Asian Journal of Economic Modelling
ISSN(e): 2312-3656
ISSN(p): 2313-2884
DOI: 10.55493/5009.v10i3.4569
Vol. 10, No. 3, 207-215.

URL: www.aessweb.com

A FUNDED PENSION SYSTEM WITH ENDOGENOUS RETIREMENT

Wataru Kobayashi[1]
Junichiro Takahata[2+]

[1]*Faculty of Policy Informatics, Chiba University of Commerce, Japan.*
Email: watakb@cuc.ac.jp Tel: +81473739956
[2]*Faculty of Economics, Dokkyo University, Japan.*
Email: jtakahat@dokkyo.ac.jp Tel: +81489426426

(+ Corresponding author)

Article History
Received: 23 April 2022
Revised: 13 July 2022
Accepted: 27 July 2022
Published: 5 August 2022

Keywords
Efficiency
Endogenous retirement
Funded pensions
Overlapping generations model
Social security
Welfare.

JEL Classification:
H21, H55, J22, J26.

ABSTRACT

This study employs an overlapping generations model to analyze the effect of public pension levels on economic variables when the labor supply of the elderly is determined endogenously. This paper focuses on the effects of a funded scheme on the economy as well as a pay-as-you-go (PAYG) scheme. First, the impact of the expansion of public pensions on the capital–labor ratio is analyzed. It is shown that the expansion of a funded pension increases the capital–labor ratio, which is contrasted with the fact that the PAYG pension is neutral to capital–labor ratio. Next, the impact of public pensions' introduction on steady-state economic welfare is evaluated. The introduction of a PAYG pension will improve economic welfare when the population growth rate is higher than the interest rate, while the introduction of a funded pension will improve economic welfare when the population growth rate is lower than the interest rate.

Contribution/Originality: This study focuses on funded pension scheme, which has been considered neutral to the capital–labor ratio. This study shows that, in a setting of endogenous retirement, a funded pension scheme has an impact on the capital–labor ratio since it has an impact not only on capital accumulation but also on labor supply.

1. INTRODUCTION

The purpose of this paper is to analyze the impact of the public pension system on resource allocation and welfare in an overlapping generations (OLG) model in which the labor supply of the elderly is endogenous. In particular, this paper focuses not only on a pay-as-you-go (PAYG) scheme but also on a funded scheme as a financial system of public pensions.

In the classic model of Diamond (1965), individuals survive for two periods and supply a certain amount of labor inelastically when young, whereas they do not supply labor when old. Hu (1979); Mihara (2005); Michel and Pestieau (2013); Miyazaki (2019) and Liu and Thøgersen (2020), on the other hand, assumed an exogenous labor supply when individuals are young and an endogenous labor supply when they are old, and analyzed the impact of public pensions on resource allocation. The pension systems considered in the literature are all financed by a PAYG scheme, which does not have a reserve fund.

As is well known, a funded scheme is one of the two major financial schemes for public pensions, along with a PAYG scheme. Nevertheless, it has not been analyzed since it was shown by Samuelson (1975) that a funded scheme is neutral to resource allocation when labor supply is given exogenously. The model used by Samuelson (1975) is one in which the labor supply is given exogenously, but even if the labor supply is endogenous, it is considered that a funded scheme would be still neutral to resource allocation if households correctly recognize that the discounted present value of the proportionate benefits received is equal to the contribution paid proportional to their labor income[1]. However, funded pension schemes may affect the labor supply, which may have an impact on resource allocation in a whole economy where the funded pension benefits include a fixed portion that is not proportional to income, as studied by Frassi et al. (2019), or where such schemes may prevent individuals from receiving pension benefits during employment, as per Michel and Pestieau (2013), others assume PAYG pension schemes.

In this paper, the focus is on a funded pension scheme as well as on a PAYG pension scheme. Most assumptions of PAYG schemes studied by Michel and Pestieau (2013), among others, are maintained in this study. The pension system assumed is one in which benefits will be paid only after individuals retire. This paper is structured as follows: Section 2 describes the model and derives resource allocation in a steady-state market equilibrium; Section 3 derives a socially optimal resource allocation and compares it to that of market equilibrium in the absence of pensions; Section 4 analyzes the impact of the introduction of pensions for each financial scheme on welfare; and Section 5 presents the conclusions.

2. MODEL

2.1. Households and Firms

Households live for two periods and earn income by supplying labor at wage rate w when they are young and old. Households supply one unit of time exogenously when young, but z unit of time endogenously when old. Households save s in the first period and receive the sum of principal and its interest, Rs, in the second period, where $R \equiv 1 + r$ and r is the interest rate. Public pensions are provided by the government based on PAYG schemes and on funded schemes. The contribution rates are τ_p and τ_f, respectively, and the same rate is levied to the first period and the second period. The amount of pension benefits is proportional to the length of leisure when households are in the second period. In this case, the amount of pension benefits for the PAYG scheme and the funded scheme per unit of leisure in period t are b_{pt} and b_{ft}, respectively. This property is considered to correspond to a system in which benefits cannot be received during the period of employment and can only be received after retirement. This is the same type of pension system as considered by Michel and Pestieau (2013); Miyazaki (2019) and Liu and Thøgersen (2020). Here, b_{pt} represents the benefit amount b_p in period t. Households spend c for their first period consumption and d for their second period consumption. The budget constraints of households born in period t (generation t) are as follows:

$$c_t = (1 - \tau_p - \tau_f)w_t - s_t, \tag{1}$$

$$d_{t+1} = R_{t+1}s_t + (1 - \tau_p - \tau_f)w_{t+1}z_{t+1} + (b_{pt+1} + b_{ft+1})(1 - z_{t+1}), \tag{2}$$

Where Equation 1 shows consumption when young is determined from the after-tax income less savings, and Equation 2 shows consumption when old is determined from the sum of after-tax income and pension benefit.

Assuming that households derive their utility from the first period consumption c, the second period consumption d, and leisure in their old period $(1 - z)$, the utility function of households of generation t is assumed to be logarithmic as follows:

$$U = U(c_t, d_{t+1}, 1 - z_{t+1}) = \ln c_t + \beta(\ln d_{t+1} + \gamma \ln(1 - z_{t+1})), \tag{3}$$

Where Equation 3 shows that utility level of generation t is increased by c_t, d_{t+1} and $1 - z_{t+1}$. The budget constraints for PAYG and funded public pensions are:

$$N_{t+1}\tau_p w_{t+1} + N_t \tau_p w_{t+1} z_{t+1} = N_t b_{pt+1}(1 - z_{t+1}),$$

[1] Frassi, Gnecco, Pammolli, and Wen (2019) explain this point as follows: "In a pure FF scheme, a variation in the contribution rate has no effect on individual labour supply, irrespective of whether the worker is high- or low-skilled. This is a clear-cut result since, in a pure FF, individuals are aware that the payment of contributions into their own account is simply another form of private savings." (p. 283).

$$R_{t+1}N_t\tau_f w_{t+1} + N_t\tau_f w_{t+1}z_{t+1} = N_t b_{ft+1}(1 - z_{t+1}).$$

Expressing these budget constraints for PAYG and funded public pensions in per capita term of the old, they can be rewritten in Equations 4 and 5, respectively, as follows:

$$(1+n)\tau_p w_{t+1} + \tau_p w_{t+1}z_{t+1} = b_{pt+1}(1 - z_{t+1}), \quad (4)$$

$$R_{t+1}\tau_f w_{t+1} + \tau_f w_{t+1}z_{t+1} = b_{ft+1}(1 - z_{t+1}). \quad (5)$$

Regarding firms' production, the production amount in period t is denoted by Y_t, the capital stock by K_t, and the labor supply by L_t. L_t, the total amount of labor supply in period t, is the sum of N_t, the population of households of generation t, and $N_{t-1}z_t$, the number of old workers of generation $t-1$, which is shown in Equation 6:

$$L_t = N_t + N_{t-1}z_t. \quad (6)$$

Suppose that the population growth rate is constant at n. The population of generation $(t+1)$, represented by N_{t+1}, is represented in Equation 7 as follows:

$$N_{t+1} = (1+n)N_t. \quad (7)$$

The production function of a company is assumed to be the Cobb–Douglas type:

$$Y_t = F(K_t, L_t) = AK_t^{\alpha}L_t^{1-\alpha}.$$

Under normalized prices and total depreciation with capital, π_t, firms' profits in period t are represented as follows:

$$\pi_t = Y_t - w_t L_t - R_t K_t.$$

From the conditions of profit maximization, we have:

$$\frac{\partial \pi_t}{\partial L_t} = A(1-\alpha)K_t^{\alpha}L_t^{-\alpha} - w_t = 0, \qquad \frac{\partial \pi_t}{\partial K_t} = A\alpha K_t^{\alpha-1}L_t^{1-\alpha} - R_t = 0.$$

Assuming that the capital per unit of labor, which we call per capita capital, is $k_t \equiv K_t/L_t$, the above conditions become Equations 8 and 9, respectively:

$$w_t = A(1-\alpha)k_t^{\alpha}, \quad (8)$$

$$R_t = A\alpha k_t^{\alpha-1}. \quad (9)$$

2.2. Market Equilibrium

Next, we consider household utility maximization. Substituting the household budget constraints (Equations 1 and 2) with the utility function (Equation 3) and differentiating them with respect to savings s_t and the labor supply z_{t+1}, the first-order conditions are obtained as follows, where an interior solution is assumed for z_{t+1}:

$$\frac{1}{(1-\tau_p-\tau_f)w_t - s_t} = \frac{\beta R_{t+1}}{R_{t+1}s_t + (1-\tau_p-\tau_f)w_{t+1}z_{t+1} + b_{pt+1}(1-z_{t+1}) + b_{ft+1}(1-z_{t+1})},$$

$$\frac{\beta\gamma}{1-z_{t+1}} = \frac{\beta\left[(1-\tau_p-\tau_f)w_{t+1} - (b_{pt+1} + b_{ft+1})\right]}{R_{t+1}s_t + (1-\tau_p-\tau_f)w_{t+1}z_{t+1} + b_{pt+1}(1-z_{t+1}) + b_{ft+1}(1-z_{t+1})}.$$

Substituting public pension budget constraints (Equations 4 and 5) into the above conditions, savings s_t and labor supply z_{t+1} are solved as follows in Equations 10 and 11, respectively:

$$s_t = \frac{\beta(1+\gamma)R_{t+1}(1-\tau_p-\tau_f)w_t - (1-\tau_p-\tau_f)w_{t+1}}{(1+\beta+\beta\gamma)R_{t+1}}, \quad (10)$$

$$z_{t+1} = \frac{(1+\beta)(1-\tau_p-\tau_f)}{1+\beta+\beta\gamma}\left[1 + \frac{R_{t+1}w_t}{w_{t+1}}\right] - \left[1 + n - \frac{R_{t+1}w_t}{w_{t+1}}\right]\tau_p - \frac{R_{t+1}w_t}{w_{t+1}}. \quad (11)$$

We consider a capital market equilibrium in this economy. Since capital is assumed to be fully depreciated, the sum of household savings and the reserve funds becomes the capital stock in the following period, so the relationship below holds:

$$K_{t+1} = N_t\left(s_t + \tau_f w_t\right).$$

After dividing by N_t, the above relationship becomes Equation 12 below, using Equation 6 and Equation 7:

$$(1 + n + z_{t+1})k_{t+1} = s_t + \tau_f w_t \tag{12}$$

Substituting household savings (Equation 10) and labor supply (Equation 11) into Equation 12, and rewriting factor prices in terms of k using Equation 8 and Equation 9, the equation of capital accumulation will become:

$$k_{t+1} = \left[\frac{\beta(1-\alpha+\gamma)\left(1-\tau_p\right) + (\alpha\beta+1)\tau_f}{\alpha(1+n)(1+\beta+\beta\gamma)\left(1-\tau_p\right) + (\alpha\beta+1)\left(1-\tau_p\right) - (\alpha\beta+1)\tau_f}\right] A\alpha k_t^{\alpha}$$

If $k_{t+1} = k_t = k$ in the above equation, we have the following steady-state per capita capital k in Equation 13:

$$k = \left[\frac{A\alpha\beta(1-\alpha+\gamma)\left(1-\tau_p\right) + A\alpha(\alpha\beta+1)\tau_f}{\alpha(1+n)(1+\beta+\beta\gamma)\left(1-\tau_p\right) + (\alpha\beta+1)\left(1-\tau_p\right) - (\alpha\beta+1)\tau_f}\right]^{\frac{1}{1-\alpha}} \tag{13}$$

If there is no funded pension in this equation, that is, if $\tau_f = 0$, the capital level is.

$$k = \left[\frac{A\alpha\beta(1-\alpha+\gamma)}{\alpha(1+n)(1+\beta+\beta\gamma) + (\alpha\beta+1)}\right]^{\frac{1}{1-\alpha}}.$$

Since τ_p does not exist in the above equation, the steady-state capital level is neutral to the contribution rate of the PAYG pension when $\tau_f = 0$. This result has already been shown by Mihara (2005) and Liu and Thøgersen (2020). On the other hand, since the coefficient of τ_f in the numerator of Equation 13 is positive and the coefficient of τ_f in the denominator is negative, the steady-state capital level increases when τ_f increases. Here, we have the following proposition.

PROPOSITION 1: When a funded scheme pension does not exist, per capita capital is unaffected by a PAYG pension contribution rate. On the other hand, regardless of the PAYG pension contribution rate, per capita capital is increased when a funded pension contribution rate increases.

Next, resource allocation in a steady state is derived. The first and second periods' consumption, labor supply, and leisure in a steady state are expressed as follows:

$$c = \frac{\left(1-\tau_p-\tau_f\right)w(1+R)}{R(1+\beta+\beta\gamma)}, \tag{14}$$

$$d = \frac{\beta\left(1-\tau_p-\tau_f\right)w(1+R)}{1+\beta+\beta\gamma},$$

$$z = \frac{(1+R)(1+\beta)\left(1-\tau_p-\tau_f\right)}{1+\beta+\beta\gamma} - R - [(1+n)-R]\tau_p,$$

$$1 - z = \frac{(1+R)\left[\beta\gamma + (1+\beta)\left(\tau_p+\tau_f\right)\right]}{1+\beta+\beta\gamma} + [(1+n)-R]\tau_p. \tag{15}$$

Where Equation 14 shows the steady state consumption when young, and Equation 15 shows the steady state leisure when old. From this, the following proposition is derived.

PROPOSITION 2: An introduction of either a PAYG pension or a funded pension increases leisure and hence decreases labor supply (See Appendix A).

Thus far, we have confirmed the steady-state economic variables and the changes in k and z due to the introduction of two types of public pensions. Summarizing the results briefly, it has been shown that an introduction of a PAYG pension does not change k and decreases z, while an introduction of a funded pension increases k and decreases z. In the following section, we compare a socially optimal resource allocation with the steady-state resource allocation without a pension. Then, the impact of the introduction of both types of pension on welfare will be confirmed.

3. SOCIAL OPTIMUM

Next, in order to consider a socially optimal resource allocation, resource constraints are considered. The resource constraint in period t is represented as follows:

$$N_t c_t + N_{t-1} d_t = Y_t - K_{t+1}. \tag{16}$$

Equation 16 argues that consumption for two generations in period t, $N_t c_t + N_{t-1} d_t$, needs to be equal to the production amount Y_t less capital for the following period K_{t+1}.

If the production amount per unit of labor, which we call per capita output, is defined as $y_t \equiv Y_t / L_t$, we obtain the following relationship after dividing Equation 16 by N_t and arranging this using Equation 7:

$$c_t + \frac{d_t}{1+n} = \left(1 + \frac{z_t}{1+n}\right) y_t - (1 + n + z_{t+1}) k_{t+1}.$$

Resource allocation that maximizes household utility in a steady state is required to satisfy the following problems:

$$\max \;\; U(c, d, 1-z) = \ln c + \beta(\ln d + \gamma \ln(1-z)),$$

$$\text{s.t. } c + \frac{d}{1+n} = \left(1 + \frac{z}{1+n}\right)[Ak^{\alpha} - (1+n)k].$$

If we denote the socially optimal capital by $\hat{k}$, and so on, the following relationships will be satisfied from the first-order conditions of the above problem:

$$A\alpha \hat{k}^{\alpha-1} = 1 + n = \frac{\hat{d}}{\beta \hat{c}}, \qquad \hat{k} = \left[\frac{A\alpha}{1+n}\right]^{\frac{1}{1-\alpha}}, \qquad \frac{1-\hat{z}}{\gamma} = \frac{\hat{d}}{\hat{w}}.$$

However, if $\hat{z} > 0$ holds from the assumption of interior solutions, then $\hat{w} = A\hat{k}^{\alpha} - (1+n)\hat{k}$. From the above relationships, labor and leisure are solved as follows:

$$\hat{z} = \frac{1 + \beta - \beta\gamma(1+n)}{1 + \beta + \beta\gamma} \text{ and } 1 - \hat{z} = \frac{\beta\gamma(2+n)}{1 + \beta + \beta\gamma}.$$

Here, we compare the steady-state resource allocation in equilibrium and the social optimum when pension does not exist. The labor supply in a steady-state equilibrium where pension is not implemented will be denoted by $\bar{z}$, and so on. Comparing the socially optimal labor supply $\hat{z}$ with labor supply $\bar{z}$ in a steady-state equilibrium without a pension, and comparing the socially optimal per capita capital $\hat{k}$ with per capita capital in a steady-state equilibrium without pension $\bar{k}$, it is shown that both are generally different:

$$\bar{z} = \frac{1 + \beta - \beta\gamma\bar{R}}{1 + \beta + \beta\gamma} \neq \frac{1 + \beta - \beta\gamma(1+n)}{1 + \beta + \beta\gamma} = \hat{z},$$

$$\bar{k} = \left[\frac{A\alpha}{\bar{R}}\right]^{\frac{1}{1-\alpha}} \neq \left[\frac{A\alpha}{1+n}\right]^{\frac{1}{1-\alpha}} = \hat{k}.$$

As is clear from these relationships, when $\bar{R} = 1 + n$ holds, the per capita capital in a steady-state equilibrium coincides with the socially optimal level and, at the same time, the labor supply in a steady-state equilibrium coincides with the socially optimal level. In addition, when the per capita capital in a steady state is greater than the optimal level, labor supply is also excessive at the same time, where $\bar{R} < 1 + n$ holds. On the contrary, when per capita capital in a steady state is less than the optimal level, labor supply is also insufficient at the same time, where $\bar{R} > 1 + n$ holds.

4. IMPACT ON WELFARE

Next, paying attention to the utility level in a steady state, the impact of the introduction of public pensions on economic welfare will be considered. First, since the optimum ratio of the first period consumption to the second period consumption is $d = \beta Rc$, the utility level in the steady state can be expressed in terms of the first period consumption c, labor supply z, and interest rate R as follows:

$$U = \beta \ln \beta R + (1+\beta) \ln c + \beta\gamma \ln(1-z).$$

At this time, the impact of the introduction of the PAYG pension and that of a funded pension on welfare are as follows, respectively:

$$\frac{\partial U}{\partial \tau_p} = \frac{\beta}{R}\frac{\partial R}{\partial \tau_p} + \frac{1+\beta}{c}\frac{\partial c}{\partial \tau_p} + \frac{\beta\gamma}{1-z}\frac{\partial (1-z)}{\partial \tau_p}, \tag{17}$$

$$\frac{\partial U}{\partial \tau_f} = \frac{\beta}{R}\frac{\partial R}{\partial \tau_f} + \frac{1+\beta}{c}\frac{\partial c}{\partial \tau_f} + \frac{\beta\gamma}{1-z}\frac{\partial (1-z)}{\partial \tau_f}, \tag{18}$$

Where Equations 17 and 18 determine the sign of steady-state welfare change when introducing PAYG and funded public pensions, respectively. Based on Equation 17 and Equation 18, we have the following proposition:

PROPOSITION 3: If $1 + n > \bar{R}$ holds, welfare can be improved by introducing a PAYG pension. Conversely, if $1 + n < \bar{R}$ holds, welfare can be improved by introducing a funded pension (See Appendix B).

The interpretation of this proposition is as follows: First, the condition $1 + n > \bar{R}$ holds, and this means that both per capita capital $\bar{k}$ and labor supply $\bar{z}$ are greater than the optimal level. Under this condition, if a PAYG pension is introduced, per capita capital $\bar{k}$ is unchanged but labor supply $\bar{z}$ is reduced, the effect of which improves welfare level.

Second, on the contrary, the condition $1 + n < \bar{R}$ means that both per capita capital $\bar{k}$ and labor supply $\bar{z}$ are initially lower than the optimal level. Under this condition, if a funded pension is introduced, per capita capital $\bar{k}$ is increased, the effect of which improves welfare, but at the same time, labor supply $\bar{z}$ is reduced, the effect of which worsens welfare. It is possible to interpret the result such that welfare is improved by a funded pension introduction as follows: when these two effects exist, the effect of welfare improvement by the former exceeds the effect of welfare deterioration by the latter.

5. SUMMARY

In this paper, the effects of a PAYG pension and a funded pension to resource allocation and welfare in an OLG model with endogenous retirement have been analyzed. The main results are summarized in the following three points.

First, a PAYG pension is neutral to per capita capital, while a funded pension increases per capita capital. This is very different from the results achieved when an exogenous labor supply is assumed, where a PAYG pension reduces per capita capital, while a funded pension is neutral.

Second, the introduction of either a PAYG or a funded pension reduces the labor supply in the second period for households. This is considered due to the assumptions that pension contributions are collected in proportion to labor income in their old period, and that pension benefits are proportional to leisure in their old period.

Third, an introduction of a PAYG pension will improve welfare in a "dynamically inefficient" case where the interest rate is lower than the population growth rate, while an introduction of a funded pension will improve welfare in a "dynamically efficient" case, on the contrary, where the interest rate is higher than the population growth rate. The former result has already been found in an economy where labor supply is exogenous, and we found that this result still holds in an economy where labor supply is endogenous. This paper also shows the latter result in an economy where labor supply is endogenous. Kobayashi and Takahata (2021) showed that a funded pension contributes to welfare improvement in an environment where labor supply is exogenous and individuals with different productivity coexist within generations, which is the effect of income redistribution within generations. This paper has clarified a different helpful function of a funded pension to improving welfare through implementing more efficient resource allocation.

In this paper, we assume a logarithmic utility function following Michel and Pestieau (2013), while Liu and Thøgersen (2020) used a CES utility function and showed that the impact of a pension depends on its parameters. Regarding the pension system, as analyzed by Mihara (2005), it is necessary to consider factors such as whether to collect contributions in the second period or whether retirement is a necessary condition for receiving pension benefits. As remaining issues, it is necessary to explore to what extent the utility function and pension system assumptions affect these results.

Funding: This study received no specific financial support.
Competing Interests: The authors declare that they have no competing interests.
Authors' Contributions: Both authors contributed equally to the conception and design of the study.

REFERENCES

Diamond, P. A. (1965). National debt in a neoclassical growth model. *The American Economic Review, 55*(5), 1126-1150.

Frassi, B., Gnecco, G., Pammolli, F., & Wen, X. (2019). Intragenerational redistribution in a funded pension system. *Journal of Pension Economics & Finance, 18*(2), 271-303.Available at: https://doi.org/10.1017/s147474721700049x.

Hu, S. C. (1979). Social security, the supply of labor, and capital accumulation. *The American Economic Review, 69*(3), 274-283.

Kobayashi, W., & Takahata, J. (2021). Financial systems and functional allocation of public Pension. *Journal of Chiba University of Commerce, 59*(1), 37-48.

Liu, P., & Thøgersen, J. (2020). Pay-as-you-go pensions and endogenous retirement. *Macroeconomic Dynamics, 24*(7), 1700-1719.Available at: https://doi.org/10.1017/s1365100518001013.

Michel, P., & Pestieau, P. (2013). Social security and early retirement in an overlapping-generations growth model. *Annals of Economics and Finance, 14-2*(B), 705-719.

Mihara, Y. (2005). Retirement decision pension system and golden rule: Is a retirement system necessity sure enough for us? *Journal of Economics the Economic Society of Osaka City University, 105*(4), 88-103.

Miyazaki, K. (2019). Optimal pay-as-you-go social security with endogenous retirement. *Macroeconomic Dynamics, 23*(2), 870-887.Available at: https://doi.org/10.1017/s1365100517000062.

Samuelson, P. A. (1975). Optimum social security in a life-cycle growth model. *International Economic Review, 16*(3), 539-544.Available at: https://doi.org/10.2307/2525994.

APPENDIX A

Proof of Proposition 2. In order to show Proposition 2, it is only necessary to prove that the derivative of leisure $(1-z)$ by τ_p or τ_f and evaluated at $\tau_p = \tau_f = 0$ is positive.

First, in general, R can be obtained as follows by substituting k in a steady state shown in Equation 13 into Equation 9:

$$R = \frac{\alpha(1+n)(1+\beta+\beta\gamma)(1-\tau_p) + (\alpha\beta+1)(1-\tau_p) - (\alpha\beta+1)\tau_f}{\beta(1-\alpha+\gamma)(1-\tau_p) + (\alpha\beta+1)\tau_f}. \tag{A1}$$

When this is differentiated by τ_p and τ_f and evaluated at $\tau_p = \tau_f = 0$, they become:

$$\frac{\partial R}{\partial \tau_p} = 0, \tag{A2}$$

$$\frac{\partial R}{\partial \tau_f} = -\frac{\alpha\beta+1}{[\beta(1-\alpha+\gamma)]^2}[\beta(1-\alpha+\gamma) + \alpha(1+n)(1+\beta+\beta\gamma) + \alpha\beta + 1]. \tag{A3}$$

Furthermore, from Equation 15, the leisure time $(1-z)$ is differentiated with respect to τ_p:

$$\frac{\partial(1-z)}{\partial \tau_p} = \left[\frac{[\beta\gamma + (1+\beta)(\tau_p+\tau_f)]}{1+\beta+\beta\gamma} - \tau_p\right]\frac{\partial R}{\partial \tau_p} + \frac{(1+R)(1+\beta)}{1+\beta+\beta\gamma} + [(1+n) - R].$$

When this is evaluated at $\tau_p = \tau_f = 0$, the first term disappears from Equation A2, and then

$$\frac{\partial(1-z)}{\partial\tau_p}=\frac{(1+\bar{R})(1+\beta)}{1+\beta+\beta\gamma}+[(1+n)-\bar{R}]. \tag{A4}$$

From Equation A1, interest rate R becomes $\bar{R}$ when $\tau_p=\tau_f=0$:

$$\bar{R}=\frac{\alpha(1+n)(1+\beta+\beta\gamma)+(\alpha\beta+1)}{\beta(1-\alpha+\gamma)}. \tag{A5}$$

Therefore, if we substitute Equation A5 into Equation A4 and arrange its terms, we obtain:

$$\frac{\partial(1-z)}{\partial\tau_p}=\frac{(1-\alpha)[1+(1+n)(1+\gamma)]}{1-\alpha+\gamma}>0.$$

Hence, it has been shown that leisure increases and labor supply decreases as τ_p rises.

On the other hand, if leisure $(1-z)$, shown as Equation 15, is differentiated with respect to τ_f, it becomes:

$$\frac{\partial(1-z)}{\partial\tau_f}=\frac{(1+R)(1+\beta)}{1+\beta+\beta\gamma}+\left[\frac{\beta\gamma}{1+\beta+\beta\gamma}-\tau_p\right]\frac{\partial R}{\partial\tau_f}.$$

Substituting Equation A3 into the above, it will become the following when $\tau_p=\tau_f=0$:

$$\frac{\partial(1-z)}{\partial\tau_f}=\frac{(1+\bar{R})(1-\alpha)}{1-\alpha+\gamma}>0.$$

Therefore, it has been shown that leisure increases and labor supply decreases as τ_f rises.

Appendix B

Proof of Proposition 3. To prove whether an introduction of a PAYG pension improves welfare or not, it is sufficient to show that $\partial U/\partial\tau_p$, represented by Equation 17, is positive. From Proposition 1, if a funded pension is not implemented, the interest rate R does not change since the level of per capita capital is neutral to τ_p. Moreover, if we differentiate the first period consumption c and leisure in the second period $(1-z)$, shown by Equation 14 and Equation 15, respectively, with respect to τ_p and evaluate at $\tau_p=\tau_f=0$, then they become as follows:

$$\frac{\partial R}{\partial\tau_p}=0, \tag{B1}$$

$$\frac{\partial c}{\partial\tau_p}=-\frac{\bar{w}}{\bar{R}}\frac{1+\bar{R}}{1+\beta+\beta\gamma}, \tag{B2}$$

$$\frac{\partial 1-z}{\partial\tau_p}=\frac{(1+\bar{R})(1+\beta)}{1+\beta+\beta\gamma}+[(1+n)-\bar{R}]. \tag{B3}$$

In addition, substituting $\tau_p=\tau_f=0$ into Equation 14 and Equation 15, we have the first period consumption $\bar{c}$ and leisure in the second period $(1-\bar{z})$ when $\tau_p=\tau_f=0$ as follows:

$$\bar{c}=\frac{\bar{w}}{\bar{R}}\frac{1+\bar{R}}{1+\beta+\beta\gamma}, \tag{B4}$$

$$1-\bar{z}=\frac{(1+\bar{R})\beta\gamma}{1+\beta+\beta\gamma}. \tag{B5}$$

Substituting Equations B1 to B5 into Equation 17 and arranging them, the impact of the introduction of a PAYG pension on welfare will become:

$$\frac{\partial U}{\partial\tau_p}=[(1+n)-\bar{R}]\frac{1+\beta+\beta\gamma}{1+\bar{R}},$$

Namely, it has been shown that $\partial U/\partial\tau_p>0$ holds true when $1+n>\bar{R}$.

Similarly, in order to prove whether the introduction of a funded pension improves welfare or not, it is sufficient to show that $\partial U/\partial\tau_f$, represented by Equation 18, is positive. If we differentiate the first period consumption c and leisure in the second period $(1-z)$, shown by Equation 14 and Equation 15, respectively, with respect to τ_f and evaluate at $\tau_p=\tau_f=0$, then they become as follows:

$$\frac{\partial c}{\partial \tau_f} = \frac{1+\bar{R}}{\bar{R}(1+\beta+\beta\gamma)}\frac{\partial w}{\partial \tau_f} - \frac{\bar{w}}{\bar{R}^2(1+\beta+\beta\gamma)}\frac{\partial R}{\partial \tau_f} - \frac{\bar{w}}{\bar{R}}\frac{1+\bar{R}}{1+\beta+\beta\gamma}, \quad \text{(B6)}$$

$$\frac{\partial 1-z}{\partial \tau_f} = \frac{\beta\gamma}{1+\beta+\beta\gamma}\frac{\partial R}{\partial \tau_f} + \frac{(1+\bar{R})(1+\beta)}{1+\beta+\beta\gamma}. \quad \text{(B7)}$$

Substituting Equation B6 and Equation B7 together with Equation B4 and Equation B5 into Equation 18, then we have:

$$\frac{\partial U}{\partial \tau_f} = \frac{\beta\bar{R}(1+\gamma)-1}{\bar{R}(1+\bar{R})}\frac{\partial R}{\partial \tau_f} + \frac{1+\beta}{\bar{w}}\frac{\partial w}{\partial \tau_f}. \quad \text{(B8)}$$

In the case of a funded pension, the level of per capita capital also changes depending on the change in τ_f. Therefore, we need derivatives of factor price with respect to τ_f as follows:

$$\frac{\partial w}{\partial \tau_f} = (1-\alpha)\alpha A k^{\alpha-1}\frac{\partial k}{\partial \tau_f}, \quad \text{(B9)}$$

$$\frac{\partial R}{\partial \tau_f} = (\alpha-1)\alpha A k^{\alpha-2}\frac{\partial k}{\partial \tau_f}. \quad \text{(B10)}$$

Substituting Equation B9 and Equation B10 into Equation B8 gives the following:

$$\frac{\partial U}{\partial \tau_f} = \left[\frac{1-\beta\bar{R}(1+\gamma)}{\bar{R}(1+\bar{R})} + \frac{1+\beta}{\bar{w}}\bar{k}\right](1-\alpha)\alpha A\bar{k}^{\alpha-2}\frac{\partial k}{\partial \tau_f}.$$

Since Proposition 1 shows $\partial k/\partial \tau_f > 0$, in order to show that $\partial U/\partial \tau_f$ is positive, it is sufficient to show that the term inside the big parentheses is positive. Considering such conditions and substituting $\bar{R}$, $\bar{w}$, $\bar{k}$, the conditions for improving welfare are expressed as follows:

$$(1+n)[\beta(1-\alpha+\gamma) - \alpha(1+\beta+\beta\gamma)] - (\alpha\beta+1) < 0. \quad \text{(B11)}$$

In order to prove Proposition 3, we need the above condition Equation B11 to be true. Using Equation A5, the condition $1+n-\bar{R} < 0$ is arranged to become:

$$1+n-\bar{R} = \frac{(1+n)[\beta(1-\alpha+\gamma) - \alpha(1+\beta+\beta\gamma)] - (\alpha\beta+1)}{\beta(1-\alpha+\gamma)} < 0.$$

From this relationship, it has been shown that $\partial U/\partial \tau_f > 0$ holds since Equation B11 is true as long as $1+n < \bar{R}$.