. We emphasized the role of government education. The model of heterogeneous households was developed on the basis of Solow-Uzawa’s neoclassical growth theory, Uzawa-Lucas model, Arrow’s learning by doing, Zhang’s creative leisure, and Walrasian general equilibrium theory. The model treats capital and human capital accumulation as endogenous. The capital accumulation and economic structure are based on the neoclassical growth theory. The human capital accumulation is due to Uzawa’s education, Arrow’s learning by doing, and Zhang’s creative leisure. The model explains income and wealth inequality between groups with government education subsidy policy. We model behavior of households by applying Zhang’s concept of disposable income and utility function. Our model reveals a complicated nonlinear dynamic interdependence between wealth accumulation, human capital accumulation, economic structural change, division of labor, and time distribution under perfect competition and government education subsidy policy. We simulated the small-open economy composed of three groups of households, the rich 1 %, the middle 69%, and the poor 20%. We found a stable equilibrium point. We carried out comparative dynamic analysis and demonstrated how a change in a parameter affects the path of economic growth. The model has many limitations when one thinks of the literature of different branches of economics. For instance, this study does not take account of social mobility in the economic system. Although we took account of the role of the government in redistributing wealth and income through education subsidy, there are many other ways that a government may affect distribution and growth. We conducted comparative dynamic analysis only with regard to a change in parameters. We may get more insights by allowing multiple parameters to be changed simultaneously or by letting parameters/shocks be changed continuously. It is also important to deal with endogenous change in preferences.

Appendix: Proving the Lemma

Equations (6), (8), and (10) imply

$$z \equiv \frac{r_0}{w} = \frac{N_m}{\bar{\beta}_m K_m}, \quad m = i, s, e, \tag{A1}$$

Where $\bar{\beta}_m \equiv \beta_m / \alpha_m$. Equations (A1) and (3) imply

$$\frac{N_i}{\bar{\beta}_i} + \frac{N_s}{\bar{\beta}_s} + \frac{N_e}{\bar{\beta}_e} = z K. \tag{A2}$$

Inserting (A1) in (6) yields

$$z = \left(\frac{r_0}{\alpha_i \bar{\tau} A_i} \right)^{1/\beta_i} \frac{1}{\beta_i}, \quad w(z) = \alpha z^{-\alpha_i}, \tag{A3}$$

Where $\alpha = \beta_i \bar{\tau} A_i \bar{\beta}_i^{-\alpha_i}$. We have

$$w_j(H_j, z) = H_j^{m_j} w. \tag{A4}$$

From (7) and (8), we get

$$p_s(z) = \frac{\bar{\beta}_s^{\alpha_s} z^{\alpha_s} w}{\beta_s \bar{\tau} A_s}. \tag{A5}$$

From (9) and (10) we obtain

$$p_e(z) = \frac{\bar{\beta}_e^{\alpha_e} w z^{\alpha_e}}{\beta_e \bar{\tau} A_e}. \tag{A6}$$

From (A4) and the definitions of \bar{y}_j , we get

$$\bar{y}_j = (1 + r) \bar{k}_j + T_0 w_j. \tag{A7}$$

Inserting $p_s c_j = \xi_j \bar{y}_j$ in (18) yields

$$\sum_{j=1}^J \xi_j \bar{N}_j \bar{y}_j = p_s F_s. \tag{A8}$$

Substitute (A7) into (A8)

$$N_s = \bar{\tau} \left(\sum_{j=1}^J \tilde{g}_j \bar{k}_j + \tilde{g} \right), \tag{A9}$$

Where we use $p_s F_s = w N_s / \bar{\tau} \beta_s$ and

$$\tilde{g}_j(z) \equiv \bar{r} \beta_s \xi_j \bar{N}_j, \quad \bar{r}(z) \equiv \frac{1+r^*}{w}, \quad \tilde{g}(z, (H_j)) \equiv \beta_s T_0 \sum_{j=1}^J H_j^{m_j} \xi_j \bar{N}_j.$$

Equations (16) and (20) imply

$$\sum_{j=1}^J \frac{\kappa_j \bar{y}_j}{p_{ej}} \bar{N}_j = \frac{w N_e}{\bar{\tau} \beta_e p_e}, \tag{A10}$$

Where we also use $F_e = w N_e / \bar{\tau} \beta_e p_e$. Inserting (A7) in (A10) yields

$$N_e = \bar{\tau} \left(\sum_{j=1}^J \bar{g}_j \bar{k}_j + \bar{g} \right), \tag{A11}$$

Where

$$\bar{g}_j = \frac{(1+r)\beta_e p_e \kappa_j \bar{N}_j}{w p_{ej}}, \quad \bar{g} \equiv \frac{\beta_e T_0 p_e}{w} \sum_{j=1}^J \frac{\kappa_j \bar{N}_j w_j}{p_{ej}}.$$

Inserting (14) in (16) yields

$$T_j = T_0 - \tilde{p}_{ej} \bar{y}_j. \tag{A12}$$

Where

$$\tilde{p}_{ej} \equiv \frac{\sigma_j}{w_j} + \frac{\kappa_j}{p_{ej}}.$$

Inserting (A6) in (A12) yields

$$T_j = \tilde{p}_j - (1+r)\bar{k}_j \tilde{p}_{ej}, \tag{A13}$$

Where

$$\tilde{p}_j = T_0 - T_0 \tilde{p}_{ej} w_j.$$

Inserting (A13) in (1), we have

$$N = n_0 - \sum_{j=1}^J n_j \bar{k}_j, \tag{A14}$$

Where

$$n_0(z, (H_j)) \equiv \sum_{j=1}^J \tilde{p}_j H_j^{m_j} \bar{N}_j, \quad n_j(z, (H_j)) \equiv (1+r)\tilde{p}_{ej} H_j^{m_j} \bar{N}_j.$$

Substitute (A9) and (A14) into (2)

$$N_i(z, (H_j), (\bar{k}_j)) = n_0 - \bar{\tau}(\tilde{g} + \bar{g}) - \sum_{j=1}^J (n_j + \bar{\tau} \hat{g}_j) \bar{k}_j, \tag{A15}$$

Where $\hat{g}_j \equiv \tilde{g}_j + \bar{g}_j$.

Substituting (6), (8) and (10) into (21) yields

$$\sum_{j=1}^J \tau_j T_{ej} \bar{N}_j = \frac{w\tau}{\bar{\tau}} \left(\frac{N_i}{\beta_i} + \frac{N_s}{\beta_s} + \frac{N_e}{\beta_e} \right). \tag{A16}$$

Insert $p_{ej} T_{ej} = \kappa_j \bar{y}_j$ in (A16)

$$\sum_{j=1}^J \hat{p}_{ej} \bar{k}_j + \hat{p}_e = \frac{N_i}{\beta_i} + \frac{N_s}{\beta_s} + \frac{N_e}{\beta_e}, \tag{A17}$$

Where we use (A7) and

$$\hat{p}_{ej} \equiv \frac{(1+r)\bar{p}_{ej}}{w}, \quad \hat{p}_e \equiv \sum_{j=1}^J \bar{p}_{ej} T_0 H_j^{m_j}, \quad \bar{p}_{ej} \equiv \frac{\bar{\tau} \tau_j \kappa_j \bar{N}_j}{\tau p_{ej}}.$$

Inserting (A9), (A11) and (A15) in (A17) yields

$$\bar{k}_1 = \varphi(z, \{\bar{k}_j\}, (H_j)) \equiv \left(\bar{n}_0 - \sum_{j=2}^J \varphi_j \bar{k}_j \right) \varphi_1^{-1}, \tag{A18}$$

Where

$$\varphi_j \equiv \hat{p}_{ej} + \frac{n_j + \bar{\tau} \hat{g}_j}{\beta_i} - \frac{\bar{\tau} \tilde{g}_j}{\beta_s} - \frac{\bar{\tau} \bar{g}_j}{\beta_e}, \quad \bar{n}_0 = \frac{n_0}{\beta_i} - \hat{p}_e - \frac{\bar{\tau} \hat{g}}{\beta_i} + \bar{\tau} \left(\frac{\tilde{g}}{\beta_s} + \frac{\bar{g}}{\beta_e} \right).$$

We can show that all the variables are expressed as functions of τ , $\{\bar{k}_j\}$ and (H_j) by the following procedure:

$\bar{\tau} = 1 - \tau \rightarrow z$ by (A3) $\rightarrow w$ by (A3) $\rightarrow w_j$ by (A4) $\rightarrow p_s$ by (A5) $\rightarrow p_e$ by (A6) $\rightarrow \bar{k}_1$ by (A18) $\rightarrow N_i$ by (A15) $\rightarrow N_e$ by (A11) $\rightarrow N_s$ by (A9) $\rightarrow \bar{y}_j$ by (A7) $\rightarrow T_j$ by (A12) $\rightarrow N$ by (2) $\rightarrow K_m, j = i, s, e$, by (A1) $\rightarrow F_m$ by the definitions $\rightarrow \bar{T}_j, \bar{T}_{ej}, c_j$, and s_j by (16) $\rightarrow K$ by (3) $\rightarrow \bar{K}$ by (4). From this procedure, (A15), (17), and (18), we get

$$\dot{\bar{k}}_1 = \bar{\Omega}_1(\tau, \{\bar{k}_j\}, (H_j)) \equiv \lambda_1 \bar{y}_1 - \varphi, \tag{A19}$$

$$\dot{\bar{k}}_j = \Lambda_j(\tau, \{\bar{k}_j\}, (H_j)) \equiv \lambda_j \bar{y}_j - \bar{k}_j, \quad j = 2, \dots, J,$$

$$\dot{H}_j = \Omega_j(\tau, \{\bar{k}_j\}, (H_j)), \quad j = 1, \dots, J, \tag{A20}$$

We take derivatives of equation (A18) with respect to t and then then combine the resulted equation with (A18). We obtain

$$\dot{\bar{k}}_1 = \frac{\partial \varphi}{\partial \tau} \dot{\tau} + \sum_{j=2}^J \Lambda_j \frac{\partial \varphi}{\partial \bar{k}_j} + \sum_{j=1}^J \Omega_j \frac{\partial \varphi}{\partial H_j}. \tag{A21}$$

Equaling the right-hand sizes of equations (A19) and (A21) implies

$$\dot{\tau} = \Lambda_1(\tau, \{\bar{k}_j\}, (H_j)) \equiv \left[\bar{\Omega}_1 - \sum_{j=2}^J \Lambda_j \frac{\partial \varphi}{\partial \bar{k}_j} - \sum_{j=1}^J \Omega_j \frac{\partial \varphi}{\partial H_j} \right] \left(\frac{\partial \varphi}{\partial \tau} \right)^{-1}. \tag{A22}$$

In summary, we proved the lemma.

Funding: This study received no specific financial support.
Competing Interests: The author declares that there are no conflicts of interests regarding the publication of this paper.

REFERENCES

Alonso-Carrera, J. and M.J. Freire-Sere, 2004. Multiple equilibria, fiscal policy and human capital accumulation. *Journal of Economic Dynamics and Control*, 28(4): 841-856. [View at Google Scholar](#) | [View at Publisher](#)

Arrow, K.J., 1962. The economic implications of learning by doing. *Review of Economic Studies*, 29(3): 155-173. [View at Google Scholar](#) | [View at Publisher](#)

Arrow, K.J., 1974. General economic equilibrium: Purpose, analytic techniques, collective choice. *American Economic Review*, 64(3): 253-272. [View at Google Scholar](#)

- Arrow, K.J. and G. Debreu, 1954. Existence of an equilibrium for a competitive economy. *Econometrica*, 22(3): 265–290. [View at Google Scholar](#) | [View at Publisher](#)
- Arrow, K.J. and F.H. Hahn, 1971. *General competitive analysis*. San Francisco: Holden-Day, Inc.
- Azariadis, C., 1993. *Intertemporal macroeconomics*. Oxford: Blackwell.
- Banerjee, A.V., 2004. Educational policy and the economics of the family. *Journal of Development Economics*, 74(1): 3–32. [View at Google Scholar](#) | [View at Publisher](#)
- Barro, R.B., 2001. Human capital and growth. *American Economic Review*, 91(2): 12–17. [View at Google Scholar](#)
- Barro, R.J. and X. Sala-i-Martin, 1995. *Economic growth*. New York: McGraw-Hill, Inc.
- Becker, G., 1981. *A treatise on the family*. Cambridge, MA: Harvard University Press.
- Behrman, J., R. Pollak and P. Taubman, 1982. Parental preferences and provision for progeny. *Journal of Political Economy*, 90(1): 52–73. [View at Google Scholar](#) | [View at Publisher](#)
- Benigno, G. and P. Benigno, 2003. Price stability in open economies. *Review of Economic Studies*, 70(4): 743–764. [View at Google Scholar](#) | [View at Publisher](#)
- Blankenau, F. and N.B. Simpson, 2004. Public education expenditures and growth. *Journal of Development Economics*, 73(2): 583–605. [View at Google Scholar](#) | [View at Publisher](#)
- Booth, A.L. and M.G. Coles, 2010. Tax policy and returns to education. *Labour Economics*, 17(1): 291–301. [View at Google Scholar](#) | [View at Publisher](#)
- Bovenberg, A.L. and B. Jacobs, 2005. Redistribution and education subsidies are siamese twins. *Journal of Public Economics*, 89(11–12): 2005–2035. [View at Google Scholar](#) | [View at Publisher](#)
- Burmeister, E. and A.R. Dobell, 1970. *Mathematical theories of economic growth*. London: Collier Macmillan Publishers.
- Caselli, F. and J. Ventura, 2000. A representative consumer theory of distribution. *American Economic Review*, 90(4): 909–926. [View at Google Scholar](#) | [View at Publisher](#)
- Castelló-Climent, A. and A. Hidalgo-Cabrillana, 2012. The role of education quality and quantity in the process of economic development. *Economics of Education Review*, 31(4): 391–409. [View at Google Scholar](#) | [View at Publisher](#)
- Chakraborty, B. and M.R. Gupta, 2009. Human capital, inequality, endogenous growth and education subsidy: A theoretical analysis. *Research Economics*, 63(2): 77–90. [View at Google Scholar](#) | [View at Publisher](#)
- Chatterjee, S., 1994. Transitional dynamics and the distribution of wealth in a neoclassical growth model. *Journal of Public Economics*, 54(1): 97–119. [View at Google Scholar](#) | [View at Publisher](#)
- Cox, D., 1987. Motives for private income transfers. *Journal of Political Economy*, 95(3): 508–546. [View at Google Scholar](#) | [View at Publisher](#)
- De Hek, P.A., 2005. On taxation in a two-sector endogenous growth model with endogenous labor supply. *Journal of Economic Dynamics and Control*, 30(4): 655–685. [View at Google Scholar](#) | [View at Publisher](#)
- Debreu, G., 1959. *Theory of value: An axiomatic analysis of equilibrium*. London: Yale University Press.
- Fernandez, R. and R. Rogerson, 1998. Public education and income distribution: A dynamic quantitative evaluation of education finance reform. *American Economic Review*, 88(4): 813–833. [View at Google Scholar](#)
- Florida, R., C. Mellander and K. Stolarick, 2008. Inside the black box of regional development – human capital, the creative class and tolerance. *Journal of Economic Geography*, 8(5): 615–649. [View at Google Scholar](#) | [View at Publisher](#)
- Forbes, K., 2000. A reassessment of the relationship between inequality and growth. *American Economic Review*, 90(4): 869–870. [View at Google Scholar](#) | [View at Publisher](#)
- Gale, D., 1955. The law of supply and demand. *Mathematica Scandinavica*, 3: 33–44.
- Gali, J. and T. Monacelli, 2005. Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies*, 72(3): 707–734. [View at Google Scholar](#) | [View at Publisher](#)
- Galindev, R., 2011. Leisure goods, education attainment and fertility choice. *Journal of Economic Growth*, 16(2): 157–181. [View at Google Scholar](#) | [View at Publisher](#)

- Hanushek, E. and D. Kimko, 2000. Schooling, labor-force quality and the growth of nations. *American Economic Review*, 90(5): 1184-1208. [View at Google Scholar](#) | [View at Publisher](#)
- Heckman, J.J., 1976. A life-cycle model of earnings, learning, and consumption. *Journal of Political Economy*, 84(2): S9-S44. [View at Google Scholar](#) | [View at Publisher](#)
- Jones, L.E., R.E. Manuelli and P.E. Rossi, 1993. Optimal taxation in models of endogenous growth. *Journal of Political Economy*, 101(3): 485-517. [View at Google Scholar](#)
- Kollmann, R., 2001. The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: A quantitative investigation. *Journal of International Economics*, 55(2): 243-262. [View at Google Scholar](#) | [View at Publisher](#)
- Kollmann, R., 2002. Monetary policy rules in the open economy: Effects on welfare and business cycles. *Journal of Monetary Economics*, 49(5): 899-1015. [View at Google Scholar](#) | [View at Publisher](#)
- Krueger, A.B. and M. Lindahl, 2001. Education for growth: Why and for whom. *Journal of Economic Literature*, 39(4): 1101-1136. [View at Google Scholar](#) | [View at Publisher](#)
- Lane, P.R., 2001. The new open economy macroeconomics: A survey. *Journal of International Economics*, 54(2): 235-266. [View at Google Scholar](#) | [View at Publisher](#)
- Lazear, E., 1977. Education: Consumption or production. *Journal of Political Economy*, 85(3): 569-597. [View at Google Scholar](#)
- Lucas, R.E., 1988. On the mechanics of economic development. *Journal of Monetary Economics*, 22(1): 3-42. [View at Google Scholar](#)
- Malchow-Møller, N., S. Nielsen and J.R. Skaksen, 2011. Taxes, tuition fees, and education for pleasure. *Journal of Public Economic Theory*, 13(2): 189-215. [View at Google Scholar](#)
- Maliar, L. and S. Maliar, 2001. Heterogeneity in capital and skills in a neoclassical stochastic growth model. *Journal of Economic Dynamics and Control*, 25(9): 1367-1397. [View at Google Scholar](#) | [View at Publisher](#)
- Mas-Colell, A., M.D. Whinston and J.R. Green, 1995. *Microeconomic theory*. New York: Oxford University Press.
- McKenzie, L.W., 1959. On the existence of general equilibrium for a competitive market. *Econometrica*, 27(1): 54-71. [View at Google Scholar](#) | [View at Publisher](#)
- Mino, K., 1996. Analysis of a two-sector model of endogenous growth with capital income taxation. *International Economic Review*, 37(1): 227-251. [View at Google Scholar](#) | [View at Publisher](#)
- Mino, K., 2001. Optimal taxation in dynamic economies with increasing returns. *Japan and the World Economy*, 13(3): 235-253. [View at Google Scholar](#) | [View at Publisher](#)
- Nikaido, H., 1956. On the classical multilateral exchange problem. *Metroeconomica*, 8(2): 135-145. [View at Google Scholar](#) | [View at Publisher](#)
- Nikaido, H., 1968. *Convex structures and economic theory*. New York: Academic Press.
- Obstfeld, M. and K. Rogoff, 1996. *Foundations of international macroeconomics*. Mass., Cambridge: MIT Press.
- Pasinetti, L.L., 1960. A mathematical formulation of the Ricardian system. *Review of Economic Studies*, 27(2): 78-98. [View at Google Scholar](#) | [View at Publisher](#)
- Pasinetti, L.L., 1974. *Growth and income distribution - essays in economic theory*. Cambridge: Cambridge University Press.
- Penalosa, C.G. and S.J. Turnovsky, 2006. Growth and income inequality: A canonical model. *Economic Theory*, 28(1): 25-49. [View at Google Scholar](#) | [View at Publisher](#)
- Samuelson, P.A., 1959. A modern treatment of the ricardian economy: I. The pricing of goods and labor and land services. *Quarterly Journal of Economics*, 73(1): 1-35. [View at Google Scholar](#) | [View at Publisher](#)
- Sano, K. and Y. Tomoda, 2010. Optimal public education policy in a two sector model. *Economic Modelling*, 27(5): 991-995. [View at Google Scholar](#) | [View at Publisher](#)
- Solow, R., 1956. A contribution to the theory of growth. *Quarterly Journal of Economics*, 70(1): 65-94. [View at Google Scholar](#)
- Stokey, N.L. and S. Rebelo, 1995. Growth effects of flat-rate taxes. *Journal of Political Economy*, 103(3): 519-550. [View at Google Scholar](#) | [View at Publisher](#)
- Turnovsky, S.J. and C.G. Penalosa, 2006. Distributional dynamics in a neoclassical growth model: The role of elastic labor supply. *Journal of Economic Dynamics & Control*, 32(5): 1399-1431.

- Uzawa, H., 1961. On a two-sector model of economic growth. *Review of Economic Studies*, 29(1): 47-70. [View at Google Scholar](#) | [View at Publisher](#)
- Uzawa, H., 1965. Optimal technical change in an aggregative model of economic growth. *International Economic Review*, 6(1): 18-31. [View at Google Scholar](#) | [View at Publisher](#)
- Walras, L., 1874. *Elements of pure economics*, translated from the French by W. Jaffé, 1954. London: Allen and Unwin.
- Zeng, D.-Z. and Z. Xiwei, 2011. Tourism and industrial agglomeration. *Japanese Economic Review*, 62(4): 537-561. [View at Google Scholar](#) | [View at Publisher](#)
- Zhang, J., 2003. Optimal debt, endogenous fertility, and human capital externalities in a model with altruistic bequests. *Journal of Public Economics*, 87(7-8): 1825-1835. [View at Google Scholar](#) | [View at Publisher](#)
- Zhang, W.B., 2007. Economic growth with learning by producing, learning by education, and learning by consuming. *Interdisciplinary Description of Complex Systems*, 5(1): 21-38. [View at Google Scholar](#)
- Zhang, W.B., 2013. Income and wealth distribution with physical and human capital accumulation: Extending the Uzawa-Lucas model to a heterogeneous households economy. *Latin American Journal of Economics*, 50(2): 257-287. [View at Google Scholar](#) | [View at Publisher](#)
- Zhang, W.B., 2016. Impact of education subsidies and taxation on wealth and human capital accumulation. *Eastern European Business and Economics Journal*, 2(3): 222-247. [View at Google Scholar](#)

Views and opinions expressed in this article are the views and opinions of the author(s), Asian Journal of Economic Modelling shall not be responsible or answerable for any loss, damage or liability etc. caused in relation to/arising out of the use of the content.