



## COMPARISON OF THE LEE-CARTER AND ARCH IN MODELLING AND FORECASTING MORTALITY IN ZIMBABWE



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### ABSTRACT

*This paper aims to determine which model best fits the mortality profile of Zimbabwe. The models that being compared were the Lee-Carter and ARCH and the period under review was from 1983-2004 using mortality rates of five years age interval. The parameters of the Lee-Carter model were estimated using singular value decomposition. The box Jenkins approach was used to determine the order of lags for the ARIMA. An ARIMA (1, 1, 1) was used to forecast the overall mortality levels. To determine which model best fit the mortality profile of Zimbabwe goodness of fit using the Kolmogorov-Simonov and root mean square error was used. The result suggested that the Lee-Carter model provided a better fit as compared to ARCH. However both models failed to fit for ages between 50-59 years. Both models were used to forecast, but selecting the one that had the best fit of the two models on a particular age group interval for the period 2005-2014. This paper concludes that generally the Lee-Carter model provided a better fit for the mortality profile of Zimbabwe.*

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### Contribution/ Originality

The study contributes to the existing literature on the best methods of modeling and forecasting mortality, interestingly with focus Zimbabwe, a developing economy. The study is the first of its kind in Zimbabwe on this branch of knowledge.

### 1. INTRODUCTION

The study of mortality is of utmost importance as it provides an insight of the health of individuals not only in Zimbabwe but also indicates the social well-being of individuals across the globe. According to the [World Health Organisation \(2009\)](#) the leading global risks for mortality in the world are high blood pressure (responsible for 13% of deaths globally), tobacco use (9%), high blood glucose (6%), physical inactivity (6%), and overweight and obesity (5%). These risks are responsible for raising the risk of chronic diseases such as heart disease, diabetes and cancers. They affect countries across all income groups hence the study of mortality not only is important for health and life insurance but also for health ministries and non-governmental organization as well.

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From financial point of view, considering today's competitive term life insurance market there is little opportunity to arrive at competitive premiums without some type of mortality improvement built into the pricing. It is noted that a small change in the mortality forecast improvement can have a disproportionately large impact on pricing and profitability. In researches that were conducted in USA a one percent incremental increase in the mortality forecasts improvement can reduce the present value of claims from a block of term life business by up to nine percent. It can be noted that that for pricing actuaries their concern is of having a better forecasts of mortality which will aid them ascertaining the premium that have to be paid that will cuter for unexpected losses in the future.

Building a bona fide mortality management solution are not easy [Collins et al. \(2011\)](#) but the value-creating potential is too big to ignore. In the short run, a mortality management solution improves operational efficiencies and mortality results. In the long run, it captures data that provide essential insights for building competitive products and meeting evolving regulatory demands and to have a bona fide management system there is a need to develop a robust model that give better forecasts of mortality and benefits of profitability and pricing is better attained.

The better prediction of mortality is essential because it is these uncertainty that the model would be used to come up with better mitigations of the basis risk that that arise from mortality and longevity risk on pension expenditure and the impact on social security reforms . It is essential to have better understanding of mortality forecasts as from these forecast we come up with longevity risk and this create a platform for other sophisticated derivative instruments like longevity derivatives and bonds to be introduced.

One of the most commonly used models to forecast age-specific mortality rates is based on the [Lee and Carter \(1992\)](#) model. The model has attracted a lot of attention and has become a benchmark for mortality modelling and life table predictions although the model has also been subject to criticism. Even though the model was mainly intended to describe the statistical variation in all-cause mortality in the United States and similar developed countries, the model is now widely used to predict all-cause and cause specific mortality for a large range of developed and less developed countries.

## 2. LITERATURE REVIEW

In actuarial science, force of mortality represents the instantaneous rate of mortality at a certain age measured on an annualized basis the force of mortality is assumed to follow various distribution from gompertz, uniform, balduccimakeham and exponential to mention a few. It is suggested that the force of mortality can be calculated as derivative of the underlying distribution divided by the underlying distribution.

$$\mu_{x:t} = \frac{s'(x)}{s(x)}$$

Where  $\mu$  the force of mortality is  $s'(x)$  is the derivative of the underling distribution and  $s(x)$  is the underling distribution.

[Gompertz \(1825\)](#) after observing a law of geometrical progression (i.e. exponential rise) in death rates based on 19th century people in England, Sweden, and France between ages 20 and 60, came up with a law of modelling mortality and he suggested that the force of mortality. In terms of actuarial notation, this formula can be expressed as:

$$\mu_{x:t} = BC^x$$

Where  $B$  and  $C$  are model parameters they should be greater than zero,  $x > 0$  is the age, and  $\mu_{x:t}$  is the force of mortality at age  $x$ . Gompertz Law is an exponential function, and it is regularly consider a reasonable assumption for middle ages and older ages. If the mortality rates of the elderly age is assumed to follow the Gompertz law and the

estimation of the parameters  $B$  and  $C$  is not that complex, then the mortality projection would be more straightforward and rather than using regular non-parametric methods for forecasting. This is one of the main reasons why a parametric approach has been used overwhelmingly in mortality projections, especially for elderly mortality rates. However the Gompertz law has received much attention, there are still no standard methods to test if a set of yearly age-specific mortality data satisfies this law.

The Uniform or De Moivre's Model is another parametric model that could be used to model mortality (Finan, 2011). The force of mortality can be found by;

$$\mu(x) = \frac{1}{\omega - x}$$

Where  $\omega$  the maximum or terminal age and  $x$  is the age at death random variable.

The Modified Gompertz Model or The Makeham's Model this was proposed. Makeham's is a refinement to the Gompertz law where he suggested that a positive constant was to be added to the Gompertz law to come up with an model below for modelling force of mortality:

$$\mu(x) = A + BC^x$$

Where  $B > 0; A > -B; C > 1$  and  $x > 0$

The Weibull law of mortality is defined by the hazard function

$$\mu(x) = kx^n$$

Where  $k > 0, n > 0, x \geq 0$ . That is, the death rate is proportional to a power of age. Notice that the exponential model is a special case of Weibull model where  $n = 0$ .

Form the reseach by Eglé *et al.* (2012) they employed lee-carter mortality forecasting they comparing the suitability of the Lee-Carter model for different countries namely France Lithuania and Belarus and their period of study was from 1970-2005 they used the age specific mortality rates, they obtained that the model almost accurately describes and predicts mortality for France. Only short-term (1-2 years) forecasts for the Belarusian and Lithuanian mortality are accurate, while the accuracy of longer-term (4-5 years) forecasts is lower, especially for men. From these results, they concluded that the Lee-Carter method is most suitable for populations with a clear upward or downward mortality trend over time. They also suggested that for further improvement in results they should be more researches done using the lee-carter extensions.

According to Emilio (2012) in their research on measuring the Impact of Longevity Risk on Pension Systems: The Case of Italy, they wanted to project the mortality using the lee-carter model. They fitted the model to Italian mortality rates for the period 1965-1999 and generating forecasts for the subsequent nine years, up to 2008. However, they made extensions to their model by considering the cohorts ranging from 20 to 110 years. In their findings they found that the model performs very well in explaining mortality from 40 to 100 years, however the power of the lee carter model significantly drops as the observations on mortality rates become more volatile at very old ages. They concluded that the overall performance of the lee-carter model was overall satisfactory, as the realized ex-post data almost never violate the 95 per cent confidence bounds.

Giacometti *et al.* (2009) compared the Lee-Carter model and AR-ARCH model for forecasting mortality. They obtained data of mortalities from 1960-2006 for the ages between 40-90 of Italian and they fitted the data on the lee-carter and ARCH(1) they first fitted the data from 1960-2004 on the two models and they forecasted the mortalities

from 2004-2006 they then used the Kolmogorov Smirnov to compare which model then provided the best fit and they found that the ARCH(1) model with *t*-student innovations provides the best fit among the models investigated because it is able to capture the non-Gaussian behaviour of the dynamics associated with the time and age processes. Moreover, this model is capable of enlarging the mortality matrix in two dimensions time and age. From the literature review it can be noted that lee-carter model is one of the model used in modelling and forecasting of mortality rates in various countries however other researcher used the lee-carter extension to model mortality rates.

### 3. RESEARCH METHODOLOGY

First the collected data will implemented in both the Lee-Carter and ARCH and parameters for both models will be obtained. Then forecasts for both models will be projected, for the Lee-Carter model projections will be obtained using ARIMA then finally the forecasted mortality rates will be compared to actual mortality rates. A number of test will be conducted actual mortality rate against the forecasted rates to determine the weather the model fit the actual data using RSME (root squared mean error) and finally K-S (Kolmogorov-Simonov) test then the results from the two test of the model will be compared to determine which model can be used in modeling and forecasting mortality. The research targeted central mortality rate Zimbabwe the age specific mortality rates from ages 0-85 years the source of the data was obtained from ZIMSTATS and ministry of health, the sampling period was from 1983 to 2014. The software Matlab 8.2 was used for analysis.

#### 3.1. Central Mortality Rates

<sup>1</sup>Mortality is the death rate for age *x* in year *t*, i.e. the ratio between the total number of deaths in the population of age *x* in year *t* and the total population of age *x* in year *t* and the mortality is calculated as

$$m_{x;t} = \frac{D_{x;t}}{N_{x;t}}$$

Where *D*: is the number of deaths in particular age at a particular time

*N*: is the total population of the age being investigated at particular time.

#### 3.2. The Lee-Carter Model

Lee and Carter (1992) the model is as follow:

$$\log m_{x;t} = \alpha + \beta k + \varepsilon$$

Under the condition that  $\sum k = 0$  and  $\sum b = 1$

Where:  $m_{x;t}$  is the matrix of the central death rates at age  $x \{x = x_1; x_2 \dots \dots \dots x_n\}$  in years *t*

$\{t = t; t + 1; \dots \dots \dots t + T\}$

$$m_{x;t} = \begin{pmatrix} m_{1;1} & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & m_{x;t} \end{pmatrix}$$

The  $\varepsilon$  represents the deviation of the model from the observed log-central death rates and is expected to be Gaussian  $\varepsilon \sim N(0; \sigma^2)$ . The parameter  $\alpha$  is the estimates, are given by the averages of the force of mortality over the time period

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<sup>1</sup>EgleIgnataviciute, Rasa Mikalauskaite-Arminiene, Jonas Šiaulyš LEE-CARTER MORTALITY FORECASTING Lithuanian Journal of Statistics November 2012

$$\alpha = \frac{1}{N} \sum_{t=1}^n \mu_{x;t}$$

The  $\beta$  measures the response at age  $x$  to change in the overall level of mortality over time,  $k$  represents the overall level of mortality in year  $t$

The constants  $\beta$  and  $k$  are uniquely estimated using singular value decomposition (SDV)

### 3.3. Singular Value Decomposition

Lee and Carter used the singular value decomposition to estimate the parameters by first subtracting the average of force of mortality from the log of mortality.

$$\log(m_{x;t}) - \alpha = Y_{x;t}$$

Therefore

$$SVD(Y_{x;t}) = \sum \rho_i U_{x;i} V_{i;t}$$

Where  $\{\rho_1 > \rho_2 > \dots > \rho_n\}$  are order of the singular values of  $Y_{x;t}$ ,  $U_{x;i}$  and  $V_{i;t}$  are left and right singular vectors.

### ARIMA

The estimated  $k$  is then modelled and forecasted using the ARIMA in most G7 countries they used of (0;1;0) but however the most suitable way of determining the order of lags is though using the box Jenkins approach but first the order of (0;1;0) which lee and carter used in forecasting in 1992 which is as follows.

$$k_t = k_{t-1} + d + \epsilon$$

Where  $d$  the drift parameter and  $\epsilon$  is the error term.

### 3.4. Box Jenkins Approach

The box Jenkins approach was used to determine the order of legs for the ARIMA model the process shall be from the carried out in the following as follows:

1. Establish the stationarity of kappa time series. If kappa series is not stationary, successively difference your series to attain stationarity.
2. Identify a (stationary) conditional mean model for kappa data. The sample ACF and PACF functions were used as selection procedure for the legs. But the failure of the PACF and ACF to give decisive order of legs then Akaike or Bayesian information criterion was used using the AIC matrix the combination that gave the lowest value was the optimal order of legs for the ARIMA model.
3. Then parameters for the ARIMA model are estimate.
4. Goodness-of-fit test was conducted to checks to ensure the model describes kappa data adequately. Residuals were not normally distributed then  $t$  innovations were superimposed on the on the model on the ARIMA.
5. The model was used to generate forecasts.

**ARCH**

We analyse separately rows of the mortality table. As a first step, we analyse the data by rows; that is, we fix a specific age  $x$  and consider a time series process as follows;

$$\mu_{x;t} = p_x(t) + \alpha_1 \mu_{x;t-1} + \varepsilon$$

$$\delta_{x;t}^2 = \beta_0 + \beta_1 \varepsilon_{x;t-1}^2$$

Where  $\varepsilon_{x;t}$  is identical and independently distributed with mean zero and constant

**3.5. Simulation**

Data from 1983 to 2004 is used to obtain parameter of the model the two respective models are used to forecast the mortality five years ahead from 2005 to 2014 the forecasted mortalities are then compared with the actual mortality rated the to analyze if the forecasted data fit to the actual the following test will be conducted RSME (root squared mean error), MAPE (mean absolute percentage error) and finally K-S (Kolmogorov-Simonov) which will be used as test of goodness of fit.

**RSME**

$$RSME = \sqrt{\frac{\sum (\hat{s}(t) - s(t))^2}{N}}$$

**3.6. Kolmogorov-Smirnov**

This test the goodness of fit of the models being investigated namely the lee-carter and the ARCH ascertain which model can best model the Zimbabwean mortality using the following test:

$$F_n(X) = \frac{1}{N} \sum_{i=1} I(X_i < x)$$

The asymptotic distribution of the KS. Statistic is available and the critical value at 95% confidence level is 1.36.

**4. DATA ANALYSIS AND RESULTS**

Bayesian information criterion matrix

|        | MA (1)  | MA (2)  | MA (3)  | MA (4)  |
|--------|---------|---------|---------|---------|
| AR (1) | 25.2913 | 25.9575 | 29.8567 | 34.5961 |
| AR (2) | 28.3670 | 28.7214 | 32.3142 | 28.7488 |
| AR (3) | 31.7236 | 27.6958 | 36.5606 | 33.3378 |
| AR (4) | 34.9238 | 29.9061 | 39.0513 | 39.1833 |

From the matrix above it can be noted that the most suitable order of legs for the moving average and the autoregressive are 1 and 1 respectively since the correlation between the two legs produced the lowest Bayesian value of 25.2913 as compared to other orders.

According to Lee and Carter (1992) to forecast kappa an ARIMA (0, 1, 0) should be used but may differ with different countries because of the mortality profiles.

Estimation using Gaussian distribution

| <b>ARIMA (0, 1, 0) Model:</b>                  |           |           |             |
|--|-----------|-----------|-------------|
| Conditional Probability Distribution: Gaussian |           |           |             |
| Parameter                                      | Value     | Error     | t-Statistic |
| Constant                                       | -0.301732 | 0.0626157 | -4.8188     |
| Variance                                       | 0.108719  | 0.0363283 | 2.9926      |

Estimation using t distribution

| <b>ARIMA (0, 1, 0) Model:</b>           |           |           |             |
|---|-----------|-----------|-------------|
| Conditional Probability Distribution: t |           |           |             |
| Parameter                               | Value     | Error     | t-Statistic |
| Constant                                | -0.302623 | 0.0650251 | -4.65394    |
| Variance                                | 0.108789  | 0.0642196 | 1.69402     |
| DoF                                     | 200       | 26965.1   | 0.00741701  |

Bayesian information criterion for Gaussian and t distribution respectively

Bayesian information criterion =

|          |                 |
|----------|-----------------|
| Gaussian | t- distribution |
| 29.0773  | 25.9797         |

From of the Box Jenkins approach result it can be noted that the ARIMA estimation is suitable for kappa estimation at ARIMA (0, 1, 0) and at ARIMA (1, 1, 1) but the most suitable model is the ARIMA (1, 1, 1) since it produced the smallest Bayesian value of 25.2913 as compared to that defined by Lee and Carter (1992) with innovations of ARIMA (0, 1, 0) which produced a value of 25.9797. Therefore it can be concluded that the ARIMA (1, 1, 1) is the most suitable for estimating and forecasting the kappa (k) in LEE-CARTER model.

FORECASTED KAPPA ( $k$ ) using the ARIMA (1, 1, 1)

| Year Kappa |              |
|------------|--------------|
| 2005       | -4.290214291 |
| 2006       | -4.789526953 |
| 2007       | -5.252830259 |
| 2008       | -5.693648493 |
| 2009       | -6.120426529 |
| 2010       | -6.538437542 |
| 2011       | -6.950974223 |
| 2012       | -7.360092604 |
| 2013       | -7.767076521 |
| 2014       | -8.172727629 |

**FITNESS OF THE MODEL**

$H_0$ : The lee-carter model fit and forecast the mortality rates of Zimbabwe.

$H_1$ : The lee-carter model does not fit and forecast the mortality rates of Zimbabwe.

Lee-Carter Fitness Summary

| <b>LEE-CARTER</b>   |               |               |             |                   |
|---------------------|---------------|---------------|-------------|-------------------|
| <b>age interval</b> | <b>K-S@5%</b> | <b>K-S@1%</b> | <b>RSME</b> | <b>conclusion</b> |
| 0-4                 | h=0           | h=0           | 0.000384478 | Fit               |
| 5-9                 | h=0           | h=0           | 0.00001529  | Fit               |
| 10-14               | h=0           | h=0           | 0.000012602 | Fit               |
| 15-19               | h=0           | h=0           | 0.000101453 | Fit               |
| 20-24               | h=0           | h=0           | 0.000094583 | Fit               |
| 25-29               | h=0           | h=0           | 0.000066852 | Fit               |
| 30-34               | h=0           | h=0           | 0.000156056 | Fit               |
| 35-39               | h=0           | h=0           | 0.000181462 | Fit               |
| 40-44               | h=0           | h=0           | 0.000208938 | fit               |
| 45-49               | h=0           | h=0           | 0.000172338 | fit               |
| 50-54               | h=1           | h=1           | 0.000818121 | not fit           |
| 55-59               | h=1           | h=1           | 0.001669677 | not fit           |
| 60-64               | h=0           | h=0           | 0.001435886 | fit               |
| 65-69               | h=0           | h=0           | 0.000820139 | fit               |
| 70-74               | h=0           | h=0           | 0.001407179 | fit               |
| 75-79               | h=1           | h=1           | 0.008174681 | not fit           |
| 80-84               | h=1           | h=1           | 0.016175251 | not fit           |

From the table above, the fitted lee-carter model to the different age groups fitted across fifteen age groups and the model failed to fit for 75-79, 80-84, 50-54 and 55-59. In generally the lee-carter model fitted for age groups from ages 0-50 of which the most productive age the model fitted.

Arch Fitness Summary

| <b>AR(1) ARCH(1)</b> |               |               |             |                   |
|----------------------|---------------|---------------|-------------|-------------------|
| <b>age interval</b>  | <b>K-S@5%</b> | <b>K-S@1%</b> | <b>RSME</b> | <b>conclusion</b> |
| 0-4                  | h=0           | h=0           | 0           | fit               |
| 5-9                  | h=0           | h=0           | 4.3E-05     | fit               |
| 10-14                | h=0           | h=0           | 2.2E-05     | fit               |
| 15-19                | h=0           | h=0           | 4.2E-05     | fit               |
| 20-24                | h=0           | h=0           | 0.00012     | fit               |
| 25-29                | h=1           | h=1           | 0.00024     | not fit           |
| 30-34                | h=1           | h=1           | 0.00032     | not fit           |
| 35-39                | h=1           | h=1           | 0.00042     | not fit           |
| 40-44                | h=0           | h=0           | 0.0002      | fit               |
| 45-49                | h=0           | h=0           | 0.00014     | fit               |
| 50-54                | h=1           | h=0           | 0.00061     | not fit           |
| 55-59                | h=1           | h=0           | 0.00121     | not fit           |
| 60-64                | h=1           | h=1           | 0.00258     | not fit           |
| 65-69                | h=0           | h=0           | 0.00074     | fit               |
| 70-74                | h=0           | h=0           | 0.00069     | fit               |
| 75-79                | h=1           | h=0           | 0.00602     | not fit           |
| 80-84                | h=1           | h=1           | 0.01281     | not fit           |

From the table above, fitting the ARCH model to the different age groups succeeded across eleven age groups and the model failed to fit for 75-79, 80-84, 50-54, 55-59, 30-34 and 35-39. In general the ARCH model fitted for age groups more than half age groups under review of which the model failed to forecast for the most productive age groups which may be set back to actuarial modelling as the model cannot be used in pricing life insurance products.



Comparison of the Two Models

| LEE-CARTER   |        |        |             |            | AR(1) ARCH(1) |        |        |           |            |
|--------------|--------|--------|-------------|------------|---------------|--------|--------|-----------|------------|
| age interval | K-S@5% | K-S@1% | RSME        | conclusion | age interval  | K-S@5% | K-S@1% | RSME      | conclusion |
| 0-4          | h=0    | h=0    | 0.000384478 | fit        | 0-4           | h=0    | h=0    | 0         | better fit |
| 5-9          | h=0    | h=0    | 0.00001529  | better fit | 5-9           | h=0    | h=0    | 0.0000426 | fit        |
| 10-14        | h=0    | h=0    | 0.000012602 | better fit | 10-14         | h=0    | h=0    | 0.0000219 | fit        |
| 15-19        | h=0    | h=0    | 0.000101453 | fit        | 15-19         | h=0    | h=0    | 0.0000423 | better fit |
| 20-24        | h=0    | h=0    | 0.000094583 | better fit | 20-24         | h=0    | h=0    | 0.0001203 | fit        |
| 25-29        | h=0    | h=0    | 0.000066852 | better fit | 25-29         | h=1    | h=1    | 0.0002362 | not fit    |
| 30-34        | h=0    | h=0    | 0.000156056 | better fit | 30-34         | h=1    | h=1    | 0.0003243 | not fit    |
| 35-39        | h=0    | h=0    | 0.000181462 | better fit | 35-39         | h=1    | h=1    | 0.0004208 | not fit    |
| 40-44        | h=0    | h=0    | 0.000208938 | fit        | 40-44         | h=0    | h=0    | 0.0002009 | better fit |
| 45-49        | h=0    | h=0    | 0.000172338 | fit        | 45-49         | h=0    | h=0    | 0.0001364 | better fit |
| 50-54        | h=1    | h=1    | 0.000818121 | not fit    | 50-54         | h=1    | h=0    | 0.0006101 | not fit    |
| 55-59        | h=1    | h=1    | 0.001669677 | not fit    | 55-59         | h=1    | h=0    | 0.0012111 | not fit    |
| 60-64        | h=0    | h=0    | 0.001435886 | better fit | 60-64         | h=1    | h=1    | 0.0025767 | not fit    |
| 65-69        | h=0    | h=0    | 0.000820139 | fit        | 65-69         | h=0    | h=0    | 0.0007377 | better fit |
| 70-74        | h=0    | h=0    | 0.001407179 | fit        | 70-74         | h=0    | h=0    | 0.0006915 | better fit |
| 75-79        | h=1    | h=1    | 0.008174681 | not fit    | 75-79         | h=1    | h=0    | 0.006016  | not fit    |
| 80-84        | h=1    | h=1    | 0.016175251 | not fit    | 80-84         | h=1    | h=1    | 0.0128097 | not fit    |

Key: h=0 accepted that the model fit, h=1 does not accept that the model fit

$H_0$ : Both models best fit and forecast the mortality rates of Zimbabwe.

$H_1$ : None of the two models best fit and forecast the mortality rates of Zimbabwe.

When comparing the two models the lee-carter model fitted 15 age groups and failed to fit for four age groups namely 80-84,75-79, 55-59 and 50-54 however on the other hand the ARCH model was able to fit for eleven age groups and failed to fit for 25-39,50-59 and 75-85 of which tis model failed age group that is considered to be important age which is 29-39 as they form a large potion contributors to life insurance.

Where the two models fitted a particular age group again the lee-carter proved to be more efficient in terms of fitting as the model produce relatively lower magnitude of error across seven age groups as compared to ARCH which was efficient across six age groups as a result it used when estimating mortality rates for ages that are considered to be the most effective for economic growth as it produce lower magnitude of error.

## 5. CONCLUSION

When comparing the two models, the Le-Carter generally provided better fitting as compared to the ARCH. The lee-carter fitted many age groups under review. The lee-carter produced lower error as compared to the ARCH, finally lee-carter fitted for the most productive age groups from 20-50 years. In conclusion both models can only be used to forecast from 0-75 years beyond that age both models cannot be used for mortality projections.

## APPENDIX

Expected Future Mortality Rates

| AGE/YR | YR 2015 | YR 2016 | YR 2017 | YR 2018 | YR 2019 | YR 2020 | YR 2021 | YR 2022 | YR 2023 | YR 2024 | model |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|-------|
| 0-4    | 0.00112 | 0.00108 | 0.00104 | 0.00100 | 0.00096 | 0.00093 | 0.00090 | 0.00087 | 0.00084 | 0.00082 | ARCH  |
| 5-9    | 0.00011 | 0.00011 | 0.00010 | 0.00010 | 0.00009 | 0.00009 | 0.00008 | 0.00008 | 0.00008 | 0.00007 | L-C   |
| 10-14  | 0.00014 | 0.00013 | 0.00013 | 0.00012 | 0.00012 | 0.00011 | 0.00011 | 0.00010 | 0.00010 | 0.00010 | L-C   |
| 15-19  | 0.00056 | 0.00054 | 0.00052 | 0.00050 | 0.00049 | 0.00047 | 0.00045 | 0.00044 | 0.00042 | 0.00041 | ARCH  |
| 20-24  | 0.00082 | 0.00080 | 0.00078 | 0.00076 | 0.00075 | 0.00073 | 0.00071 | 0.00069 | 0.00068 | 0.00066 | L-C   |
| 25-29  | 0.00081 | 0.00080 | 0.00078 | 0.00077 | 0.00075 | 0.00074 | 0.00072 | 0.00071 | 0.00069 | 0.00068 | L-C   |
| 30-34  | 0.00102 | 0.00101 | 0.00099 | 0.00097 | 0.00095 | 0.00094 | 0.00092 | 0.00090 | 0.00089 | 0.00087 | L-C   |
| 35-39  | 0.00153 | 0.00151 | 0.00149 | 0.00147 | 0.00145 | 0.00142 | 0.00140 | 0.00138 | 0.00136 | 0.00134 | L-C   |
| 40-44  | 0.00227 | 0.00222 | 0.00217 | 0.00212 | 0.00206 | 0.00201 | 0.00196 | 0.00191 | 0.00185 | 0.00180 | ARCH  |
| 45-49  | 0.00381 | 0.00374 | 0.00366 | 0.00359 | 0.00351 | 0.00344 | 0.00336 | 0.00329 | 0.00322 | 0.00314 | ARCH  |
| 50-54  | 0.00610 | 0.00602 | 0.00593 | 0.00585 | 0.00576 | 0.00568 | 0.00559 | 0.00551 | 0.00542 | 0.00534 | L-C   |
| 55-59  | 0.00962 | 0.00949 | 0.00936 | 0.00922 | 0.00908 | 0.00894 | 0.00880 | 0.00866 | 0.00853 | 0.00839 | L-C   |
| 60-64  | 0.01525 | 0.01503 | 0.01480 | 0.01457 | 0.01433 | 0.01410 | 0.01387 | 0.01363 | 0.01341 | 0.01318 | L-C   |
| 65-69  | 0.02386 | 0.02348 | 0.02307 | 0.02266 | 0.02225 | 0.02184 | 0.02143 | 0.02103 | 0.02063 | 0.02024 | ARCH  |
| 70-74  | 0.03731 | 0.03665 | 0.03596 | 0.03525 | 0.03455 | 0.03385 | 0.03316 | 0.03247 | 0.03180 | 0.03114 | ARCH  |
| 75-79  | 0.06164 | 0.06064 | 0.05959 | 0.05852 | 0.05745 | 0.05638 | 0.05532 | 0.05427 | 0.05325 | 0.05223 | L-C   |
| 80-84  | 0.10403 | 0.10264 | 0.10119 | 0.09970 | 0.09821 | 0.09671 | 0.09523 | 0.09376 | 0.09231 | 0.09087 | L-C   |

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