Asian Journal of Economic Modelling
ISSN(e): 2312-3656
ISSN(p): 2313-2884
DOI: 10.18488/5009.v10i1.4402
Vol. 10, No. 1, 1-16.

URL: www.aessweb.com

THE REGULATORY COMMITMENT PROBLEM, INDUSTRY STRUCTURES AND INVESTMENT INCENTIVES IN NETWORK QUALITY UPGRADES

Ines Ben Dkhil[1+]
Khaireddine Jebsi[2]
Edmond Baranes[3]

[1]*Faculty of Economics and Management of Nabeul, University of Carthage, Tunisia.*
Email: bendkhilines@gmail.com Tel: +21622848556
[2]*Faculty of Economics and Management Science and Laboratory of Strategic Management of Innovation and Regional Integration, Sousse University, Sousse, Tunisia.*
Email: khairy.jebsi@topnet.tn Tel: +21673301808
[3]*University of Montpellier, UFR Economie, France.*
Email: edmond.baranes@umontpellier.fr Tel: 0434432444


(+ Corresponding author)

Article History
Received: 19 November 2021
Revised: 21 December 2021
Accepted: 3 January 2022
Published: 17 January 2022

Keywords
Regulatory commitment
Investment in network upgrade
Vertically differentiated services
Non-ad-hoc specifications
Positive spillover sensitivity.

JEL Classification:
L96; L51.

ABSTRACT

This paper examines how the timing of access price regulation and the incumbent firm's structure affect the investment incentives relating to network upgrades. We consider a general setting with non-ad-hoc specifications for the service quality and investment fixed cost functions, and we compare different possible scenarios for vertically integrated industry structures and the timing of regulatory actions. First, we show that the competition-investment trade-off may be solved when the regulator can fix the access price before the integrated network provider's investment decision. Second, we show that the sole requirement of vertical separation on the incumbent firm is no guarantee for the viability of service-based competition, since foreclosure cannot be avoided in the absence of access price regulation. Third, we show that monopoly is socially preferable to retail competition when the investment spillover is high, and the regulator cannot commit ex-ante to the access price.

Contribution/ Originality: This study is one of few to have modeled the effects of investment incentives on network upgrades and the regulatory commitment problem by considering non-ad-hoc specifications for the service quality and investment fixed cost functions.

1. INTRODUCTION

In many countries, existing telecommunication infrastructures need to be upgraded to respond to the growing demand for ultra-fast broadband services (to make possible online interactive applications and activities). Over the last two decades, regulation has played a pivotal role in creating competition in telecommunication markets. Still, it seems that this comes at the price of delaying investment in next-generation network (NGN) infrastructures. This problem, known as the competition-investment dilemma in economic literature, is due to the natural monopoly characteristic of the telecommunication industry. Recent studies recommend relaxing regulation to mitigate this regulatory trade-off and spur innovation (Briglauer, Cambini, & Grajek, 2018; Ben Dkhil & Jebsi, 2020).[1] In practice, however, the regulator's task is more complex because it is mainly constrained by the regulator's lack of

[1] See Abrardi and Cambini (2019) for a recent survey on the regulation-broadband deployment relationship.

ex-ante commitment to an appropriate access price level that would fully cover the ex-ante sunk costs and risks of investments. Economists have identified three primary sources of the regulatory commitment problem: (1) information asymmetry (investors are motivated to exaggerate their true investment costs to boost their access returns) (Gans & King, 2003); (2) the irreversibility of investment in infrastructure; and (3) the investment cycle length relative to the duration of the regulatory contract (see (Foros, 2004; Kotakorpi, 2006)). Gans and King (2004) show that if the regulator can commit ex-ante with a reasonable linear access charge, the "truncation" of the expected investor's returns is avoided. Therefore, socially efficient investment occurs. Brito, Pereira, and Vareda (2010) and Vareda (2010) consider the credible regulatory hypothesis to be admissible for short periods, and they show that two-part tariffs may offer a remedy for the regulatory commitment problem. Avenali, Matteucci, and Reverberi (2015) found that under the assumption of the regulator's ability to commit, both consumer and investment welfare are improved. In recent years, several regulatory measures[2] (such as the incumbent obligation of public information - accounting and technical information, network characteristics, terms and conditions for supply, etc.) have attempted to reduce the commitment problem.

This paper studies the investment incentives for network upgrades under different conditions regarding the timing of the regulatory intervention and industry structures (vertical separation versus vertical integration of the incumbent firm). Our setup has two key distinctive features in comparison with previous research. First, while previous studies consider *ad-hoc* specifications for the investment functions, we assume for the purposes of generalization that the quality of end-user services depends implicitly on the access network quality and that the investment cost function is implicitly defined in the access quality. Second, following (Laffont, Gremaq, Tirole, & Geras, 1996), the determination of the optimal access pricing takes into account both demand and supply features in both upstream and downstream markets by considering two additional game stages for the end-user equilibrium prices and qualities. We consider a vertical differentiation model with variable quality costs and symmetric quality choices under Bertrand competition[3] to model the supply and demand behaviors at the downstream level. We show that under the assumption of credible regulatory commitment, the competition-investment trade-off[4] is solved when the network provider is integrated, and investment spillovers are low. We find that vertical separation of facility-based firms does not guarantee competition since foreclosure may occur in the absence of access price regulation. Third, when the regulator cannot commit, we show that monopoly is socially preferable to retail competition when the investment spillover is high. More generally, in this case, investment incitation decreases in response to positive spillover sensitivity. The rest of this paper is organized as follows. Section II presents the model. Section III summarizes and discusses the main findings, and section IV concludes.

2. THE MODEL: MAIN ASSUMPTIONS AND SCENARIOS

We consider a vertically related industry (see Figure 1), where an essential input (the network access service) is provided by an upstream monopoly (NAP[5]), at a price a, with quality k. One unit of the final retail service[6] (e.g., internet connection) necessitates just one unit of the access service (access to the network). In the downstream

[2] See Ben Dkhil and Jebsi (2020) for international survey and data on these reforms and their effects.

[3] See Motta (1993) for a complete description of the different versions of the vertical differentiation model.

[4] Literature on regulation and investment in the telecommunication industry points out the trade-off between promoting competition through access regulation in the short run in order to enhance welfare (static objective), and encouraging incumbents to upgrade the existent network infrastructures in the longer term (dynamic objective) (see (Bourreau, Doğan, & Manant, 2010; Friederiszick, Grajek, & Röller, 2008; Laffont & Tirole, 2000)).

[5] Abbreviation of Network Access Provider.

[6] In the rest of this paper, the retail service will be simply referred to as « service ».

market, the two internet service providers (ISPs) compete "à la Bertrand" and have different abilities to provide two vertically differentiated services (high- and low-quality services), although they use the same access service.[7] By making this last assumption, we follow Sarmento and Brandão (2009) and Kotakorpi (2006) and Foros (2004) who argue that retailers differ in their ability to take advantage of investments, given their different experiences as retail service providers. Furthermore, as in Manenti and Scialà (2013), we consider that this high (or low) service quality, induced by investment in the network access quality, is the source of a positive spillover effect. In particular, we assume that a high (or low) positive spillover effect occurs when the independent rival in the downstream market is the provider with the high (or low) service quality.

2.1. Industry Structures

We consider three potential industry structures. We call

- VI1, the first case of Vertical Integration: it refers to the case where the NAP is integrated with the high-quality downstream firm (the ISP_1).
- VI2, the second case of Vertical Integration: it refers to the case where the NAP is integrated with the low-quality downstream firm (the ISP_2).
- VS, the case of Vertical Separation: it refers to the case where the NAP is vertically separated.[8]

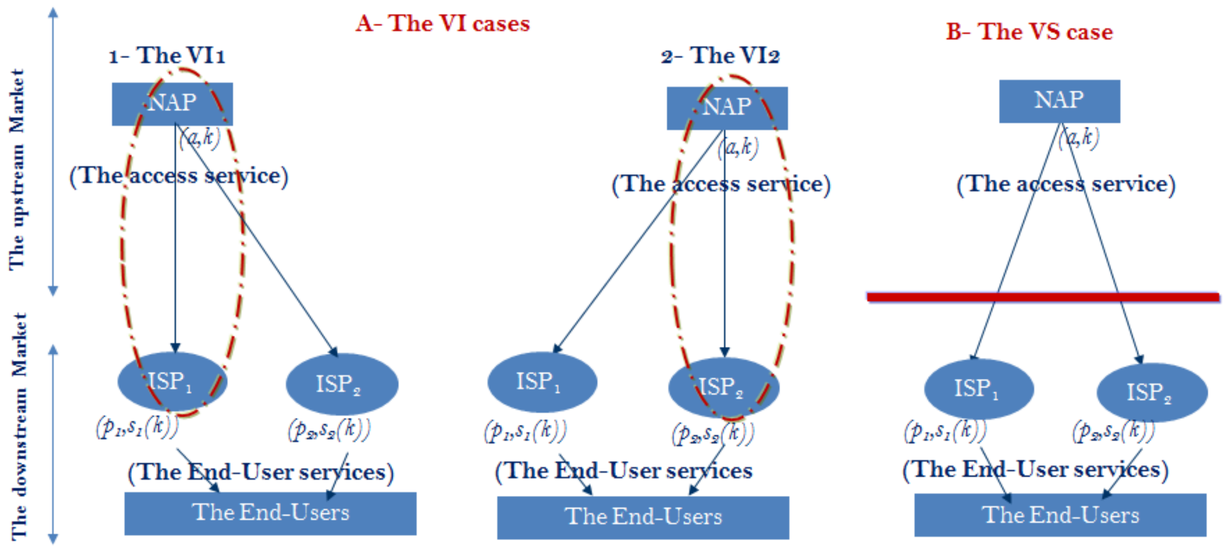

Figure 1. Industry structures.

2.2. Non-Ad-Hoc Specifications for Quality and Investment Cost Functions

The NAP undertakes an investment $I(k)$ to upgrade the network. To allow generalization, the NAP's fixed cost $I(k)$ is assumed to be increasing, convex and implicitly defined in the network access quality k. We further assume that the quality, $s_i / i \in \{1, 2\}$, of the service provided by the downstream firm i depends positively and implicitly on k, which is the same for the two retail rivals. $s_i(k)$ is assumed to be increasing and convex

[7] The quality degradation or sabotage problem (non-price discrimination) is not considered here.

[8] We assume that the current regulations demand that the network infrastructure owner is prohibited from operating in end-user service markets. Therefore, under VS, the scope of the network infrastructure owner's activity is limited to the provision of the access service.

($s'_i(k) > 0, s''_i(k) < 0$). For each level of k, there is a range of service qualities; $s_i \in [\underline{s}(k), \bar{s}(k)] \forall k > 0$, where $\underline{s}(k)$ and $\bar{s}(k)$ are respectively the lowest and highest available quality.[9]

2.3. Demand Structure

We use the standard formulation of the vertical differentiation model to present the consumers' behaviors at the downstream level. A consumer with a taste parameter θ enjoys the (indirect) utility $v_i(\theta) = \theta s_i(k) - p_i$ if he buys the service i at a price p_i and zero otherwise.

θ is uniformly distributed along the interval $[0,1]$, with density 1. Without loss of generality, we assume that $s_1(k) > s_2(k)$. θ_m denotes the taste parameter of the marginal consumer who is indifferent between the two differentiated services, so that $v_1(\theta_m) = v_2(\theta_m) \Leftrightarrow \theta_m = \frac{p_1 - p_2}{s_1(k) - s_2(k)}$. θ_c refers to a consumer who is indifferent between buying service 2 or not buying at all, so that $v_2(\theta_m) = 0 \Leftrightarrow \theta_c = \frac{p_2}{s_2(k)}$.The market is assumed to be uncovered, and demands are structured such that: consumers with taste $\theta \in [\theta_m, 1]$ will purchase service 1, those with taste $\theta \in [\theta_c, \theta_m[$ will purchase service 2 and those with taste $\theta \in [0, \theta_c[$ do not purchase either of the differentiated services. Demands can then be written as follows

$$\begin{cases} D_1 = 1 - \dfrac{p_1 - p_2}{s_1(k) - s_2(k)} \\ D_2 = \dfrac{p_1 - p_2}{s_1(k) - s_2(k)} - \dfrac{p_2}{s_2(k)} \\ with\ D_i > 0 \end{cases}$$

When D_i=0, we assume that the total demand is reduced to demand addressed to the firm $j \neq i$ $(i, j \in \{1,2\}$, which is the following $D = D_j = 1 - \frac{p_j}{s_j}$.

The inverse demands can be written as follows:

$$\begin{cases} p_1 = s_1(k) - \left(D_1 s_1(k) + D_2 s_2(k)\right) \\ p_2 = s_2(k) - s_2(k)(D_1 + D_2) \\ with\ p_i > 0 \end{cases}$$

[9] This assumption of positive dependence between the qualities of access and final services is real since the deployment of fiber optic NGN technologies improves the quality of final service by improving its parameters (debit, error rate, latency, and jitter). The assumption of convexity implies that there is a minimum level of infrastructure quality, denoted by k_0, such that $k > k_0 > 0$, and that is considered necessary to allow a minimum level of service quality $s_i(k_0) \in [\underline{s}_0 = \underline{s}(k_0), s_0 = \bar{s}(k_0)]$

By canceling D_1 and D_2 in the inverse demands, we derive the reserve prices $p_i = s_i(k); i \in \{1,2\}$, and we can hence deduce the expression of the consumers' surplus as follows

$$CS = (s_1 - p_1)\frac{D_1}{2} + (s_2 - p_2)\frac{D_2}{2}$$

2.4. Supply Structure

π^{NAP} and π^{iNAP} denote the NAP's profits respectively, when the NAP is separated and integrated into the ISP$_i$, and π_i is the ISP$_i$'s profit ($i \in \{1,2\}$). Hence, the firms' profits corresponding to each industry configuration are shown in Table 1.

Table 1. Firms' profits within each industry structure.

Industry structure	VS	VI1	VI2
Firms' profits	$\pi^{NAP} = a(D_1 + D_2) - I(k)$ $\pi_1 = (p_1 - cs_1(k) - a)D_1$ $\pi_2 = (p_2 - cs_2(k) - a)D_2$ (wih c ∈]0,1[is a parameter)	$\pi^{1NAP} = aD_2 + (p_1 - cs_1(k))D_1 - I(k)$ $\pi_2 = (p_2 - cs_2(k) - a)D_2$	$\pi^{2NAP} = aD_1 + (p_2 - cs_2(k))D_2 - I(k)$ $\pi_1 = (p_1 - cs_1(k) - a)D_1$

$cs_i(k)$ is the marginal cost of service i's provision. It is assumed to be an increasing proportion of the service quality provided by firm i. For simplicity, the marginal cost for the network provision is normalized to zero in our setting.

2.5. Games

For each of the industry structures, we consider three games with perfect information, each composed of four stages: the two last stages are devoted to the price and quality choices at the downstream level, the price subgame (stage 4) and the quality subgame (stage 3), while the two preliminary stages are devoted to the access price and the network access quality (investment) choices, and depend on the game considered:

1) Under the unregulated access game, the network access provider sets both the access price and the network access quality.
2) Under the credible regulatory commitment game, the regulator determines the access price (stage 1) before the NAP's investment decision (stage 2).
3) Under the no-credible commitment game, the regulator determines the access price (stage 2) after the NAP's investment decision (stage 1).

We solve the games of the nine scenarios that result from this setting by backward induction.

3. MAIN RESULTS: COMPARISONS AND DISCUSSIONS

3.1. Retail Price and Quality Subgames

The Nash equilibrium outcomes in the price subgame take the following general form[10]

[10] For identical levels of quality and access price, given this general form we can derive the following comparison between equilibrium prices under the three industry structures considered here : $p_1^{(VI1)} = p_1^{(VS)} = p_1(\theta = 1) > p_1^{(VI2)} = p_1(\theta = 2)$,and

$$\begin{cases} p_1 = a + cs_1(k) + \dfrac{\Delta s[2bs_1(k) - a]}{4s_1(k) - s_2(k)} \\ p_2 = a + cs_2(k) + \dfrac{\Delta s[bs_2(k) - 2\theta a]}{4s_1(k) - s_2(k)} \end{cases}$$

$$where\ \Delta s = s_1(k) - s_2(k), b = 1 - c\ and\ \theta = \begin{cases} 1\ (under\ VS\ and\ VI1) \\ 2\ (under\ VI2) \end{cases}$$

The Nash equilibrium in the quality subgame exhibits a corner solution (which corresponds to the best available quality $\bar{s}(k)$) for the high-quality firm,[11] while the rival's best reply will be a function of $\bar{s}(k)$.). Formally, we get

$$s_1(k) = s_1^{VS} = s_1^{VI1} = s_1^{VI2} = \bar{s}(k)$$

$$\begin{cases} s_2^{(VS)} = s_2^{(VI1)} = \dfrac{\bar{s}(k)\left[(2b\bar{s}(k) - 3a) + \sqrt{\left(11a + 2b\bar{s}(k)\right)^2 - 144a^2}\right]}{7b\bar{s}(k) - 4a} \\ s_2^{(VI2)} = \dfrac{4\bar{s}(k)[b\bar{s}(k) + 5a]}{7b\bar{s}(k) - a} \end{cases}$$

(In remainder of the paper, we use the superscripts *VS*, *VI1*, *VI2* to distinguish between equilibrium results and properties under the different industry structures.)

The equilibriums of the above quality subgame prevail when the following nonnegative rival demand condition is met $a < \bar{a}^{VI2} = \frac{b\bar{s}(k)}{7} < \bar{a}^{VS} = \bar{a}^{VI1} = \frac{b\bar{s}(k)}{2}$. In other words, the violation of this last condition removes the competition at the retail level.

3.2. Access Price and Network Quality Choices

3.2.1. The Unregulated Access Game

Proposition 1 summarizes the main result of the unregulated game.

Proposition 1. *Access price regulation is necessary to prevent foreclosure and ensure competition at the retail level under both vertical integration and vertical separation structures.*

Proof. See the appendix for the proofs of all Propositions and Lemma.

$p_2^{(VI1)} = p_2^{(VS)} = p_2(\theta = 1) > p_2^{(VI2)} = p_1(\theta = 2)$. Note that these equilibrium results prevail under the condition that both rivals are active (i.e., $D_i > 0$). By replacing the equilibrium prices in D_i, we get the following nonnegative demand conditions:

$s_1(k) > s_2(k) > \frac{2a}{b}\ (for\ both\ VI1\ and\ VS\ cases) and\ s_1(k) > \frac{a}{b}\ (for\ the\ VI2\ case)$. Note that under these last conditions, the second-order conditions are satisfied.

[11] By considering the nonnegative demand conditions, we can easily verify that the separated (integrated) ISP_1's profits are monotonically increasing with s_1 :

$VS: \frac{d\pi_1}{ds_1} = \frac{(2bs_1(k)-a)\left[b\left(8s_1^2(k)-6s_1(k)s_2(k)+4s_2^2(k)\right)+a\left(4s_1(k)-7s_2(k)\right)\right]}{\left(4s_1(k)-s_2(k)\right)^2} > 0$; $VI1: \frac{d\pi^{1VI1}}{ds_1} = \frac{(8s_1(k)b^2-ab)s_2^2(k)+\left(-12s_1^2(k)b^2-20bs_2(k)+a^2\right)s_2(k)+16s_1^3(k)b^2+20s_1(k)a^2}{\left(4s_1(k)-s_2(k)\right)^2} > 0$; $VI2: \frac{d\pi_1}{ds_1} = \frac{(4bs_1(k)-a)\left[2bs_1^2(k)-(2bs_1(k)+7a)s_2(k)+4bs_2^2(k)+4s_1(k)a\right]}{\left(4s_1(k)-s_2(k)\right)^2} > 0$

3.2.2. The Credible Access Game

The next Lemma provides some fundamental properties of the NAP's optimal network access quality choice function $k^{(.)}(a)$.

Lemma 1. *(i) When* a *is fixed at the marginal cost of the network provision, the separated NAP's investment choice* $k^{(VS)}(a)$ *reaches a minimum while the integrated NAP's investment choice* $k^{(VI.)}(a)$ *reaches a maximum. (ii)* $k^{(VS)}$ $(k^{(VI2)})$ *is increasing (decreasing) with the access price. (iii)* $k^{(VI1)}$ *is decreasing in the neighborhood of the marginal cost of the network provision.*

The characterization of the optimal access price at the first stage of the credible regulatory game under the VI structure yields the following finding.

Preposition 2. *The private investment choice is maximized when the regulator can commit on the cost-based access price before the integrated NAP's investment decision.*

Proof. Immediately from Lemma 1.

Based on Lemma 1 and Proposition 1, we can state the following corollary.

Corollary 1. *The competition-investment dilemma may be solved when the regulator can commit to the cost-based regime before the integrated NAP's investment decision*

The integrated NAP profits from its ability to take advantage of investment and makes revenue by choosing the highest level of access quality since its access activity is not profitable. In this case, the cost-based regime not only ensures competition, but also raises the NAP's investment level, thereby solving the competition-investment dilemma.

3.2.3. The No-Credible Game

Proposition 3 provides the main result of the no-credible game.

Proposition 3. *When the regulator cannot commit to the access price before the NAP's investment decision, the regulated access price exceeds the marginal cost of the network provision. In the particular case that investment spillover is high, the regulator raises the access price leading to a monopoly.*

When the regulator cannot commit and the investment spillover is high, retail competition may be socially undesirable because the integrated NAP (with low service quality) is less motivated to invest since it does not take sufficient advantage of its own investment.

Going back to the no-commitment game's first stage, the following result emerges from the NAP's profit equilibrium expressions, as stated in proposition 4.

Proposition 4. *If the regulator cannot commit, the NAP's investment incitation depends on the degree of sensitivity of the highest service quality improvement in response to a slight increase in the amount of investment undertaken (**positive spillover sensitivity**). Formally, investment in next generation networks occurs when the positive spillover sensitivity* $\varepsilon_{\bar{s}/I}$ *exceeds a certain threshold* $\tau^{(.)}$*, which depends on the industry structures as we can see below:*

$$\varepsilon_{\bar{s}/I} = \frac{\Delta \bar{s}(k)}{\Delta I(k)} > \tau^{(.)} = \frac{I_0}{b^2 \bar{s}_0 \beta^{(i)}}, with\ i \in \{VS, VI1, VI2\}$$

and $\beta^{(VS)} = 0.02626640759 < \beta^{(VI2)} = 0.1428571429 < \beta^{(VI1)} = 0.1569241318$

$$\Leftrightarrow \tau^{(VS)} > \tau^{(VI2)} > \tau^{(VI1)}$$

This last proposition states that under the no regulatory commitment, the amount of investment undertaken by the NAP decreases according to the positive spillover sensitivity. Under VI1, the NAP's investment decision is less sensitive to the spillover effect. This is because in this case the integrated NAP is the biggest beneficiary of its own investment since it offers the highest service quality. On the contrary, under VS, the NAP is less incented to invest because it is the last beneficiary of its own investment, and the most significant proportion of benefits gained by the separated NAP's investment will be shared between the independent retailers (which explains the highest spillover sensitivity threshold $\tau^{(VS)}$ above). VI2 is the intermediate case between the two extremes (VI1 and VS) since the integrated NAP benefits from its own investment but these benefits are lower than the rival ones.

3.2.4. The Games' Comparison Results

The comparison of the main results of the different scenarios is summarized in the next proposition.

Proposition 5. *(i) The separated NAP's investment incentive is at its highest level in the absence of access price regulation. (ii) Vertical separation undermines investment incentives. (iii) Investment spillovers reduce investment incentive. (vi) The private investment choice is always below the socially optimal level.*

4. CONCLUSION

In this paper, we provided a general setup of the possible interplays between a vertically (separated) integrated monopoly network provider, a regulator and one (two) independent retailer(s) by considering non-*ad-hoc* specifications for the investment cost function and service quality functions. This article's main message is that a monopoly access provider's decision to invest in network upgrades is primarily constrained by the regulator's ability to commit and the degree of positive investment spillover. We show that an ex-ante cost-based regime can solve the competition investment dilemma under certain conditions. In particular, this dilemma is solved when the commitment ability assumption is met; the regulator chooses to set the access price at cost, pushing the integrated network provider to invest maximally to compensate its losses in the upstream market. Furthermore, we show that when the regulator cannot commit, investment in network upgrades decreases along with positive spillover sensitivity. In the particular case that the spillover effect is relatively high, retail competition is not socially desirable because the integrated network owner does not invest sufficiently. We also show that the vertical separation requirement does not improve welfare or investment incentives and cannot even guarantee retail competition.

Funding: This study received no specific financial support.
Competing Interests: The authors declare that they have no competing interests.
Authors Contribution: All authors contributed equally to the conception and design of the study.

REFERENCES

Abrardi, L., & Cambini, C. (2019). Ultra-fast broadband investment and adoption: A survey. *Telecommunications Policy, 43*(3), 183-198.

Avenali, A., Matteucci, G., & Reverberi, P. (2015). Can access regulation promote broadband investment and consumer welfare? *International Journal of Technology, Policy and Management, 15*(4), 357-377.Available at: https://doi.org/10.1504/ijtpm.2015.072795.

Ben Dkhil, I., & Jebsi, K. (2020). Access regulation and broadband deployment: Evidence from a worldwide dataset. *International Journal of the Economics of Business, 27*(2), 203-253.

Bourreau, M., Doğan, P., & Manant, M. (2010). A critical review of the "ladder of investment" approach. *Telecommunications Policy, 34*(11), 683-696.Available at: https://doi.org/10.1016/j.telpol.2010.09.002.

Briglauer, W., Cambini, C., & Grajek, M. (2018). Speeding up the internet: Regulation and investment in the European fiber optic infrastructure. *International Journal of Industrial Organization, 61*, 613-652.Available at: https://doi.org/10.1016/j.ijindorg.2018.01.006.

Brito, D., Pereira, P., & Vareda, J. (2010). Can two-part tariffs promote efficient investment on next generation networks? *International Journal of Industrial Organization, 28*(3), 323-333.

Foros, O. (2004). Strategic investments with spillovers, vertical integration and foreclosure in the broadband access market. *International Journal of Industrial Organization, 22*(1), 1-24.Available at: https://doi.org/10.1016/s0167-7187(03)00079-1.

Friederiszick, H., Grajek, M., & Röller, L.-H. (2008). Analyzing the relationship between regulation and investment in the telecom sector (pp. 1-35). Berlin,Germany: Citeseer.

Gans, J., & King, S. (2003). Access holidays for network infrastructure investment. *Agenda: A Journal of Policy Analysis and Reform, 10*(2), 163-178.Available at: https://doi.org/10.22459/ag.10.02.2003.05.

Gans, J. S., & King, S. P. (2004). Access holidays and the timing of infrastructure investment. *Economic Record, 80*(248), 89-100.Available at: https://doi.org/10.1111/j.1475-4932.2004.00127.x.

Kotakorpi, K. (2006). Access price regulation, investment and entry in telecommunications. *International Journal of Industrial Organization, 24*(5), 1013-1020.Available at: https://doi.org/10.1016/j.ijindorg.2005.11.007.

Laffont, J.-J., Gremaq, I., Tirole, J., & Geras, I. (1996). Creating competition through interconnection: Theory and practice. *Journal of Regulatory Economics, 10*(3), 227-256.

Laffont., J. J., & Tirole, J. (2000). *Competition in telecommunications*. Cambridge: MIT Press.

Manenti, F. M., & Scialà, A. (2013). Access regulation, entry and investments in telecommunications. *Telecommunications Policy, 37*(6-7), 450-468.Available at: https://doi.org/10.1016/j.telpol.2012.11.002.

Motta, M. (1993). Endogenous quality choice: Price vs. quantity competition. *The journal of Industrial Economics, 41*, 113-131.

Sarmento, P., & Brandão, A. (2009). Next generation access networks: The effects of vertical spillovers on access and innovation. FEP Working Papers No 321.

Vareda, J. (2010). Access regulation and the incumbent investment in quality-upgrades and in cost-reduction. *Telecommunications Policy, 34*(11), 697-710.Available at: https://doi.org/10.1016/j.telpol.2010.09.003.

APPENDIX

Proof of Proposition 1.

$k^{(.)}(a)$ denotes the optimal network access quality, which corresponds to the NAP's profit – maximizing the network access quality level after substituting the equilibriums of the previous stages. The FOC and the SOC corresponding to the NAP's maximization problem at the first stage of the unregulated game are respectively as follows

$$\frac{d\pi^{(1)NAP}(\bar{s}(k),a)}{da} = \frac{\partial\pi^{(1)NAP}}{\partial a} + \frac{\partial\pi^{(1)NAP}}{\partial\bar{S}}\left(\frac{\partial\bar{S}}{\partial k}\right)\left(\frac{dk^{(.)}}{da}\right) \qquad (A.1)$$

$$\frac{d^2\pi^{(1)NAP}(\bar{s}(k),a)}{da^2} = \frac{\partial^2\pi^{(1)NAP}}{\partial a^2} + \frac{\partial^2\pi^{(1)NAP}}{\partial a\partial k}\frac{dk^{(.)}}{da} + \frac{\partial\pi^{(1)NAP}}{\partial k}\frac{d^2k^{(.)}}{da^2} \qquad (A.2)$$

Both the second and third terms on the right-hand side of equations $(A.1)$ and $(A.2)$ are zero by the Envelope theorem. We show below that the NAP's optimal access price $a^{(i)NAP} \geq \bar{a}^{(.)}$, which violates the nonnegative rival demand condition regardless of the industry structure.

In the case of VS: By canceling $\frac{\partial \pi^{NAP}}{\partial a}$, we get two local maximums at $a = a_1^{NAP} = 0.6382575342\,\bar{s}(k^{(VS)}) > \bar{a}^{(VS)}$ and $a = a_2^{NAP} = 0.9246695295\bar{s}(k^{(VS)}) > \bar{a}^{(VS)}$ ($\left.\frac{\partial^2 \pi^{NAP}}{\partial a^2}\right|_{a=a_1^{NAP}} = \frac{-1.177366353}{\bar{s}(k^{(VS)})} < 0$; and $\left.\frac{\partial^2 \pi^{NAP}}{\partial a^2}\right|_{a=a_2^{NAP}} = \frac{-1.063662607}{\bar{s}(k^{(VS)})} < 0$)

For the VI1 case: We get a global maximum at $\bar{a}^{(VI1)}$ ($\frac{\partial \pi^{1NAP}(\bar{s}^{(VI1)})}{\partial a} = 0$; $\left.\frac{\partial^2 \pi^{1NAP}}{\partial a^2}\right|_{a=\bar{s}^{(VI1)}} = \frac{-8}{9\bar{s}(k^{(VI1)})} < 0$

For the VI2 case: We get $\frac{\partial \pi^{2NAP}}{\partial a} = \frac{34b\left(\bar{s}(k^{(VI2)})\right)+2a}{48b\left(\bar{s}(k^{(VI2)})\right)} > 0$ and $\frac{\partial^2 \pi^{2NAP}}{\partial a^2} = \frac{1}{24\bar{s}(k^{(VI2)})} > 0$. Therefore, the maximum is reached at the upper bound of the interval $[0, \bar{a}^{VI2}]$, so that $a^{2NAP} = \bar{a}^{(VI2)}$.

Proof of Lemma 1. **(i)** We derive the main properties of NAP's optimal network access quality choice $k^{(.)}(a)$ by applying the implicit function theorem as follows

$$\frac{dk^{(.)}(a)}{da} = -\frac{\partial^2 \pi^{(i)NAP}(\bar{s}(k),a)}{\partial k \partial a} \Big/ \frac{\partial^2 \pi^{(i)NAP}}{\partial (k)^2} \qquad (A.3)$$

We determine the extrema a^* of $k^{(.)}(a)$ by solving $\left(\frac{dk^{(.)}(a)}{da} = 0 : \text{the FOC}\right)$. As $\left.\frac{\partial^2 \pi^{(i)NAP}}{\partial k^2}\right|_{k=k^{(.)}(a)} < 0$ and $\frac{\partial^2 \pi^{(i)NAP}(\bar{s}(k),a)}{\partial k \partial a} = \underbrace{\left(\frac{d\bar{s}(k)}{dk}\right)}_{>0 \text{ by assumption}} \left(\frac{\partial^2 \pi^{(i)NAP}}{\partial \bar{s} \partial a}\right)$, *the FOC* can be reduced to $(A.4)$ as

$$\frac{\partial^2 \pi^{(i)NAP}(a)}{\partial \bar{s} \partial a} = 0 \qquad (A.4)$$

The nature of a^* is identified by applying the SOC (i.e., by studying the $sign\left(\left.\frac{d^2 k^{(.)}(a)}{da^2}\right|_{a=a^*}\right)$). Differentiating $(A.1)$ with respect to a , we get $\frac{d^2 k^{(.)}(a)}{da^2} = -\left(\frac{\partial^3 \pi^{(i)NAP}}{\partial k \partial a^2} \cdot \frac{\partial^2 \pi^{(i)NAP}}{\partial k^2} + \frac{\partial^3 \pi^{(i)NAP}}{\partial k^2 \partial a} \cdot \frac{\partial^2 \pi^{(i)NAP}}{\partial k \partial a}\right) \Big/ \left(\frac{\partial^2 \pi^{(i)NAP}}{\partial k^2}\right)^2$, which can be reduced to $\frac{d^2 k^{(.)}(a)}{da^2} = -\left(\frac{\partial^3 \pi^{(i)NAP}}{\partial k \partial a^2} \Big/ \frac{\partial^2 \pi^{(i)NAP}}{\partial k^2}\right) + \left(\frac{\partial^3 \pi^{(i)NAP}}{\partial k^2 \partial a} \cdot \frac{dk^{(.)}(a)}{da}\right)$(after simplification and the replacement of the second term of $(A.1)$ by $\frac{dk^{(.)}(a)}{da}$). As $\left.\frac{dk^{(.)}(a)}{da}\right|_{a=a^*} = 0$ and $\left.\frac{\partial^2 \pi^{(i)NAP}}{\partial k^2}\right|_{k=k^{(.)}(a)} < 0$, we can write that:

$$sign\left(\left.\frac{d^2 k^{(.)}(a)}{da^2}\right|_{a=a^*}\right) \equiv sign\left(\left.\frac{\partial^3 \pi^{(i)NAP}}{\partial k \partial a^2}\right|_{a=a^*}\right) \equiv sign\left(\left.\frac{\partial^3 \pi^{(i)NAP}}{\partial \bar{s} \partial a^2}\right|_{a=a^*}\right) \qquad (A.5)$$

Using Maple software, we get the following results

- **For the VS case**: $\frac{\partial^2 \pi^{NAP}(a^*)}{\partial \bar{s} \partial a} = 0$ with $a^* = 0$; and $\left.\frac{\partial^3 \pi^{NAP}}{\partial \bar{s} \partial a^2}\right|_{a=a^*=0} = \frac{15}{8(\bar{s}(k))^2} > 0$. Therefore, $a^* = 0$ is a minimum of the separated NAP's network quality choice function $k^{(VS)}(a)$. We can easily verify that $\frac{\partial^2 \pi^{NAP}}{\partial \bar{s} \partial a} = \frac{\partial^2(aD(\bar{s}(k),a))}{\partial \bar{s} \partial a} > 0 \text{ for all } a \in [0, \bar{a}^{(VS)}[$.

- **For the VI1 case**: $\frac{\partial^2 \pi^{1NAP}(a^*)}{\partial \bar{s} \partial a}$ =0 with a^* =0 or $a^* \approx 0.0221b\bar{s}(k)$; and $\frac{\partial^3 \pi^{1NAP}}{\partial \bar{s} \partial a^2}\Big|_{a=a^*=0}$ $=-\frac{43}{96(\bar{s}(k))^2} < 0$ *and* $\frac{\partial^3 \pi^{1NAP}}{\partial \bar{s} \partial a^2}\Big|_{a=a^* \approx 0.0221b\bar{s}(k)} \approx \frac{0.3331}{(\bar{s}(k))^2} > 0$. Therefore, a^*=0 is a maximum while $a^* \approx 0.0221b\bar{s}(k)$ is a minimum of the NAP's network access quality choice function $k^{(VI1)}(a)$.

  The function $\frac{\partial^2 \pi^{1NAP}(a)}{\partial \bar{s} \partial a}$ is continuous on the interval $[0, \bar{a}^{(VI1)}[$ and reaches a local maximum at a=0 and a minimum at $a \approx 0.0221b\bar{s}$. Therefore,
  $\frac{\partial^2 \pi^{1NAP}(a)}{\partial \bar{s} \partial a} < 0$ *for* $a \in]0, 0.0221b\bar{s}[$ *and*, $\frac{\partial^2 \pi^{1NAP}(a)}{\partial \bar{s} \partial a} > 0$ *for* $a \in]0.0221b\bar{s}(k), \bar{a}^{(VI1)}[$ It follows that the NAP's network access quality choice function $k^{(VI1)}(a)$ is decreasing on $]0, 0.0221b\bar{s}[$ and increasing on $]0.0221b\bar{s}, \bar{a}^{(VI1)}[$.

- **For the VI2 case**: $\frac{\partial^2 \pi^{2NAP}(a^*)}{\partial \bar{s} \partial a} = -\frac{a}{24\bar{s}^2}$ =0 for a^* =0 and $\frac{\partial^3 \pi^{2NAP}}{\partial \bar{s} \partial a^2}\Big|_{a=a^*=0} = -\frac{1}{24\bar{s}^2} < 0$. Therefore, a^*=0 corresponds to a maximum of the NAP's network access quality choice function $k^{(VI2)}(a)$. We have $\frac{\partial^2 \pi^{2NAP}}{\partial \bar{s} \partial a} = -\frac{a}{24\bar{s}^2} < 0 \; (for\; a < \bar{a}^{(VI2)})$.

Proof of Proposition 3.

Under the no credible regulatory game, the regulator determines the access price $a^{nc(i)}$ that maximizes the total welfare $W^{(i)}$ after anticipating the equilibrium prices and qualities at the retail level. We show below that the socially optimal access price $a^{nc(i)}$ exceeds the marginal cost of the network provision regardless of the industry structure.

For the VS and VI1 cases: We get a global maximum at

$$a^{nc(VS)} = a^{nc(VI1)} = 0.03101650843b\bar{s}(k) > 0 \left(a^{nc(VS/VI1)} < \bar{a}^{(VS/VI1)}; and \; \frac{\partial^2 W^{(VS/VI1)}}{\partial a^2}\Big|_{a=a^{nc(VS/VI1)}} = \frac{-1.159827555}{\bar{s}(k)} < 0\right)$$

For the VI2 case: The welfare is increasing with a: $\frac{dW^{(VI2)}}{da} = \frac{5b\bar{s}(k)+4a}{24\bar{s}(k)} > 0$. Consequently, it is socially desirable that the regulator raises the access price above $\bar{a}^{(VI2)}$, leading the NAP's rival to exit the market and recreating a monopoly at the retail level.

Proof of Proposition 4.

For VI2 case: Based on the last result of the previous regulatory subgame, we recursively resolve the retail price and quality subgames. Maximizing the monopoly NAP's profits by
$\pi^{m2NAP} = \left(1 - \frac{p}{s(k)}\right)(p - cs(k)) - I(k)$ with respect to $p, s \in \left[\underline{s}(k), \frac{4}{7}\bar{s}(k)\right]$, where $\underline{s}(k) = \varepsilon\bar{s}(k), 0 < \varepsilon < \frac{4}{7}$), we get $p^{m2} = \frac{1}{2}(2-b)s(k)$. Substituting the latter back into the monopoly NAP's

profit, we obtain $\pi^{m2NAP} = \frac{b^2 s(k)}{4} - I(k)$. The NAP's profit increases with the retail service quality s; and therefore at the equilibrium we get $s^{m2NAP}(k) = \frac{4}{7}\bar{s}(k)$ and $\pi^{m2NAP} = \frac{b^2 s(k)}{7} - I(k)$

For the VS and VI1 cases: Substituting $a^{nc(VS/VI1)}$ back into the corresponding NAP's profit functions, we get respectively $\pi^{(VS)NAP} = 0.02626640759 b^2 \bar{s}(k) - I(k)$ and $\pi^{(VI1)NAP} = 0.1569241318 b^2 \bar{s}(k) - I(k)$.

Finally, we can write the equilibrium NAP's profits whatever the industry structure is as follows

$$\pi^{(i)NAP} = \beta^{(i)} b^2 s(k) - I(k)$$

With $\beta^{(VS)} = 0.02626640759 < \beta^{VI2} = \frac{1}{7} \equiv 0.1428571429 < \beta^{(VI1)} = 0.1569241318$

The NAP invests only if

$$\pi^{(i)NAP}(k) > \pi^{(i)NAP}(k) \Leftrightarrow \beta^{(i)} b^2 \bar{s}(k) - I(k) > \beta^{(i)} b^2 \bar{s}_0 - I_0$$

By arranging the last inequality, we can write

$$\varepsilon_{\bar{s}/I} = \frac{\frac{\bar{s}(k)-\bar{s}_0}{\bar{s}_0}}{\frac{I(k)-I_0}{I_0}} = \frac{\Delta \bar{s}(k)}{\Delta I(k)} > \tau^{(i)} = \frac{I_0}{b^2 \bar{s}_0 \beta^{(i)}}$$

Proof of Proposition 5.

i. Credible versus No-credible scenario

We focus now on comparing the NAP's investment incentive between the two regulated scenarios. Let $a^{c(i)}$ denote the socially optimal access price at the credible game. In this case, the regulator moves first and sets $a^{c(i)}$ to maximize social welfare after anticipating $k^{(i)}(a)$ and the outcomes of the price and quality subgames. Formally, $a^{c(i)}$ is the solution of the following committed regulator's problem

$$\max_{a \in [0, \bar{s}^{(i)} - \varepsilon]} W^{(i)}\left(a, k^{(i)}(a)\right)$$

The corresponding FOC and *SOC* are respectively as follows

$$\frac{dW^{(i)}}{da} = \frac{\partial W^{(i)}}{\partial a} + \frac{\partial W^{(i)}}{\partial k}\frac{dk^{(i)}}{da} \qquad (A.6)$$

$$\frac{d^2 W^{(i)}}{da^2} = \frac{\partial^2 W^{(i)}}{\partial a^2} + \frac{\partial^2 W^{(i)}}{\partial k \partial a}\frac{dk^{(i)}}{da} + \frac{\partial W^{(i)}}{\partial k}\frac{d^2 k^{(i)}}{da^2} \qquad (A.7)$$

For the VS case: The second term on the right hand of (A.6) is non-negative as $\frac{\partial W^{(VS)}}{\partial k} = \underbrace{\frac{\partial \pi^{NAP}}{\partial k}}_{=0\text{ (by the Envelope theorem)}} + \underbrace{\frac{\partial(\pi_1 + \pi_2 + CS)}{\partial k}}_{>0}$ and $\frac{dk^{(VS)}}{da} > 0$ (see Lemma 1)). It follows that $\frac{dW^{(VS)}}{da} > \frac{\partial W^{(VS)}}{\partial a}$; and therefore the regulator should set $a^{c(VS)} > a^{nc(VS)}$ and the NAP invests more when the regulator can move first.

For the VI cases: The second term on the right hand of (A.6) is non-negative as $\frac{\partial W^{(VI1)}}{\partial k} = \underbrace{\frac{\partial \pi^{iNAP}}{\partial k}}_{=0\text{ (by the Envelope theorem)}} + \underbrace{\frac{\partial(\pi_j + CS)}{\partial k}}_{>0}$ while $k^{(VI1)}$ decreases with a $\left(\frac{dk^{(VI1)}}{da} < 0\right)$ and reaches a maximum when a is fixed at the network cost provision, i.e., $a = 0$ (see Lemma 1). It follows that $\frac{dw^{(VIi)}}{da} < \frac{\partial w^{(VIi)}}{\partial a}$; and therefore $a^{c(VIi)} < a^{nc(VIi)}$. Furthermore, the joint surplus of the consumers and the NAP's rival reaches a maximum at $a = 0$ ($\frac{\partial(\pi_1 + cs)}{\partial a} = \frac{a - 4b\bar{s}(k)}{s\bar{s}(k)} < 0 \; for \; a < a^{nc(VI2)}$. It follows that $a^{c(VIi)}$ should be fixed at cost and in this case the NAP invests more when the regulator can move first.

Credible versus Unregulated scenario

The outcomes of the three last subgames of the two scenarios are the same. Therefore, the final value of investment function $k^{(i)}(a)$ (with $i \in \{VS, VI1, VI2\}$) depends on the choice of a at the first stage and the behavior of $k^{(i)}(a)$ .This is shown in more detail in the following comparative table:

Table 2. Comparison of access quality subgame's equilibriums (Credible versus Unregulated scenarios).

The behavior of $k^{(i)}(a)$ (see Lemma 1)	Credible scenario (see the last proof)	Unregulated scenario (see proposition 1)	Conclusion
$k^{(VS)}(a)$ is increasing	$\bar{a} > a^{c(VS)} > a^{nc(VS)}$	$a^*_{NAP} = \bar{a}$	$k^{(VS)}(a^*_{NAP}) > k^{(VS)}(a^{c(VS)})$
$k^{(VI1)}(a)$ is decreasing in the neighborhood of the marginal cost of the network provision.	$a^{c(VI1)} = 0$	$a^*_{NAP} = \bar{a}$	$k^{(VI1)}(a^*_{NAP}) > k^{(VI1)}(0)$
$k^{(VI2)}(a)$ is decreasing	$a^{c(VI2)} = 0$	$a^*_{NAP} = \bar{a}$	$k^{(VI2)}(a^*_{NAP}) < k^{(VI2)}(0)$

ii.

- Under the no credible game, we have

$$\pi^{1NAP} - \pi^{NAP} = \left(\beta^{(VI1)} - \beta^{(VS)}\right) b^2 \bar{s}(k) > 0$$

$$\pi^{2NAP} - \pi^{NAP} = \left(\beta^{(VI2)} - \beta^{(VS)}\right) b^2 \bar{s}(k) > 0 \text{ (see proposition 4)}$$

- At the third stage of the credible and deregulation games, we have

$$\frac{d\pi^{1NAP}}{dK} - \frac{d\pi^{NAP}}{dK} \frac{d(\pi^{1NAP} - \pi^{NAP})}{d\bar{s}} \frac{d\bar{s}}{dk} = \frac{4(P_1 R + P_2)}{(26b\bar{s} - R - 13a)C} \frac{d\bar{s}}{dk} > 0 \; \forall a \in \left[0, \bar{a}^{(VS/VI1)} = \frac{b\bar{s}(k)}{2}\right[;with:$$

$$P_1 = 4032b^7s^7 + 4032ab^6s^6 + 27752a^2b^5s^5 - 37164a^3b^4s^4 - 11566a^4b^3s^3 + 31835a^5b^2s^2 - 14448a^6bs + 2072a^7 > 0$$

(Indeed, $in \; P_1$, the coefficient of the monomial with the highest degree is positive, while P_1 admits two single positive real roots (1.880175370 $b\bar{s}$, 2.337576010 $b\bar{s}$) and a double root (2 $b\bar{s}$) that are superior to $\bar{a}^{(VS/VI1)}$)

$R = \sqrt{4b^2s^2 + 44abs - 23a^2}$ defined $\forall a \in \left[0, \bar{a}^{(VS/VI1)} = \frac{b\bar{s}(k)}{2}\right[$

$C = s^2(26bs - R - 13a)^2(2bs + R - 3a^2)^2 R > 0 \; \forall a \in \left[0, \bar{a}^{(VS/VI1)} = \frac{b\bar{s}(k)}{2}\right[$

$$P_2 = 8064b^8s^8 + 52416ab^7s^7 - 583120a^2b^6s^6 + 807296a^3b^5s^5 - 249184a^4b^4s^4 - 187580a^5b^3s^3 + 154847a^6b^2s^2 - 38752a^7bs + 3128a^8$$

In P_2 , the coefficient of the monomial with the highest degree is positive, and P_2 admits three single positive real roots (0.2129277944 $b\bar{s}$, 1.879962659 $b\bar{s}$, 2.056412648 $b\bar{s}$, 5.988869622 $b\bar{s}$) and a double root (2 $b\bar{s}$). The first root of P_2 (0.2129277944 bs) is inferior to $\bar{a}^{(VS/VI1)}$. Therefore, we cannot determine the sign of P_2. Assume that $P_2 < 0$, the sign of $P_1 R + P_2$ is the same as the sign of

$(P_1R)^2-(P_2)^2=$
$-576a^2\left(56\,b^5\bar{s}^5+644ab^4\bar{s}^4-222\,a^2b^3\bar{s}^3-753\,a^3b^2\bar{s}^2+16\,a^4\,b\bar{s}+184a^5\right)(-2\,b\bar{s}+a)^4(7\,b\bar{s}+4\,a)^5=-576a^2\,P_3\,(-2\,b\bar{s}+a)^4\underbrace{(-7\,b\bar{s}+4\,a)^5}_{<0}$ >0.

Indeed, $P_3=56\,b^5\bar{s}^5+644ab^4\bar{s}^4-222\,a^2b^3\bar{s}^3-753\,a^3b^2\bar{s}^2+16\,a^4\,b\bar{s}+184a^5>0$ (In P_3, the coefficient of the monomial with the highest degree is positive, and P_3 admits two single positive real roots (0.9271524007 $b\bar{s}>\bar{a}^{(VS/VI1)}$, 1.880068438$b\bar{s}>\bar{a}^{(VS/VI1)}$)

$$\frac{d\pi^{2NAP}}{dK}-\frac{d\pi^{NAP}}{dK}=\frac{4(P_4R+P_5)}{6C}\frac{d\bar{s}}{dk}\;>0\;\forall a\in\left[0,\bar{a}^{(VI2)}=\frac{b\bar{s}(k)}{7}\right[;with:$$

$$P_4=576b^6\bar{s}^6+1152ab^5\bar{s}^5-27260a^2b^4\bar{s}^4+8852a^3b^3\bar{s}^3-36531a^4b^2\bar{s}^2+41452a^5b\bar{s}-11041a^6>0$$

(Indeed, $in\;P_4$, the coefficient of the monomial with the highest degree is negative, while P_4 admits three single positive real roots (2. $b\bar{s}$, 0.1703316300 $b\bar{s}$, 2.110692484 $b\bar{s}$) that are superior to $\bar{a}^{(VI2)}$)

$$P_5=1152b^7\bar{s}^7+8640ab^6\bar{s}^6-101000a^2b^5\bar{s}^5-217812a^3b^4\bar{s}^4+1457306a^4b^3\bar{s}^3-1639535a^5b^2\bar{s}^2+685710a^6b\bar{s}-98509a^7$$ >0

(Indeed, $in\;P_5$, the coefficient of the monomial with the highest degree is negative, while P_5 admits five single positive real roots (2. $b\bar{s}$, 0.1529625323 $b\bar{s}$, 0.4581435822 $b\bar{s}$, 2.017637634 $b\bar{s}$, 2.610892233 $b\bar{s}$) that are superior to $\bar{a}^{(VI2)}$)

iii.

- Under the no credible game, we have
  $\pi^{1NAP}-\pi^{2NAP}=\left(\beta^{(VI1)}-\beta^{(VI2)}\right)b^2\bar{s}(k)>0$ (see proposition 4)
- At the third stage of the credible and deregulation games, we have

$$\frac{d(\pi^{1NAP}-\pi^{2NAP})}{dk}=\frac{d(\pi^{1NAP}-\pi^{2NAP})}{d\bar{s}}\frac{d\bar{s}}{dk}=\frac{(2b\bar{s}-a)(P_6R+P_7)}{3(26b\bar{s}-R-13a)C}\frac{d\bar{s}}{dk}>0\;\forall a\in\left[0,\bar{a}^{(VI2)}=\frac{b\bar{s}(k)}{7}\right[;with:$$

$$P_6=20736b^6\bar{s}^6+31104ab^5\bar{s}^5+337748a^2b^4\bar{s}^4-254696a^3b^3\bar{s}^3+433803a^4b^2\bar{s}^2-390136a^5b\bar{s}+96157a^6>0$$

(Indeed, $in\;P_5$, the coefficient of the monomial with the highest degree is positive, while P_5 admits two single positive real roots (2.013128316 $b\bar{s}$, 2.255187082 $b\bar{s}$) that are superior to $\bar{a}^{(VI2)}$)

$$P_7=-3234816b^8\bar{s}^8+151911936ab^7\bar{s}^7+820647936a^2b^6\bar{s}^6-2956022944a^3b^5\bar{s}^5+10869549328a^4b^4\bar{s}^4-15262267984a^5b^3\bar{s}^3+9495070760a^6b^2\bar{s}^2-2727470042a^7b\bar{s}+296790689a^8$$ >0

(Indeed, $in\ P_6$, the coefficient of the monomial with the highest degree is positive, while P_6 admits four single positive real roots (2. $b\bar{s}$, 0.2260541253 $b\bar{s}$, 2.069771779 $b\bar{s}$, 3.316642901 $b\bar{s}$) that are superior to $\bar{a}^{(VI2)}$)

iv. We get that:

$$\frac{dW^{(VS)}}{dk} - \frac{d\pi^{NAP}}{dk} = \frac{d(\pi_1+\pi_2+CS)}{d\bar{s}}\frac{d\bar{s}}{dk} = \frac{2(P_8R+P_9)}{(26b\bar{s}-R-13a)C}\frac{d\bar{s}}{dk} > 0\ \forall a \in \left[0, \bar{a}^{(VS)} = \frac{b\bar{s}(k)}{2}\right[;\ with:$$

$$P_8 = 25344b^7\bar{s}^7 + 25344ab^6\bar{s}^6 - 73560a^2b^5\bar{s}^5 + 122940a^3b^4\bar{s}^4 - 285770a^4b^3\bar{s}^3 + 283833a^5b^2\bar{s}^2 - 115584a^6b\bar{s} + 16576a^7 > 0$$

(Indeed, $in\ P_8$, the coefficient of the monomial with the highest degree is positive, while P_8 admits four single positive real roots (2. $b\bar{s}$, 0.9840118912 $b\bar{s}$, 2.025827562 $b\bar{s}$, 2.470991247 $b\bar{s}$) that are superior to $\bar{a}^{(VS)}$)

$$P_9 = 50688b^8\bar{s}^8 + 329472ab^7\bar{s}^7 - 2394192a^2b^6\bar{s}^6 + 3311952a^3b^5\bar{s}^5 - 516704a^4b^4\bar{s}^4 - 1722308a^5b^3\bar{s}^3 + 123482a^6b^2\bar{s}^2 - 310016a^7b\bar{s} + 25024a^8$$

(we cannot determine the sign of P_9, which admits a double root (2. $b\bar{s}$) and four single positive real roots (0.3499713248 $b\bar{s}$, 0.9634627461 $b\bar{s}$, 2.128913864 $b\bar{s}$, 6.294051922 $b\bar{s}$). The last three roots are superior to $\bar{a}^{(VS)}$ while the first root is inferior to $\bar{a}^{(VS)}$. Therefore, we should consider two cases. In the first case, we assume that $P_9 > 0$, therefore $P_8R + P_9 > 0$. In the second case, we assume $P_9 < 0$, therefore the sign of $P_8R + P_9$ is the same as the sign of

$$(P_8R)^2 - {P_9}^2 = -4608a^2\left(-264\,b^6\bar{s}^6 - 2904\,a\,b^5\bar{s}^5 + 1954\,a^2\,b^4\bar{s}^4 + 4220\,a^3\,b^3\bar{s}^3 - 1611\,a^4\,b^2\bar{s}^2 - 2816\,a^5b\bar{s} + 1472\,a^6\right)(-2\,b\bar{s} + a)^3(-7\,b\bar{s} + 4\,a)^5 = -4608a^2P_{10}\underbrace{(-2\,b\bar{s} + a)^3}_{<0}\underbrace{(-7\,b\bar{s} + 4\,a)^5}_{<0} > 0.$$

With

$$P_{10} = -264\,b^6\bar{s}^6 - 2904\,a\,b^5\bar{s}^5 + 1954\,a^2\,b^4\bar{s}^4 + 4220\,a^3\,b^3\bar{s}^3 - 1611\,a^4\,b^2\bar{s}^2 - 2816\,a^5b\bar{s} + 1472\,a^6 < 0$$

(indeed, $in\ P_{10}$, the coefficient of the monomial with the highest degree is positive, while P_{10} admits a single positive real root (0.9745018709 $b\bar{s}$> $\bar{a}^{(VS)}$)

$$\frac{dW^{(VI1)}}{dk} - \frac{d\pi^{1NAP}}{dk} = \frac{d(\pi_2+CS)}{d\bar{s}}\frac{d\bar{s}}{dk} = \frac{2(P_{11}R+P_{12})}{(26b\bar{s}-R-13a)C}\frac{d\bar{s}}{dk} > 0\ \forall a \in \left[0, \bar{a}^{(VI1)} = \frac{b\bar{s}(k)}{2}\right[;\ \text{with:}$$

$$P_{11} = 17280b^7\bar{s}^7 + 17280ab^6\bar{s}^6 - 129064a^2b^5\bar{s}^5 + 197268a^3b^4\bar{s}^4 - 262638a^4b^3\bar{s}^3 + 220163a^5b^2\bar{s}^2 - 86688a^6b\bar{s} + 12432a^7 > 0$$

(Indeed, $in\ P_{11}$, the coefficient of the monomial with the highest degree is positive, while P_{11} admits four single positive real roots (2. $b\bar{s}$, 0.6692204437 $b\bar{s}$, 2.026626450 $b\bar{s}$, 2.490636749 $b\bar{s}$) that are superior to $\bar{a}^{(VI1)}$)

$$P_{12} = 34560b^8\bar{s}^8 + 224640ab^7\bar{s}^7 - 1227952a^2b^6\bar{s}^6 + 1697360a^3b^5\bar{s}^5 - 18336a^4b^4\bar{s}^4 - 1347148a^5b^3\bar{s}^3 + 925127a^6b^2\bar{s}^2 - 232512a^7b\bar{s} + 18768a^8 > 0$$

(Indeed, $in\ P_{12}$, the coefficient of the monomial with the highest degree is positive, while P_{12} admits two single positive real roots (2.133570253 $b\bar{s}$, 6.383851489 $b\bar{s}$) and a double positive real root (2. $b\bar{s}$), that are superior to $\bar{a}^{(VI1)}$)

$$\frac{dW^{(VI2)}}{dk} - \frac{d\pi^{2NAP}}{dk} = \frac{d(\pi_1 + CS)}{d\bar{s}}\frac{d\bar{s}}{dk} = \frac{7b^2\bar{s}^2 - a^2}{16\bar{s}^2} > 0\ \forall a \in \left[0, \bar{a}^{(VI2)} = \frac{b\bar{s}(k)}{7}\right[$$