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THE ROLE OF A CHILD TAX POLICY: POPULATION, HUMAN CAPITAL, AND ECONOMIC GROWTH

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ABSTRACT

This study analyzes the impact of a child tax policy on the population, human capital, and economic growth in an overlapping generations model with endogenous fertility and human capital. The model shows that the child tax policy inhibits population growth and promotes human capital accumulation, generating a trade-off between the quantity and quality of children. The reduced population and increased human capital both contribute to per capita income. The balance between the negative impact of the child tax policy on the population and the positive impact on per capita income ultimately determines the impact on aggregate growth. The results show that the effect on aggregate growth rate is positive when parents' preference for children is large enough, and negative otherwise. In addition, with the pay-as-you-go pension system, I find that, under feasible conditions, the effect of the child tax policy on pensions is the same as that on total growth.

Contribution/ Originality: This study is the first to qualitatively analyze the impact of China's oft-debated child tax policy on its aggregate growth rate. I introduced a child tax variable in an overlapping-generations model with endogenous fertility and human capital accumulation; the findings highlight the policy benefits as well as consequences.

1. INTRODUCTION

Numerous studies have examined the negative relationship between population growth and economic development since the Malthusian population trap in 1798. For example, Li and Zhang (2007) used panel data from China's 28 provinces over 20 years to examine the relationship between the two and found a negative correlation. In addition, Dao (2012) used statistical modeling on data from 43 developing countries and found a negative effect of population growth on per capita economic growth. However, many scholars, such as Romer (1986); Romer (1990) and Jones (1999), questioned Malthus' view of the exogenous treatment of technological progress, arguing that if technological progress was endogenous, the relationship between population and economic development would become neutral or even positive. More recently, Sethy and Sahoo (2015) used India's time series data from 1970 to 2010 to find that population promotes economic development. Therefore, there is no consistent conclusion about the relationship between population and economic development.

In many developing countries, excessive population growth has greatly burdened economic growth and caused several other problems (Ahmed & Ahmad, 2016; Guga, Alikaj, & Zeneli, 2015; Headey & Hodge, 2009). Among the developing countries, China had implemented a child tax policy that sets a specific birth quota for each family (Huang, Lei, & Sun, 2021; Zhang, 2017). Therefore, to prevent excessive population growth, families whose number of children exceeds the birth quota must pay fines proportional to their incomes. After experiencing the Great Famine around 1960, China ushered in an explosive increase in the birth rate in 1962, which directly led to the promulgation of population control documents related to family planning by the Chinese government (Wang, Zhao, & Zhao, 2017; Zhang, 2017). As a result, since 1965, the fertility rate in China experienced a sharp decline soon after the implementation of the policy. In 1979, the Chinese government enacted the strictest one-child policy in history, subsequently relaxing the birth quota in 2015 and further in 2021 (Wang, 2021). In 2021, China's birth rate reached a record low since the Great Famine. According to data from the National Bureau of Statistics, as of the end of 2021, China's population was 1.4 billion, with 10.6 million babies born in 2021, or 7.52 births per 1,000 people.

The impact of China's child tax policy has sparked intense debate among scholars. The most discussed is the policy's impact on the population, and the conclusions so far remain controversial. The policy's strict control of the population appears to have severely limited the growth of the fertility rate. However, there are many empirical studies demonstrating that in countries without population control, economic development can also lead to a reduction in the birth rate (Whyte, Feng, & Cai, 2015; Zhang, 2017). In addition, there is considerable research on the impact of a child tax policy on the quality of children. According to Wang et al. (2017), human capital in China has increased dramatically over the past three decades. Many researchers have empirically found a quantity–quality trade-off for children in China (Huang, 2022; Li, Zhang, & Zhu, 2008; Li & Zhang, 2017; Qin, Zhuang, & Yang, 2017). Furthermore, some macroeconomic studies have shown that a child tax policy has a positive effect on human capital (Gu, 2022; Liao, 2013). Conversely, several empirical studies have demonstrated that a child tax policy does not have a significant effect on increasing human capital (Li & Zhang, 2017; Rosenzweig & Zhang, 2009), which makes the impact of a child policy on the quantity–quality trade-off of children unclear.

Although there is considerable empirical research on the child tax policy, theoretical research is relatively limited, especially studies that consider the child tax variable in the model. For example, Fanti and Gori (2009) indicated that a child tax can both promote population growth and per capita income but did not consider the education investment in children and human capital accumulation. Furthermore, Fanti and Gori (2014) incorporated the public pay-as-you-go (PAYG) social security system and education expenditures for children into the model and revealed that a child tax positively affects capital accumulation and education expenditures but reduces the fertility rate, thus verifying the quantity–quality trade-off of children. However, the accumulation of human capital was not considered in their model. In addition, Zhu, Whalley, and Zhao (2014) and Gu (2022) both focused on China's one-child policy and found that the policy reduces birth rates but increases each child's human capital. Especially considering China's national conditions, both models assumed that parents' old age support comes from the transfer of children, rather than the pension system. Zhu et al. (2014) also found that the positive effect of China's child tax policy on human capital can compensate for the negative effect on the population and ultimately increase overall output. In contrast, Gu (2022) used quantitative analysis to find a negative impact on total output. However, neither of them introduced the child tax variable in the model when they modeled the one-child policy. Zhu et al. (2014) assumed that population growth was exogenous, whereas Gu (2022) gave a binding constraint on the population growth rate to model the one-child policy, both of which are inconsistent with the contents of the policy. China's child tax policy stipulates that if a couple has more children than a prescribed quota, they are required to pay a penalty proportional to their income. It is not that people can only have children within the quota set by the policy. In this study, the economy grows endogenously, and the fertility rate and human capital are both endogenously determined. The child tax variable and the PAYG pension system are introduced into the model, which not only analyzes the impact of child tax policy on the fertility rate and per capita variables but also qualitatively examines the impact on the aggregate growth rate in an overlapping generations (OLG) model originally proposed by Diamond (1965) in which individuals live a finite length of time, long enough to overlap with at least one period of another agent's life.

The main contributions of this study are as follows: First, this study is the first to qualitatively analyze the impact of the child tax policy on the aggregate growth rate with the introduction of a child tax variable.[1] Second, when verifying whether there is a trade-off between the quantity and quality of children under the influence of child tax policy, this study includes both in the utility function under the consideration of human capital accumulation; that is, both are endogenously determined.[2] By analyzing the impact of the introduction of the child tax on the pension system, etc., it reflects the importance of the length of policy implementation. Therefore, this study calls on policymakers to consider not only the benefits but also the consequences of policies and to make predictions and implement measures as soon as possible.

The major findings of this study are as follows: In the long run, imposing taxes on childbirth reduces population growth and increases education expenditure, thereby promoting the accumulation of human capital and validating the trade-off between the quantity and quality of children. In addition, it promotes a balanced-growth rate. If parents do not have a high preference for children, the negative impact of introducing a child tax on population growth will be greater than the positive impact on the balanced-growth rate, resulting in a decline in the aggregate growth rate. Under the assumption that the policy is implemented for one period, the impact on the aggregate growth rate also determines the impact on pensions. When discussing the impact on pensions, this study refers to the one-child policy implemented in China for 36 years; given the negative impact of the policy on the fertility rate, if the policy is implemented for too long, it will bring about problems such as accelerating the process of low birth rate and aging and the collapse of the pension system. Therefore, the assumption that the policy is implemented for only one period has practical significance.

The remainder of this paper is structured as follows: Section 2 introduces the model setting, and Section 3 analyzes and discusses the results. Section 4 concludes the paper.

2. THE MODEL

This study considered a closed economy with endogenous fertility (n_t), where individuals are identical and there is no gender difference. Individuals live for three periods: childhood, young adulthood, and old age. In the childhood period, individuals are exclusively educated and accumulate human capital. In young adulthood, individuals rear n_t children and work full-time (time is normalized to one). In the last period, individuals do not work, and they live on pensions financed by the taxation of young people in the same period. n_t and N_t represent the child population and the young adult population in period t , respectively, and N_{t+1} represent the older adult population in period $t+1$; therefore, $N_{t+1} = n_t N_t$.

2.1. Individuals

This study adopted the model by De La Croix and Doepke (2003) and considered an economy in which individuals born in the period t derive utility from consumption in their youth and old age, denoted as $c_{1,t}$ and $c_{2,t+1}$, respectively. In addition, they obtain utility from the number and quality of their children born in period t, denoted as n_t and h_{t+1}, respectively. Therefore, the lifetime utility of a representative individual born in period $t-1$ is given as:

$$U_t = ln(c_{1,t}) + \beta\, ln(c_{2,t+1}) + \gamma\, ln(n_t h_{t+1}), \tag{1}$$

where $0 < \beta < 1$ is the time discount factor and $0 < \gamma < 1$ is the altruism factor, which shows parents' preference for children.

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[1] Both Zhu et al. (2014) and Gu (2022) did not introduce the child tax variable into the model, and Gu (2022) did not qualitatively analyze the impact on the aggregate variable. Fanti and Gori (2009); Fanti. and Gori (2014) introduced the child tax into the model but did not analyze the effect on the aggregate variables.

[2] In analyzing the impact of the child tax policy, previous studies do not consider human capital in the utility function. Although Fanti. and Gori (2014) added children's educational expenditure to the utility function, they did not consider the accumulation of human capital. In contrast, Zhu et al. (2014) assumed fertility to be exogenous.

It is assumed that human capital depends on the investment in education in childhood and human capital in the previous generations. Following De La Croix and Michel (2007), this study considered a Cobb–Douglas formulation as follows:

$$h_{t+1} = \psi {e_t}^{\eta} {h_t}^{1-\eta}, \tag{2}$$

where $\psi > 0$ is a productivity parameter, $0 < \eta < 1$ can be interpreted as the contribution of education investment on human capital, $0 < 1 - \eta < 1$ indicates the intergenerational transfers of human capital toward children, e_t is the education investment made by the parents, and h_t represents the parents' human capital.

The young generation (N_t) in the working-age group joins the labor market and supplies one labor unit while receiving the wage w_t per unit of effective labor. The wage income, denoted as $w_t h_t$, is consumed at a young age, invested on children, and saved to support consumption upon getting old. In addition, this study assumes that the government levies payroll taxes on the young generation and adopts a PAYG method to fund pensions for older adults in the same period. A constant per child tax, which is proportional to the young individuals' wage income, is levied by the government to transfer to the same generation as a lump-sum subsidy. Denoting the gross interest rate from period t to $t + 1$ by R_{t+1}, the lifetime budget constraints are given by:

$$c_{1,t} + s_t + e_t n_t + (z + q) w_t h_t n_t = w_t h_t (1 - \tau) + T_t, \tag{3}$$

$$c_{2,t+1} = R_{t+1} s_t + p_{t+1}, \tag{4}$$

where $c_{1,t}$ and s_t are the consumption and savings of young adults in period t, respectively; $c_{2,t+1}$ represents their consumption in old age; e_t is the education expenditure for each child; $z w_t h_t$ is the goods cost for raising each child; $q w_t h_t$ represents the child tax paid for each child; and n_t is the number of children. Child tax is assumed to be a certain percentage of the current working generation' wage income, which is in line with China's child tax policy, and $0 < q < 1$ indicates this percentage. In addition, T_t is a lump-sum subsidy financed from the child tax, τ is a constant tax rate of social security, and p_{t+1} is the pension paid to older adults in period $t + 1$.

Maximizing the utility subject to the budget constraints, the first-order conditions can be inferred as the following:

$$s_t = \frac{\beta R_{t+1} [w_t h_t (1 - \tau) + T_t] - (1 + \gamma) p_{t+1}}{R_{t+1} (1 + \beta + \gamma)} \tag{5}$$

$$n_t = \frac{(1 - \eta) \gamma \{R_{t+1} [w_t h_t (1 - \tau) + T_t] + p_{t+1}\}}{w_t h_t R_{t+1} (1 + \beta + \gamma)(z + q)} \tag{6}$$

$$e_t = \frac{\eta (z + q)}{(1 - \eta)} w_t h_t \tag{7}$$

2.2. Firms

This study assumed a competitive market with identical firms, where the production function is a Cobb–Douglas production function: $Y_t = A {K_t}^{\alpha} {L_t}^{1-\alpha}$, where Y_t, K_t, and $L_t = h_t N_t$ indicate the total output, physical capital, and labor input, respectively, in period t; $A > 0$ denotes the scale parameter, and $0 < \alpha < 1$ denotes the distributive capital share. Denoting $k_t = K_t / N_t$ as per capita capital and $y_t = Y_t / N_t$ as per capita output, the intensive form production function is: $y_t = A {k_t}^{\alpha} {h_t}^{1-\alpha}$. Capital fully depreciates at the end of each period, and the price of final products is normalized to 1. According to the conditions for profit maximization, the wage rate and capital rental rate can be expressed as follows:

$$R_t = \alpha A {k_t}^{\alpha - 1} {h_t}^{1-\alpha} \tag{8}$$

$$w_t = (1 - \alpha) A {k_t}^{\alpha} {h_t}^{-\alpha} \tag{9}$$

2.3. Government

The government adopts the PAYG method to finance social security benefits by imposing a payroll tax at a flat rate τ on the young working generation and paying it to older adults in the same period. Simultaneously, the government levies a child tax on the young generation and returns it as a lump sum subsidy. Therefore, the government budget constraints are as follows:

$$\tau w_{t+1} h_{t+1} n_t = p_{t+1}, \tag{10}$$
$$T_t = q n_t w_t h_t, \tag{11}$$

where $N_{t+1} = n_t N_t$, as mentioned earlier, and Equation 10 and 11 do not affect an individual's fertility decisions.

By inserting the values of Equation 10 and 11 into Equation 5 and 6, respectively to eliminate p_{t+1} and T_t, the demand function for children and the savings function become:

$$n_t = \frac{(1-\eta)\gamma R_{t+1} w_t h_t (1-\tau)}{w_t h_t R_{t+1}[(1+\beta+\gamma)z + (1+\beta+\eta\gamma)q] - (1-\eta)\gamma\tau w_{t+1} h_{t+1}} \tag{12}$$

$$s_t = \frac{w_t h_t (1-\tau)[\beta(z+q) w_t h_t R_{t+1} - (1-\eta)\gamma\tau w_{t+1} h_{t+1}]}{w_t h_t R_{t+1}[(1+\beta+\gamma)z + (1+\beta+\eta\gamma)q] - (1-\eta)\gamma\tau w_{t+1} h_{t+1}} \tag{13}$$

2.4. Market Equilibrium

Given that $N_{t+1} = n_t N_t$, the equilibrium condition in the capital market is given by:

$$k_{t+1} = \frac{s_t}{n_t} \tag{14}$$

By inserting the values of Equation 8, 9, 12, and 13 into Equation 14, the following equation can be obtained:

$$k_{t+1} = \frac{\alpha\beta(z+q)(1-\alpha)A}{\gamma(1-\eta)[\alpha+\tau(1-\alpha)]} k_t{}^{\alpha} h_t{}^{1-\alpha} \tag{15}$$

By inserting the value of Equation 9 into Equation 7 to eliminate w_t, the human capital accumulation equation can be rewritten as follows:

$$h_{t+1} = \psi \left[\frac{\eta(z+q)(1-\alpha)A}{1-\eta}\right]^{\eta} k_t{}^{\alpha\eta} h_t{}^{1-\alpha\eta} \tag{16}$$

3. EFFECTS OF THE CHILD TAX POLICY

3.1. Fertility

To analyze the ultimate impact of the child tax on the fertility rate in the long run, Equation 8, 9, 12, and 15 are combined to obtain:

$$n_{(q)} = \frac{(1-\eta)\gamma(1-\tau)[\alpha+\tau(1-\alpha)]}{\{(1+\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha\}z + \{(1+\eta\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha\}q} \tag{17}$$

From Equation 17, the following proposition can be advanced:

Proposition 1.

A child tax policy decreases the fertility rate in the long run.

Proof. The derivative of Equation 17 with respect to the child tax rate q is given by:

$$\frac{\partial n_{(q)}}{\partial q} = -\frac{DF}{(Ez+Fq)^2} < 0, \tag{18}$$

where $D = (1-\eta)\gamma(1-\tau)[\alpha+\tau(1-\alpha)]$, $E = (1+\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha$, $F = (1+\eta\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha$.

As the above condition is true for any $q \geq 0$, n is a decreasing function of q. Then, $n_{(q=0)} > n_{(q>0)}$; $n_{(q=0)}$ and $n_{(q>0)}$ represent the fertility rate before and after implementing the child tax policy, respectively. Therefore, introducing a child tax always reduces the fertility rate. Proposition 1 holds.

The economic intuition underlying Proposition 1 is straightforward. Introducing a child tax into the economy affects the fertility rate through two channels. First, the channel of the substitution effect indicates that a child tax increases the rearing cost of children, which leads to a negative effect on the fertility rate. Second, the channel of the income effect indicates that in the short term, individuals receive a lump sum subsidy fund from the child tax, which increases their current income and savings, thereby promoting capital accumulation. In the long term, a reduction in the fertility rate reduces the population of the whole economy and thus increases per capita income. In addition, as will be verified in section 3.4, the existence of a quantity–quality trade-off leads to further accumulation of human capital, thereby increasing the per capita income. Increased income has a positive effect on the fertility rate. Evidently, the negative substitution effect is greater than the positive income effect, resulting in a final negative effect of the child tax on the fertility rate.

3.2. The Balanced-Growth Rate

Equation 15 and 16 are rewritten to obtain the following equations:

$$\frac{k_{t+1}}{k_t} = \frac{\alpha\beta(z+q)(1-\alpha)A}{\gamma(1-\eta)[\alpha+\tau(1-\alpha)]}\left(\frac{k_t}{h_t}\right)^{\alpha-1} \tag{19}$$

$$\frac{h_{t+1}}{h_t} = \psi\left[\frac{\eta(z+q)(1-\alpha)A}{1-\eta}\right]^{\eta}\left(\frac{k_t}{h_t}\right)^{\alpha\eta} \tag{20}$$

Equation 19 and 20 describe the dynamics of the economy. Denoting $x_t = k_t/h_t$ and combining the above two equations, the dynamics of x_t can be expressed as Equation 21:

$$x_{t+1} = \frac{\alpha\beta(z+q)(1-\alpha)A}{\psi\gamma(1-\eta)[\alpha+\tau(1-\alpha)]}\left[\frac{1-\eta}{\eta(z+q)(1-\alpha)A}\right]^{\eta} {x_t}^{\alpha(1-\eta)} \tag{21}$$

Then, a stationary x on the balanced-growth path can be deduced as Equation 22:

$$x = \left\{\frac{\alpha\beta}{\psi\gamma[\alpha+\tau(1-\alpha)]\eta^{\eta}}\left[\frac{(1-\alpha)A}{1-\eta}\right]^{1-\eta}\right\}^{\frac{1}{1-\alpha(1-\eta)}}(z+q)^{\frac{1-\eta}{1-\alpha(1-\eta)}} \tag{22}$$

Defining the balanced-growth rate as $g = k_{t+1}/k_t = h_{t+1}/h_t = y_{t+1}/y_t$, the following equation is obtained:

$$g_{(q)} = \left\{\frac{(1-\alpha)A\psi^{\frac{(1-\alpha)}{\eta}}\eta^{(1-\alpha)}}{1-\eta}\left[\frac{\alpha\beta}{\gamma[\alpha+\tau(1-\alpha)]}\right]^{\alpha}\right\}^{\frac{\eta}{1-\alpha(1-\eta)}}(z+q)^{\frac{\eta}{1-\alpha(1-\eta)}} \tag{23}$$

It is obvious that there is no stable steady-state in this model and the economy grows endogenously. Equation 23 shows that the per capita output and physical and human capital all grow at a constant rate g.

From Equation 23, the following proposition can be advanced:

Proposition 2.

A child tax policy may be used as a tool to increase the balanced-growth rate.

Proof.

The derivative of Equation 23 with respect to the child tax rate q is given by :

$$\frac{\partial g_{(q)}}{\partial q} = g_{(q)}\frac{\eta}{1-\alpha(1-\eta)}(z+q)^{-1} > 0 \tag{24}$$

As the above condition is always true for any $q \geq 0$, the value of $g_{(q)}$ increases with an increase in the value of q. Then, it is easy to infer that $g_{(q>0)} > g_{(q=0)}$; $g_{(q>0)}$ and $g_{(q=0)}$represent the balanced-growth rate after and before implementing the child tax policy, respectively, which shows that the introduction of a child tax promotes the balanced-growth rate and thus increases the long-term growth. Proposition 2 holds.

The economic intuition underlying the proposition is straightforward. First, the introduction of the child tax reduces the fertility rate. The decline in the fertility rate, on the one hand, leads to a decrease in the total population of the entire economy, thereby increasing per capita output; on the other hand, a reduced birth rate implies a decrease in spending, which increases the accumulation of per capita capital and human capital. Second, the child tax is given back to the working generation in the form of a subsidy, thereby increasing their current incomes and facilitating short-term savings and capital accumulation.

3.3. Aggregate Growth Rate

G is defined as the aggregate growth rate, where $G = K_{t+1}/K_t = H_{t+1}/H_t = Y_{t+1}/Y_t = gn$. To analyze how child tax affects the aggregate growth rate, a generic function of G can be written as follows:

$$G_{(q)} = g_{(q)} n_{(q)} \tag{25}$$

The derivative of Equation 25 with respect to the child tax rate q is as follows:

$$\frac{dG_{(q)}}{dq} = \underbrace{\frac{\partial g_{(q)}}{\partial q}}_{+} n + \underbrace{\frac{\partial n_{(q)}}{\partial q}}_{-} g \tag{26}$$

The value of Equation 26 presents the effect of a child tax on the aggregate growth rate. According to Equation 18 and 24, the first and second terms on the right side of Equation 26 are positive and negative, respectively. Therefore, the final effect of introducing a child tax on the aggregate growth rate is ambiguous. There are two opposing effects when child tax increases: (a) a positive effect on the balanced-growth rate and (b) a negative effect on the fertility rate. To figure out which effect dominates, Equation 17 and 23 are combined with Equation 25 to obtain the following equation:

$$G_{(q)} = g_{(q)} n_{(q)} = H(z+q)^{\frac{\eta}{1-\alpha(1-\eta)}} \frac{D}{Ez + Fq} \tag{27}$$

$$H = \left\{ \frac{(1-\alpha)A\psi^{\frac{(1-\alpha)}{\eta}} \eta^{(1-\alpha)}}{1-\eta} \left[\frac{\alpha\beta}{\gamma[\alpha + \tau(1-\alpha)]} \right]^{\alpha} \right\}^{\frac{\eta}{1-\alpha(1-\eta)}}$$

$$D = (1-\eta)\gamma(1-\tau)[\alpha + \tau(1-\alpha)]$$

$$E = (1+\gamma)[\alpha + \tau(1-\alpha)] + \beta\alpha$$

$$F = (1+\eta\gamma)[\alpha + \tau(1-\alpha)] + \beta\alpha$$

Therefore, the following proposition holds:

Proposition 3.

Unless parents' preference for children is large enough ($\gamma > \gamma_0$), the introduction of a child tax would reduce the aggregate growth rate.

Proof. Equation 27 is differentiated with respect to the child tax rate q as follows:

$$\frac{\partial G_{(q)}}{\partial q} = G_{(q)} \frac{\{\eta E - [1-\alpha(1-\eta)]F\}z - [(1-\alpha)(1-\eta)]Fq}{[1-\alpha(1-\eta)](z+q)(Ez+Fq)},$$

where $E = (1+\gamma)[\alpha + \tau(1-\alpha)] + \beta\alpha$, $F = (1+\eta\gamma)[\alpha + \tau(1-\alpha)] + \beta\alpha$.

Then, the derivative at $q = 0$ reads:

$$\frac{\partial G_t}{\partial q}_{(q=0)} = G_{t(q=0)} \frac{\eta E - [1-\alpha(1-\eta)]F}{[1-\alpha(1-\eta)]Ez}$$

It is obvious that $\frac{\partial G_t}{\partial q}_{(q=0)} > 0$ only holds when $\gamma > \gamma_0$,

where

$$\gamma_0 = \frac{(1-\alpha)[\beta\alpha + \alpha + \tau(1-\alpha)]}{\alpha\eta[\alpha + \tau(1-\alpha)]}$$

Therefore, the marginal introduction of a child tax can boost the aggregate growth rate only when $\gamma > \gamma_0$ holds. Accordingly, Proposition 3 holds.

Proposition 3 indicates that the ultimate impact of a child tax on the aggregate growth rate depends on parents' preference toward children. When parents' preference for children remains at a high level ($\gamma > \gamma_0$), the positive effect shown by Proposition 2 is predominant; otherwise, the negative effect on the fertility rate shown by Proposition 1 is predominant.

The economic intuition is as follows: When parents' preference for children increases, the negative effect of introducing a child tax on the fertility rate can be alleviated. The principle is obvious; the more individuals like children, the more children they want. In addition, the derivative of g with respect to γ is always negative according to Equation 23 (see Appendix A). An increase in γ could also indirectly weaken the positive effect of introducing a child tax on g. When a critical point is exceeded ($\gamma > \gamma_0$) by introducing a child tax, the positive effect on the balanced-growth rate exceeds the negative effect on the fertility rate, resulting in a positive final effect on the aggregate growth rate. Evidently, the direct positive effect on the fertility rate of parents' preference for children exceeds the indirect negative effect on the balanced-growth rate when $\gamma > \gamma_0$.

3.4. The Quantity–Quality Trade-off of Children

Substituting Equation 9 into Equation 7, the function of education expenditure can be rearranged as follows:

$$e_t = \frac{\eta(z+q)}{(1-\eta)} w_t h_t = \frac{\eta(z+q)(1-\alpha)}{(1-\eta)} A{k_t}^{\alpha}{h_t}^{1-\alpha} = \frac{\eta(z+q)(1-\alpha)}{(1-\eta)} y_t \tag{28}$$

According to Equation 28, education expenditure per child can be interpreted as a certain portion of the income. The factors affecting the education expenditure per child can be separated into two parts: a) the part $(\eta(z+q))/((1-\eta))$ increases with the introduction of a child tax; and b) the income part $w_t h_t$, grows along with per capita output y_t and shares the same growth rate g, which could be promoted by the introduction of a child tax, according to Proposition 2. Therefore, the child tax policy will lead to a trade-off between the quantity and quality of children.

The underlying economic intuition can be interpreted as follows: On the one hand, the child tax makes individuals exogenously resistant to having more children, thereby reducing the overall cost of raising children and allowing more resources to be spent on each child, as reflected in an increased share of education expenditure per child $(\eta(z+q))/((1-\eta))$. On the other hand, as introduced in Proposition 1, the introduction of a child tax could increase per capita income and thus increase the education expenditure per child. The increased expenditure on education per child promotes human capital accumulation. Therefore, the child tax policy has a positive effect on human capital accumulation.

3.5. Pension

Next, the effect on pensions is examined. First, the pension function is rewritten as follows:

$$p_{t+1} = \tau w_{t+1} h_{t+1} n_t = \tau(1-\alpha) A{k_{t+1}}^{\alpha}{h_{t+1}}^{1-\alpha} n_t = \tau(1-\alpha) y_{t+1} n_t$$

Pensions are derived from the taxes paid by the current working generation and depend on the size of the working population and their income levels. From the above equation, it is easy to see that the per capita pension for older adults in period $t+1$ (p_{t+1}) depends on the number of children in period t (n_t) and their income level ($w_{t+1}h_{t+1}$). In addition, according to Propositions 1 and 2, introducing a child tax has a negative effect on the fertility rate (n_t) and a positive effect on the per capita output (y_{t+1}), which makes the effect on the pension ambiguous.

The balanced-growth rate g is substituted and the pension function is rearranged as follows:

$$p_{t+1} = \tau w_{t+1} h_{t+1} n_t = \tau(1-\alpha) A{k_{t+1}}^{\alpha}{h_{t+1}}^{1-\alpha} n_t = \tau(1-\alpha) y_{t+1} n_t = \tau(1-\alpha) y_1 \frac{y_2}{y_1} \dots \frac{y_t}{y_{t-1}} \frac{y_{t+1}}{y_t} n_t$$
$$= \tau(1-\alpha) y_1 {g_{(q)}}^t n_{(q)}$$

It is assumed here that the child tax policy is implemented from period 1; then, the per capita output in period 1 (y_1) remains unchanged. t represents the time span of child tax policy implementation. To evaluate the ultimate effect of the child tax on pensions, it is necessary to verify the effect of q on ${g_{(q)}}^t n_{(q)}$.

According to Appendix B, the following condition can be obtained:

$$\frac{\partial {g_{(q)}}^t n_{(q)}}{\partial q} > 0 \text{ as } t > t_0, \text{ while } t \text{ is an integer},$$

where

$$t_0 = \frac{[1-\alpha(1-\eta)](z+q)\{(1+\eta\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha)}{\{(1+\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha\}z\eta + \{(1+\eta\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha\}q\eta}$$

From the above condition, it can be seen that only when the time span of the child tax policy implementation is long enough ($t > t_0$) will it be conducive to the growth of pensions. From a practical point of view, for example, China's one-child policy, which started in 1979 and ended in 2015 has spanned a total of 36 years, which is approximately equivalent to one period. Then, assuming $t = 1$ here, the equation of pensions can be rearranged as $p_{t+1} = \tau(1-\alpha)y_1 g_{(q)} n_{(q)}$. According to Proposition 3, unless $\gamma > \gamma_0$, p_{t+1} will decrease with the introduction of the child tax; based on the huge impact of the child tax policy on the fertility rate, it is unlikely that the policy will be implemented for too long. Otherwise, it will lead to the problem of low birth rates and an aging population. In China, for example, after the child tax policy that ended in 2021, the birth rate in 2021 hit a record low since the Great Famine. The effect on pensions is therefore similar to the effect on the aggregate growth rate; i.e., the introduction of a child tax will reduce pensions if parents' preference for children cannot be maintained at a high level.

The underlying economic significance is also well understood. As verified above, the introduction of a child tax has led to low birth rates and aging and promoted economic development, thus affecting the pension system. The impact on pensions depends on the length of time for which the policy is implemented and parents' preferences for children. The longer the policy is implemented, the greater the promotion effect on economic development, and the higher the incomes of the working population, the more conducive it is to the improvement of pensions. In contrast, the longer the policy is implemented, the faster the process of low birthrates and an aging population, which puts the pension system at risk of collapse. In the short term, such as the single period hypothesized above, the impact of introducing a child tax on pensions depends on parents' preference for children, as explained in Proposition 3.

4. CONCLUSIONS

This study has shown the effects of the child tax on population and economic development in an OLG model with endogenous fertility. It set up a model with the PAYG social security system and considered human capital accumulation. The main findings of this study are that the child tax policy will reduce population growth and promote economic development in the long run, and the combined impact of the two will determine the ultimate impact on the aggregate growth rate. When parents' preference for children is large enough, the positive effect of introducing a child tax on economic growth is dominant, thereby promoting the overall development of the economy. However, when parents' preference for children is not sufficient to mitigate the negative effects of declining fertility rates, it leads to a lower aggregate growth rate. When the policy is implemented for only one period, as in China's one-child policy, the ultimate impact on the aggregate growth rate also determines the impact on the pension system. It is precisely because the imposition of a child tax will reduce the fertility rate, which will accelerate the process of low fertility and aging and cause the collapse of the pension system. Therefore, the length of policy implementation should be determined on the basis of balancing the relationship between economic development and population growth. It has also been verified above that the introduction of the child tax reduces the number of children, but it also increases the investment in education for each child, thereby promoting the accumulation of human capital, which verifies the trade-off between the quantity and quality of children.

This study analyzes the various economic effects of the introduction of the child tax policy. However, this series of effects will not necessarily disappear when the policy is abolished. This is not only because the specific contents of the policy will affect people's behavior but also because when a policy is implemented for a long time, the ideas conveyed by it change people's concepts and social norms, which are then difficult to alter. Even if the policy is lifted, the perceptions of the affected generation are unlikely to change. For example, China's one-child policy and family planning policy mainly promote fewer and better births, which means reducing the number of children born and paying more attention to the quality of children. Owing to this promotion by the government, people nationwide have begun to pay attention to children's education. This is also well explained by the trade-off between the quantity and quality of children in the model. After the Chinese government eased birth restrictions in 2015, the birth rate did not increase significantly. In 2021, the Chinese government completely abolished the child tax policy. Simultaneously, China's birth rate reached a record low after declining for five consecutive years since 2016. China's recent policy of banning after-school tutoring also shows that the Chinese government is trying to reduce the overemphasis on children's education brought about by the child tax policy, so as to avoid lower fertility rates, which are still being affected despite the policy being lifted. Further research is needed on the subsequent impact of the cancellation of the policy and I also call for more research on this. I hope this study can give some reference to those countries that need population control.

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APPENDICES

A) The effect of parents' preference for children γ *on the fertility rate* n *and the balanced-growth rate* g.

Firstly, recall the equations of fertility and balanced-growth rate as follows.

$$n = \frac{D}{Ez + Fq} \tag{1}$$

$$g = H(z+q)^{\frac{\eta}{1-\alpha(1-\eta)}} \tag{2}$$

where

$$H = \left\{ \frac{(1-\alpha)A\psi^{\frac{(1-\alpha)}{\eta}}\eta^{(1-\alpha)}}{1-\eta} \left[\frac{\alpha\beta}{\gamma[\alpha+\tau(1-\alpha)]} \right]^{\alpha} \right\}^{\frac{\eta}{1-\alpha(1-\eta)}}$$

$$D = (1-\eta)\gamma(1-\tau)[\alpha+\tau(1-\alpha)]$$

$$E = (1+\gamma)[\alpha+\tau(1-\alpha)] + \beta\alpha$$

$$F = (1+\eta\gamma)[\alpha+\tau(1-\alpha)] + \beta\alpha$$

The derivatives of the equations of n and g with respect to the parent's preference for the child (γ) then read:

$$\frac{\partial n}{\partial \gamma} = n_{(\gamma)} \left\{ \frac{1}{\gamma} - \frac{[\alpha+\tau(1-\alpha)](z+\eta q)}{Ez+Fq} \right\} > 0 \qquad A-1$$

$$\frac{\partial g}{\partial \gamma} = -\frac{\alpha\eta}{\gamma[1-\alpha(1-\eta)]} g_{(\gamma)} < 0 \qquad A-2$$

It is obvious that $A-2$ holds. The proof of $A-1$ is as follows:

$$\frac{1}{\gamma}=\frac{[\alpha+\tau(1-\alpha)](z+\eta q)}{\gamma[\alpha+\tau(1-\alpha)](z+\eta q)}=\frac{[\alpha+\tau(1-\alpha)](z+\eta q)}{[\alpha+\tau(1-\alpha)]\gamma z+[\alpha+\tau(1-\alpha)]\eta q}$$
$$>\frac{[\alpha+\tau(1-\alpha)](z+\eta q)}{[\alpha+\tau(1-\alpha)](1+\gamma)z+[\alpha+\tau(1-\alpha)](1+\eta\gamma)q}$$
$$>\frac{[\alpha+\tau(1-\alpha)](z+\eta q)}{\{[\alpha+\tau(1-\alpha)](1+\gamma)+\beta\alpha\}z+\{[\alpha+\tau(1-\alpha)](1+\eta\gamma)+\beta\alpha\}q}=\frac{[\alpha+\tau(1-\alpha)](z+\eta q)}{Ez+Fq}$$

Therefore, $A-1$ holds.

$A-1$ and $A-2$ show that parents' preference for children (γ) has a positive effect on the fertility rate (n) and a 2 effect on the balanced-growth rate (g), respectively.

B) The effect of q *on* $g_{(q)}{}^{t}n_{(q)}$.

Again, recall the equations of fertility and balanced-growth rate as follows.

$$n=\frac{D}{Ez+Fq} \tag{3}$$

$$g=H(z+q)^{\frac{\eta}{1-\alpha(1-\eta)}} \tag{4}$$

where

$$H=\left\{\frac{(1-\alpha)A\psi^{\frac{(1-\alpha)}{\eta}}\eta^{(1-\alpha)}}{1-\eta}\left[\frac{\alpha\beta}{\gamma[\alpha+\tau(1-\alpha)]}\right]^{\alpha}\right\}^{\frac{\eta}{1-\alpha(1-\eta)}}$$
$$D=(1-\eta)\gamma(1-\tau)[\alpha+\tau(1-\alpha)]$$
$$E=(1+\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha$$
$$F=(1+\eta\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha$$

Then the derivative of the equation of $g_{(q)}{}^{t}n_{(q)}$ with respect to q becomes:

$$\frac{\partial g_{(q)}{}^{t}n_{(q)}}{\partial q}=g_{(q)}{}^{t}n_{(q)}\left[\frac{t\eta}{[1-\alpha(1-\eta)](z+q)}-\frac{F}{Ez+Fq}\right]$$

Therefore,

$$\frac{\partial g_{(q)}{}^{t}n_{(q)}}{\partial q}>0$$

as

$$t>\frac{[1-\alpha(1-\eta)](z+q)\{(1+\eta\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha\}}{\{(1+\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha\}z\eta+\{(1+\eta\gamma)[\alpha+\tau(1-\alpha)]+\beta\alpha\}q\eta}$$

for any $q\geq 0$ and t is an integer.