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LOG-ELLIPTICAL TAIL DISTRIBUTIONS OF POWER PRICES – EVIDENCE FROM GERMANY AND FRANCE

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ABSTRACT

Electricity price distributions exhibit high volatility, positive and negative price spikes and consequently fat tails. As risk management is mainly concerned with extreme outcomes that exceed a defined safety threshold value, the exceedances, we can focus on the tails of the distributions. With Augmented Dickey-Fuller-, Durbin-Watson-, and Autoregressive Conditional Heteroskedasticity Lagrange multiplier tests we show for daily German and French power prices that the log exceedances in the expected Pareto tails of electricity price distributions can be assumed as stationary, independent and identically distributed. We can therefore assume, that the prerequisites of the applied peak over threshold extreme value theory and statistical inference are fulfilled. With the further results of the here suitable Andersen-Darling test we conclude, that for daily German and French power prices it can be justified to recommend the log logistic or the lognormal distribution as alternatives in power price risk management.

Contribution/ Originality: This study shows that in practical risk management the Pareto exceedance distributions of power prices can be adequately approximated by the lognormal or the log logistic distribution. This provides a lot of advantages in practical risk and portfolio management of power price risk exposures.

1. INTRODUCTION

Electricity price risk constitutes the main risk of utilities. Non-storability of electricity, outages of power plants, demand inelasticity, dependency on transmission and distribution grid capacities, thus leading to high volatility of power prices and positive and negative price spikes and consequently to fat tails (Escribano, Ignacio Peña, & Villaplana, 2011; Weron, 2009) are major reasons why it is so complicated but at the same time highly relevant to model electricity price distributions. As risk management is mainly concerned with extreme outcomes, that exceed a defined safety threshold value, we can circumvent the challenge of estimating complete power price distributions and instead focus on the tails of the distributions. Depending on the considered investment realized price risk can materialize through extremely high or extremely low prices, so that generally power risk management has to consider both tail distributions.

The peak over threshold method of extreme value theory, pot evt, can be used to describe the exceedance distribution, where each data point is calculated as the difference of its original observed value and the defined threshold value under the condition, that the original value has exceeded the threshold value (Davison & Smith, 1990; Embrechts, Klüppelberg, & Mikosch, 1997). Although the Pareto distribution, which under certain assumptions is the asymptotic exceedance distribution for fat tailed distributions, is easy to define in closed form, it is rather uncommon in practical risk management. Additionally, there can be serious problems in estimating its shape and scale parameter with the usually proposed estimation methods (Castillo & Hadi, 1997). According to Hosking and Wallis (1987) the following methods have their disadvantages: maximum likelihood asymptotic assumptions seem only to be valid for more than 500 observations, method of moment estimation can deliver unreliable estimators, and the probability weighted method of moment estimation is only reliable for a certain value interval of the shape parameter. Furthermore, for heavier fat tails it can happen that the variance or even the expected value of the Pareto distribution is not finite, thus making a parametric risk management impossible and laws of large numbers do not apply (Malevergne, Pisarenko, & Sornette, 2011).

Using the property that Pareto exceedance distributions can be approximated by the lognormal or the log logistic distribution, the two latter being easily defined and always providing two finite moments, we will show that it can be justified to recommend them as alternative distributional assumptions in power price risk management. This study is therefore organized as follows. Based on general findings of pot evt and statistical theory we will first show, why the tails of power price distributions theoretically can also be described either by the lognormal or by the log logistic distribution. In the next section we present the results of the applied findings for daily German and French power prices finding strong support for this hypothesis. Final remarks conclude the study.

2. METHODS

Balkema and De Haan (1974) and Pickands III (1975) have shown, that with a random variable Y and the exceedances $X = Y - u$, with threshold value u, for a large class of distribution functions F, the conditional excess distribution function is defined as:

$$F_u(x) = P(Y - u \leq x \mid Y > u) \quad \text{with } 0 \leq x \leq Y_F - u \text{ and } Y_F \leq \infty \text{ is the right endpoint of F}$$

which for a large threshold value u ($u \rightarrow \infty$) is well approximated by $F_u(x) \approx G_{k,\alpha}(x)$ where $G_{k,\alpha}(x)$ is the Generalized Pareto Distribution, GPD, function. $G_{k,\alpha}(x) = 1 - (1 - kx/\alpha)^{1/k}$ for $k \neq 0$ and $G_{k,\alpha}(x) = 1 - \exp(-x/\alpha)$ for $k = 0$, where k is the shape parameter and α is the scale parameter with $0 \leq x < \infty$ for $k \leq 0$ and $0 \leq x \leq \alpha/k$ for $k > 0$. The more k is negative the heavier is the tail. To apply pot evt to negative exceedances, we simply multiply the negative exceedances by minus 1 and then use standard theory.

As electricity price distributions are generally assumed to be fat tailed, we would correspondingly assume the exceedances as Pareto distributed with scale parameter $k < 0$.

Crucial for pot evt to be valid is the identification of an adequately large u, thereby handling the trade-off, that for a threshold value too high, too few exceedances are identified to estimate the extreme value distribution in a statistically reliable sense, and for a threshold value too low the distributional implications of the theory do not hold.

In practical statistical analysis it is often impossible to distinguish whether empirical tail distributions follow a Pareto or a lognormal distribution since they in general and especially for high standard deviations show a similar behavior (e.g. (Bee, Riccaboni, & Schiavo, 2013; Eeckhout, 2009; Malevergne et al., 2011; Mitzenmacher, 2003)). As the log logistic distribution is in relation to the logistic distribution as is the lognormal to the normal, meaning a random variable is log logistic distributed, when the log of the random variable is logistic distributed, and the normal distribution can be very good approximated by the logistic (e.g. Bowling, Khasawneh, Kaewkuekool, and Cho (2009)) so can the lognormal by the log logistic. Therefore, it can be expected that the log logistic distribution is also a good approximation to describe empirical tails of fat tailed distributions. This can be helpful, when the considered tail behavior is fatter than modeled by the lognormal, since the logistic distribution provides a higher kurtosis than the normal distribution.

We investigate German and French daily electricity price index data of the physical electricity indices, Phelix, of the European Energy Exchange in Leipzig, Germany for base and peak load prices between June 2000 for Germany (December 2001 for France) and December 2016. This leads to 6000 (5400) observations for Germany

(France) for each complete empirical price distribution. The daily base (peak) load index value consists of the arithmetic mean of 24 (12, from 8:00 am to 8:00 pm) hourly spot market clearing prices for the corresponding delivery hour the next day, measured in €/megawatt hour, €/mwh.

In order to identify an adequate high respectively low threshold value for which the implications of pot evt hold and assuming E(Y) < ∞ we investigate the mean excess function e(u) for increasing threshold values u (Davison & Smith, 1990; Embrechts et al., 1997; Gilli & Késllezi, 2006; McNeil, 1997) which for nonnegative identical and independently distributed random variables Y is generally defined as e(u) = E(Y - u | Y > u). For log normal distributed exceedances or empirical Pareto distributions the empirical mean excess function should show a concave behaviour from a certain threshold point onwards (Embrechts et al., 1997).

Before we can test the tail distributional assumptions we have to investigate if the empirical exceedances are stationary, independent and identically distributed, so that we can assume that expected value and variance of the underlying theoretical distributions are finite, and that we can use standard statistical inference and at the same time the assumptions of standard pot evt are fulfilled (Embrechts et al., 1997). To save an explicit unit root testing strategy, we over-parameterized the Augmented Dickey-Fuller test equation with a deterministic trend and intercept. As this weakens the power of the test, a rejection of the null hypothesis is, therefore, a conservative indication.

$$\Delta E_t = \alpha^* E_{t-1} + \sum_{i=1}^{p-1} \alpha_i^* \Delta E_{t-i} + \mu + \theta t + \epsilon_t \qquad \epsilon_t \sim i.i.d.(0, \sigma^2)$$

with ΔE_t = difference of log exceedances at time t and t-1, α^* = difference between coefficient of pure autoregressive process p, and 1, α_i^* = coefficient of i^{th} lag term, p = number of lags of assumed autoregressive process, μ = mean, θ = coefficient of deterministic trend, t = time parameter, ε_t = disturbance term, *i.i.d.*= identical and independently distributed, $(0, \sigma^2)$ = distributed with expected value of 0 and variance σ^2.

The shrinkage of exceedances in each series to equidistant observations in time, in contrast to reality where they appear with random timely distances, potentially imposes an artificial lag structure on the data of the unit root test which is addressed by a conservative lag structure determined by the Schwarz information criterium.

To test for correlations in the tails we calculate the Durbin-Watson test statistics and conduct ARCH (Autoregressive Conditional Heteroskedasticity) Lagrange multiplier tests of the residuals of regressions of the log exceedances on a constant.

Subsequently, we derive estimators of the first two moments of the empirical tail distributions by maximum likelihood estimation and conduct Anderson-Darling tests for the alternative null hypothesis that log exceedances are normal or logistic distributed. The Anderson-Darling test is here suitable as its weighting function gives greater importance to observations in the tail (Stephens, 1979) and leads to statistically reliable results also for small sample sizes (Lewis, 1961).

3. RESULTS

In each of our German and French base and peak load power price distributions the mean excess functions show a concave shape before the most extreme exceedances show a typical diffuse outlier behavior. Within the usual tolerance that exists to determine an adequate threshold value we can in each of our German and French cases identify threshold values that lead to a 0,5% tail fraction of the complete empirical distribution. Table 1 shows the descriptive statistics of the 0,5% tail distributions and the results of the performed unit root tests.

Table 1. Descriptive statistics and Augmented Dickey-Fuller test results of non-ordered log exceedances of 0,5% tail distributions.

	Exceedances						
	threshold value	min value	max value	ADF test[1]		DW[2]	ARCH-LM[3]
	(€/mwh)	(€/mwh)	(€/mwh)	lags[4]	p value[5]	test value	p value
Germany (30 observations)							
Base load pos. exceedances	104.85	0.06	196.69	0	0.0469	1.6827	0.8633
Peak load pos. exceedances	144.00	0.77	399.72	0	0.0000	1.8655	0.0264
Base load neg. exceedances*	7.50	0.10	64.37	1	0.0740**	1.9927	0.3815
Peak load neg. exceedances*	8.95	0.03	45.41	0	0.0006	2.0440	0.8222
France (27 observations)							
Base load pos. exceedances	120.20	0.59	492.57	3	0.0023	1.8965	0.4657
Peak load pos. exceedances	172.70	0.22	973.91	0	0.0017	2.0005	0.4756
Base load neg. exceedances*	8.70	0.02	49.69	4	0.0009	2.2952	0.5525
Peak load neg. exceedances*	8.57	0.01	28.85	0	0.0009	1.8105	0.6067

Note: 1 All Augmented Dickey-Fuller tests with deterministic intercept and trend, critical MacKinnon test value at 5%.
for German (French) log prices: -3,5742 (-3,5950); 2 Durbin-Watson test value, 3 ARCH Lagrange multiplier test (1 lag).
4 Number of lags according to Schwarz information criterium (SIC); 5 probability test value.
* absolute negative exceedance values ** becomes 0,0021 for fixed lag length of 0.

The null hypotheses, that a tail distribution exhibits a unit root, can be rejected on a level of significance of 5% in all but one case, where we have to fix the underlying lag length to get the expected result.

Durbin-Watson and ARCH Lagrange multiplier tests show in nearly all cases no evidence of linear correlation or a correlation of the increments.

Table 2 shows the descriptive sample statistics and the tail distribution tests for German and French log power price exceedances.

In all German cases the null hypothesis of the Anderson-Darling test of a logistic tail distribution cannot be rejected on the grounds of a high level of significance.

For French negative exceedances the probability values to accept the tested distributional assumptions are not too convincing although from a general perspective of test theory acceptable. Considering that power price distributions are generally right skewed, so that the estimation of the negative exceedances is more sensitive to outliers, we received more convincing results in two alternative estimations after omitting extreme outliers, although the significance of the sample mean for base load exceedances in this single case remains rather disappointing. Considering that the logistic distribution also fits the French base load positive exceedances quite well, changing the sample μ only from 2,9743 to 2,9396 and the sample standard deviation from 1,6103 to 1,6704, we can summarize, that the logistic distribution can also serve as an adequate representation of French log power price exceedances.

Table 2. Descriptive statistics and tail distribution tests of German and French log power price exceedances.

		Sample							ML[2] estimated	
Germany	**Threshold value (€/Mwh)**	**Mean**	**Standard Deviation**	**Skewness**	**Kurtosis**	**Normal** Ad p value[1]	**Logistic**	**Distri-Bution***	**M**	**Scale**
Base load positive exceedances	104.85	2.6718	1.677	-1.1846	5.1671	0.1399	>0.25	logistic	2.8095 (0.2775)	0.8791 (0.1338)
Peak load positive exceedances	144.00	3.5966	1.3211	-0.5559	3.8819	0.5696	>0.25	logistic	3.6281 (0.2286)	0.7202 (0.1093)
Base load -(negative exceedances)	7.50	1.5336	1.4744	-0.5155	3.3061	0.4665	>0.25	logistic	1.6101 (0.2530)	0.8030 (0.1238)
Peak load -(negative exceedances)	8.95	1.2777	1.7921	-1.0346	3.5702	0.0225	>0.05	logistic	1.4886 (0.3031)	0.9612 (0.1487)
France									μ	σ / scale
Base load positive exceedances	120.2	2.9743	1.6103	0.0882	2.5112	0.9211	>0.25	normal**	2.9743 (0.3099)	1.6103 (0.2233)
Peak load positive exceedances	172.7	3.3984	1.9456	-0.3579	2.9866	0.9224	>0.25	logistic*	3.4458 (0.3654)	1.0879 (0.1739)
Base load -(negative exceedances)	8.70	-0.4700	1.6751	-0.1124	3.6480	0.0207	>0.01	normal**	-0.4700 (0.3224)	1.6751 (0.2323)
Peak load -(negative exceedances)	8.57	-0.676	1.5712	-0.3044	4.9052	0.0052	>0.025	logistic***	-0.6361 (0.2460)	0.7667 (0.1295)
France negative exceedances (outlier corrected)									M	scale
Base load -(negative exceedances)[3]	8.70	0.0526	0.6828	-0.4048	2.9716	0.856	>0.25	logistic	0.0814 (0.1477)	0.3783 (0.0702)
Peak load -(negative exceedances)[4]	8.57	-0.5248	0.8831	0.5484	3.2003	0.1954	>0.25	logistic	-0.5822 (0.1693)	0.4797 (0.0826)

Note: 1 Anderson-Darling test probability value; 2 Maximum likelihood; standard error in brackets; 3 corrected for 7 outliers = 20 remaining observations;
4 corrected for 3 outliers = 24 remaining observations
* decision based on AD test result and higher significance of ML estimated μ, ** decision based on normality test results and higher significance of ML estimated μ.
*** decision based on AD test result, sample kurtosis and higher significance of ML estimated μ

Generally, the high significance of the estimated sample location and scale parameters, which are both important in a moment-based risk management approach, is a salient result.

Figure 1 shows two examples of the goodness of fit between empirical sample and the theoretical assumed logistic distribution. In line with our theoretical arguments, we see that the normal distribution could serve as an adequate approximation of the empirical distribution as well.

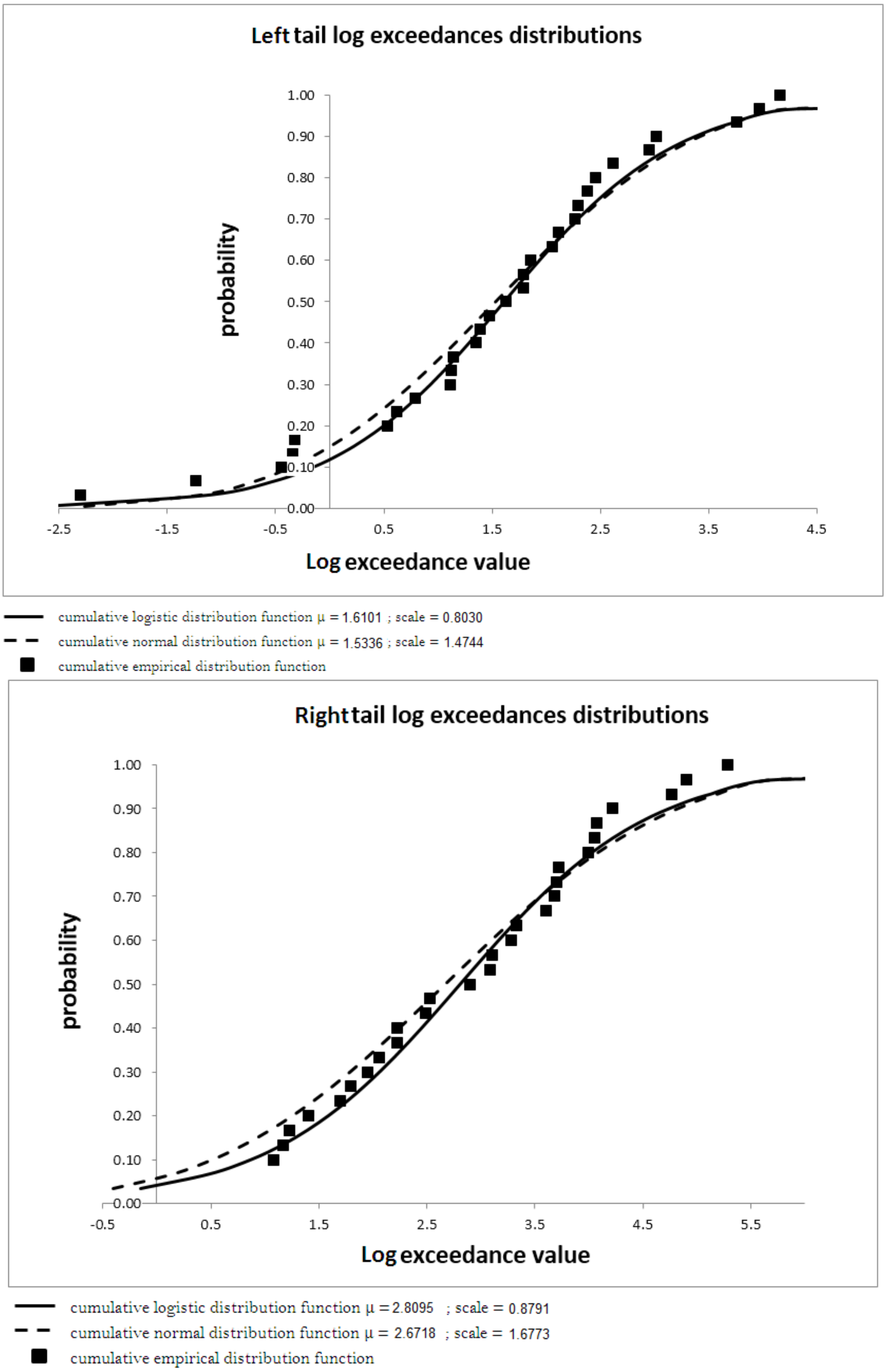

Figure 1. German base load negative (left) and positive (right) log power price exceedances distributions.

4. CONCLUDING REMARKS

Being able to assume log power price exceedances as logistic or normal distributed is a salient result and provides a lot of advantages in practical risk and portfolio management. Both distributions with their finite two moments can easily be described, in case of the logistic even in closed form, and allow for the application of established methods from financial portfolio and risk theory. This comprises especially the aggregation of expected values referring to laws of large numbers and the application of μ-scale or μ-σ principles and moment defined parametric quantile risk measures like value at risk or conditional value at risk. In our illustrative German and French examples, we have seen that the sample means either under a logistic or under a normal assumption for the log price exceedances differ only slightly and are generally highly significant. This is also true for all of our examples for the estimated scale parameters. Generally, when estimating the tail distributions, a thorough statistical analysis of the underlying assumptions and the achieved results is necessary, especially when outlier adjustments or omissions are considered which can quickly come under the suspicion of provoking a desired result.

Based on our analysis there is strong evidence that for daily German and French power price index distributions the assumption of log logistic or log normal tail distributions can be justified, thus providing a suitable distributional alternative in electricity price risk management. A generalization of these results especially to alternative power price series, hourly market clearing spot prices and to power price series of other markets would of course require further empirical work.

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