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Senior high school students thinking levels in permutation and combination using solo taxonomy

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ABSTRACT

Although the Ghanaian mathematics curriculum emphasizes critical thinking as a core competence, students still appear to lack this skill. In this mixed-method study, the Structure of the Observed Learning Outcome (SOLO) taxonomy was used to assess senior high school students' thinking levels in permutation and combination. A sample of 256 males and 104 females was randomly selected from three senior high schools for the study. The data were collected using tests and interviews, and analyzed descriptively and inferentially using Kruskal-Wallis tests. The results showed that while only one-fifth of the students reached the higher relational and extended abstract thinking levels, the majority (73.9%) remained at the lower levels of pre-structural, uni-structural, and multi-structural thinking. These students struggled to apply basic counting and multiplication principles in solving higher-order thinking problems. The Kruskal-Wallis H test further revealed statistically significant differences in thinking levels across the study programmes. General Science students demonstrated the highest thinking levels, followed by General Agriculture and Business students. The study concluded that students' thinking levels in permutation and combination were low. It is recommended that teachers, textbook authors, and curriculum developers adopt representations and activity-based teaching strategies to help students develop a conceptual understanding of the topic.

Contribution/ Originality: This current study analyzes the thinking levels of Ghanaian SHS students based on the SOLO taxonomy to identify the very low levels at which they understand basic concepts and also highlights significant differences in levels of understanding across various programs of study, suggesting possible pedagogical interventions.

1. INTRODUCTION

Every day, people engage in mental actions often referred to as thinking. Almost every problem in human life necessitates the use of thinking to solve it. Remembering, considering, giving reasons, reflecting, and deciding on something to solve a problem are mental actions in the human mind (Celik & Ozdemir, 2020). In the face of 21st-century challenges, every individual should possess the thinking skills necessary to solve every problem. When an individual thinks about solving a mathematical problem, it means that the person is engaging in mathematical

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thinking. Schoenfeld and Sloane (2016) state that mathematical thinking is the ability to think and make judgments independently.

The process of mathematical thinking involves translating incoming external information into symbols that are then translated into calculations in accordance with established mathematical rules. Moreover, mathematical thinking is an integral part of learning about and learning through mathematics. It entails more than just acquiring a set of skills. Students engage in mathematical thinking by identifying and posing issues, as well as selecting and implementing relevant techniques to solve them. Conjecturing and proving, applying and verifying, generalizing, employing mathematical models, conveying ideas and answers, and reflecting on learning are all part of the process (PISA (Programme for International Student Assessment), 2006). Having high mathematical thinking skills helps students solve math problems with low math anxiety and demonstrates positive attitudes towards mathematics (Kargar, Tarmizi, & Bayat, 2010). Therefore, in Ghana, the ability to think mathematically and solve problems based on mathematical reasoning is a crucial educational goal.

In the Ghanaian education system, mathematics is a compulsory subject studied in primary, secondary, and higher education because mathematics can develop students' thinking abilities. According to Yayuk and As' ari (2020), Mathematics can develop the ability to think logically, critically, creatively, systematically and solve problems.

Ghana's Common Core Programme (CCP) curriculum emphasizes critical thinking and problem-solving among learners. Consequently, teachers are required to engage their students in mental processes beyond memorization (MOE, 2020). Young minds will undoubtedly require critical thinking and problem-solving skills far beyond their school years**.** To keep up with the ever-changing technological advances, Greenhill (2010) students must obtain, understand, and analyze information on a much wider scale and use this information to solve problems. Therefore, it is the responsibility of educators to equip students with the strategies and skills they need to think critically and solve problems to cope with the changing world.

Critical thinking skills are an important aspect of teaching and learning. Thinking skills are fundamental to the educational process, and a person's thoughts can affect the ability to learn, as well as the speed and effectiveness of learning (Suhartoyo, 2017). Therefore, thinking skills are associated with the learning process. Students who are trained to think critically demonstrate a positive impact on the development of their education. The findings of Enciso, Enciso, and Daza's (2017) study reported an improvement in reading comprehension and the average grades of students; therefore, an increase in the resolution of problems in Mathematics and Science has been observed, following a training program focused on thinking skills. This demonstrates that thinking skills are essential for students to solve problems in their learning process, thereby fostering competitive student thinking, developing students' intellect, and helping to avoid errors in reasoning.

At the SHS level, mathematics is categorized into two: Core mathematics and Elective mathematics. Areas covered under elective mathematics include Algebra, Trigonometry, Coordinate Geometry, Calculus, Vectors and Mechanics, Matrices and Transformations, Logic, and Probability and Statistics. According to CRDD (2010), Core Mathematics and English Language are prerequisites for Elective Mathematics. They also added that subjects like Physics and Technical Drawing may also enhance the study of this subject.

Combinatorial mathematics deals with the study of permutations and combinations, and the enumeration of sets of elements. It is a topic in elective mathematics that finds its application in probability. It is taught in the second semester of SHS 2. Permutation and combination are crucial aspects of introductory statistics courses, according to Ihsan and Karjanto (2019). Besides, permutations and combinations are materials that form the basis for learning topics in discrete mathematics. This demonstrates that the concepts of permutation and combination are essential at both basic and advanced levels of education.

Additionally, the concept of permutation and combination underlie the topic of combinatorial analysis (Abrahamson & Cendak, 2006). In the field of combinatorics, permutations and combinations are also useful in other

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fields. Ihsan and Karjanto (2019) reveal that permutations and combinations are important parts of the introductory statistics course in some universities. That is to say, if students are able to attain a conceptual understanding of this topic, their understanding of courses that have permutations and combinations as prerequisites will be enhanced.

According to Busadee and Laosinchai (2013) in studying permutations and combinations, students need real-world problems in the form of more relevant ones to stimulate learning, retain knowledge, and develop thinking skills among students. However, in most classrooms, permutations and combinations are typically taught by requiring students to memorise formulae, categorise problems, and solve each problem by using the appropriate formula. Students who learn these topics in this fashion may be successful in the short term, performing well on tests, but often lack the conceptual understanding required to solve real-world problems.

Many researchers have clearly established that students have difficulty with even basic counting and multiplication tasks, which are fundamental principles in permutation and combination. For example, Abrahamson and Cendak (2006) stated that students only counted H and T when they were asked to find the number of ways to arrange the letters HTHT. Ihsan and Karjanto (2019) revealed that permutations and combinations are difficult matters and prerequisites for learning advanced mathematics, especially those related to material opportunity and statistics. Moreover, over-counting and uncertainty about whether order matters are also variables that contribute to such challenges. (Annin & Lai, 2010). Counting problems, with their enormous numerical answers, might be difficult to verify, according to Eizenberg and Zaslavsky (2004).

Lockwood and Gibson (2014) acknowledge that students' difficulties can be attributed to their inability to interpret word problems by thinking critically. They added that even though permutation and combination problems stated in context improve mathematical thinking and problem-solving skills among students, they are very challenging for students. In addition, the key point in assessing reasoning in combination is identifying the difficulties students face in solving combination problems. Students often use trial and error without any recursive procedure, leading to the formation of all possibilities, which results in non-systematic enumeration (McGalliard, 2012).

This fact proves that the level of students' mathematical ability, particularly in combinatorics, is still low. This low ability indicates that students have difficulties, or their thinking levels are low.

SOLO Taxonomy (Structure of Observed Learning Outcomes) provides a simple, reliable, and robust model that is used to classify students' ability to respond to a problem into five different levels: pre-structural, uni-structural, multi-structural, relational, and extended abstract (Putri, Mardiyana, & Saputro, 2017).

At the pre-structural level of understanding, the task is inappropriately approached, and the student has missed the point or needs help to start. The next two levels, unistructural and multi-structural, are associated with bringing in information (surface understanding). At the uni-structural level, one aspect of the task is identified, and student understanding is disconnected and limited. The jump to the multi-structural level is quantitative. At the multi-structural level, several aspects of the task are known, but their relationships to each other and the whole are missed. The progression to relational and extended abstract outcomes is qualitative. At the relational level, the aspects are linked and integrated, contributing to a deeper and more coherent understanding of the whole. At the extended abstract level, the new understanding at the relational level is rethought at another conceptual level, viewed in a new way, and used as the basis for prediction, generalization, reflection, or creation of new understanding (Hook & Mills, 2011). The authors added that the taxonomy is widely used for designing curriculum outcomes and assessment tasks that become progressively more difficult as students advance through their education. In recent times, many researchers have conducted studies on students' thinking levels in Algebra and Geometry. For example, Claudia, Kusmayadi, and Fitriana (2020) classified students' responses in solving linear program problems based on the SOLO taxonomy. Also, Apawu, Owusu-Ansah, and Akayuure (2018) used the SOLO taxonomy as the framework to classify algebraic thinking levels of junior high school students on entering SHS. Furthermore, Caniglia and Meadows (2018) employed the SOLO taxonomy to classify the strategies pre-

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service teachers use in solving "one-question problems" in geometry. In addition, Putri et al. (2017) analyzed students' thinking levels in geometry using the SOLO taxonomy. Finally, Armah, Cofie, and Okpoti (2017) used the Van Hiele levels of geometric thinking to assess Ghanaian pre-service teachers before their departure for the Student Internship Programme (Teaching Practice) at basic schools. However, studies on students' thinking levels in permutations and combinations are scarce.

Therefore, in this study, the researchers investigated the thinking levels of SHS students in solving problems related to permutation and combination based on the SOLO taxonomy. Although critical thinking is one of the most esteemed goals in education, many educators remain unconcerned about the critical thinking skills and abilities demonstrated by students (Afriansyah, Herman, & Dahlan, 2021). The Ghanaian educational curriculum emphasizes critical thinking and problem-solving as the core competencies of the CCP, which require teachers to elevate their students' mental workflow beyond just memorization (MOE, 2020). However, it appears students are still deficient in critical thinking, and as a result, they make errors in mathematical problem-solving. This is because most of the teaching and assessments teachers carry out are centered on correct answers, symbol manipulations, rote skills, and little or no application of mathematical concepts (Kirvan, Rakes, & Zamora, 2015).

Permutation, combination, and related concepts continue to be important components of the school mathematics curriculum. This topic serves as a prerequisite for advanced courses such as probability, discrete mathematics, combinatorics, and others. It also promotes visualization, algebraic fluency, and attention to accurate calculations.

However, in many high school mathematics sequences, permutations and combinations are often omitted entirely. When they are discussed, they are not usually given sufficient time for students to develop an appreciation and mastery of these topics (United States Academic Decathlon, 2018). Ben-Hur (2006) pointed out that the difficulty faced by high school students in understanding permutation and combination concepts is a significant issue in probability lessons. Additionally, since problems involving permutations and combinations are typically presented as word problems, students need to think critically to solve such problems (Salman, 2002). The author added that students' ability to solve problems in these forms is still weak since they lack critical thinking.

Furthermore, in Ghana, a report by West African Examination Council (WAEC) (2016) stated that many students find it challenging to apply the concepts of permutation and combination when calculating simple probabilities, and most students tend to skip these questions. The author added that the few students who attempt such questions often make errors. This suggests potential issues related to teaching methods or students' understanding of the topic. Finally, the researcher's experience, interactions, and discussions with students and mathematics teachers confirm that most students encounter difficulties in this area.

Considering the importance of this topic in mathematics, it is clear that knowledge of this subject is crucial for understanding advanced mathematical concepts. However, the fact remains that in Ghana, apart from the chief examiner's report highlighting the challenges and how students avoid questions, there are limited studies examining students' thinking levels and the errors they make in permutations and combinations.

2. METHODOLOGY

The study employed descriptive research with both quantitative and qualitative approaches. Simple random and purposive sampling were used to select 360 students from SHS A, SHS B, and SHS C (those who had studied the topic) who participated in this study. The data required for this study were gathered using tests and interviews. The test (Permutation and Combination Test (PCT)) was used to collect quantitative data, while the interviews were used to collect qualitative data.

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3. RESULTS

3.1. Overall Scores of SHS Students in the PCT

This section presents the descriptive statistics of the overall scores obtained by students in the PCT. Table 1 presents the total scores obtained by students in the PCT. These statistics were gathered to give the researchers an overview of students' thinking in the test.

Table 1. Total scores of students in the PCT.

Score	Number of students (N)	%
0 – 2	20	5.6
3 – 4	66	18.3
5 – 6	34	9.4
7 – 8	96	26.7
9 – 10	53	14.7
11 – 12	22	6.1
13 – 14	17	4.7
15 – 16	19	5.3
17 – 18	12	3.3
19 – 20	0	0.0
21 – 22	7	1.9
23 – 24	14	3.9
Mean = 9.22	SD = 5.789	Max. = 24, Min. = 0

The results in Table 1 show that 269 (74.7%) of the students obtained less than half of the total score, while about 91 (25.3%) obtained half or more than half of the total score. This indicates that the general thinking level of SHS students in the PCT was low. In addition, descriptive statistics gathered indicate that out of a total score of twenty-four (24) marks, the mean score of students was 9.22, which was less than half of the total marks, and the standard deviation was 5.789. The standard deviation, which is greater than one (1), is an indication that the mean is not a true representative of the thinking of students and that the scores obtained by most students were low.

In an attempt to measure students' levels of thinking in permutation and combination, six (6) items were administered to the 360 students who participated in the study. The first three (3) sub-questions under item 1 required students to use basic counting and multiplication principles to find the number of different possible routes. These items were designed to measure the unistructural level of thinking in permutation and combination based on the SOLO taxonomy. The total marks were 3, and the unistructural level was considered to be attained if a participant scored 1 or more. A participant who scored less than 1 (scored 0) was not categorized at this level. Table 2 presents the overall frequencies of students who were thinking correctly and incorrectly under the uni-structural level. Students who did not provide responses to some of the items were labeled as "No Response," as shown in Table 2.

Table 2. Item analysis for item 1 (Uni-structural Level).

Item	Correct N (%)	Wrong N (%)	No response N (%)
1a	349 (96.9)	11 (3.1)	0 (0.0)
1b	346 (96.1)	14 (3.9)	0 (0.0)
1c	343 (95.3)	10 (2.8)	7 (1.9)

From Table 2, it can be seen that all the students responded to item 1a. Out of 360 students, 349 (96.9%) answered the question correctly, while 11 (3.1%) answered incorrectly. Similarly, in item 1b, all students provided responses. Of these, 346 (96.1%) were correct, while 14 (3.9%) were incorrect. Finally, in item 1c, 343 (95.3%) students answered correctly, 10 (2.8%) answered incorrectly, and 7 (1.9%) did not attempt the item. Overall, it can

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be observed that students' understanding of item 1 was high, as the majority answered correctly. This indicates that most students demonstrated strong skills in basic counting and multiplication principles.

A similar analysis was conducted for the three sub-questions under item 2 (2a, 2b, 2c), which required students to utilize the concepts of basic counting, the multiplication principle, and factorials to determine the number of arrangements and selections possible. The questions aimed to assess the multi-structural level of the SOLO taxonomy. Table 3 presents the overall frequencies of students who were thinking correctly and incorrectly under the multi-structural level. Similarly, students who did not provide responses to some of the items were labeled as "No Response," and those who partially responded correctly were labeled as "Partially Correct," as shown in Table 3.

Table 3. Item analysis for item 2 (Multi-structural level).

Item	Correct N (%)	Partially correct N (%)	Completely wrong N (%)	No response N (%)
2a	249 (69.2)	6 (1.7)	105 (29.2)	0 (0.0)
2b	254 (70.6)	7 (1.9)	98 (27.2)	1 (0.3)
2c	130 (36.1)	13 (3.6)	207 (57.5)	10 (2.8)

Statistics gathered in Table 3 indicate that in item 2a, 249 (69.2%) of the responses were correct, 6 (1.7%) were partially correct, and 105 (29.2%) were completely wrong. Item 2a was responded to by all the students. In item 2b, 254 (70.6%) of the responses were correct, 7 (1.9%) were partially correct, 98 (27.2%) were completely wrong, while 1 (0.3%) student failed to respond to item 2b. Finally, in item 2c, 130 (36.1%) of the responses were correct, 13 (3.6%) were partially correct, 207 (57.5%) were completely wrong, and 10 (2.8%) of the students did not attempt the item. In general, students' thinking in item 2 was high since the majority of the students were able to answer the question correctly.

Again, items 3, 4, and 5 were intended to measure the relational level of the SOLO taxonomy, which required students to apply permutations or combinations to solve problems and also justify why the chosen procedure works. The analysis was conducted accordingly. Table 4 presents the overall frequencies of students who were thinking correctly and incorrectly at the relational level. Students who did not respond to some items were labeled as "No Response," and those who responded partially correctly were labeled as "Partially Correct," as shown below.

Table 4. Item analysis for items 3, 4 and 5 (Relational level).

Item	Correct N (%)	Partially correct N (%)	Completely wrong N (%)	No response N (%)
3	92 (25.6)	3 (0.8)	243 (67.5)	22 (6.1)
4	85 (23.6)	7 (1.9)	253 (70.3)	11 (3.1)
5	59 (16.4)	9 (2.5)	271 (75.3)	21 (5.8)

Table 4 indicates that 22 (6.1%) of the students failed to attempt item 3, 243 (67.5%) answered it incorrectly, 92 (25.6%) answered it correctly, and 3 (0.8%) students provided partially correct answers. For item 4, 11 (3.1%) of the students failed to attempt the question, 253 (70.3%) answered it incorrectly, 85 (23.6%) answered it correctly, and 7 (1.9%) students provided partially correct answers. Finally, for item 5, 21 (5.8%) of the students failed to attempt the question, 271 (75.3%) answered it incorrectly, 59 (16.4%) answered it correctly, and 9 (2.5%) students provided partially correct answers. Generally, students' performance on items 3, 4, and 5 was low since the majority of students were unable to answer the questions correctly.

Lastly, item 6, intended to measure the extended abstract level of the SOLO taxonomy, was analyzed. The question required students to extend their knowledge in PC to solve real-life problems related to probability. Table 5 presents the overall frequencies of students who were thinking correctly and incorrectly under the extended

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abstract level. Students who did not provide responses to some of the items were labeled as "No Response," and those who partially responded correctly were labeled as "Partially Correct," as shown below.

Table 5. Item analysis for items 6 (Extended abstract level).

Item	Correct N (%)	Partially correct N (%)	Completely wrong N (%)	No response N (%)
6	16 (4.4)	5 (1.4)	324 (90.0)	15 (4.2)

It is indicated in Table 5 that 16 (4.4%) of the students correctly answered the question, 5 (1.4%) provided partially correct answers, 324 (90.0%) answered the question incorrectly, and 15 (4.2%) did not attempt the question at all. The overall understanding of students in item 6 was very limited since the majority of the participants were unable to answer the item correctly by extending the ideas of permutation and combination in calculating simple probability.

3.2. Overall Classification of Students on Levels of SOLO Taxonomy

Table 6 presents the number of students reaching the various levels of the SOLO taxonomy.

Table 6. Levels of the SOLO taxonomy reached by students.

Levels	Pre-structural	Uni-structural	Multi-structural	Relational	Extended abstract	Total
N (%)	11 (3.1%)	68 (18.9%)	187 (51.9%)	73 (20.3%)	21 (5.8%)	360 (100.0%)

Statistics gathered in Table 6 indicate that 11 (3.1%) out of the 360 students reached the pre-structural level. This means these students had a limited understanding of the concepts and, as a result, they were unable to solve any of the items correctly. Also, 68 (18.9%) of the students reached the uni-structural level, meaning these students were able to solve either one, two, or all of the questions in item 1 but were unable to answer the rest. Furthermore, 187 (51.9%) students reached the multi-structural level, meaning these students attained the uni-structural level and, in addition, they were able to score 4 or more in item 2 but were unable to solve the rest of the items (items 3, 4, and 5). That is to say, they were able to use the idea of basic counting, the multiplication principle, and factorial to find a possible number of selections and arrangements. Moreover, 73 (20.3%) of the students reached the relational level. This means these students were able to attain the multi-structural level and also score 6 or more out of the total marks (9), which were allocated to the items that measured the relational level. These students were able to identify a procedure for solving the given problems and justify why that procedure worked. Finally, 21 (5.8%) of the students reached the extended abstract level. This means these students attained the relational level and were able to score 3 or more out of the total marks that were allocated to item 6. That is to say, students at this level were able to extend their knowledge to calculate simple probabilities of events occurring.

In order to gain more insight into the thinking levels demonstrated by students, the researcher interviewed four students, each representing one of the levels of the SOLO taxonomy (uni-structural, multi-structural, relational, and extended abstract). Below are some excerpts from the interviews regarding how the students approached various test items.

3.3. Uni-Structural

Students who reached this level of the SOLO taxonomy were able to use basic counting and the multiplication principle to find the number of different roads asked in item 1, but were not able to apply these concepts to solve questions involving linear and cyclic permutations. The researcher interviewed one student with ID A112 at this level to explain his working processes.

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Researcher: *How did you solve the questions?*

A112: *In question 1, I used multiplication to determine the number of roads connecting the towns. For questions 2a, 2b, and 2c, n was not given, so it was difficult to solve. In question 3, since the committee consists of 3 members, the answer will be 3 factorial, and for question 4, it will be 4 factorial because the password consists of 4 letters. I couldn't do questions 5 and 6.*

It is clear that the student was successful in answering item 1 but was unable to answer the rest of the questions. The student could not identify the total number of persons (things) in the context of the questions in items 2, 3, and 4. For items 5 and 6, the student did not tackle them. This is a clear indication that student A112 was operating at the unistructural level of the SOLO taxonomy since the student had challenges in using multiple ideas and connecting these ideas.

3.4. Multi-Structural

Students who reached this level of the SOLO taxonomy were able to attain the unistructural level, and in addition, students were able to apply basic concepts of multiplication to solve questions involving linear and cyclic permutations. The researcher interviewed one student with ID C008 at this level to explain his working processes.

Researcher: *How did you solve the questions?*

C008: *In question 1, I multiplied the number of roads connecting the towns. From question 2, each town is represented by one person so in all, there will be 5 people and n will be equal to 5. For 2a, the number of arrangements in a row will be 5 factorial. Questions 2b and 2c involve circular arrangements, so the formula is* $(n-1)!$ *where* $n = 5$*. Questions 3 and 4 are a combination, while questions 5 and 6 are permutations.*

Researcher: *Why?*

C008: *I can't explain, but the answer to question 3 is* ${}^{3}C_{1}$*, question 4 is* ${}^{4}C_{1}$*, question 5 is* ${}^{20}P_{5}$ *and question 6 is* ${}^{11}P_{5}$*.*

From the excerpt above, the student was able to answer questions 2a and 2b correctly, but was unable to answer the rest of the items correctly, even though he tackled all the items. For item 2c, the student considered it a cyclic permutation instead of a combination without replacement. In item 3, the student was able to identify that it was a combination, but could not justify his response and was also unable to solve it since he was unable to determine the total number of students in the class. For the rest of the items, the student was unable to identify whether the problem could be answered using permutation or combination. It can be seen from the student's responses that he had difficulties in distinguishing between permutation and combination; as a result, the student was only guessing. Even though the student was able to gather multiple ideas, he had difficulty connecting these ideas by establishing justifications and relationships in items 3, 4, and 5. This clearly shows that student C008 was operating at the multi-structural level of the SOLO taxonomy.

3.5. Relational

Students who reached this level of the SOLO taxonomy were able to attain the multi-structural level, and in addition, students were able to solve and justify questions involving permutation and combination with restrictions and without restrictions. The researcher interviewed one student with ID A013 at this level to explain her working processes. The responses provided by the student on how items 1 and 2 were answered shared similarities with the responses provided by Student C008 at the multi-structural level. However, the student responded differently to how item 2c was answered.

Researcher: *How did you solve the questions?*

A013: *For question 2c, it involves a combination since in a handshake order does not matter. Therefore, the formula will be* ${}^{5}C_{1}$*. Question 3 also involves a combination because in forming the three-member committee from the 19 students in the class, the order in which the 3 students are selected does not matter. As a result, the answer to this*

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question is $^{19}C_3$. *In question 4, it involves permutation because in passwords, the order is important. There are 26 letters in the English alphabet and the password consists of 4 letters so the answer will be* $^{26}P_4$. *Also, a combination can be used to solve question 5 since the 5 students to be chosen from the 20 students, of which the class captain is included, can be done in any way. The answer to this question will be* $^{20}C_5$. *Question 6 also involves combination, so the formula will be* $\frac{\left(\frac{2}{5}+\frac{3}{6}\right)}{5+6}$.

From the student's response above, it is clear that the student was able to distinguish between permutation and combination with justifications as well. However, in items 2c, 5, and 6, the student answered the questions incorrectly. In item 2c, the student was unable to identify that two people can shake hands at a time. For item 5, the student did not understand the condition attached to the question. As a result, the student was unable to answer question 6 since question 5 was a transition to question 6. Even though the student was able to gather multiple ideas and connect these ideas, the student was unable to extend these ideas to other areas, thus using the ideas of permutation and combination in calculating simple probability. This clearly shows that the student was operating at the relational level of the SOLO taxonomy.

3.6. Extended Abstract

Students who reached this level of the SOLO taxonomy were able to attain the relational level, and in addition, the students were able to solve and justify questions involving permutation and combination with restrictions and without restrictions. These students were able to extend these ideas in calculating simple probabilities. The researcher interviewed one student with ID B087 at this level to explain his working processes.

Researcher: *How did you solve the questions?*

B087: *For question 1a, I counted by tracing the different possible routes, but I later realized multiplication could be used so I applied it. In question 2, each town was represented by one person, so there were 5 people in total. Question 2a was talking about a linear arrangement, so I solved it as* $5!$ *or* $^5P_5$*. For question 2b, I used* $(5-1)!$ *Because it was talking about circular arrangements. I used a diagram to solve question 2c by counting the possible number of ways the handshakes can be done.* $^5C_2$ *also gave me the same answer so I think that will also work.*

Researcher: What makes you think so?

B087: *This is because two people can shake hands at a time, so it's like selecting two items from five. Question 3 was solved using the combination formula since order is not important in selecting the students to form the committee. For question 4, n wasn't given directly, but it is 26 since there are 26 letters in the English alphabet, so I used permutation in solving it, since order matters. In question 5, it is a combination, but there is a condition that the class captain must be included, so the captain is an automatic member. Therefore, it will be left with 19 students for us to select the remaining 4 students from, so I solved it as* $1 \times {}^{19}C_4$.

Researcher: *Why didn't you add, but rather multiply?*

B087: *It sounds like the class captain and the other students. Because of the "and", I multiplied. For question 6, I first found the total number of students as* $^{11}C_5$ *since there are 11 people in all, and 5 people are to be selected and then, I found the probability as* $({}^5C_2 \times {}^6C_3)/{}^{11}C_5$ *since 2 men are to be selected from 5 men and 3 women are to be selected from 6 women.*

Student B087 is a typical example of a student operating at the extended abstract level, as the student was able to solve all the items correctly and also provided justifications for his chosen procedures for solving the items. In item 2c, the student demonstrated a high level of conceptual understanding by devising multiple strategies for solving the item. In an effort to gain deeper insight into students' understanding of permutation and combination,

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the researcher further inquired how students determine whether a problem can be solved using permutation or combination. Additionally, the researcher asked if students employ any alternative strategies beyond the standard formulas. Excerpts of the students' responses are presented below.

A112: *I sometimes guess because I find it difficult to distinguish between the two. I only know about the formula.*

C008: *When I see "arrange" in the question, I use permutation and when I see "select or choose", I use combination. The formula is what is in the textbook.*

A013, B087: *I sometimes look out for keywords and sometimes analyze the question if I don't see those keywords. Apart from the formula, my teacher said listing can be used, but it is time-consuming so he didn't teach it.*

It is clear from the students' responses that they are familiar with the formula approach to solving questions, since that is what is presented in most textbooks. In situations where they are unable to quote the appropriate formulas, they face challenges.

4. DISCUSSION

The purpose of this study was to investigate senior high school students' thinking levels in permutation and combination. The study found that senior high school students' thinking in permutation and combination could be classified into the five levels of the SOLO taxonomy, which are pre-structural, uni-structural, multi-structural, relational, and extended abstract. In most classrooms where there is diversity in students' thinking, these classifications will help teachers to understand the various levels of students' thinking in order to provide the necessary assistance to students at the lower levels to reach the extended level of abstraction. It is in this context that many studies, including Apawu et al. (2018); Claudia et al. (2020) and Putri et al. (2017) SOLO taxonomy to classify students' thinking levels in various areas of mathematics, such as Algebra and Geometry. Despite numerous studies, there is a scarcity of research on how the SOLO taxonomy can be applied to classify students' thinking in permutation and combination, which are prerequisites for advanced courses such as discrete mathematics and probability. Therefore, this study is novel.

Furthermore, it was found in this study that the majority of the students, representing 73.9%, reached the lower levels of the SOLO taxonomy (pre-structural, uni-structural, and multi-structural), with a few representing 26.1% at the higher levels (relational and extended abstract). When the answers of the students at the lower levels, especially those at the multi-structural level, were examined, it was detected that they understood basic counting and multiplication principles but could not apply these concepts in finding the number of ways arrangements and selection can be done. In addition, these students could not distinguish between permutation and combination, which resulted in incorrect answers. Even those who were able to distinguish could not apply a valid procedure to solve the given problems. Again, students' responses during the interview revealed that the majority of the students solved the problems by recalling formulas. Those who were unable to quote these formulas could not devise any other strategy to solve the items, and even the students who were successful in using appropriate formulas to solve the items could not list the possible outcomes that correspond to their answers.

This is in line with Busadee and Laosinchai (2013), who said that students who learn these topics in this fashion (memorization of formulas) may be successful in the short term, performing well on tests, but often lack the conceptual understanding required to solve real-world problems. This situation creates some inconsistencies in the answers of students at the multi-structural level. As a matter of fact, in the study by Göktepe and Özdemir (2013) examining the spatial visualization skills of pre-service mathematics teachers with the SOLO model, it was found that the pre-service teachers were predominantly at the multi-structural level. Additionally, in the aforementioned study, it was stated that pre-service teachers could use data in their answers, but their inability to grasp the relationship between data caused some deficiencies. According to Callingham, Pegg, and Wright (2009) as the transition from the unistructural thinking level to the extended abstract thinking level increases, the level of

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individuals using and associating data and viewing data from a broader perspective also increases. Additionally, the authors stated that the most challenging transition is from the relational level to the extended abstract level.

This situation supports why there are fewer students who reached the extended abstract level in the study conducted. According to Biggs and Tang (2009), the number of details in student responses indicates the quantitative aspect of structural complexity, whereas the level of associating the details with each other reflects its qualitative aspect. The authors added that pre-structural, uni-structural, and multi-structural thinking levels are expressed as quantitative learning, while the relational and extended abstract thinking levels are expressed as qualitative learning. Therefore, considering that most of the students in this study are at the levels of pre-structural, uni-structural, and multi-structural thinking, it can be said that most of the answers reflect quantitative learning. In addition, since the majority of the students reached the first three levels (pre-structural, uni-structural, and multi-structural) of the SOLO taxonomy, it can be said that students' thinking in permutation and combination is low (surface thinking).

This is supported by Hughes (2017) the statement indicates that the first three levels (pre-structural, uni-structural, and multi-structural) are surface-level thinking, requiring students to find information and gather ideas. The last two levels (relational and extended abstract) involve deeper thinking, where students process the information they have collected in previous levels. This suggests that most students in the study can locate information and gather ideas, but encounter challenges in integrating and organizing this information and these ideas. In Ghana, the Common Core Programme curriculum expects teachers to engage students in mental processing beyond memorization. Therefore, teaching approaches and assessments should be structured to promote critical thinking, enabling students to reach the higher level of extended abstraction.

5. CONCLUSION

The study concluded that SHS students' thinking levels in permutation and combination are low. The researcher's interaction with the students during the interviews revealed that the students have a limited understanding of the concepts related to permutation and combination. As a result, students are likely to perform poorly in advanced courses that have permutations and combinations as prerequisites.

6. RECOMMENDATIONS

The researchers recommend that SHS teachers, textbook authors, and curriculum developers combine routine and non-routine problems in teaching and learning. This will help students realize that permutations and combinations are not solely about formulas, but a variety of approaches can usually be used to arrive at a correct solution, just as a variety of approaches can be used to arrive at an incorrect solution. In addition, teachers must make use of representations (tree diagrams, paper chain activities, etc.) and other activity-based teaching strategies in the classroom to help students develop a conceptual understanding of the topic before introducing students to the various formulas.

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