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An instructional approach in mathematics learning for fourth-grade students based on collaborative learning and inquiry-based learning

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ABSTRACT

This study combines collaborative learning and inquiry-based learning, analyzes the characteristics of fourth-grade mathematics textbook, and designs an instructional model based on collaborative learning and inquiry-based learning. Based on this instrcutional model, this study adopts a quasi-experimental design, involving 142 fourth-grade students from a public primary school in Chengdu, China, using pre-tests and post-tests to measure the effectiveness of the instrcutional model. The main research results indicate that students who receive instrcutional approach based on collaborative learning and inquiry-based learning show significant improvement in analytical thinking and reasoning skills compared to students receiving traditional teaching approach. The main contribution of this study lies in demonstrating that instrcutional approach based on collaborative learning and inquiry-based learning can effectively enhance the mathematical analytical thinking and reasoning skills of fourth-grade students, providing a replicable framework for educational practice.

Contribution/ Originality: The originality of this study lies in the unique application of a combined instructional approach utilizing both collaborative learning and inquiry-based learning to enhance analytical thinking and reasoning skills in mathematics for fourth-grade students.

1. INTRODUCTION

There are no doubts that cognitive skills like analytical thinking and reasoning are elemental for the intellectual success of a student and in a person's everyday problem-solving (Murphy, Rowe, Ramani, & Silverman, 2014; Nunes et al., 2007). Analytical and logical thinking as well as higher-order thinking skills are 21st-century skills classified as such since youth nowadays requires those skills in preparation for the future (Ananiadou & Claro, 2009; Kwangmuang, Jarutkamolpong, Sangboonraung, & Daungtod, 2021). Fourth grade is a critical time for students to develop these skills throughout their educational careers. A study proved that age 9 is a critical period for children's brain development and cognitive function development (Gale, O'Callaghan, Godfrey, Law, & Martyn, 2004). Therefore, for educators, it is crucial to identify and utilize stimulating learning environments conducive to fostering the development of students' analytical thinking and reasoning skills during the fourth grade period. Meanwhile, Relevant studies have shown that collaborative learning and inquiry-based learning can effectively strengthen students' analytical thinking and reasoning skills (Jensen & Lawson, 2011; Nuangchalerm & Thammasena, 2009; Sasanti, Hamtasin, & Thongsuk, 2024). However, research on instructional approach based on collaborative learning and inquiry-based learning to enhance the mathematical analytical thinking and reasoning skills of fourth-grade primary school students is relatively scarce. Therefore, our research will focus on improving effective instructional approach that combine inquiry-based learning with collaborative learning, aiming to enhance fourth-grade students' analytical thinking and reasoning skills in mathematics learning and provide generalizable reference for practical teaching.

2. LITERATURE REVIEW

2.1. Importance of Analytical thinking and Reasoning Skills

A range of studies have highlighted the importance of reasoning skills in the learning of mathematics. Amir-Mofidi, Amiripour, and Bijan-Zadeh (2012) and Hasanah, Tafrilyanto, and Aini (2019) underscore the pivotal role of reasoning in comprehending and resolving mathematical problems. Tajudin and Chinnappan (2015) investigated the correlation between reasoning skill and mathematical problem-solving performance among Malaysian students, concluding that reasoning skill significantly enhances problem-solving. Singley and Bunge (2014) proposes early integration of reasoning skill into elementary mathematics curriculum to augment students' mathematical proficiency and facilitate seamless transitions to higher-level mathematics courses. In an experiment, Lestari (2019) observed that middle school students who engaged in problem-solving methods exhibited superior mathematical reasoning skill in contrast to those using traditional methods. Recent studies provide additional evidence supporting the advantages of problem-based learning in nurturing students' reasoning skill, as indicated by Mahmud and Mohd Drus (2023) investigation, which suggests that elementary mathematics teachers employing diverse oral questioning can effectively foster students' development of mathematical reasoning skill.

2.2. Concepts, Values, Models of Collaborative Learning

Collaborative learning is an instructional method in education in which groups of students collaborate to tackle challenges, achieve objectives, or produce outcomes (Baker, 2015; Laal & Ghodsi, 2012). Learners share responsibility not only for their own learning but also for the progress of their peers. Consequently, the success of one learner contributes to the success of their peers (Gokhale, 1995).

Collaborative learning (CL) is the process in which groups of students work together to come up with solutions to problems. This type of learning has been shown to be beneficial in the areas of higher order cognitive skills with the research showing this to be true by Vasodavan, DeWitt, and Alias (2021) and Loes and Pascarella (2017). Following this type of teaching method does not only stimulate deep learning but also active participation of the students in the learning process. Collaboration works to high efficiency is noticeable when students work in pairs and a person assumes the role of listener while the other one finds relevant discussions pertinent to the investigation at hand. This dynamic experience again, allows partners to supplement these crucial cognitive skills through idea generation, constructive dialogue, feedback acceptance, and meaningful responses to questions and comments (Johnson, 1971; King, 2007; Peterson & Swing, 1985).

The traditional collaborative learning model encompasses activities such as Jigsaw, Group Investigation (GI), Co-op, Student Team Achievement Division (STAD), Team Games Tournaments (TGT), and Learning Together (LT) (Balfakih, 2003; De Vries & Slavin, 1978; Doymus, 2008; Johnson & Johnson, 2002; Kagan, 1985; Supriyati & Utama, 2015). Building upon the traditional collaborative learning models, Suh and Lee (2006) devised a comprehensive collaborative model encompassing task preparation, individual learning, team learning, and task evaluation. Suh (2011) integrated aspects of learning environments and modern technology to present four collaborative learning scenarios: (1) Project-based collaborative learning employing presentation and communication tools; (2) Story-based collaborative learning via role-playing games; (3) Collaborative games employing interactive carpets; (4) Inquiry-based collaborative learning featuring immersive display.

2.3. Concepts, Values, Models of Inquiry Learning

The educational approach of inquiry-based education is centered on enhancing student active participation in the whole process unfolding from exploring ideas to investigating and finally discovering and formulating inquires. As Kirschner shows, the term inquiry-based education occurs when studying a phenomenon: formulating questions, collecting and analyzing data and eventually drawing conclusions (Kirschner, Sweller, & Clark, 2006). This educational method vehemently supports the constructivist approach which indicates that the learning is a highly experiential process and flourishes only through active involvement and interaction with the surrounding influences. Application of scientific evidence supports the fact that learner-centered education promotes depth of learning and deep ability of have reasoned thinking. In an investigation on the change of understanding and problem-solving skills compared to students who were involved in inquiry-driven education, it was discovered that they had such higher levels (Blumenfeld et al., 1991). For example, in the same way a meta-analysis of 164 studies done by Zafra-Gómez, Román-Martínez, and Gómez-Miranda (2015) also showed that there was a significant difference in learning outcomes after students were involved in activities revolving around inquiry.

The inquiry-based learning includes transformative processes and regulative processes (De Jong & Njoo, 1992; Njoo & de Jong, 1993) whereas the former are targeted to improve inquiry skills, and the latter are oriented to governing and supporting particular transformative processes (De Jong, 2006). Numerous models of inquiry based learning exist, including the inquiry cycle theory (De Jong, 2006; White & Frederiksen, 1998) seven stages of inquiry (Pedaste & Sarapuu, 2006) 5E learning cycle model (Bybee et al., 2006) modified inquiry theoretical model (Maeeots & Pedaste, 2014) the modern inquiry-based learning framework (Pedaste et al., 2015) and the POEE (Predict, Observe, Explain and Evaluate)non-guided inquiry learning model (Al Mamun, Lawrie, & Wright, 2020).

2.4. Concept of Instructional Approach

An instructional approach is a set of teaching methods and strategies spanning the teacher-centered and student-centered techniques, with individual elements such as lecture, direct instruction, and group discussion (Akimenko, 2016). It aspires to fulfill the needs of all sort of learners with the use of technology (Damodharan & Rao, 2009) and a planned action of instruction that concerns the people, tools, and environment subsystem (Isman, 2011). A common notion in experiential learning is instructional design, which deals with the planning and preparation of a comprehensive and productive learning featuring elements like instructional sequence activities, content outlines, instructional methods, media, and instructional tools (Batubara, 2018). As well as this, the method highlights the pertinence of setting learning objectives and achievement (DeLong & Winter, 2001) and designing the lessons to get success (Basri, 2017). Lastly, the essence of game-based learning is interactive and adaptive collaboration between the teacher, students and means of instruction (Ono, 2006).

2.5. Current Status of the Study

Prior researches have mainly concentrated on investigating the determinants of collaborative inquiry-based learning, with a few examples being social annotation (SoAn) tools as a facilitative agent (Chan & Pow, 2020) computer technology related support (Lämsä, Hämäläinen, Koskinen, & Viiri, 2018; Pietarinen, Palonen, & Vauras, 2021) and the association of learning methodologies and high-order thinking (Lu, Pang, & Shadiev, 2021). It should be mentioned, however, that some research has shown collaborative inquiry-based learning to be the only factor directly and indirectly influencing higher order thinking skills (Liu, Liu, Wang, Li, & Xu, 2023). Nonetheless, there remains a gap in research regarding the enhancement of fourth-grade students' mathematical analytical thinking and reasoning skills through the design of instructional activities rooted in collaborative and inquiry-based learning. This study aims to refine an instructional approach based on collaborative and inquiry-based learning to enhance the mathematical analytical thinking and reasoning skills of fourth-grade elementary school students, thereby offering a practical and reproducible instructional framework.

3. RESEARCH METHODOLOGY

3.1. Research Design

This study aims to investigate the effects of instructional approach based on collaborative learning and inquiry-based learning on the analytical thinking and reasoning skills of fourth-grade students in primary school. The study is divided into three stages:

The first stage is the design of instructional model. We thoroughly reviewed the relevant literature on collaborative learning, inquiry-based learning, analytical thinking skills, and reasoning skills, and collaborated with teachers to interpret the fourth-grade mathematics textbooks. Based on the characteristics of the fourth-grade mathematics textbooks and the basic elements of collaborative learning and inquiry-based learning, we designed instructional model and developed assessment scales and test papers for evaluating skills. The second stage is the validation and revision phase. We conducted a one-week collaborative learning and inquiry-based instructional model for the validation group, and assessed them using skill assessment scales and test papers. Through the analysis of research data, we assessed the reliability and validity of the research tools and revised the instructional model accordingly. The final stage is the implementation phase. We employed an equivalent control group design with a significance level (α) set at 0.05. Both the experimental and control groups underwent pre-tests to assess their basic analytical thinking and reasoning skills for subsequent comparative analysis. The study lasted for one month, during which a post-test was conducted to evaluate the treatment effects of the instructional approach.

3.2. Research Variables

This study investigates the effects of various teaching approaches on the mathematical analytical thinking and reasoning skills of fourth-grade students. The independent variable in this study is the teaching approach, which is divided into two groups: an experimental group that implements an instructional approach based on collaborative learning and inquiry-based learning, and a control group that employs a traditional teaching approach. The dependent variables comprise the mathematical analytical thinking and reasoning skills exhibited by fourth-grade students.

3.3. Participants

Research sample will be drawn from 142 fourth-grade students at Chengdu Xiti Road Primary School in China, using a cluster random sampling method. A total of 75 students will be selected, and they will be randomly assigned to the experimental group, control group, and validation group, with 25 students in each group.

3.4. Treatment of the Research

In this study, we implemented several measures to ensure the reliability of our findings. Firstly, we conducted random sampling to ensure the representativeness of the sample. Secondly, we provided standardized training to teachers involved in the assessment to maintain consistent evaluation standards. Additionally, to mitigate the influence of individual teaching levels and styles on the experimental outcomes, we selected teachers with similar seniority and provided uniform training prior to the study. Lastly, we used identical test papers for both pre-test and post-test assessments, ensuring a sufficient time gap between them to minimize the pre-test's impact on post-test results.

3.5. Research Questions

The two research questions examined in this study are:

1. Is there a significant differences in analytical thinking and reasoning skills among students engaged in the instructional approach based on inquiry-based and collaborative learning before and after participation?
2. Is there a significant difference in analytical thinking and reasoning skills between students engaged in the

instructional approach based on inquiry-based and collaborative learning compared to those participating in traditional teaching approach?

3.6. Definition of Terms

3.6.1. Instructional Approach Based on Collaborative Learning and Inquiry-Based Learning

Instructional approach focuses on structured and systematic ways and manners of teaching that are employed by teachers with the purpose of content teaching, organizational forms of teaching, activity design in teaching, and teaching resources. This study does a collaborative and inquiry-based learning instructional method by engaging students to learn from group work, discussions, problem-solving, and practical experiences. Teachers play a directing role by acting as facilitators through teaching process and this way transform an environment which is favorable for setting up the tasks collaboratively, raising of questions, exploration of phenomena and solving of problems.

3.6.2. Traditional Teaching Approach

The traditional teaching approach is teacher-centered, focusing on imparting knowledge to students in a structured manner through methods such as lectures and direct guidance.

3.6.3. Analytical Thinking Skill

Analytical thinking skill is the ability of student to take analytical approach to a problem solving processes, which includes making logical deduction, inferences, reasoning and critical evaluation. This study will employ two methods to assess students' analytical thinking skills: Evaluating the students' skill to use of analytical thinking in solving the mathematical problems by means of evaluation in test paper, and completely analyzing students' quickness in analytical thinking during the learning process by competency performance scale.

3.6.4. Reasoning Skill

Reasoning skill is the capacity of an individual to engage in logical reasoning, judgment, and inference based on existing information, logical rules, and experiences. This study will adopt two approaches to evaluate students' reasoning skill: assessing their application of reasoning skills in solving mathematical problems through test paper and comprehensively evaluating students' actual performance in reasoning skill during learning through a competency performance scale.

3.7. Research Instruments

3.7.1. Instructional Model Based on Collaborative and Inquiry-Based Learning

3.7.1.1. Contextual Introduction

Objective: The aim is to introduce scenarios to captivate students' attention and provoke their curiosity, prompting them to unearth and ponder over issues derived from real-life situations, thereby cultivating their analytical skills in problem observation and definition. Activity: The teacher introduces a scenario, enabling students to observe, analyze, and translate scenario information into mathematical concepts, and propose potential mathematical problems and solution strategies, eg. what insights have we gleaned from this scenario? What valuable mathematical insights can we derive from this data? What mathematical inquiries can be raised based on this mathematical information, and what methodologies could be employed to address them?

3.7.1.2. Collaborative Inquiry

Objective: Foster intellectual collisions among students and enhance the development of reasoning skills and analytical thinking through group cooperation and exploration.

Activity: Utilize questions raised in the initial stage to facilitate collaborative exploration in groups. This involves analyzing the problem, suggesting hypotheses, devising solutions, anticipating outcomes, and implementing plans to verify hypotheses.

3.7.1.3. Communication and Sharing

Objective: Enhance students' analytical thinking skills, learn how to articulate their ideas, evaluate learning outcomes, and demonstrate their reasoning processes.

Activity: Groups present and report their learning outcomes, engage in exchange and sharing with other groups, and simultaneously evaluate the ideas of other groups.

3.7.1.4. Deep Collaborative Inquiry

Objective: Guide students to apply and expand upon previous learning strategies to address more complex problems, thereby enhancing their analytical thinking and reasoning skills.

Activity: Groups delve into in-depth collaborative inquiry learning regarding the second question, which builds upon the first one, encouraging students to engage in more complex data analysis and problem-solving, potentially involving higher-level mathematical operations and logical reasoning.

3.7.1.5. Communication and Sharing

Objective: Facilitate in-depth communication and sharing among students, as well as the skills to integrate knowledge.

Activity: Groups engage in sharing detailed processes of problem-solving, focusing on how to adjust strategies to overcome encountered difficulties.

3.7.1.6. Reflection and Conclusion

Objective: Enhance students' understanding of the learning content, cultivate the habit of summarizing and reflecting on the learning process, and promote the improvement of meta-cognitive skills.

Activity: Reviewing the learning content, the teacher guides students to summarize the strategies they have gained for problem-solving and evaluate the effectiveness of these methods. For instance: What have you learned today? Which method do you find most useful?

Figure 1 Illustrates an instructional model based on collaborative and inquiry-based learning.

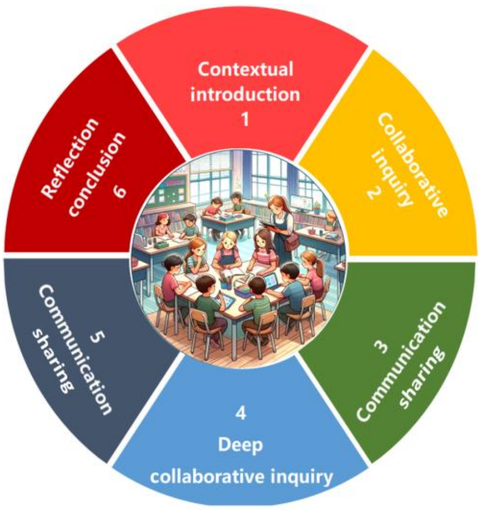

Figure 1. Instructional model based on collaborative and inquiry-based learning.

3.7.2. Assessment Scale

The assessment scale for reasoning skill comprises four dimensions. It assesses students' skills in identifying, employing systematic thinking, and strategically applying them to solve mathematical problems. The critical thinking dimension scrutinizes students' capacity to analyze and evaluate mathematical information and evidence, employ logical reasoning, and make predictions based on data.

The communication dimension assesses students' comprehension of mathematical reasoning and the correct use of mathematical language signs. The application concept dimension assesses the students' competence in applying mathematical concepts in real life situations, relationship between the concepts and solving problems. Consequently, the score scale that varies from 1 point (novice) to 5 points (master), indicates the evolution of students from failing to solve the mathematical problems, through expecting the problems and then applying the advanced tactics.

The analytical thinking skill assessment scale comprises four dimensions: data interpretation, testing students' ability to understand data provided in graphs, charts, and tables and their ability to infer, recognize trends, and patterns. Logical reasoning tests the use of both deductive and inductive reasoning, the building of logical relationships, and the analysis of mathematical arguments. Pattern recognition evaluates recognition and generalisation of patterns in numerical sequences, pictures and equations and also their application in mathematical problem solving.

Problem decomposition assessment splits complex mathematical problems into parts, carries out a systematic problem analysis and solves these parts sequentially. The score basis places the performance criteria on a scale from 1 point (novice) to 5 points (excellent), where 1 point (novice) represents difficulties in interpreting visual data types, and 5 points (excellent) marks outstanding interpretation of complicated data and trends, patterns, or outliers.

3.7.3. Test-Paper

The papers consists of 25 multiple-choice questions aimed at assessing students' analytical thinking and reasoning skills. The questions cover six aspects: Number Sense and Operations, Patterns and Sequences, Logic and Problem Solving, Measurement and Geometry, Fractions and Decimals, Data and Statistics.

Through assessing these aspects, it evaluates students' mastery of analytical thinking and reasoning skills in mathematical learning.

3.8. Grading Procedure

The assessment of analytical thinking and reasoning skills will include both test papers and evaluation scales. Each student's assessment score consists of the paper-test score (out of 25) and the scale assessment score (out of 60), totaling 85 points.

Assessments will be carried out for students in both the experimental and control groups after the initial and final teaching sessions. In the experimental group, the 25 students will be divided into five groups designated as Group 1 through Group 5. Meanwhile, in the control group, each of the 25 students will be assigned a unique number from 1 to 25 (refer to Figure 2). The assessment team comprises 5 mathematics teachers. In the experimental group, each teacher is assigned to assess one group, whereas in the control group, each teacher evaluates five students.

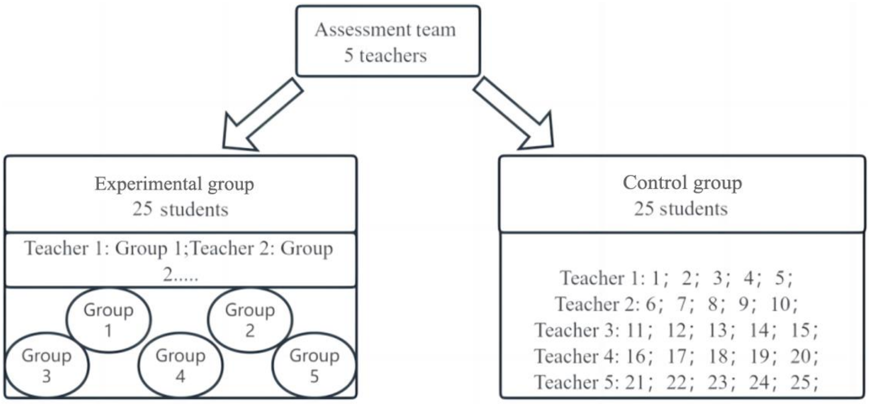

Figure 2. Scale assessment process.

3.9. Ethical Issues

Prior to commencing the study, we obtained approval from the Ethics Review Committee of Nakhon Phanom University and Chengdu City Xiti Road Primary School. Subsequently, we provided comprehensive research details to school administrators and teachers involved in the intervention study, ensuring their consent. Furthermore, we ensured that parents were well-informed by providing ample information and obtaining their explicit consent.

4. RESULTS

4.1. Participants

The research samples were drawn from 142 fourth-grade students at Chengdu Xiti Road Primary School in China, employing a cluster random sampling method. Seventy-five students were selected and randomly assigned to three groups: experimental, control, and validation, each comprising 25 students. Participants had an average age of approximately 9.5 years, with ages ranging from 9 to 10 years. The gender distribution of the participants was as follows:

Table 1 presents participants information.

Table 1. Participant's information.

Items	Number	Number (Male)	Number (Female)
Sample size	75	36	39
Experimental group	25	14	11
Control group	25	12	13
Validation group	25	10	15

4.2. Instrument Reliability

Twenty-five students from the validation group participated in testing the assessment scale. The primary aim of this process is to assess the reliability of the research instrument, and to support this, we provided precise test results. This study employed Cronbach's α and KMO (Kaiser-Meyer-Olkin is a statistical measure used to assess the suitability of data for factor analysis or principal component analysis) tests to assess the reliability and validity of the scale measuring analytical thinking and reasoning skills. The results showed Cronbach's α coefficients of 0.953 and 0.943, and KMO values of 0.821 and 0.785, respectively, for the scale. Consequently, the scales demonstrated satisfactory internal consistency and validity.

4.3. Analysis of Homogeneity

Initially, an independent sample t-test was conducted on the pretest scores of the two treatment groups. The mean pretest score of students in the experimental group was comparable to that of the control group, showing no significant disparity. As depicted in Table 2 the t-test outcome (t=-0.24, t=-0.75, p>0.05) did not achieve statistical significance, indicating that pretest differences between treatment groups were negligible.

Table 2. Comparison of pre-test.

Item classification	Group	M	SD	t	df	p
Analytical thinking skills	Experimental group	64.2	8.12	-0.24	49	0.81
	Control group	64.7	7.35			
Reasoning skills	Experimental group	64	7	-0.75	49	0.46
	Control group	65.4	5.79			

4.4. Hypothesis Testing

4.4.1. Hypothesis Testing I

Is there a significant differences in analytical thinking and reasoning skills among students engaged in the instructional approach based on collaborative learning and inquiry-based before and after participation?

Table 3. Comparison of pre-test and post-test in the experiment group.

Students' scores in different tests	M	SD	t	df	p
Pre-test of analytical thinking skills	64.2	8.12	-13.03	24	0.000*
Post-test of analytical thinking skills	70.8	5.94			
Pre-test of reasoning skills	64	7	-16.55	24	0.000*
Post-test of reasoning skills	71.4	4.95			

Note: *P<0.05.

A paired T-test was conducted on the same group of students to assess whether there was a significant difference in the average test scores between two tests administered at different time points. The paired T-test results in Table 3 indicate a significant difference in scores between the two tests: t (24) = -13.03, t (24) = -16.55, p < 0.05. On average, students scored higher on the second test compared to the first, indicating an improvement in performance. With mean differences (MD) of 6.6 and 7.4, and pooled standard deviations of approximately 7.03 and 5.95 respectively, the effect sizes (d) were calculated to be 2.61 and 3.31, indicating a large effect size following Cohen (1988) criteria. Consequently, students engaged in the instructional approach based on collaborative learning and inquiry-based learning exhibited higher levels of analytical thinking and reasoning skills compared to before the intervention.

4.4.2. Hypothesis Testing II

Is there a significant difference in analytical thinking and reasoning skills between students engaged in the instructional approach based on collaborative learning and inquiry-based learning compared to those participating in traditional teaching approach?

Table 4. Comparison of post-test.

Item classification	Group	M	SD	t	df	p
Analytical thinking skills	Experimental group	70.8	5.94	2.30	49	0.026
	Control group	66.8	6.48			
Reasoning skills	Experimental group	71.4	4.95	2.66	49	0.011
	Control group	67.6	5.05			

Note: P<0.05.

An independent samples T-test was conducted to investigate significant differences in primary students' analytical thinking and reasoning skills among different instructional approaches: Instructional approach based on collaborative learning and inquiry-based learning versus Traditional teaching approach. Table 4 illustrates a significant disparity in test scores for analytical thinking and reasoning skills among the sampled students based on the different teaching approaches, with $t(49) = 2.30$, $t(49) = 2.66$, $p < 0.05$. On average, students enrolled in the collaborative learning and inquiry-based learning approach outperformed those in the traditional teaching approach in analytical thinking and reasoning skills. The mean differences (MD) were calculated to be 4 and 3.8 respectively, with pooled standard deviations of approximately 6.21 and 5. Consequently, the effect sizes (d) were calculated to be 0.65 and 0.59, which align with Cohen (1988) criteria for a medium effect size. Thus, students engaged in the instructional approach based on collaborative learning and inquiry-based learning demonstrated higher levels of analytical thinking and reasoning skills compared to those involved in the traditional teaching approach.

5. DISCUSSION

5.1. Summary of the Study

This research employed pretest-posttest control group quasi-experimental design to evaluate the effects of instructional methods which are collaborative learning-based and inquiry-based learning on the mathematical analytical thinking and reasoning skills of stage four primary school students. The results of this research showed that instructional strategy based on cooperative learning and inquiry-based learning greatly improved the mathematical analytical thinking and reasoning skills of fourth graders, consistent with previous literature (Jensen & Lawson, 2011; Nuangchalerm & Thammasena, 2009). Similarly, preceding literature has highlighted the effectiveness of inquiry-based learning and collaborative teaching approaches (Loes & Pascarella, 2017; Vasodavan et al., 2021; Zafra-Gómez et al., 2015). At the age of 9-10 students who are fourth graders reach a crucial period of cognitive development. The studies show that children in the middle childhood period (9-11 years) can easily perform complicated cognitive tasks and are also illustrate greater cognitive control and flexibility (Canada, Hancock, & Riggins, 2021; Cragg & Nation, 2009). This is while mathematics is recognized as one of those subjects that help in developing higher order thinking skills (Ahmad et al., 2017). Several studies stress the need for the higher-order thinking skills, such as critical thinking, problem-solving, and reasoning in mathematics education (Chang, Hwang, Chang, & Wang, 2021; Dolapcioglu & Doğanay, 2022; Kwangmuang et al., 2021). The study is specifically target on the critical period of children's cognitive development with mathematics as the background, to improve the analysis thinking and reasoning skills of the fourth grade students through the collaborative and inquiry based learning activity. This study partially addresses a gap in current research on the analytical thinking and reasoning skills of fourth grade students in the area of mathematics.

5.2. The Six-Step Instructional Model

In this study, we developed a six-step instructional model based on collaborative learning and inquiry-based learning, including "Contextual Introduction; Collaborative Inquiry; Communication and Sharing; Deep Collaborative Inquiry; Communication and Sharing; Reflection and Conclusion". This instructional model is based on the characteristics of collaborative learning and inquiry-based learning, emphasizing group collaboration; problem-oriented, student-centered, teacher-guided; collaboration and inquiry as the main learning methods, emphasizing communication among students and reflection and summary after learning.

In general, analytical thinking and reasoning skills are components of higher-order thinking skills (Pujiastuti & Haryadi, 2023). Higher-order thinking skills are the capacities to analyze, synthesize, evaluate, create, and apply information. Higher-order thinking development occurs through social interaction over time (Almerich, Suárez-Rodríguez, Díaz-García, & Cebrián-Cifuentes, 2020; Cheng, Hwang, & Lai, 2020). This is consistent with the sociocultural theory view that individuals learn and develop cognitive competencies through social interaction and tools within cultural contexts (De Valenzuela, 2014). Constructivism is based on the idea that the learners do not receive the knowledge passively but they construct their own knowledge systems through active investigation and practice (Bada & Olusegun, 2015). In this model of the instructional activity, students gain a deeper understanding of mathematical concepts by doing and solving real-world problems, rather than just accepting mathematical rules and formulas. This active investigatory learning process makes students to think more critically (Lu et al., 2021). The model of instructional activity starts with situation introduction as a beginning of learning; the fourth-grade students, which are in the concrete operational stage (9-12 years-old) begin to acquire mathematical concepts still relying on concrete objects or situations for understanding (Börnert-Ringleb & Wilbert, 2018). Knowledge should be learned by students within particular contexts in order to understand and memorize (Fyfe, McNeil, & Borjas, 2015). Introducing mathematical ideas into everyday life helps students see the need for mathematics and motivates them to do the research and such in a more in-depth way.

5.3. Instructional Recommendations

Scenarios should be contextualized to better engage students' interest, aligning with their daily experiences like family, school, community, or hobbies (Silseth & Erstad, 2018; Turner et al., 2024). Furthermore, scenarios should integrate mathematical concepts, including time, space, quantity, and change, facilitating students' comprehension of abstract mathematical concepts through real-life experiences. In the organization of students for collaborative inquiry learning, rules are crucial to assign particular tasks to each group member, encouraging the sharing of ideas and cooperation (Barkley, Major, & Cross, 2014). Subsequently in the phase of deep exploration, teachers should direct learners to probe further concerning the initial question. For example, with the help of guiding questions, teachers should observe and guide students, engaged in collaborative inquiry learning, who come across challenges (Dobber, Zwart, Tanis, & van Oers, 2017; Lehtinen & Viiri, 2017) such as "What perspective should we approach this problem from?" "What information or data is required for our analysis?". This promotes students' involvement in conversation and cooperation. During discussions, educators should promote the articulation of students' thoughts and foster active listening and critiquing of classmates' opinions. What is your stance on this matter? What approach did you take to solve the problem? Did you face any challenges during the discussion? "How were you able to handle them?" improves good communication and sharing.

In the deep exploration stage, educators should guide students in probing the previous question further. For instance, through posing difficult problems, the students will be led to explore complex mathematical concepts and problems (Crespo & Sinclair, 2008). In addition, students need to be inspired to try different techniques in solving the problems and develop the ability to work as a team to tackle the challenges. In subsequent sharing sessions, instructors are encouraged to guide students in reflection on their problem solving processes and to discussions on fine tuning active strategies. For example, the teachers can create a situation in which students from different groups are prompted to ask questions, give comments, and discuss in depth, thus helping themselves understand each other's cognitive processes (Stein, Engle, Smith, & Hughes, 2008). In concluding and reflection phase, educators need to help the students to reflect on what they learned and to evaluate effectiveness of various methodologies (Van den Heuvel-Panhuizen & Drijvers, 2020; Wheatley, 1992). These include the consideration of real-world problems, and the development of extra learning approaches.

In teaching practice, teachers should constantly change activity design to follow the student's situation and amendments in terms of guidance and feedback. In addition, the faculty are urged to use technological instruments, including virtual meeting tools or online platforms, to facilitate students' communication and cooperation. Importantly, teachers should act as role models and supporters that inspire the student's curiosity, motivation, and active partaking in inquiry-based learning.

Not all instructional content necessitates the application of collaborative inquiry-based learning. On one hand, the perspectives of Kirschner et al. emphasize the limitations of working memory and the benefits of direct teaching methods in alleviating the load on working memory (Kirschner et al., 2006). On the other hand, the perspectives of Hmelo-Silver et al. underscore the support and guidance provided in constructivist learning environments to assist students in managing the load on working memory (Hmelo-Silver, Duncan, & Chinn, 2007). An analysis conducted by Chinn et al. raises questions about the efficacy of direct teaching in fostering students' engagement in real-world inquiry skills. Chinn et al. emphasized the necessity for students to practice and explore in complex real-world settings to develop their reasoning skills in the digital media age. Direct teaching might not offer adequate opportunities for students to confront this complexity and challenge and learn how to adapt and respond. Instead, involving students in inquiry within real-world tasks and contexts enables them to better comprehend and address these intricate pieces of information (Chinn, Barzilai, & Duncan, 2020a, 2020b). However, it's important to note that this doesn't imply that direct instructions lack value in other domains or specific contexts. Different learning tasks and disciplines may require a range of teaching methods and strategies. Some learning tasks may be more suitable for direct teaching, while others may be better suited for collaborative inquiry-based learning.

5.4. Limitations and Future Directions

This study is subject to several limitations, primarily stemming from the small sample size, the relatively brief duration of the educational intervention. Experimental data indicate that the instructional approach based on collaborative learning and inquiry-based learning exert a notable influence on students' analytical thinking and reasoning skills. However, students exhibited moderate effect sizes in analytical thinking and reasoning skills. Schmidt, Van Der Molen, Te Windel, and Wijnen (2009) in a meta-analysis study on problem-based learning, highlighted that problem-based learning, when compared to more traditional teaching approach primarily reliant on direct instruction, resulted in a modest increase in medical knowledge but a significant enhancement in medical skills. Furthermore, Kapur (2016) argued that direct teaching fosters short-term learning but is less advantageous for long-term memory retention and skill transfer to unfamiliar problems. Therefore, it can be inferred that the duration of educational interventions plays a crucial role in determining the effectiveness of collaborative inquiry-based learning in cultivating student skills. In future research endeavors, we aim to address these limitations by increasing the sample size, and conducting a comprehensive evaluation of our instructional approach. Additionally, we intend to prolong the duration of the educational intervention. These planned enhancements would facilitate a more thorough examination of the effectiveness of the instructional approach.

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